Data-driven Turbulence Prediction with Latent Diffusion Models

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Outline

Basic

Packages & Setup

Customized Citation Format

VT Theme Setup

Shades for blocks in VT Theme

Macros & Utilities

Surrogate Modelling

Limitations of Traditional Models I

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Limitations of Traditional Models II

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Diffusion Models: Idea

- Forward process (Diffusion process): add noise
  - Add i.i.d. Gaussian noise to data samples over many steps
  - Progressively degrade the data into pure noise (prior distribution, often i.i.d. Gaussian)
  - Explicitly prescribed, no modeling needed
- Backward/Reverse process (Denoising process): remove noise
  - Added noise approximated by a neural network
  - Recover the original data by gradually removing the noise using neural network, starting from pure noise

Diffusion Models: Overall Architecture

- Forward Process:
  - Given a data sample x_0, each forward step adds a small amount of Gaussian noise:

$$x_t = \sqrt{1-\beta_t}x_{t-1}+\sqrt{\beta_t}\epsilon, \quad \epsilon \sim \mathcal{N}(0, I),\ t \in [1, T\boldsymbol{x}_t = \sqrt{1-\beta_t}\boldsymbol{x}_{t-1}+\sqrt{\beta_t}\boldsymbol{\epsilon},$$

  - β_t: noise schedule, ϵ: sampled Gaussian noise
- Model Training:
  - Get x_t from x_0.
  - Train a model ϵ_θ to predict the added noise at a specified time step t.
  - Approximate backwards process using

$$x_{t-1} = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon_\theta(x_t, t)\right) + \sqrt{\beta_t}\epsilon$$

where $\alpha_t = 1-\beta_t, \quad \bar{\alpha}_t = \prod_{i=1}^{t}\alpha_i$

Diffusion Models: Conditional Diffusion Models

- *Diffusion model*: learn the probabilistic distribution $p(\boldsymbol{x})$
- *Conditional diffusion model*: learn the conditional probability of $p(\boldsymbol{x}|\boldsymbol{y})$
- What $\boldsymbol{y}$ (conditions) can be:
  - Class labels, text prompt, attributes, or any conditioning signal ...
- When you ask ChatGPT questions
  - You input text $\boldsymbol{y}$.
  - ChatGPT give you answers by sampling $p(\boldsymbol{x}|\boldsymbol{y})$, either text or images.

Conditional Diffusion Models for Flow Field Prediction

- In fluid dynamics, $\boldsymbol{y}$ (conditions) can be:
  - Labels: Reynolds number ...
  - Constraints: physical laws
  - Incomplete information: sparse measurements
  - Integral information without details: lift, drag
- Matches the distribution of the training data, unlike deterministic models.

Conditional Diffusion Models for Data Assimilation

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Generative Prediction of Urban-scale Extreme Events: Project Idea I

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Generative Prediction of Urban-scale Extreme Events: Project Idea II

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Generative Prediction of Urban-scale Extreme Events: Results

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Generative Modeling of Kolmogorov Flow: Kolmogorov Method

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Generative Modeling of Kolmogorov Flow: Data Assimilation Method

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Generative Modeling of Kolmogorov Flow: Data Assimilation Results

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Generative Modeling of Kolmogorov Flow: State Results

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Generative Modeling for Windfarm Prediction: Reduced Order Modeling

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Generative Modeling for Windfarm Prediction: Neural Network Framework

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ITLR-DDSim

Generative Modeling for Windfarm Prediction: Results I

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Generative Modeling for Windfarm Prediction: Results II

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Summary

Thank you!

Thank you for your attention!

References