Channel Estimation Using a Chirp Signal and the Fractional Fourier Transform

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***Abstract*— The Fractional Fourier Transform (FrFT) is a useful tool that has many applications, such as interference mitigation for communications and radar target echo separation. In this paper, we present a new use, which is estimating an unknown multipath channel, by sending a short chirp signal through the channel. The multiple received chirps in multipath are rotated to the proper FrFT dimension where they become high power tones, whose amplitudes and delays are easily estimated by determining which values in the rotated spectrum exceed a given threshold γ, which is also easily computed. These are then mapped back to the original time domain. This method is enabled because of the nature of the FrFT and its ability to pull signals, especially chirp signals, out of noise. We present the signal and multipath model, and then describe how the FrFT is used to obtain the channel estimates. Through simulations, we show that this is a very accurate method, providing root mean-square error (RMSE) estimates of both channel coefficients and delays at least an order magnitude below that of existing methods, even at signal-to-noise ratios (SNRs) as low as 0 dB. It is also very low in complexity, because all coefficient amplitude and delays are estimated simultaneously with few computations; it therefore offers a promising channel estimation solution for existing and future terrestrial communications systems, including $4G/5G$ cellular systems requiring high data rate applications.**

Keywords - channel estimation, chirp, Fractional Fourier Transform, multipath

I. Introduction

The Fractional Fourier Transform (FrFT) has been applied in numerous fields of research, including optics [1], quantum mechanics, image processing, and communications ([4]). It is a powerful tool that can extract signals from noise or separate two signals that overlap in time and/or frequency. FrFT application lends itself particularly to the problem of separating multiple overlapping radar chirp signals. Chirps become tones in the proper FrFT domain, and hence can be readily extracted or notched [7]. Applying that principle in this paper, we use the FrFT to the problem of estimating an unknown RF terrestrial multipath channel, using a short transmitted chirp signal. This chirp is known as a pilot signal, which is a known signal that can help in processing the signal at the receiver, by aiding with acquisition and channel estimation. The multipath channel produces several delayed and scaled chirp echoes at the receiver, which can be converted into tones by the FrFT, and the amplitude and spacing of the tones provides the characteristics of the multipath channel coefficients. This occurs because multipath involves a signal bouncing off of nearby objects, resulting in multiple, non-coherent versions of the signal arriving at the receiver.

Past work on channel estimation has been numerous, as it is a significant issue in systems where multipath is a problem, primarily cellular systems but also low elevation angle satellite systems. We compare our results to the lower bound derived in [3]. Most methods of channel estimation use some form of pilot, which is a known portion of the signal that aids the receiver in estimating the channel. The most recent and most promising methods include least-squares (LS) estimation, and minimum mean-square error (MMSE) based estimation methods [5]. Improved methods based on LS and MMSE are presented in [8]. We will compare the performance of the proposed chirp pilot method with the results presented in these papers to demonstrate the improvement in the proposed technique.

In addition, the authors in [9] proposed a method using the FrFT for underwater acoustics, wherein Doppler, delays and rotations have to be estimated successively because these parameters differ for each component. For our problem of terrestrial multipath estimation, Doppler differences between successive multipath components is negligible, and this results in a simpler and more computationally efficient algorithm where each component may be estimated simultaneously using just one FrFT rotation.

While it must be understood that chirps are not currently used as pilots, the performance results suggest that they could make good alternatives for accurate channel estimation in future, e.g. $5G$, systems. Combined with the accuracy and simplicity of the proposed method, they can provide rapid channel estimation to enable operation with the high data rates that future systems will need to support.

An outline of the paper is as follows: Section II gives background information on the FrFT and its relation to the Wigner Distribution (WD), which is a useful visual representation of the FrFT. Section III describes the transmitted chirp signal model and the multipath channel model. Section IV presents the best-known conventional methods for channel estimation using LS and MMSE, as well as a brief description of a lower bound on the root mean-square

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estimation (RMSE) error. Section V describes the proposed method for computing the channel coefficient amplitudes and associated delays, using the FrFT [1]. Section VI gives some numerical examples to quantify the performance of the method in noise. Finally, we conclude with a summary and remarks on future work in Section VII.

II. Background: The Fractional Fourier Transform (FrFT)

The time-frequency plane representation of a signal, $x(t)$, and noise, $n(t)$, is shown in Fig. 1. This is also called the Wigner Distribution (WD), and it is a useful visualization tool for the FrFT. The time and frequency axes are orthogonal to each other in this representation. The time axis may be rotated using a rotational parameter 'a', where $0 \leq a < 2$ to determine the new time axis (t_a) and new frequency axis (f_a). This is done using the FrFT and provides a new way of looking at a signal. The figure shows an example of using the FrFT to remove noise from the signal. By rotating to the new time axis t_a, we can make the signal and noise more separable, enabling a filter (shown in red in Fig. 1) to easily extract the signal and completely suppress the noise. Filtering in time or frequency alone would not have resulted in such complete noise suppression.

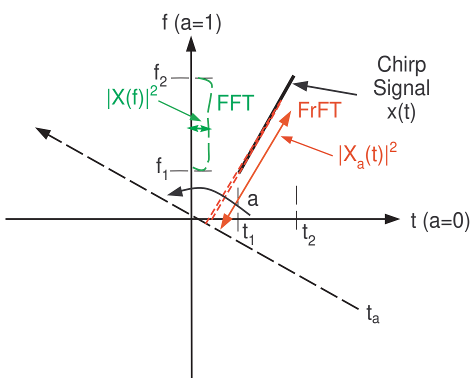

Fig. 1. WD Representation of Signal x(t) and Noise n(t)

In discrete time, we model the signal $x(t)$ as an $N \times 1$ vector $\mathbf{x}$. Its $N \times 1$ dimension FrFT $\mathbf{X}_a$ is defined as

$$\mathbf{X}_a = \mathbf{F}^a \mathbf{x}, \tag{1}$$

where $\mathbf{F}^a$ is an $N \times N$ matrix whose elements are given by [7])

$$\mathbf{F}^a[m,n] = \sum_{k=0, k\neq(N-1+(N)_2)}^{N} u_k[m] e^{-j\frac{\pi}{2}ka} u_k[n]. \tag{2}$$

[1]Patent Pending

Here, $u_k[m]$ and $u_k[n]$ are the eigenvectors of the matrix $\mathbf{S}$, defined by [2]

$$\mathbf{S} = \begin{bmatrix} C_0 & 1 & 0 & \dots & 1 \\ 1 & C_1 & 1 & \dots & 0 \\ 0 & 1 & C_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & C_{N-1} \end{bmatrix}, \tag{3}$$

and

$$C_n = 2cos(\frac{2\pi}{N}n) - 4, \tag{4}$$

where $l_1 = max(0, n-(N-1))$ and $l_2 = min(n, N-1)$, and $n = 0, 1, ..., N-1$. Here, 'a' is the variable rotational parameter, ranging from $a = 0$ to $a = 2$, and is varied using a step size Δa. Typically, Δa is chosen to be 0.1 in a low noise (and clutter) environment, or 0.01 if a signal to be extracted is weak or the noise is strong; even smaller step sizes for very dense environments can be considered.

It is well-known that the projection of the WD of a signal $\mathbf{x}$ onto the time axis t_a gives the energy of the signal, which is equivalent to taking the magnitude squared in the FrFT domain 'a'. In other words, we compute the quantity in Eq. (1), take its magnitude squared, i.e. $|\mathbf{X}_a|^2$, and we obtain the energy in the rotated signal. This is shown by the red curve in Fig. 2. It is analogous to taking the magnitude squared of a traditional Fast Fourier Transform (FFT) to obtain energy of a signal in the frequency domain. This quantity is conventionally called power spectral density (PSD), and a similar expression for PSD in the FrFT domain 'a' is

$$PSD_{\mathbf{x}}(a) = |\mathbf{X}_a|^2. \tag{5}$$

Note that $PSD_{\mathbf{x}}(a)$ is easily computed from Eq. (1), via an FrFT, whereas determining the WD of the signal and finding its projection is far more difficult in practice. There are numerous existing methods for computing the FrFT efficiently, for example [2]. Next, we describe the chirp signal, its FrFT, and the channel model.

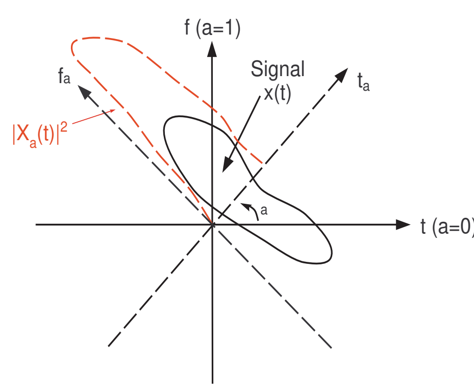

Fig. 2. WD Representation of Signal x(t); Energy of Signal $x(t)$ Computed Using $|\mathbf{X}_a(t)|^2$ in Red

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III. Signal and Channel Models

Let $x(t)$ be a chirp signal whose frequency linearly increases over a time interval, t_0 to t_1, and over the frequency interval, f_0 to f_1. In the right FrFT domain, the chirp becomes a narrow, high powered tone. See Fig. 3 for a diagram, where the lines show the magnitude squared of the FFT (green) and FrFT (red). The PSD computed by taking the magnitude squared of an FFT of the chirp is spread out over frequency, giving it a smaller magnitude (shown by the double arrow green line). At the proper rotational value, the FrFT compresses the signal into a tone, resulting in an increased magnitude of its PSD, computed using Eq. (5) (shown by the double arrow red line). This enables us to easily pull the chirp out of noise and interference (not shown in the figure).

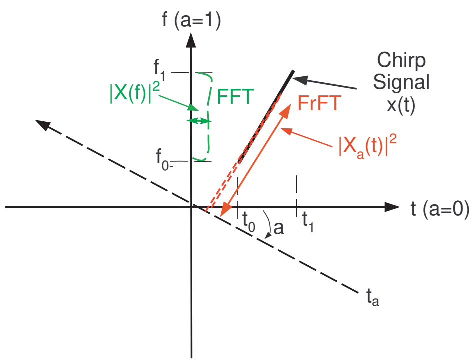

Fig. 3. WD Representation of Chirp Signal x(t)

We model the transmitted chirp signal as a real signal, denoted by

$$x(t) = cos(2\pi f_0 t) \cdot cos(\pi K t^2), \tag{6}$$

where f_0 is the initial frequency of the chirp [Hz], and K is its rate of change of frequency [Hz/second]. We assume that the received signal $y(t)$ is the transmitted signal $x(t)$ distorted by a Rayleigh distributed multipath channel $c(t)$ and additive white Gaussian noise (AWGN). The AWGN is modeled using a desired signal-to-noise ratio (SNR). Our objective is to use our knowledge of the transmitted chirp signal parameters f_0 and K, as well as the received signal $y(t)$, to estimate the channel. This is described after we discuss the conventional methods for estimating the channel.

IV. Conventional Channel Estimation Techniques

We limit our attention to optimum techniques presented in the recent literature. Specifically, we consider least-squares (LS) and minimum mean-square error (MMSE) based estimation, which are well-known conventional techniques. We also present the lower bound on estimation error.

A. Least-Squares Channel Estimation

We follow [5] and write the received signal in discrete time vector form as

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{n}, \tag{7}$$

where the vectors $\mathbf{x}$ and $\mathbf{n}$ are formed by collecting N samples of the signal, and the columns of the $N \times N$ matrix $\mathbf{C}$ are formed from the real channel vector, $\mathbf{c} = [c_1 \; c_2, \; \ldots, \; c_L]^T$, where L is the number of non-zero channel coefficients, using

$$\mathbf{C} = \begin{bmatrix} c_1 & 0 & \ldots & 0 \\ c_2 & c_1 & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ c_L & c_{L-1} & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \end{bmatrix} \tag{8}$$

Typically, L is less than or equal to seven. The delay spread of the channel, in units of seconds, is the number of samples between c_L and c_1 divided by the sampling rate f_s [samples/second] (used to capture the signal at the receiver). Note that for most conventional methods, the pilot signal $\mathbf{x}$ takes the form of a known sequence of bits. In the LS technique, we seek the channel that minimizes the following cost function

$$\Gamma = (\mathbf{y} - \mathbf{C}\mathbf{x})^H (\mathbf{y} - \mathbf{C}\mathbf{x}). \tag{9}$$

Taking the derivative of Γ with respect to the channel matrix $\mathbf{C}$, setting it to zero and solving, we obtain the following solution for the channel estimates (for details see [5], [6], and [8]),

$$\hat{\mathbf{C}}_{LS} = (\mathbf{x}^H \mathbf{x})^{-1} (\mathbf{x}^H \mathbf{y}). \tag{10}$$

The root mean-square error (RMSE) between the true channel and its estimates is computed using

$$RMSE_{LS} = ||\mathbf{C} - \hat{\mathbf{C}}_{\mathbf{LS}}||_2. \tag{11}$$

B. Minimum Mean-Square Error (MMSE) Channel Estimation

The MMSE criteria minimizes the error in the channel estimates, given by

$$\epsilon = |\mathbf{C} - \hat{\mathbf{C}}|^2. \tag{12}$$

The solution is readily computed to be ([5] and [6])

$$\hat{\mathbf{C}}_{MMSE} = [\mathbf{x}^H \mathbf{x} + \frac{\sigma_n^2}{\sigma_c^2}]^{-1} (\mathbf{x}^H \mathbf{y}). \tag{13}$$

Note that the MMSE solution requires knowledge of the statistics of the channel, specifically the variance of the noise and channel, given by σ_n and σ_h, respectively. Similar to LS, the RMSE for this method is computed by

$$RMSE_{MMSE} = ||\mathbf{C} - \hat{\mathbf{C}}_{\mathbf{MMSE}}||_2. \tag{14}$$

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C. Lower Bound

The lower bound (LB) in [3] is derived from the MMSE estimate by letting the channel be a single path. The RMSE in this case is written as

$$RMSE_{LB} = \sqrt{\frac{1}{1+SNR^{-1}}}, \tag{15}$$

where SNR is the signal-to-noise ratio and determines the noise power, σ_n^2. We will compare all three of these RMSE values to that of the proposed algorithm, discussed next.

V. Proposed Chirp and FrFT Based Channel Estimation Algorithm

Consider the WD of a chirp signal traversing a simple two-tap channel, where the first tap is due to the direct path and the second is due to a multipath bounce. This is shown in Fig. 4, in which the direct path is the solid green line and the multipath is the dashed green line. The chirp initial frequency is f_0 and its final frequency is f_1. The slope, or rate of change of frequency is K, defined as

$$K = (f_0 - f_1)/(t_0 - t_1) = \Delta f_1/\Delta t_1, \tag{16}$$

where $\Delta f_1 = f_0 - f_1$ and $\Delta t_1 = t_0 - t_1$. We must estimate the amplitude of the multipath component shown by the dashed green line, as well as its time delay, given by Δt_2. Note that if we rotate to the FrFT time axis $t_{a_{opt}}$, we can easily compute the magnitude of the multipath signal by finding the peak value in the second tone from Eq. (5). The axis is found through a search over all 'a', choosing the one that gives the strongest amplitude of the signal's FrFT [7]. Starting with $a = 0$, we compute the FrFT of the received discrete time signal, $\mathbf{y}$, obtained by collecting N samples of $y(t)$. Thus, we obtain

$$\mathbf{Y}_a = \mathbf{F}^a \mathbf{y}, \tag{17}$$

and we choose the maximum value over the new axis t_a

$$Y_{max,a} = max(|\mathbf{Y}_a|). \tag{18}$$

Note that the result is a scalar value. After performing this operation over the range of all 'a', $a = 0{:}\Delta a{:}2$, we obtain the vector $\mathbf{Y}_{max,a}$ and compute the value of 'a' that gives the strongest peak, i.e.

$$a_{opt} = arg\ \underset{a}{max}\ \mathbf{Y}_{max,a}. \tag{19}$$

We then rotate to this optimum 'a', a_{opt} with

$$\mathbf{Y}_{a_{opt}} = \mathbf{F}^{a_{opt}} \mathbf{y}, \tag{20}$$

which converts all the chirps to tones. Note that computation of a_{opt} only needs to be done once, as it is the same for all components. On the contrary, in [9], it must be done separately for each component.

The channel coefficient amplitudes for all the multipath components may now be found directly. First we compute

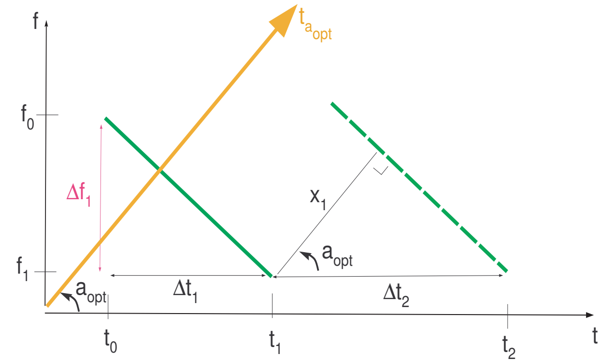

Fig. 4. WD of Direct Path - Solid Green Line and Single Multipath Component - Dashed Green Line

a threshold γ based on the rotated spectrum's noise floor, which may be easily computed from the first few samples,

$$\gamma = var(\mathbf{Y}_{a_{opt}}(1:\delta)), \tag{21}$$

where δ is the number of samples, which could be set to $\delta = 100$, or some other small number. Next, we determine the values of $\mathbf{Y}_{a_{opt}}$ that fall above the threshold. These are the amplitude estimates of the channel coefficients,

$$\hat{\mathbf{c}}_{Proposed} = \mathbf{Y}_{a_{opt}}(|\mathbf{Y}_{a_{opt}}| > \gamma). \tag{22}$$

We mention that we can ignore samples that are adjacent to maximum values, as these may be part of the same multipath component. In general, we consider that only components separated by at least a few samples (determined by the expected delay spread and nature of the channel) are valid ones. Hence, all coefficients are obtained simultaneously.

We can also compute the time delay between the direct path and the multipath component by measuring x_1 from the rotated spectrum by finding the distance between the peaks (refer to Fig. 4). To compute the true delay, we must calculate Δt_2 from x_1. From the figure, we see that this may be easily computed from the geometry using

$$\Delta t_2 = x_1/cos(a_{opt}). \tag{23}$$

Defining $l = 1$ as the direct path, the time delay between multipath component l and $l-1$, $l = 2, 3, ..., L$ can be more generally written as

$$\Delta t_l = t_l - t_{l-1} = x_{l-1}/cos(a_{opt}), \tag{24}$$

where x_{l-1} is the measured time delay between multipath component l and $l-1$ along the optimum FrFT axis t_a. These are also computed simultaneously from the rotated spectrum, providing rapid estimation of the channel. Defining the vector, $\hat{\boldsymbol{\Delta}\mathbf{t}} = [\Delta t_2\ \Delta t_3\ \ldots\ \Delta t_L]$, we can

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write RMSEs of the channel amplitudes and delays for the proposed technique as

$$RMSE_{c,Proposed} = ||\mathbf{c} - \hat{\mathbf{c}}_{Proposed}||_2 \tag{25}$$

and

$$RMSE_{\Delta t,Proposed} = ||\mathbf{\Delta t} - \hat{\mathbf{\Delta t}}||_2. \tag{26}$$

The $RMSE_{\Delta t,Proposed}$ is not compared to that of the conventional methods, because they were not computed.

VI. Numerical Examples

The transmitted chirp signal has the form of Eq. (6), where $f_0 = 500$ kHz and $K = -50$ MHz/s (minus indicates it is decreasing). A sampling rate of $f_s = 1$ Msps is used to generate the time domain signal for $N = 1,024$ samples, and we choose $\Delta a = 0.01$. Fig. 5 shows the received signal's Wigner Distribution (WD), in the presence of a multipath channel with seven taps, at SNR = 20 dB; the multipath components are easily seen. Hence, there is a direct path plus six multipath components. The total delay spread is about 5.5 ms and the separation between the two closest paths is about 0.43 ms. In this example, the channel coefficients are $\mathbf{c} = [c_1\, c_2\, c_3\, c_4\, c_5\, c_6\, c_7] = [0.64\, 0.51\, 0.41\, 0.3\, 0.21\, 0.13\, 0.09]$. The time deltas are $\mathbf{\Delta t} = [\Delta t_2\ \Delta t_3\ \Delta t_4\ \Delta t_5\ \Delta t_6\ \Delta t_7] = [0.43\ 1.29\ 0.66\ 1.08\ 1.47\ 0.57]$ ms.

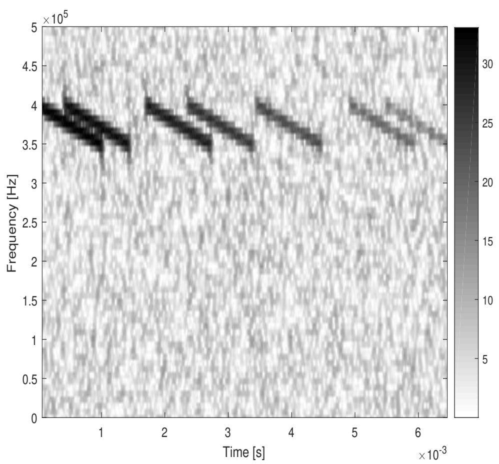

Fig. 5. WD of Received Chirp Signal, $y(t)$; $L = 7$ Tap Multipath Channel

Fig. 6 shows the WD rotated with Eq. (20), using $a_{opt} = 0.8$ from Eq. (19). We see from the figure that the chirps have been rotated to a dimension where they are now tones. We can easily compute the threshold, which is $\gamma = 0.25$ from this data. Finding the amplitude of each peak above the threshold gives the channel coefficients, and times between peaks gives the x_l's. The time delays are then computed from Eq. (24). The RMSE of the estimates is negligible in this example due to high SNR. Note that a_{opt} only needs to be computed once for a given set of chirp parameters.

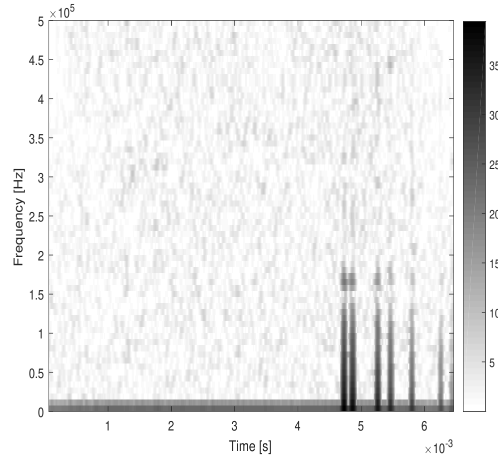

Fig. 6. $\mathbf{Y}_{a_{opt}}$: WD of Received Chirp Signal Rotated to Best 'a', a_{opt}; $L = 7$ Tap Multipath Channel

In Fig. 7 we compute the RMSE between the true channel coefficient amplitudes and their estimates, as well as the true channel delays and their estimates. We plot this over a range of SNR, for $L = 3, 5$, and 7 tap channels, where the channel coefficients randomly change every trial and the results averaged over $M = 1,000$ trials. We see that the RMSE < 0.1 when SNR > 5 dB; however, even at -10 dB SNR, RMSE < 0.3. The time delays are very accurate (between about 10^{-6} and 10^{-2}) over the full SNR range, -10 to 20 dB. The channel amplitude RMSE is seen to come very close to the lower bound, taken from Eq. (15) as briefly discussed above and derived in [3]. We also show results from conventional LS and MMSE based algorithms for comparison, computed from Eqs. (11) and (14) ([5] and [8]). The authors in those references show MSE, so notice that we have take the square root of the MSE to convert it to RMSE. Also, the results are very similar in those two papers, so we only show one curve for each method. The length of the channel is $L = 5$ for these cases, so only a single curve exists for the LS and MMSE algorithms. However, comparing to our $L = 5$ case, we observe that the proposed method improves upon LS and MMSE by about $5 - 8$ dB. This is due to the fact that channel coefficients are pulled out of noise by the FrFT rotation, making estimation of the coefficients more accurate, especially from weak multipath components.

The RMSE plateau seen in the delay estimates is due to the noise in the compressed, single sample, tone. We point out that the algorithm degrades gracefully as N is decreased or L is increased beyond 7 taps. Also, with this example, because $a_{opt} = 0.8$, using a search step size of $\Delta a = 0.1$ does not degrade performance at all. Hence, a

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careful selection of chirp parameters can allow a smaller search space and hence reduce computational time of the algorithm even further.

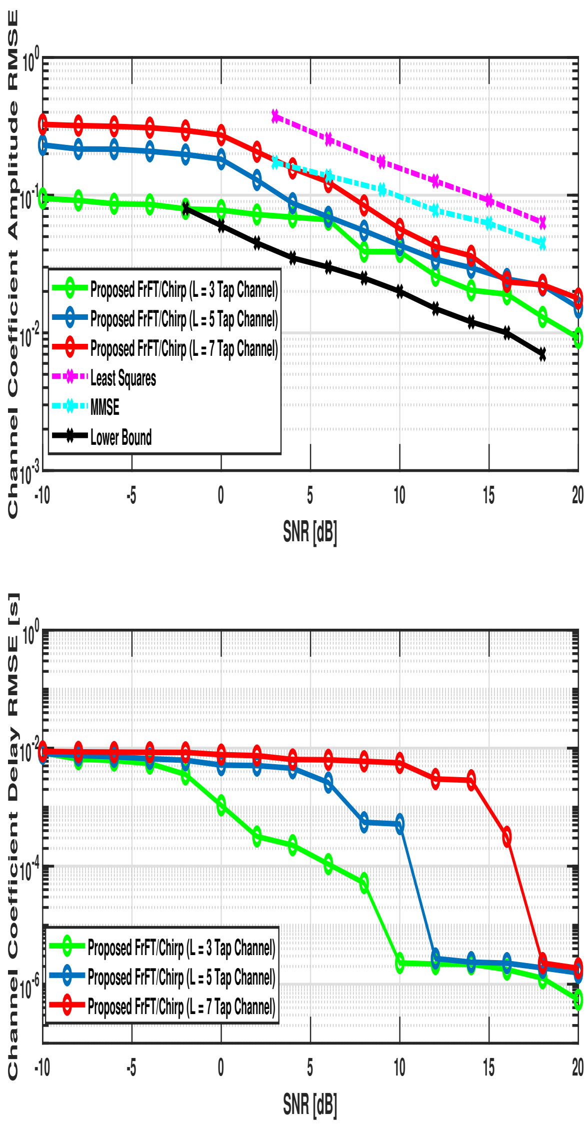

Fig. 7. Channel Coefficient Amplitude and Time Delay Root-Mean Square Errors (RMSEs) Using Proposed Method; LS, MMSE, and Lower Bound (LB) Shown for Comparison; $L = 3$, 5, and 7 Tap Channels

VII. Conclusion

This paper presents a very simple and robust algorithm for accurately computing the multipath delay profile in an RF channel. A short chirp is transmitted as a pilot. Its received components are converted to tones using the Fractional Fourier Transform, from which we can easily compute the channel tap magnitudes and delays. This involves measuring peaks above a threshold in the rotated spectrum, as well as the time between the peaks, and mapping the time in the rotated plane back to the original time via a simple formula. The technique is shown to work accurately with up to $L = 7$ multipath coefficients, providing $5 - 8$ dB improvement over conventional methods such as MMSE and coming within about 2 dB of the lower bound; it works at low signal-to-noise ratios (SNRs) because converting the chirps to tones pulls signals out of the noise. For the same reason, it requires few samples to compute, by sending a short chirp at the beginning (e.g. in the header) of a transmission to measure the multipath. This technique is therefore simple, effective, robust, and does not require any additional overhead than required by any existing method relying on pilot signals. It could be very useful for terrestrial communications links that suffer from multipath, e.g. in future $5G$ communications links. Future work includes testing the proposed method over a high data rate link to ensure its ability to perform estimates in real-time in a dynamic, non-stationary channel.

Acknowledgment

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