A sectoral approach to measuring output gap: Evidence from 20 US industries over 1948-2019

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Abstract

The existing output gap estimations rely on aggregate output data, assume constant utilization over industries (see Coibion et al. (2017) and Owyang et al. (2018)). However, each sector has its cycle, and missing sectoral dynamics creates underestimation of the economy-wide slack. By using a new peak to peak method, motivated by the business cycle analysis of Mitchell (1946), I estimate output gaps of 20 US industries over 1948-2019. For the last cycles of industries, I use a generalized Poisson model to predict the next peaks' timing and magnitude by Bayesian statistical inference. I find a generally negative output gap at the aggregate except for the second half of the 1960s, and a persistent excess-capacity since 1990. For 2019, I estimate a negative output gap of 2.5%, coming from the under-utilization in utilites, manufacturing, wholesale trade, transportation-warehousing, finance-insurance and education, while the Congressional Budget Office and Hodrick–Prescott filter indicate overheating since 2017. My findings show that secular stagnation is not an inevitable future of the US economy and suggest demand supporting policies to eliminate the chronic output gap.

Keywords: sectoral output gap, generalized Poisson model, Bayesian statistics, secular stagnation.

JEL Classification: E32, E60, C11

1. Introduction

The slow recovery of the US economy from the Great Recession created concerns about demand-side secular stagnation (Summers, 2014); however, headline output gap estimations did not show a persistent economic slack over the last business cycle. The reason was frequent downward revisions to potential output instead of bounding aggregate demand. The Federal Reserve started to raise policy rates in late 2016, believing that the economy was approaching its full-capacity even though inflation did not pass the long-run target rate. Pushing breaks despite the concerns about a lasting economic slack raised questions about the reliability of current output gap estimations (Coibion et al., 2017).

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A characteristic feature of the existing output gap estimations is their reliance on aggregate data and their assumption of the same utilization rate for each industry. However, each sector has its cycle, which does not necessarily match the business cycle, as discussed by Mitchell (1946). Cycles of manufacturing, retail trade, transportation, arts-entertainment, and food-accommodation follow business cycle fluctuations closely. In contrast, others present less-cyclicality than the business cycle, such as information, real estate, education, health care, and government. The magnitudes of sectoral cycles are also different from the macroeconomic cycles as industries do not grow in balance with GDP. Information, finance, real estate, health care, and professional-business services have grown faster than GDP since 1980. In contrast, the growth of agriculture, mining, and utilities has been lower than the macroeconomic growth rate post-1980. Missing the length and the magnitude of sectoral cycles creates unreliable capacity measures at the aggregate, which generally underestimate the US economy's output gap.

This paper estimates the US output gap over 1948-2019 by aggregating 20 industries' output gaps. I use the reference cycle peak to peak method, which takes peak years of sectors from the reference cycle relatives of Mitchell (1946). These are calculated by dividing the value-added of industries by their average over successive business cycle trough years. I define trend output as the value-added in peaks of reference cycle relatives and interpolate it to other years by assuming a constant growth rate between any two peaks. The economic rationale of this method is smoothing short-run macroeconomic cycles to focus on long-run trends of the industries and the economy. The advantage of the reference cycle peak to peak compared to the direct peak to peak, which takes peak years from the value-added series, is turning long cycles of industries into shorter run fluctuations around business cycle averages.

Creating macroeconomic fluctuations in value-added of industries, which present less-cyclicality than the business cycle, makes their output gap estimations more reliable than the direct peak to peak. For example, the value-added of information reached its first peak after 1980, and the health industry did not make a peak since the early 1980s. Using the direct peak to peak will require extrapolations over more than 30 years for these industries. On the other hand, the reference cycle relatives have several peak years due to recurrent macroeconomic fluctuations. For industries following the business cycle, the reference cycle peak to peak, and the direct peak to peak give similar results because peak years of the value-added and reference cycle relatives generally overlap.

However, the reference cycle peak to peak might create unreliable estimates for the last cycles as it extrapolates the growth rate of the trend between the last two peaks to the years following the last peak. To increase the precision of the capacity estimates for the last cycles, I use a generalized Poisson model. Another contribution of the paper is predicting the timing and the magnitude of the next peaks of 20 US industries by Bayesian statistical inference. I define sectoral waiting times between peak years and the annual growth rate from one peak to another as model parameters and estimate their marginal posterior distributions by Random Walk Metropolis-Hastings (RWMH) simulations. Results for 20 US industries over 1948-2019 show that the mean waiting times over sectors are very close to each other with a mean around five years and a standard deviation of less than one year. Mean annual growth rates range quite widely from 1% to 7%, with a mean of 3% and a standard deviation of 2%, capturing stagnant and growing sectors of the economy.

The main advantage of the reference cycle peak to peak compared to aggregate output gap estimations is its reliance on sectoral output measures. I weight sectoral output gaps by the value-added shares of industries to calculate the economy-wide output gap. The reference cycle method does not estimate a positive aggregate output gap (except the second half of the 1960s), and it does not necessarily eliminate output gaps at each macroeconomic cycle. For example, it shows a permanent excess-capacity in the US economy since the 1990 peak unlike the Congressional Budget Office (CBO) and Hodrick–Prescott (HP) filter. The reference cycle method estimates a negative output gap of 2.5% in 2019, whereas CBO and HP filter indicate overheating since 2017. Unlike the headline measures, the reference cycle method addresses concerns about a lasting economic slack in the last business cycle even though the unemployment rate declined to historically low levels. In addition to the economy-wide output gap, the reference cycle peak to peak also gives output gaps for the industries that lack independent capacity measures such as services as the Federal Reserve Board's survey data only covers manufacturing, mining, and utilities. In 2019, the reference cycle method estimates a negative output gap for utilities, manufacturing, wholesale trade, transportation-warehousing, finance-insurance and education.

The paper is organized into five sections. In section 2.1, I describe the reference cycle peak to peak in detail on manufacturing and healthcare, and compare the results for these two industries to the direct peak to peak. I introduce the generalized Poisson model to solve the last cycle problem in section 2.2 and explain the Bayesian statistical inference used to estimate model parameters. I report output gaps at the aggregate and industry levels over 1948-2019 in section 2.3 together with the marginal posterior distributions of waiting times and growth rate between peak years for 20 industries. Section 3 reviews the existing output gap estimations and compares them to the reference cycle method. In section 4, I discuss the contemporary macroeconomic significance of my findings in connection with the secular stagnation hypothesis of Summers (2014). Section 5 is the conclusion.

2. Sectoral estimation of the output gap

I decompose the US economy into 20 industries: Agriculture, mining, utilities, construction, manufacturing, wholesale trade, retail trade, transportation and warehousing, information, finance, real estate, professional services, management of companies, administrative services, education, healthcare and social assistance, arts-entertainment, food-accommodation, other services, and government. I use BEA's double-deflated real value-added data over 1948-2019 to estimate each industry's trend output and the output gap. I assume that each sector is at its trend in 1948, a peak year of GDP. I use the reference cycle relatives to get peak years for estimating the trend output of industries.

2.1 The reference cycle peak to peak

The reference cycle method takes its motivation from Mitchell (1946)'s business cycle analysis. Mitchell (1946, p.24) defines an industry's reference cycles as the segment of the industry output that lies between successive business cycle trough years. Reference cycle average $\bar{y}_{(t_j,t_{j+1})}$ over the (t_j, t_{j+1}) business cycle trough years is calculated by:

$$\bar{y}_{(t_j,t_{j+1})} = \sum_{t=t_j}^{t_{j+1}} y_t/(t_{j+1} - t_j)$$

I convert the log real value-added y_t to percentages of reference cycle averages, called reference cycle relatives, $y_{t,rel}$ over the (t_j, t_{j+1}) business cycle trough years in the following way:

$$y_{t,rel} = (e^{y_t - \bar{y}_{(t_j,t_{j+1})}}) * 100$$

I take the business cycle trough years from NBER and convert real value-added of 20 US industries over 1948-2019 into their reference cycle relatives. For the last business cycle, I use averages over 2009-2019. Figure 1 shows how the reference cycle relatives are obtained for manufacturing and healthcare over 1948-2019. The panels on the left show the real value-added of manufacturing and healthcare together with their reference cycle averages, and the panels on the right give reference cycle relatives of the two industries. Shaded regions are NBER recession periods.

I take peak years of reference cycle relatives $y_{t,rel}$, and define real value-added at those peak years as the industry's trend output. I linearly interpolate the trend to the years between any two peaks. I refer to this method as the reference cycle peak to peak to distinguish it from the direct peak to peak, which takes peak years from the real value-added series.

Suppose an industry follows the business cycle fluctuations closely such as manufacturing. In that case, the reference cycle peak to peak and the direct peak to peak will give similar results because peak years of the industry value-added and GDP will overlap. This method's advantage compared to the direct peak to peak is making output gaps of less-cyclical industries more reliable. For example, value-added of information, administrative services, and education reached their first peaks in the 1970s or the early 1980s. The direct peak to peak gives unrealistic output gaps for these industries over more than 30 years. However, reference cycle relatives see many peaks during this period due to successive business cycles. The problem with healthcare and the government is even more prominent when peak to peak is directly applied to value-added. The government had only one peak in 2010, and until then, the direct peak to peak gives persistent overheating, which does not make economic sense. Healthcare value-added had its last two peaks in the 1970s, and when the growth rate between these two peaks is extrapolated until 2019, a continuous excess-capacity, which reaches more than 50%, is observed. Figure 2 below shows output gaps of manufacturing and healthcare estimated by the direct peak to peak and the reference cycle peak to peak with extrapolating the trend between the last two peaks to the years following the last peak.

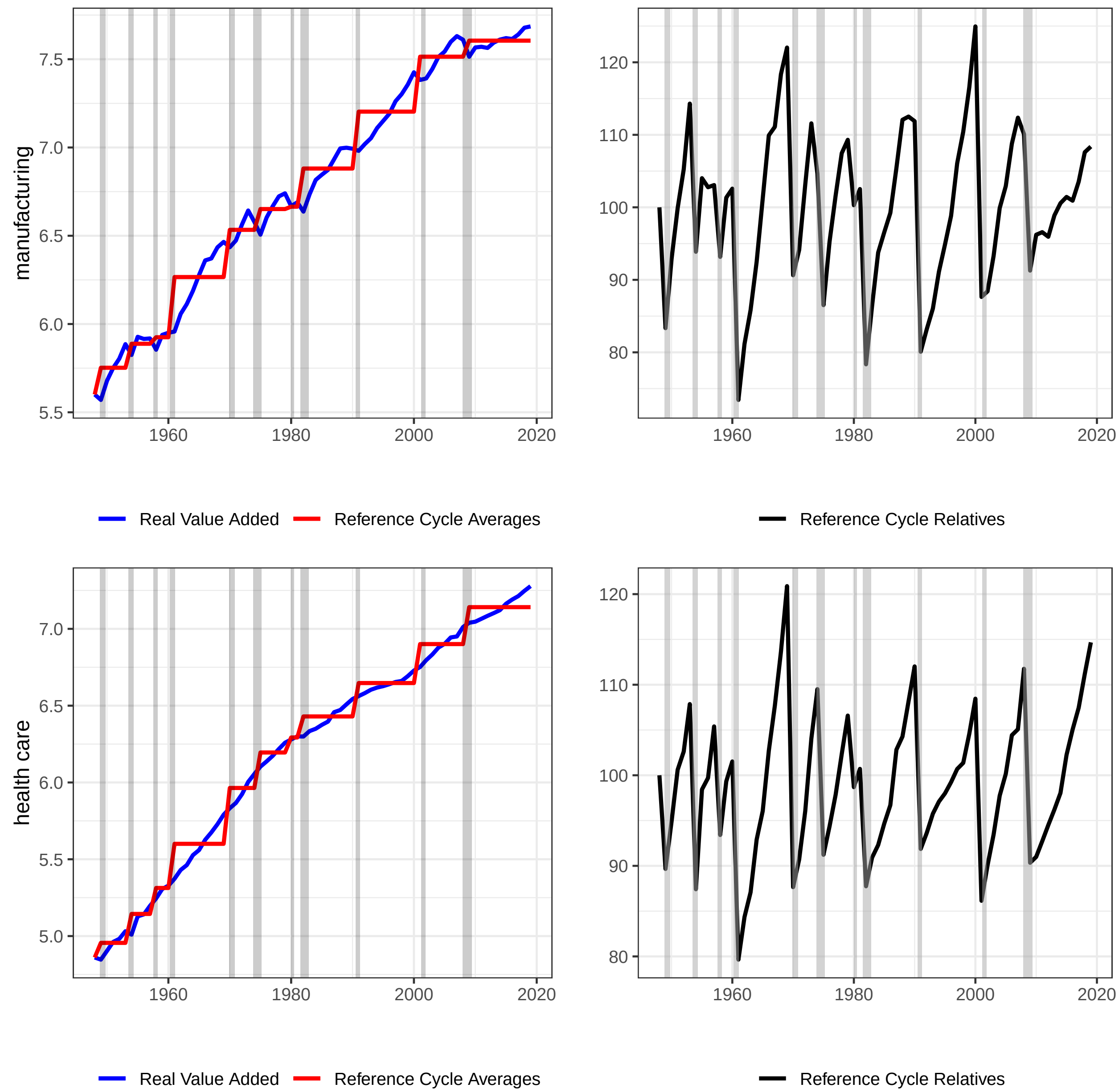


Figure 1: **Reference cycle averages and relatives of manufacturing and healthcare over 1948-2019. Reference cycle averages are average value added between business cycle trough years. Reference cycle relatives are real value added divided by reference cycle averages.**

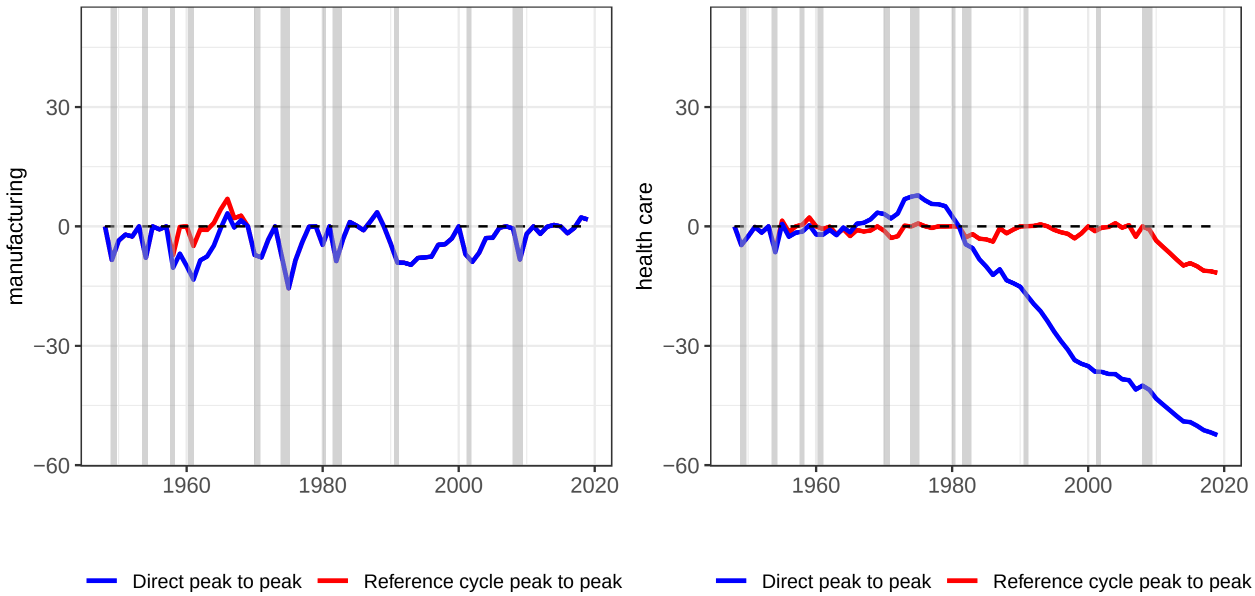


Figure 2: **Output gaps of manufacturing and healthcare by the direct peak to peak and the reference cycle method over 1948-2019. The two methods give almost identical results for manufacturing; however, for healthcare there is a significant difference.**

The two methods give almost identical utilization rates for manufacturing; however, there is a significant difference between them for healthcare. The reference cycle output gap of the health industry fluctuates around full-capacity over 1948-2008, but the direct peak to peak points to a permanent excess-capacity since the late 1970s. The reference cycle method improves the output gap estimates of the direct peak to peak for the less-cyclical industries by creating macroeconomic fluctuations in industry output. Moreover, it does not disturb estimates for the cyclical ones because these industries' peak years do not change significantly after transforming their value-added into reference cycle relatives. The case of manufacturing in Figure 2 shows that except for the 1960 peak, manufacturing followed the business cycle peaks closely.

On the other hand, both methods suffer from the last cycle problem because peak to peak extrapolates the growth rate of the trend between the last two peaks to the years after the last peak. The extrapolation creates unreliable trend estimates for the recent years when the period following the last peak is long. For example, applying this method to GDP means extrapolating the GDP growth rate over 2001-2007 up until 2019, which misses the Great Recession's impact on trend output. Several industries that reached their last reference cycle peaks during 2006-2008, such as retail trade, food-accommodation, real estate, professional and business services, suffer from this problem. Since the recent years is a particular interest of this research, I develop a new approach to the last cycle problem by using a generalized Poisson model to predict the timing and the magnitude of 20 US industries' next reference cycle peaks based on the information in the time series over 1948-2019.

2.2 Predicting next peaks by the generalized Poisson distribution

Poisson models are used to predict an event's next occurrence over a period based on the average number of occurrences before. One assumption of the Poisson distribution is the equality of the mean and the variance of the random variable, which is hard to meet in real-world applications. To overcome the over-and under-dispersion problem, Consul and Jain (1973) introduced a generalized Poisson model, which includes a dispersion parameter to capture the difference between mean and variance. The probability density function of the generalized Poisson distribution of a discrete random variable x with an average number of occurrence λ and a dispersion parameter μ is:

$$p(x|\lambda, \mu) = \lambda(\lambda + \mu x)^{x-1} e^{-(\lambda+\mu x)}/x!$$

where $\lambda > 0$ and $|\mu| < 1$. In my case, λ is the average waiting time between peaks of reference cycle relatives, and μ is the annual logged growth rate from one peak to another so that μx becomes the cumulative growth over the cycle. I prefer a generalized Poisson distribution instead of a classical Poisson model, which is obtained when $\mu = 0$, because I need an estimate for the growth rate over the cycle, in addition to the cycle length, to apply the reference cycle peak to peak.

I first plot histograms of the pooled sectoral waiting times and annual growth rates between peaks for GDP and 20 industries in Figure 3 with the exponential and normal fits. The minimum waiting time between peak years is two, and the maximum is twelve. The distribution of waiting times between peaks shows an exponential structure. The annual logged growth rate between peaks ranges from -10% to 13% by following an almost normal distribution.

I use Bayesian statistical inference to estimate λ and μ to predict the next peak of the reference cycle relatives. Bayesian statistics derives the posterior distribution of the model parameters conditional on the data $p(\lambda, \mu|x)$ by using the likelihood $p(x|\lambda, \mu)$, prior distribution of parameters $p(\lambda, \mu)$ and marginal distribution of data $p(x)$ through the Bayes' rule:

$$p(\lambda, \mu|x) = \frac{p(x|\lambda, \mu)p(\lambda, \mu)}{p(x)} \propto p(x|\lambda, \mu)p(\lambda, \mu)$$

To estimate the joint posterior distribution of λ and μ, I need to specify the prior distribution of the parameters. For λ, I choose a gamma prior with the mean and variance coming from the waiting times of each industry, and for μ, I use a normal prior which takes its mean and standard deviation from each sector's growth rate data.

The joint posterior distribution of parameters can be estimated analytically from the Bayes' rule; however, it is not easy to get marginal posterior distributions of λ and μ due to the cumbersome integral calculations. That's why Bayesians have developed numerical methods that allow approximating marginal posterior distributions (Särkkä, 2013, pp. 22-23). I use an RWMH algorithm, a part of the Markov Chain Monte Carlo family, to simulate marginal posterior distributions of model parameters. I report the mean of λ and μ for GDP and 20 US industries in Table 1 below, together with the waiting time and annual growth rate of

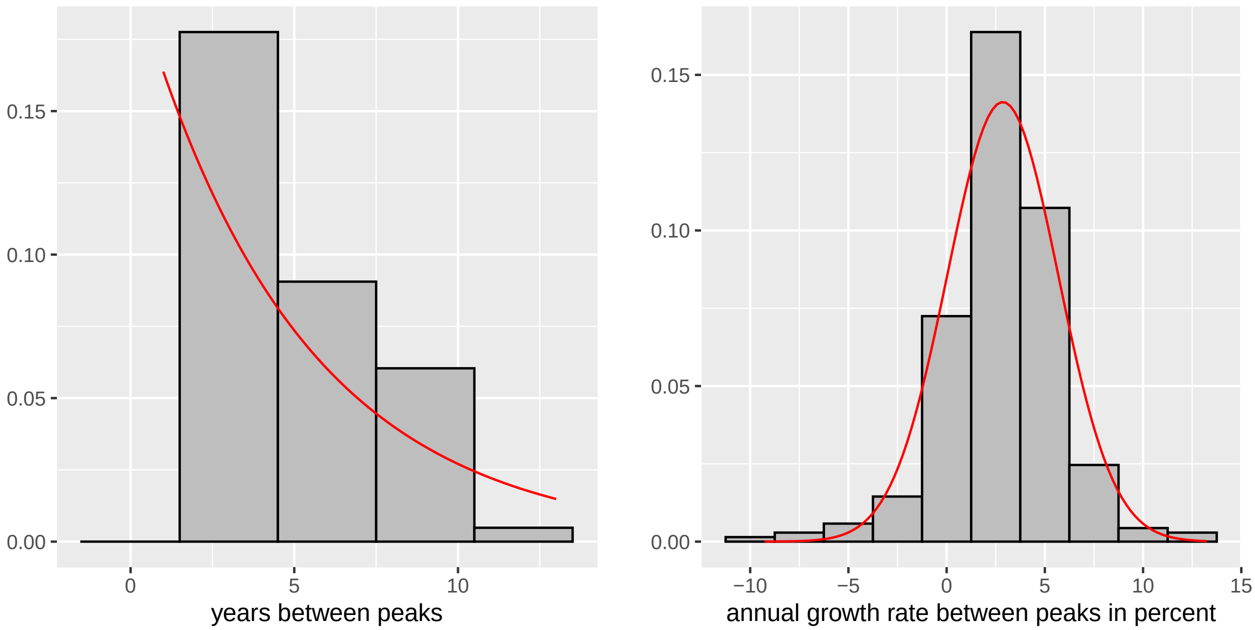


Figure 3: **Pooled sectoral waiting times and increments with the exponential and normal fits over 1948-2019. Waiting times show an exponential structure while annual growth rate between peaks is almost normally distributed.**

industries since their last reference cycle peaks. I also give predicted reference cycle peak years in the fifth column of Table 1. I give more detail about the RMWH algorithm and show the convergence plots of accepted parameter draws in the Appendix.

Generalized Poisson parameter estimates in Table 1 show that the mean waiting times of 20 US industries are quite close to each other with a mean of 4.9 and a standard deviation of 0.8. The 95% credibility interval is from 3 to 6 years. Cycle length is lowest for agriculture and highest for transportation-warehousing. Annual growth rates over the cycles, on the other hand, are widely different, with a mean of 3% and a standard deviation of 2%. The 95% credibility interval ranges from 1% to 6%. The mean cycle length is around six years for GDP, with a mean growth rate of 3%. Wholesale trade, information, finance-insurance, real estate, professional services, administration, and healthcare grow faster than GDP over the cycles, which fits well with the US economy's current trends.

Table 1: **Summary statistics of mean delay and annual growth rate between peaks over 1948-2019 together with the waiting time and growth rate since the last reference cycle peak for GDP and 20 US industries. The fifth column lists model predictions for the next reference cycle peaks.**

	λ	μ	Last delay	Last growth	Predicted peak
GDP	5.69	3.17	12.00	1.66	2019
Agriculture	2.77	1.62	3.00	0.48	2019
Mining	3.92	0.71	4.00	4.44	2019
Utilities	3.80	1.43	2.00	0.95	2021

	λ	μ	Last delay	Last growth	Predicted peak
Construction	5.43	1.97	10.00	1.25	2019
Manufacturing	4.97	2.56	4.00	1.66	2020
Wholesale Trade	4.99	4.66	4.00	0.71	2020
Retail Trade	5.54	3.18	13.00	1.05	2019
Transportation and Warehousing	6.04	2.65	1.00	-0.04	2024
Information	4.62	4.56	8.00	5.80	2019
Finance and Insurance	4.11	3.92	3.00	0.27	2020
Real Estate	5.32	3.89	11.00	1.91	2019
Professional Services	5.65	5.11	11.00	2.72	2019
Management	4.82	2.22	11.00	4.24	2019
Administration	5.55	7.07	11.00	2.26	2019
Education	4.45	2.64	3.00	-0.28	2020
Healthcare	5.81	3.69	11.00	2.42	2019
Art	4.63	2.94	5.00	1.76	2019
Accommodation	5.14	2.20	13.00	0.86	2019
Other	4.14	1.74	4.00	0.73	2019
Government	5.51	2.02	9.00	0.15	2019
Mean	4.86	3.04	7.05	1.67	NA
Standard Deviation	0.82	1.51	4.07	1.61	NA
2.5%	3.26	1.05	1.48	-0.17	NA
50%	4.98	2.64	6.50	1.15	NA
97.5%	5.93	6.14	13.00	5.15	NA

The third column of Table 1 shows the time passed since the last peaks for 20 industries. According to the mean λ values and last waiting times, industries can be classified into three groups. The first group has a mean λ value that is equal to the last delays. Agriculture, mining, arts-entertainment, and other services are in this group, and the model predicts 2019 as the next reference cycle peak year for these industries. For this group, I take last cycle growth rates in the fourth column of Table 1 as the growth rate of the trend output.[1]

The second group consists of construction, retail trade, information, real estate, professional services, management, administration, health care, accommodation, and government whose last cycle waiting times are above the mean λ values. According to the generalized Poisson model, the probability of a peak in 2019 is very close to 1 for these industries. However, an essential feature of the Poisson distribution is that the average waiting time does not depend on the time passed since the event's last occurrence. It means that the next peak's timing does not change, whether it is estimated in 2019 or 2009, given that the last peak was before 2009. However, I do not use this property of the model to predict the next reference cycle peak of the industries because it leads to unrealistic output gaps in the last cycles. The

[1] I also estimated the conditional posterior distribution $\lambda|\mu$ with μ equals the last cycle growth rate of industries to use this additional information in the series. However, last cycle growth did not change posterior distributions significantly. Since the mean values of $\lambda|\mu$ are very close to the mean values of λ reported in Table 1, I do not report them separately.

problem occurs in the estimation of the trend output growth rate in the last cycle. One approach is to take the mean μ as the growth rate of the trend output and project the next peak based on the mean λ from 2019. In this case, almost all industries in the third group will observe large negative output gaps because the cycle lengths are long and mean μ is higher than last cycle growth rates (except information and management). For example, for administration annual output gap will be 5%, and the cycle length will be 17 years. Another approach is to take the mean λ times the mean μ as the cumulative growth over the last cycle and divide it by the last waiting time plus mean λ to get the growth rate of the trend output. However, long waiting times for the second group industries will cause overheating this time, reaching almost 35% for management and 30% for information. Instead, similar to the first group, I take 2019 as the last peak year for reference cycle relatives of these industries.

For the third group, the mean λ is greater than the last delay. Utilities, manufacturing, wholesale trade, transportation-warehousing, finance-insurance, and education are in this group. The model predicts the next reference cycle peak year for transportation-warehousing as 2024, for utilities as 2021, and for manufacturing, wholesale trade, finance-insurance, and education as 2020 based on the information in the series over 1948-2019.[2] For these industries, I take the mean μ as the trend output growth rate in their last cycles. The difference between the mean μ and last cycle growth rates in the fourth column of Table 1 creates negative output gaps of these industries.

2.3 Aggregate and sectoral output gaps over 1948-2019

After I estimate output gaps of 20 industries over 1948-2019, I weight them by each industry's value-added share and get the economy-wide output gap. Figure 4 below gives output gaps by the reference cycle method, CBO, and HP filtered series (with the smoothing parameter $\lambda = 100$). The reference cycle output gap differs from CBO and HP filter on two grounds.

Firstly, it does not estimate positive output gaps, except for the 1960-1969 business cycle.[3] This feature of the reference cycle output gap comes from the sectoral underpinnings of the method. Although sectoral overheating is not ruled out, the reference cycle peak to peak limits movements above full-capacity because peak years are defined at the maximum output levels. There are some periods when value-added grows fast first and slows down before reaching a local maximum, therefore moving above the linear trend between peaks; however, relatively short cycle lengths make persistent overheating harder. As I discuss below, sectoral output gaps rarely become positive and do not exceed 10%. Also, it is not likely to observe overheating in most industries simultaneously that would create a positive output gap at the aggregate. As Table 1 above shows, sectoral waiting times between peaks are not identical; therefore, as some industries experience overheating others will still be below their peaks.

[2]Generalized Poisson model does not take the information coming from the pandemic related recession in early 2020 into account. The impact of the recession on industry reference cycle relatives can be evaluated after 2020 real value-added by industry data are released by the BEA.

[3]The overheating in the late 1960s comes mainly from manufacturing, which had a high impact on aggregate output gap until 1980 as it produced almost 25% of the value-added over 1948-1980.

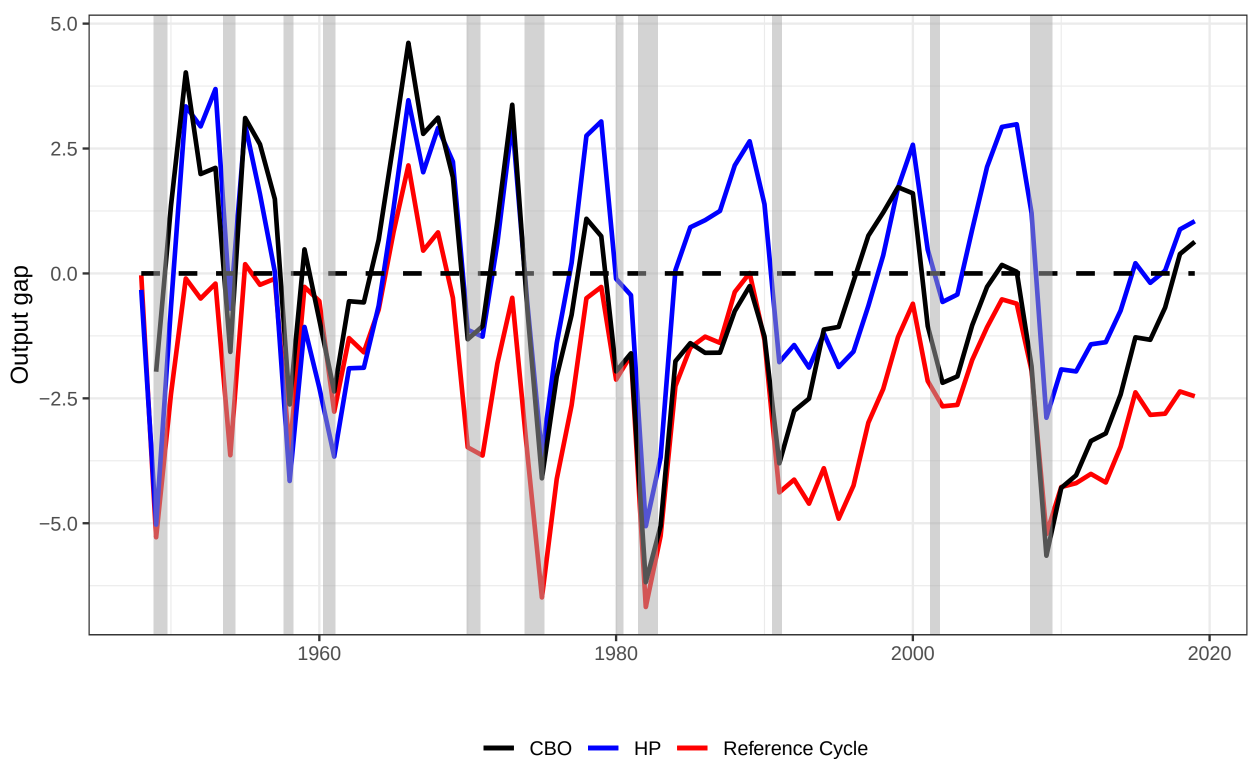
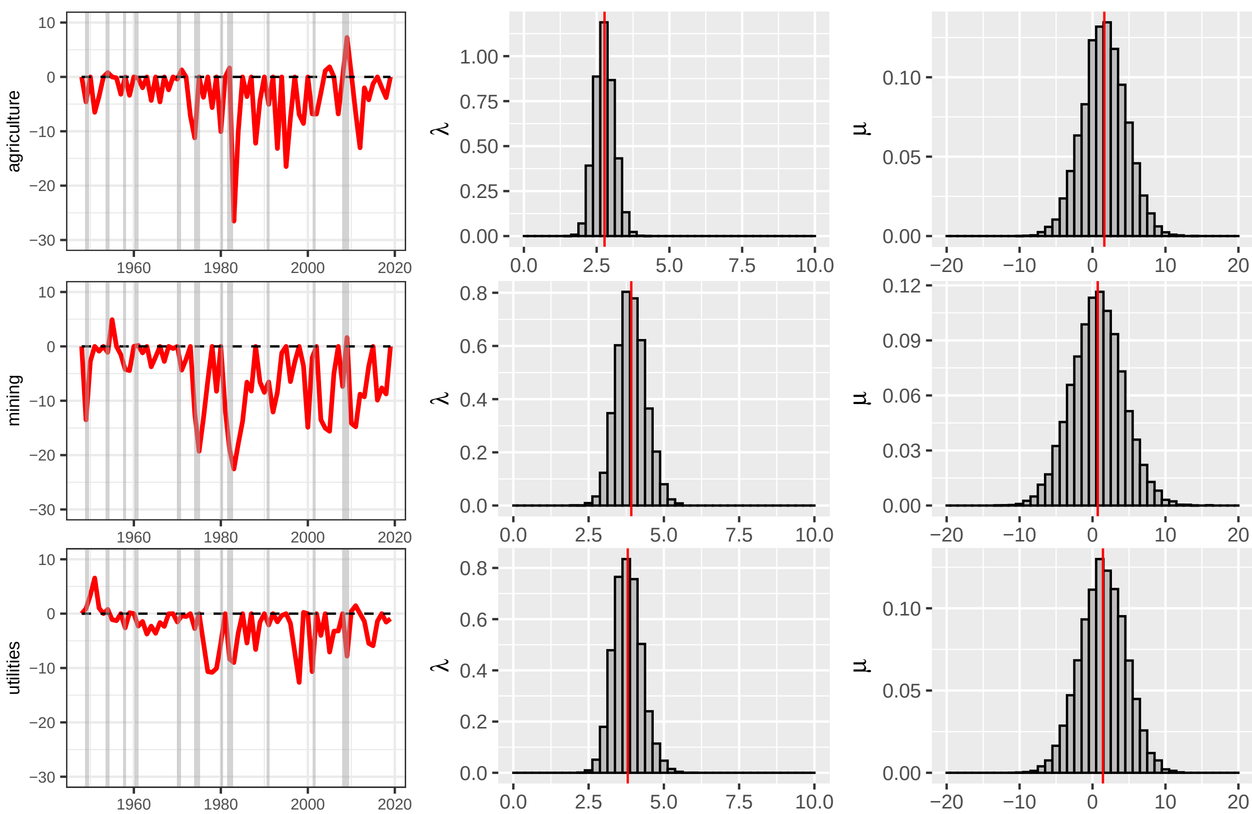
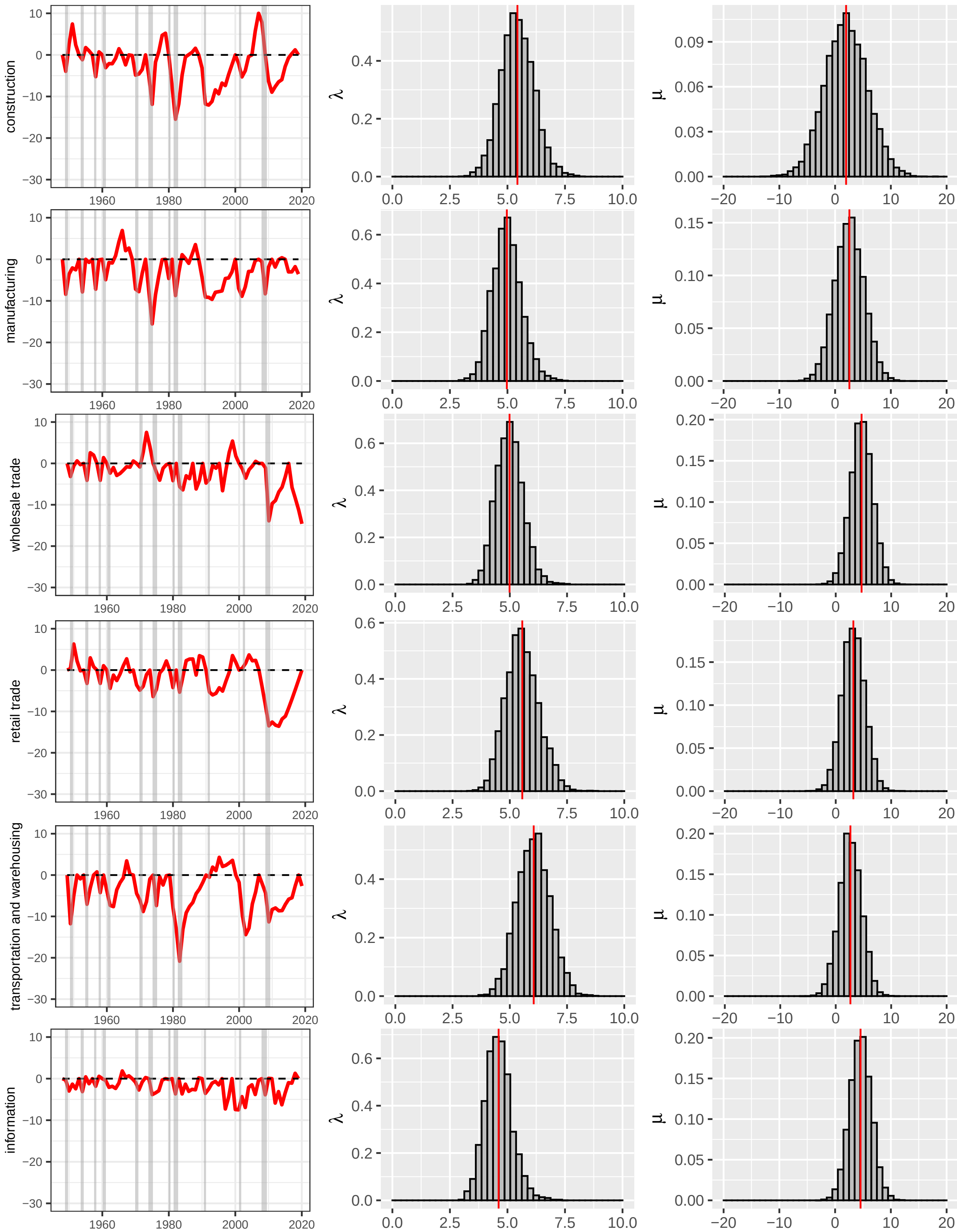


Figure 4: **Output gaps estimated by the reference cycle method, CBO, and HP filter over 1948-2019. The reference cycle method estimates negative output gaps except for the second half of the 1960s. The reference cycle output gap decouples from the CBO and the HP filter after 1990 by showing a persistent excess-capacity in the US economy. In 2019, it finds a negative output gap of 2.5 percent unlike positive output gap estmiates of the CBO and HP filter.**

Secondly, the reference cycle output gap decouples from CBO and HP filter after the 1990 peak by showing a permanent excess-capacity in the US economy over the last 30 years. It indicates that the US economy entered into a recession in 2001, 2008 and 2020 without utilizing all available resources. After the Great Recession, the reference cycle output gap shows a sluggish recovery, whereas CBO and HP filter indicate overheating since 2017. In 2019, there was still an excess-capacity of 2.5% according to the reference cycle method. This finding is more in line with the US economy's aggregate demand problems in the last decades than the results of CBO methodology and HP filter, which necessarily eliminate output gaps in each business cycle, thus putting all the weight to potential output adjustments.

In addition to aggregate output gaps, I discuss 20 industry output gaps estimated by the reference cycle method. I add posterior distributions of waiting times and annual growth rates between peak years to the output gap plots of 20 industries in Figure 5 below. Sectoral output gaps are generally negative, and they never pass 10% when they are positive. Moving above full-capacity output is possible; however, permanent overheating is not observed in any of the 20 industries. Excess-capacity is more likely, but output gaps are temporary, as full-capacity is attained in the peak year of each cycle. This property makes reference cycle peak to peak a good candidate for capturing trend output of industries.




construction
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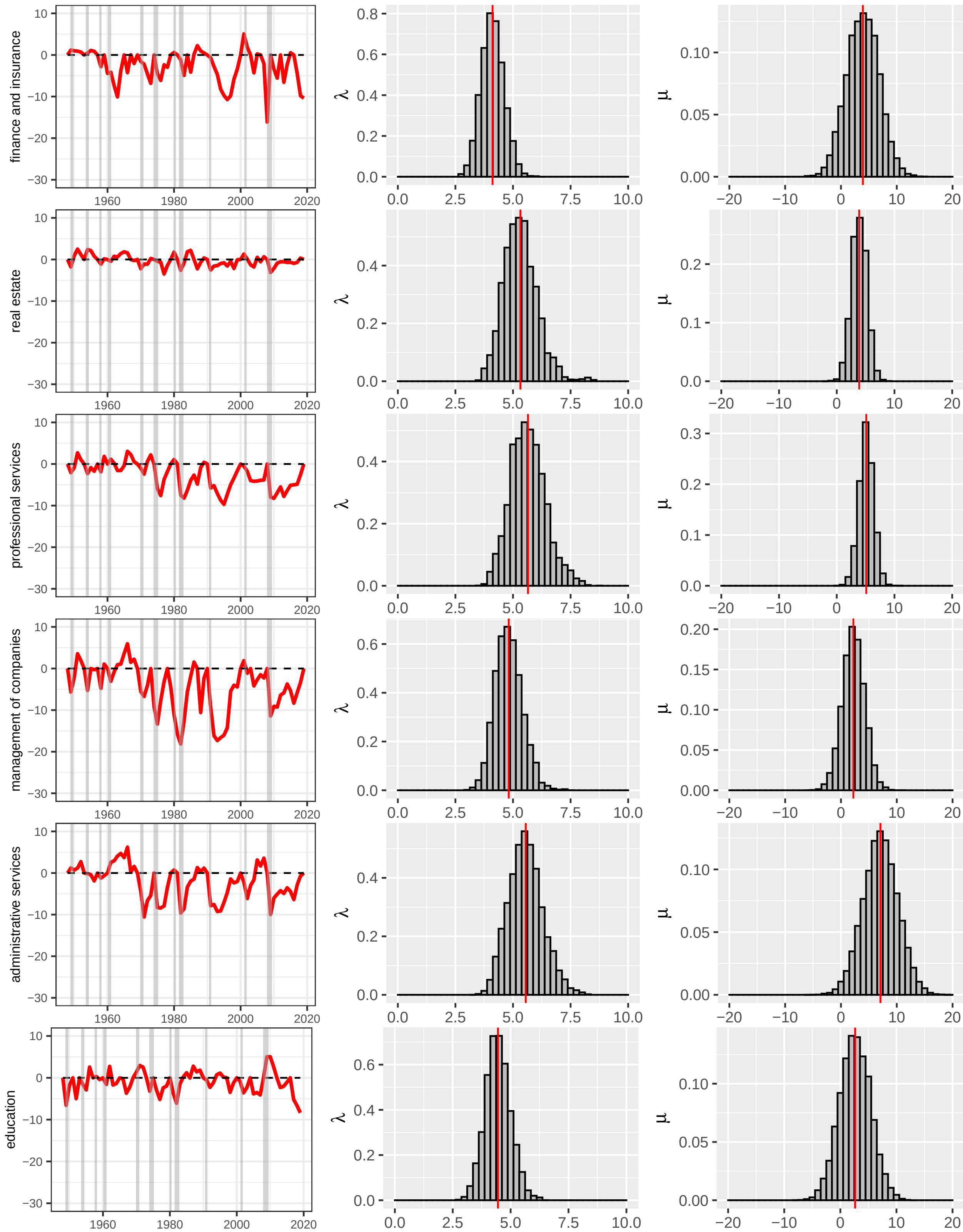
finance and insurance
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education
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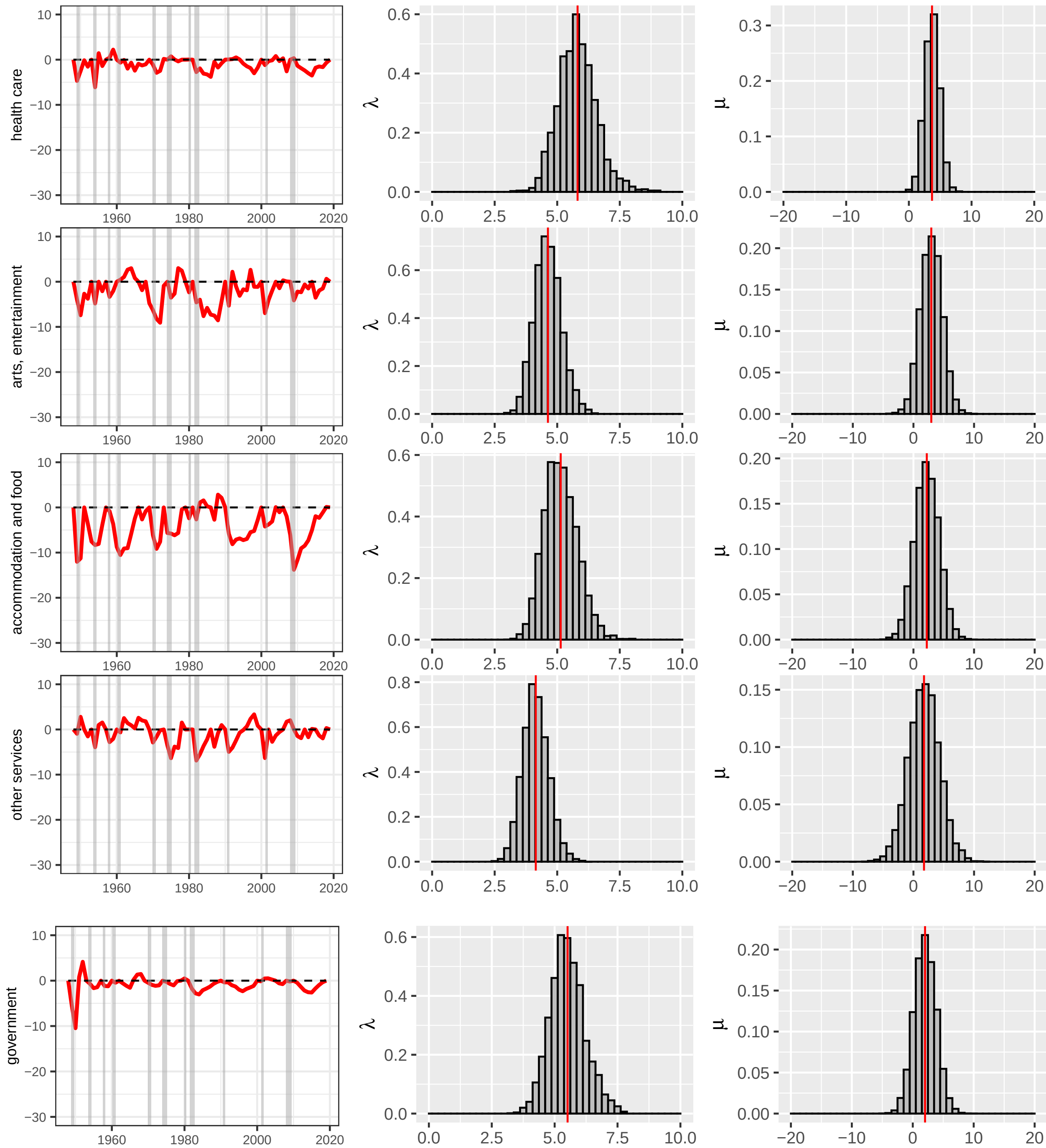

Figure 5: **Output gaps, posterior distribution of waiting times and annual growth rates for 20 US industries over 1948-2019.**

Agriculture, mining, and utilities have noisy output gaps because of the short waiting time between peaks (less than four years) and volatility in the value-added series. On the other hand, information, real estate, healthcare, and government show little variation in their output gaps with relatively long cycles. Construction, manufacturing, retail trade, transportation-warehousing, professional-business services, arts-entertainment, and food-accommodation follow business cycle fluctuations closely but with different magnitudes. Construction, transportation-warehousing, and management of companies show sharp ups and downs in their output gaps, particularly after the 1980 recession. In contrast, manufacturing, professional and administrative services fluctuate in the (-10,0) band over the same period. Finance-insurance has a long cycle from the mid-1980s until 2001, and after that, its cycle length declines and volatility increases. Its output gap is more than -10% in 2019. Unlike other industries, the utilization rate of education rises during the Great Recession, which then declines and shows an excess capacity of almost 10% in 2019.

3. Discussion of existing output gap measures

This section reviews the literature on output gap estimations for the US and compares the existing measures to my estimates. In the literature, there is not an agreed-upon definition of what potential output means. Blecker and Setterfield (2019) discuss two definitions: Full-employment output and full-capacity output. Full-employment output assumes that potential is constrained by the availability of labor, whereas full-capacity assumes a labor-abundant economy in which capital is the constraining factor of production. Full utilization of capacity corresponds to the output level, which minimizes the costs of production. Producing above the full capacity output is possible but not desired due to rising overtime payments and more frequent break down of the machinery, as discussed by Blecker and Setterfield (2019). Therefore, if a firm operates above its full capacity for a sustained period, it will invest more to increase its productive capacity. The reference cycle method supports this view by rarely estimating positive output gaps at the industry and aggregate levels.

Shaikh and Moudud (2004, p.2) define potential output as “the desired level of output from given plant and equipment” instead of an engineering capacity that refers to mechanical limits of production, such as operating seven days, 24 hours. Shaikh and Moudud (2004) also mention that economic capacity does not necessarily require full-employment of the labor force, and, therefore, these two concepts are not identical in contrast to the assumption of the standard economic theory. Nikiforos (2016, p. 439) divides the economic capacity into preferred capacity that refers to production level, which minimizes cost, and practical (full) capacity that assumes maximum use of non-quasi-fixed inputs of production. The Federal Reserve defines full-capacity in their survey as the level of output “under normal and realistic operating conditions,” as reported by Nikiforos (2016, p. 446), which fits the practical (full) economic capacity. On the other hand, Fontanari et al. (2019) interpret the potential output as the full-employment output that corresponds to a target level of unemployment, and León-Ledesma and Thirlwall (2002) define the growth rate of the potential (natural rate of growth) as the rate which leaves the unemployment rate constant.

As its definition, there is no consensus on a standard approach to estimating the potential output. One strand of the literature focuses on the survey data of capacity utilization such as the Federal Reserve Board (FRB) and Shaikh (1987) or average workweek of capital estimations either based on the changes in the shift structure as used by Nikiforos (2016) or actual electricity consumption over a theoretical maximum, as calculated by Foss (1963).[4]

[4]FRB utilization is de-trended by construction, as Nikiforos (2016) argues; therefore, it is not an appropriate

Although these methods rely on independent measures of capacity, their scope is limited because they cover only manufacturing, mining, and utilities, which produce around 15% of the US value-added today. For services, there are no independent capacity measures because of the lack of survey data. Another strand of the literature estimates the potential output by using the aggregate output data and applying either a theoretical model such as a production function or statistical filters such as the HP filter to derive the trend of the output series.

CBO uses the production function approach to estimate the potential GDP of the US economy. Shackleton (2018, p. 3) gives the potential output definition of CBO as "a measure of maximum sustainable output—the level of real GDP in a given year that is consistent with steady growth and a stable rate of inflation." CBO uses a sectoral approach to estimating the potential output where the sectors are coming from the BEA's decomposition of the economy: Households, non-profit institutions, non-farm business sector, farms, and government (federal and state&local). Shackleton (2018, p. 8) defines the general methodology as first estimating potential labor force and potential employment, then distributing it to the sectors of the BEA, and estimating the potential output of each industry by its production function.

For the non-farm business that makes almost 75% of the US economy, CBO uses a Solow growth model that combines potential labor hours (L_t^*), capital service (K_t), and potential total factor productivity (A_t^*) under a Cobb-Douglas production technology (constant c is needed because right-hand side variables are index values).

$$log(GDP_t^*) = log(A_t^*) + (1 - \alpha_t^*)log(L_t^*) + \alpha_t^* log(K_t) + c$$

CBO estimates a variable's trend by using the employment gap as a proxy for business cycle fluctuations. The employment gap is defined as the percent difference between actual employment and potential employment, calculated as potential labor force times one minus the natural rate of unemployment. CBO adjusts for cyclical movements of a series by a methodology called piecewise log-linear regression with time trends.[5] Potential labor hours (L_t^*) and total factor productivity (A_t^*) are estimated in this way; however, capital input (K_t) is left as it is because marginal productivities of different types of capital "already represent the potential contribution of capital to output." (Owyang et al., 2018, pp. 300) Lastly, CBO

measure for investigating long-run movements in capacity utilization. On the other hand, the average workweek of capital gives how many hours capital is utilized over a week by referring to an engineering capacity. It can show short and long-run changes in utilization better than FRB; however, it does not give information about how intense capital is utilized, such as production speed (Nikiforos, 2016, p. 447). Gahn et al. (2020) discusses the national emergency rate of the Survey of Plant Capacity, based on plant managers' comparisons of their actual output to national emergency output that includes full-utilization of capital at maximum intensity.

[5] CBO regresses the variable of interest in the log form on actual and lagged employment gap and dummy variables for each business cycle. The dummy variable takes zero before a particular business cycle peak is reached and increases by one unit each year until the following peak. This is defined as the piecewise log-linear regression with time trends or sticks (Shackleton, 2018, p. 15). After estimating coefficients of the time dummies, CBO sets actual and lagged employment gaps to zero and uses the fitted values of coefficients to derive the trend component. The estimated trend rises linearly over each business cycle and jumps at the beginning of the following cycle.

smooths the capital share by HP filter and weights growth rates of inputs by $(1 - \alpha^*)$ and α^* respectively.

For farms and non-profit institutions, CBO assumes that labor is the only factor of production, and estimates potential output as potential labor hours times potential labor productivity. For the households, the only production is the housing service of the owner-occupied dwellings. The government production function takes inputs of labor hours and depreciation of capital as factors of production together with labor productivity.

The CBO estimation's main drawback is that potential output, which is believed to be determined by supply-side factors such as productivity and labor force growth, constantly converges to actual output by eliminating output gaps, therefore, obscuring possible under-utilization of resources. One of the reasons behind the convergence is the Cobb-Douglas production function. Felipe and McCombie (2014) argue that aggregate production function is a direct transformation of the national accounting definition of gross value-added as compensation of employees plus gross operating surplus. When market values of labor and capital inputs are used for aggregation, the Cobb-Douglas production function necessarily fits data because this is how data are compiled in national accounts.

Another reason for the convergence is the natural rate of unemployment, which is used several times to estimate the employment gap, which is at the center of cyclical adjustments through the piecewise log-linear regressions. The natural rate is used to estimate labor force participation rates of different groups, calculate potential labor force, and determine potential employment as $(1 - u_t^*)$ times potential labor force. The employment gap acts as a center of gravitation for the CBO's output gap estimations.

However, the natural rate is unobserved and theory-dependent. It has become more controversial recently because the negative unemployment gap in the last years did not bring accelerating inflation in the US as the standard theory predicts. Using lower natural rates than the CBO is not going to solve the problem because unemployment reached historically low rates in the current business cycle, but the economic activity is still sluggish, and inflation is below the long-run target rate of the Fed. The current business cycle showed that a decline in the unemployment rate does not necessarily generate healthy economic activity due to factors ranging from the quality of newly created jobs to rising inequality that pushes aggregate demand down.

Another theoretical approach to estimate potential output is via Okun's Law, which posits an empirical relationship between the output gap and unemployment gap, calculated for a reference rate. This rate is generally taken as the natural rate of unemployment; however, Fontanari et al. (2019) use different target rates to estimate the potential output of the US over 1959-2017. They regress the first difference of unemployment rate to its first lag, GDP growth, and its first two lags (Fontanari et al., 2019, pp. 21):

$$\Delta u_t = \alpha + \gamma_1 \Delta u_{t-1} + b_1 g_t + b_2 g_{t-1} + b_3 g_{t-2} + e_t$$

They specify three different unemployment regimes, low ($u \leq 5.2$), medium ($5.3 \leq u \leq 6.6$) and high ($u \geq 6.7$), and estimate each regime's corresponding cumulative Okun coefficients

$(b^i = b^i_1 + b^i_2 + b^i_3)$. Then, by setting two target unemployment rates, $u^* = 4$ and $u^* = 3.4$, they estimate two potential output series $y^p = y(1 + \frac{1}{b^i}(u - u^*))$, with each regime's Okun coefficient, both of which give larger output gaps than CBO's estimates. Since their potential output estimates are anchored to the unemployment gap, they find a persistent excess capacity from the late 1960s until 2000, which is not observed in the reference cycle method. Moreover, since the current business cycle generated historically low unemployment rates, their estimates converge to the CBO measure in recent years, therefore, leave concerns about the current under-utilization unanswered.

León-Ledesma and Thirlwall (2002) also use Okun's law to estimate the potential output, but they interpret the growth rate, which leaves the unemployment rate constant in the Okun's equation as the natural rate of growth:

$$\Delta U = a - b(g)$$

where U is the unemployment rate, g is real GDP growth, and the natural rate becomes $g^N = a/b$. They define the natural rate as the economy's long-run growth rate, which is the sum of labor productivity growth and growth of the labor force. Alternatively, they estimate the following regression:

$$g = a_1 - b_1(\Delta U)$$

and interpret a_1 as the natural rate. For the US economy over 1961-1995, the natural growth rate is estimated as 2.99% in both regressions at the 5% significance level. However, their result is lower than the average trend growth calculated by the reference cycle method (3.50%) and CBO (3.33%) over the same period. Moreover, it is relatively low compared to actual GDP growth (3.37%), meaning that their results imply an overcapacity in the US economy over 1961-1995, which does not appear in the reference cycle output gap.

Shaikh and Moudud (2004) use a co-integration method to estimate capacity utilization of the US economy over 1960-2000 by the following regression:

$$log(Y_t) = \alpha_0 + \alpha_1 t + \alpha_2 log(K_t) + e_t$$

where Y_t is output, and K_t is the capital stock. They define potential output as part of the actual output that co-varies with the capital stock in the long-run (Shaikh and Moudud, 2004, pp. 14). Their potential output moves above actual output over 1960-1980, declines below the actual during the 1980 recession, and then moves slightly above the actual output, except the trough years of GDP, until 2000. Only the 1980 recession is recorded as an under-utilization period over 1960-2000 in the co-integration method. The reference cycle method estimates utilization rates below Shaikh and Moudud (2004)'s estimations over 1960-2000.

On the other hand, statistical filters derive potential output from the actual output by filtering out the actual output series trend by minimizing cyclical fluctuations or bringing some business cycle models to estimate the output gap. The most common filter is the HP filter that minimizes the sum of squares of the output gap plus the fluctuations in the trend through a weight parameter λ. As λ approaches zero, trend output converges to actual output, and as it rises, trend output becomes smoother. Barbosa-Filho and Taylor (2006) use HP filter to derive the potential output of the US within a Goodwin type cyclical growth framework. Owyang et al. (2018) argue that choosing the correct λ is one of the problems in using the HP filter. Another issue is the usage of past and future levels in the optimization to derive the trend output at any point in time. As Blecker and Setterfield (2019) mention, this feature of the HP filter makes it difficult to interpret any $(y_t - y_t^*)$ as an independent capacity measure at time t. Moreover, internalization of future recession information creates spurious positive output gaps before downturns in GDP (Cerra and Saxena, 2017, pp. 12), which can be seen before all recessions in Figure 4.

Coibion et al. (2017) discuss alternative potential output estimations, which impose long-run restrictions on output shocks to decompose the trend from cycles. Blanchard and Quah (1988) define two types of shocks: Demand shocks do not have any long-run impact on output and unemployment; however, supply shocks may affect output in the long-run. By using a bivariate Vector Autoregression (VAR) model of change in real output and unemployment with their eight lags, Blanchard and Quah (1988) decompose real output into its trend (with muting demand shocks) and cycles (with zero supply shock). Coibion et al. (2017) apply this method to the post-Great Recession and observe an output gap of around 6 percent in early 2017. Like Blanchard and Quah (1988), Gali (1999) uses a structural bivariate VAR model of productivity and hours by identifying technology and non-technology shocks. The restriction is that only technology shocks can change productivity growth in the long-run. Coibion et al. (2017) estimate potential output based on this method and find an output gap above 10% in 2017. Lastly, Coibion et al. (2017) discuss Cochrane (1994)'s definition of potential output with respect to the consumption share of output. By referring to the permanent income hypothesis, Cochrane (1994) argues that any shock to actual output that leaves consumption level unchanged is a transitory shock because otherwise, consumers would change their expectations and consumption decisions. Cochrane (1994) uses a bivariate VAR model of changes in consumption and output with their two lags while using the consumption/output ratio as one of the explanatory variables. By using this method, Coibion et al. (2017) estimate an output gap above 10% for 2017. All three alternative estimations of Coibion et al. (2017) point to a larger negative output gap than the reference cycle method in 2017.

There are also advanced filters, such as unobserved components (UC) and multivariate unobserved components (MUC). Unobserved components assume that actual output is the sum of potential output y_t^*, which follows a random walk with drift, and an AR(P) cyclical term c_t (Owyang et al., 2018, pp. 301):

$$y_t = y_t^* + c_t$$

$$y_t^* = \tau + y_{t-1}^* + \epsilon_t$$

$$c_t = A(L)c_{t-1} + v_t$$

Multivariate unobserved components add a theoretical relationship between potential output and inflation and allow inflation to affect the potential output levels. This method decomposes actual output y and inflation π to their trends (y^*, π^*) that follow random walks with drift and cycles ($c_t = [c_t^y, c_t^\pi]'$) which show a VAR(p) structure (Owyang et al., 2018, pp. 302):

$$\pi_t = \tau^\pi + \pi_{t-1} + \epsilon_t^\pi$$

$$y_t^* = \tau^y + y_{t-1}^* + \epsilon_t^y$$

$$c_t = A(L)c_{t-1} + v_t$$

Owyang et al. (2018) estimate the output gap of the US over 1950-2015 by using UC and MUC models. UC model shows a persistently negative output gap until the mid-1970s and a positive output gap over 1990-2009. After the last recession, the output gap declines below zero and does not recover. In 2015, the UC model predicts an excess capacity reaching almost \$1 trillion (in 2009 dollars). On the other hand, the MUC model fluctuates around full-capacity over 1950-1990, and after that, it shows a persistent overcapacity until the Great Recession. MUC output gap declines more than UC in the last business cycle and shows an excess capacity of over \$1 trillion in 2015. These findings are more than the excess capacity estimated by the reference cycle method (\$0.5 trillion) in 2015, the last year of Owyang et al. (2018). My measure also differs from the UC and MUC in its finding of excess capacity over 1991-2007.

Another advanced filter is the Kalman filter which is used by Kiefer et al. (2019) in a Goodwin type model to capture cyclical component of growth. They use a VAR(2) model with capacity utilization and labor share as dependent variables:

$$\psi_t = \alpha_0 + \alpha_1\psi_{t-1} + \alpha_2 v_{t-1} + \alpha_3\psi_{t-2} + \alpha_4 v_{t-2} + \epsilon_{\psi,t}$$

$$v_t = \beta_0 + \beta_1\psi_{t-1} + \beta_2 v_{t-1} + \beta_3\psi_{t-2} + \beta_4 v_{t-2} + \epsilon_{v,t}$$

where ψ_t is labor share and v_t is capacity utilization as the difference between the natural log of output y_t and natural log of potential output y_t^*: $v_t = y_t - y_t^*$. Growth of potential output g_t^* follows a random walk: $g_t^* = g_{t-1}^* + \epsilon_{g,t}$.

Results of Kiefer et al. (2019) follow the growth rate of CBO potential over 1950-2000 but indicate a sharper decline after 2000, which they interpret as a sign of secular stagnation that preceded the Great Recession. However, their results create a spurious output gap before the Great Recession, just like the HP filter and eliminate the under-utilization following the recession. On the other hand, the reference cycle method estimates a moderate decline in the trend after the Great Recession compared to CBO, and a persistent excess capacity since 1990.

4. Contemporary macroeconomic significance

The reference cycle method estimates the output gap of the economy without referring to full-employment or stable inflation. In this sense, it frees the output gap from an assumed theoretical relation with inflation and unemployment. According to the reference cycle method, it is possible to see high inflation without excess demand and low unemployment with sluggish demand. The 1960-1969 business cycle, which created inflation rates up to 6%, appears as an overcapacity period with the reference cycle method; however, excess capacity is estimated during the 1970s, a high inflation-high unemployment period for the US. Excess aggregate demand creates inflationary pressures such as the late 1960s, but it is not the only explanation of inflation as the 1970s show. Investigating the inflation-reference cycle output gap relation within a Phillips curve framework is part of my future research agenda.

Also, there is no reference to the natural rate argument or any other definition of full-employment in the reference cycle method. This is an advantage compared to CBO's estimations, which fluctuate around a controversial natural rate. The reference cycle method treats two full-employment economies, the 1960s, and the last business cycle, quite differently. An overcapacity is observed in the 1960s; however, there is a persistent excess capacity in the last business cycle. High unemployment pushes the output gap down as the decline of the reference cycle output gap during recessions show; however, low unemployment does not necessarily bring full-utilization when the demand side is not strong enough. This finding addresses the concerns about the economy's current slack due to the quality of jobs and rising inequality better than the conventional measures that take the unemployment gap as a proxy for the output gap.

There are several explanations of the sluggish aggregate demand in the last decades. Summers (2014) argues that US growth is in a long-term decline by referring to the secular stagnation concept of Hansen (1939), who initially used it to describe the slow growth of the post-Great Depression years. Summers (2015) explains the US economy's unsatisfactory recovery from the Great Recession by the excess savings over investment, which pushes the equilibrium interest rate below zero. Given the zero lower bound on the nominal interest rates, the adjustment occurs through a continuous decline in output, which leads to secular stagnation. Summers (2015) mentions the rise in profit share, fall in the price of investment goods, and globalization as possible reasons behind the savings glut.

Gordon (2015), on the other hand, points to productivity slowdown as the reason behind supply-side secular stagnation. He argues that potential output grew even slower than the actual output in the last recovery, thus eliminated the output gap. From a distributional perspective, Kiefer et al. (2019) also discuss a decline in potential output growth post-2000. Eichengreen (2014), however, does not give credit to the technological slowdown argument. Instead, he mentions the need for demand boosting policies and investment in infrastructure, training, and education.

Although there is a decline in the long-term growth rate of the economy since 1980, actual output was below the full-capacity level throughout the last business cycle, when CBO's output gap declined mainly due to downward revisions to potential instead of rising actual output. As I argue in the previous section, CBO's estimation methodology makes interpreting this finding harder as a supply-side stagnation. My finding of permanent excess capacity since 1990 supports the demand-side secular stagnation. Unlike the headline measures, there was a slack in the economy of 2.5%, even in the tenth year of the recovery. According to the generalized Poisson model predictions, utilities, manufacturing, wholesale trade, transportation-warehousing, finance-insurance, and education were below their trends in 2019.

Secular stagnation is the outcome of decades-long incorrect macroeconomic policies, therefore it is not an inevitable feature of the American economy. Aggregate slack in the economy opens room for short and long-run demand supporting policies. Fixing the tax code in favor of working-class Americans, increasing minimum wages, strengthening the bargaining power of labor through institutional changes, and large scale public investment on infrastructure and human capital will reduce inequality and increase output not only in the short-run but also through productivity-enhancing positive feedback effects in the long-run. As Eichengreen (2014) says, "if the US does experience secular stagnation over the next decade or two, it will be self-inflicted." (p. 41)

5. Conclusion

Current output gap estimations rely on aggregate data, assume constant utilization over industries. However, each sector has its cycle, and missing sectoral dynamics create underestimation of the output gap. In this paper, I develop a sectoral approach to trend output estimation of 20 US industries by using the reference cycle method. Trend output is defined at the peak years of reference cycle relatives of Mitchell (1946). For the years between peaks, the trend is interpolated by assuming a constant growth rate. For the years following the last peaks, a generalized Poisson model is estimated to predict the next peak's timing and magnitude. Estimated parameter values indicate that the next peak's timing is quite close to each other for 20 industries; however, its magnitude changes widely according to the economy's stagnant and growing sectors.

The reference cycle method differs from existing methods in its reliance on sectoral output measures. The aggregate output gap is derived as a weighted average of industry output gaps, generally negative over 1948-2019. The reference cycle output gap decouples from the existing measures since 1990, when the US economy has experienced demand-side problems. Unlike the HP filter, it does not estimate spurious positive output gaps before recessions, and unlike the CBO, it does not necessarily eliminate output gaps in each cycle. The reference cycle output gap was negative throughout the last business cycle even though CBO and HP filter estimated overcapacity since 2017. The reference cycle method also gives sectoral output gaps for 20 US industries over 1948-2019. In 2019, there was still an excess-capacity in utilities, manufacturing, wholesale trade, transportation-warehousing, finance-insurance, and education according to model predictions. It was possible to increase output of these industries without creating sectoral bottlenecks. My findings imply a larger room for demand boosting policies and support the demand side secular stagnation of Summers (2014).

The reference cycle method does not define full-capacity based on an assumed theoretical relation with inflation or unemployment. The economy can experience inflation before reaching the full-utilization of resources; therefore, the output can decline before arriving its trend. This is what the reference cycle method estimates for the inflationary 1970s. On the other hand, an increase in unemployment brings larger negative output gaps; however, the economy can generate full-employment without eliminating the output gap due to the quality of newly created jobs and rising inequality that pushes aggregate demand low. The reference cycle method estimates excess capacity over the last business cycle, which generated historically low unemployment rates. In this sense, the reference cycle method gives a reliable measure for aggregate demand, estimated independent of inflation and unemployment.

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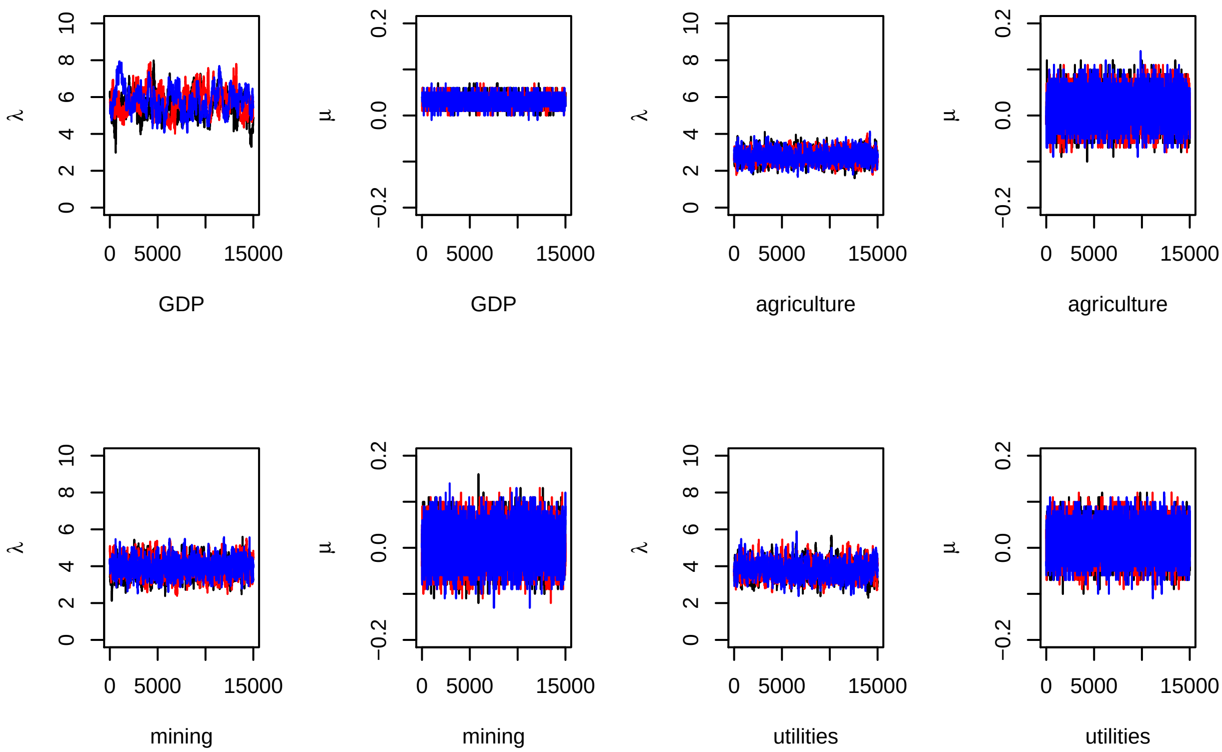
Appendix

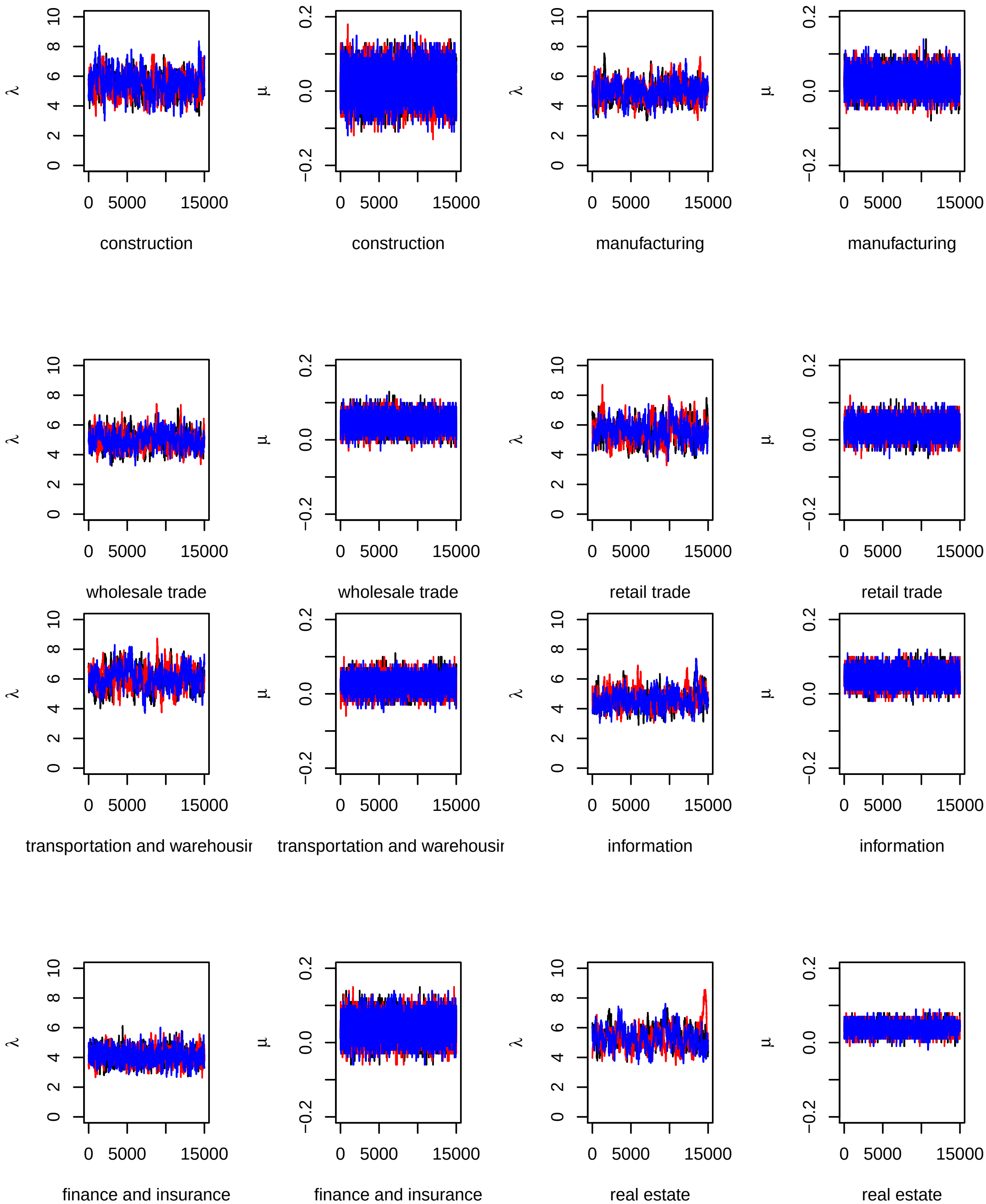
RWMH algorithm first draws candidates from a normal distribution with mean equal to the last accepted draw and with a pre-determined variance-covariance matrix. At step t, the algorithm draws a candidate (λ^t, μ^t) from $N((\lambda^{t-1}, \mu^{t-1}), \Sigma)$ and evaluates the joint posterior distribution at this candidate draw which is then compared to the posterior distribution at the last accepted draw:

$$\alpha = \frac{p(\lambda^t, \mu^t|x)}{p(\lambda^{t-1}, \mu^{t-1}|x)}$$

Then, a random number u is drawn from the uniform distribution over the $[0, 1]$ interval, and if $\alpha \geq u$, the candidate (λ^t, μ^t) is stored as the t^{th} draw. Otherwise, the last accepted draw $(\lambda^{t-1}, \mu^{t-1})$ is stored again. When this process is repeated N times, one arrives at $(\lambda^1, ..., \lambda^N)$ and $(\mu^1, ..., \mu^N)$, which converge to the marginal posterior distribution of model parameters λ and μ (Greenberg, 2012, pp. 100).

I initialize the algorithm with λ^1, which is drawn from the uniform distribution over $[3, 6]$ and μ^1, which is drawn from the uniform distribution over [-0.2,0.2]. I run the algorithm $20,000$ times with $\Sigma = (0.04, 0; 0, 0.01)$ and burn in the first $5,000$ draws. I use three chains to see whether accepted draws are mixed or not. Below, I give trace plots of parameter draws for λ and μ for GDP and 20 industries.

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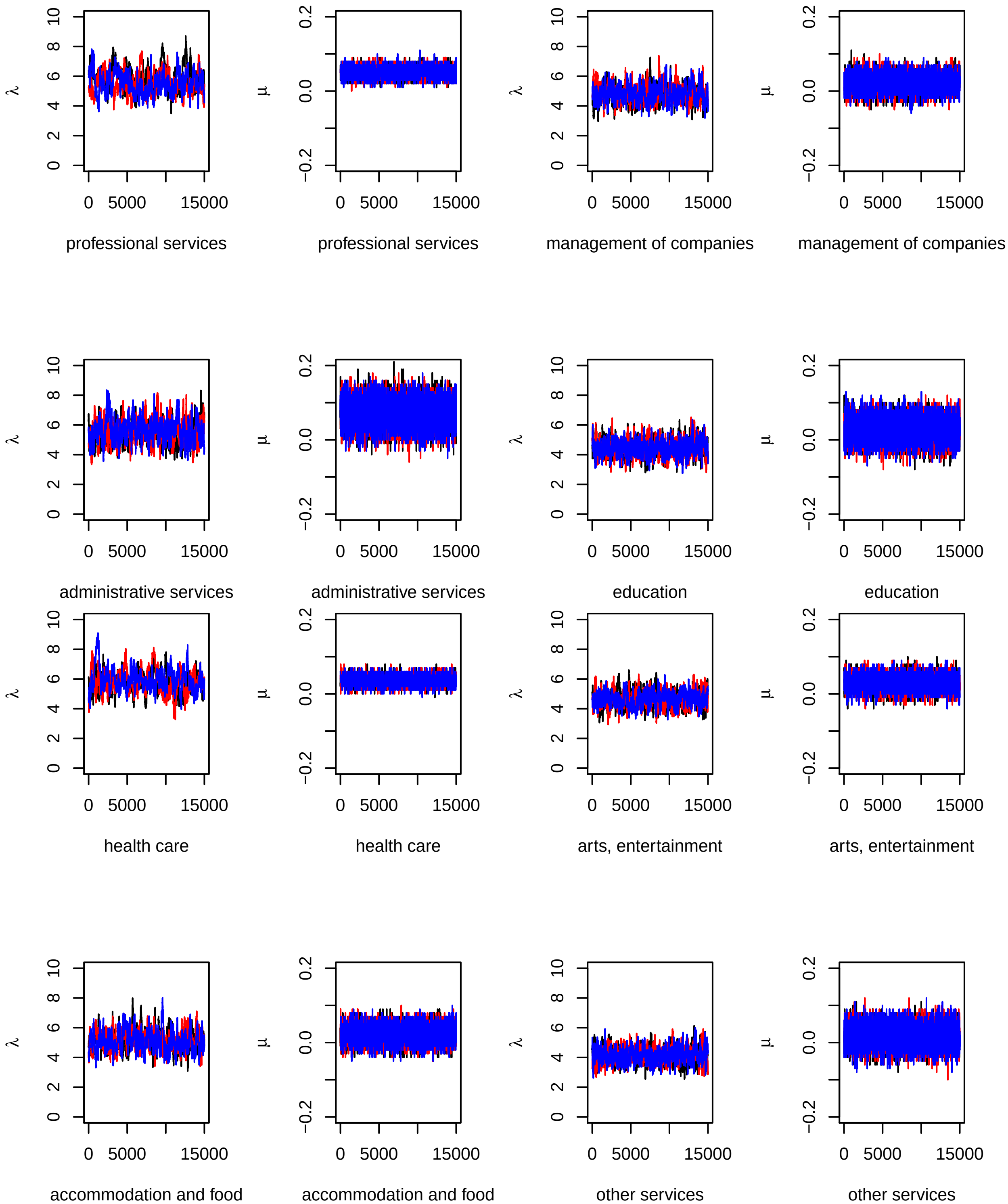
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professional services
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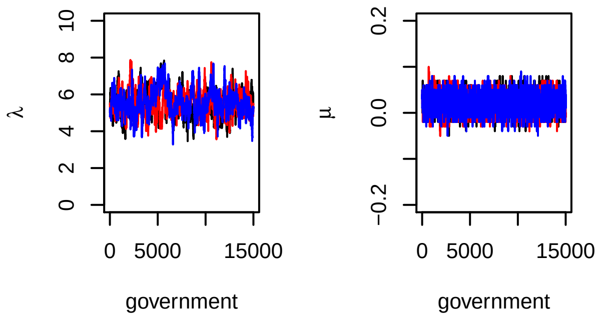
accommodation and food
other services
other services


λ
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