N1 $\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + C$

$\cancel{\frac{2^{2x+2}}{2x\ln 2}}$

N2 $\int \frac{dx}{2^{2x+2}} = \frac{1}{2}\int \frac{d(2x-2)}{2^{2x-2}} =$

$= -\frac{2^{2x-2}}{2\ln 2} + C$

N3 $\int \frac{5^x + 1}{2^x}\,dx =$

$= \int\left(\frac{5}{2}\right)^x dx + \int 2^{-x}\,dx =$

$= \frac{\left(\frac{5}{2}\right)^x}{\ln\frac{5}{2}} - \frac{1}{2^x \ln 2} + C$

N5

$\int \frac{dx}{\sin^2\left(x-\frac{\pi}{8}\right)} = \left[\begin{array}{l} t = tg\left(x-\frac{\pi}{8}\right) \\ x = arctg\, t + \frac{\pi}{8} \\ dx = \frac{1}{1+t^2}\,dt \end{array}\right] = \int \frac{tg^2\left(x-\frac{\pi}{8}\right)}{tg^2\left(x-\frac{\pi}{8}\right)+1} =$

$= \int \frac{t^2}{t^2+1} \cdot \frac{1}{1+t^2}\,dt =$

$= \int t^2\,dt = -\frac{1}{tg\left(x-\frac{\pi}{8}\right)} =$

$= -ctg\left(x-\frac{\pi}{8}\right) + C$

N6

$\int \frac{(1+\sqrt{x})^2}{x}\,dx = \int \frac{1}{x}\,dx +$

$+ \int \frac{2\sqrt{x}}{x}\,dx + \int 1\,dx =$

$= \ln|x| + 4\sqrt{x} + x +$

$+ C$

N7

$$\int \frac{\sqrt[3]{x^2} - \sqrt[4]{x}}{\sqrt{x}}\,dx = \int x^{\frac{1}{6}}dx - \int x^{-\frac{1}{4}}dx =$$

$$= \frac{6x^{7/6}}{7} - \frac{4x^{3/4}}{3} + C$$

N8

$$\int \frac{3\cdot 2^{x+1} - 2\cdot 3^x}{2^x}\,dx = \int 3\cdot 2\,dx - \int 2\cdot\left(\frac{3}{2}\right)^x dx =$$

$$= 6x - 2\frac{\left(\frac{3}{2}\right)^x}{\ln\frac{3}{2}} = 6x - \frac{2^{1-x}\cdot 3^x}{\ln 3/2} + C$$

N4

$$\int \sqrt{\frac{x+1}{x}}\,dx = \left[\begin{array}{l} t^2 = \dfrac{x+1}{x} \qquad t = \sqrt{\dfrac{x+1}{x}} \\ x+1 = xt^2 \Rightarrow x(1-t^2) = -1 \end{array}\right.$$

$$\left. x = \frac{1}{t^2-1} \qquad dx = \frac{-2t}{(t^2-1)^2}dt \right] = \int \frac{-2t^2}{(t^2-1)^2}dt =$$

$$= \left[\frac{2t^2}{(t^2-1)^2} = \frac{A\cancel{t}+B}{t^2-1} + \frac{C\cancel{t}+D}{(t^2-1)^2} = \frac{A\cancel{t}t^3 - A\cancel{t} + Bt^2 - B + C\cancel{t}+}{(t^2-1)^2} \right.$$

$$\left.\begin{array}{l} A=0 \\ -A+C=0 \\ B=2 \\ B-D=0 \Rightarrow D=2 \end{array}\right] = -\underbrace{\int \frac{2\,dt}{(t^2-1)}}_{=\mathrm{I}} - \underbrace{\int \frac{2\,dt}{(t^2-1)^2}}_{=\mathrm{II}} = -\mathrm{I} - \mathrm{II}$$

I) $\int \frac{2}{t^2-1} =$ $\ln\left|\frac{t-1}{t+1}\right|$

$\mathrm{II})$

$$-2\int \frac{-1}{(t^2-1)^2}\,dt = -2\int \frac{(-1 \pm t^2)}{(t^2-1)^2}\,dt =$$

$$= -2\left(\int \frac{dt}{t^2-1} - \overset{\overset{A}{\shortparallel}}{\int \frac{t^2\,dt}{(t^2-1)^2}}\right)$$

$A)$ $\int \frac{t\cdot t}{(t^2-1)^2}\,dt = \left[\begin{array}{ll} u=t & du=dt \\ dV=\frac{t}{(t^2-1)^2}dt & V=\int \frac{t}{(t^2-1)^2}dt = \end{array}\right.$

$$\left. = \frac{1}{2}\int \frac{d(t^2-1)}{(t^2-1)^2} = -\frac{1}{2(t^2-1)}\right] = t\cdot\left(-\frac{1}{2(t^2-1)}\right) -$$

$$-\int -\frac{1}{2(t^2-1)}\cdot dt = -\frac{t}{2(t^2-1)} + \frac{\ln\left|\frac{t-1}{t+1}\right|}{2\cdot 2}$$

$$\mathrm{I}-\mathrm{II} = \cancel{-\ln\left|\frac{t-1}{t+1}\right|} + \cancel{\ln\left|\frac{t-1}{t+1}\right|} +$$

$$+\frac{t}{2(t^2-1)} - \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|$$

$$= \frac{\ln|t+1|}{2} - \frac{\ln|t-1|}{2} + \frac{t}{t^2-1} =$$

$$= \frac{\ln\left|\frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x}}\right| - \ln\left|\frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x}}\right|}{2} + \sqrt{x+1}\sqrt{x} + C$$

=

N9

$$\int \frac{x^3}{(x-1)^2}dx = \begin{bmatrix} x=u+1 \\ x-1=u \\ dx=du \end{bmatrix} = \int \frac{(u+1)^3}{u^2}du =$$

$$= \int \frac{u^3+3u+3u^2+1}{u^2}du = \int u\,du + 3\int \frac{1}{u}du +$$

$$+3+\int \frac{1}{u^2}du = \frac{u^2}{2}+3\ln|u|+3+\frac{u^{-1}}{-1} =$$

$$= \frac{u^2}{2}+3\ln|u|+3-\frac{1}{u} =$$

$$\frac{u^3+3u^2-2}{2u}+3\ln|u| =$$

$$= \frac{(x-1)^3+6(x-1)^2-2}{2(x-1)}+3\ln|x-1| = \frac{x^3+9x+3x^2+3}{2(x-1)}$$

$$+3\ln(|x-1|)$$

№10

$$\int \frac{dx}{\sin^4 x+\cos^4 x} = \int \frac{dx}{\sin^4 x - 2\sin^2 x\cos^2 x + \cos^4 x + 2\sin^2 x\cos^2 x} =$$

$$= \int \frac{dx}{(\sin^2 x-\cos^2 x)^2 + \frac{2\sin^2\cos^2 x\cdot 2}{2}} = \int \frac{dx}{\cos^2 2x + \frac{\sin^2 2x}{2}} =$$

$$= \int \frac{2\cdot dx}{\underbrace{\cos^2 2x+\cos^2 2x + \sin^2 2x}_{1}} = \frac{1}{2}\int \frac{d(2x)}{\cos^2 2x+1} = \left[\begin{array}{l} t = tg\,2x \\ 2x = arctg\,t \end{array}\right. ;$$

$$d\,arctg\,t = d(2x) \Leftrightarrow d(2x) = \frac{1}{1+t^2}$$

$$\left.\cos^2 2x = \frac{1}{t^2+1}\right] = \int \frac{1}{\frac{1}{t^2+1}+1}\cdot\frac{1}{1+t^2}\,dt =$$

$$= \int \frac{dt}{(\sqrt{2})^2+t^2} = \frac{1}{\sqrt{2}}\,arctg\,\frac{t}{\sqrt{2}} + C = \frac{arctg\left(\frac{tg\,2x}{\sqrt{2}}\right)}{\sqrt{2}} + C$$

N11

$$\int \frac{dx}{1+\cos^2 x} = \left[\begin{aligned} & t = tg x \to x = arctg\, t \\ & dx = d\, arctg\, t = \frac{1}{t^2+1} dt \\ & \cos^2 x = \frac{1}{t^2+1} \end{aligned}\right] = \int \frac{dt}{1+\frac{1}{t^2+1}} \cdot \frac{1}{t^2+1} = \int \frac{dt}{(\sqrt{2})^2 + t^2} =$$

$$= \frac{1}{\sqrt{2}} arctg \frac{t}{\sqrt{2}} + C = \frac{arctg \frac{tg x}{\sqrt{2}}}{\sqrt{2}} + C$$

N12

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \int \frac{dx}{\sin^2 x} - \int \frac{dx}{\cos^2 x} =$$

$$= -\operatorname{ctg} x - \operatorname{tg} x + C$$

N13

$$\int \operatorname{ctg}^2 x \, dx = \int \left(\frac{1}{\sin^2 x} - 1 \right) dx = -\operatorname{ctg} x - x + C$$

N14

$$\int \frac{dx}{\cos^2 x - \underbrace{\cos 2x}_{-\cos^2 x + \sin^2 x}} = \int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$$

N15

$$\int (\arcsin x + \arccos x) dx = \frac{\pi}{2} + C$$

N15

$$\int \frac{1+\cos^2 x}{1+\cos 2x} dx = \int \frac{1+\cos^2 x}{\cancel{1}+2\cos^2 x - \cancel{1}} dx =$$

$$=\frac{1}{2}\int\frac{1}{\cos^2 x}+\frac{1}{2}\int \cancel{\frac{x}{\cos x}}\,dx=\frac{tg x+x}{2}+C$$

N17

$$\int \sin^2 3x\,dx=\frac{1}{3}\int \sin^2 3x\,d(3x)=\left[\sin^2 x=\frac{-\cos 2x+1}{2}\right]$$

$$=\frac{1}{3}\int\frac{1-\cos 6x}{2}\,d(3x)=\frac{1}{6}\left(\int 1\,d(3x)-\frac{1}{2}\int \cos 6x\,d(6x)\right)=$$

$$=\frac{3x}{6}-\frac{\sin 6x}{12}+C=\frac{6x-\sin 6x}{12}+C$$

N18

$$\int\frac{dx}{(3x+5)^3}=\frac{1}{3}\int(3x+5)^{-3}\,d(3x+5)=$$

$$=\frac{1}{3}\,\frac{(3x+5)^{-2}}{-2}=\frac{1}{-6(3x+5)^2}+C$$

N19

$$\int\sqrt[3]{x+5}\,dx=\int(x+5)^{\frac{1}{3}}\,d(x+5)=$$

$= 3\frac{(x+5)^{4/3}}{4} + C$

N20

$$\int \sin^2 x \cos x\,dx = \left[\begin{array}{l} \sin^2 x = u \to du = \sin 2x\,dx \\ \cos x\,dx = dv \to v = \sin x \end{array}\right] =$$

$$= \sin^2 x \sin x - \int \sin x \sin 2x\,dx = \sin^3 x -$$

$$-2\int \sin^2 x \cos x\,dx$$

$$3\int \sin^2 x \cos x\,dx = \frac{\sin^3 x}{3} + C$$

N21

$$\int e^{x^2} x\,dx = \int e^{x^2} d\left(\frac{x^2}{2}\right) =$$

$$= \frac{1}{2} e^{x^2} + C$$

N22

$$\int (2x-1)^{15}\,dx = \frac{(2x-1)^{16}}{2\cdot 16} + C$$

N23

$$\int \sqrt[5]{(8-3x)^3}\,dx = -\frac{1}{3}\int (8-3x)^{\frac{3}{5}}\,d(-3x) =$$

$$= -\frac{(8-3x)^{\frac{8}{5}}\cdot 5}{3\cdot 8} + C$$

N24

$$\int x\sqrt{1-x^2}\,dx = \int \sqrt{1-x^2}\,d\frac{x^2}{2} =$$

$$= \left[u = 1-x^2\right] = -\frac{1}{2}\int \sqrt{u}\,du = -\frac{1\cdot \cancel{2}\cdot(1-x^2)^{\frac{3}{2}}}{\cancel{2}\cdot 3} =$$

$$= -\frac{\sqrt{(1-x^2)^3}}{3} = \frac{-(1-x^2)\sqrt{1-x^2}}{3} =$$

$$= \frac{x^2\sqrt{1-x^2} - \sqrt{1-x^2}}{3} + C$$

N25

$$\int \frac{x^3\,dx}{\sqrt[3]{x^4+1}} = \int \frac{d\frac{x^4}{4}}{\sqrt[3]{x^4+1}} = \frac{1}{4}\int (x^4+1)^{-\frac{1}{3}}\,d(x^4+1) =$$

$$= \frac{1}{4}\cdot\frac{(x^4+1)^{8/3}\cdot 3}{4} = \frac{3\sqrt[3]{(x^4+1)^8}}{8\cancel{3}} + C$$

N26

$$\int \frac{\sin x}{\cos^2 x}\,dx = -\int \frac{1}{\cos^2 x}\,d(\cos x) =$$

$$= \frac{1}{\cos x} + C$$

N27

$$\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx = \int \sin x^{-\frac{2}{3}}\,d(\sin x) = \sqrt[3]{\sin x}\cdot 3 + C$$

N28

$$\int \frac{e^x}{e^x+1}\,dx = \int \frac{1}{e^x+1}\,d(e^x+1) =$$

$$= \ln|e^x+1| + C$$

N29

$$\int tgx \cdot dx = -\ln|\cos x| + C$$

N30

$$\int \frac{\sin 2x}{1+\cos^2 x}\,dx = \int \frac{2(\sin 2x)}{3+\cos 2x}\,dx = -2\cdot\frac{1}{2}\int \frac{d(\cos 2x)}{3+\cos 2x}$$

$$\cos^2 x = \frac{\cos 2x+1}{2}$$

$$= -\cancel{\ }\cdot\ln|3+\cos 2x| + C$$

$$\ln|3+\cos 2x| = -2\ln|\cos^2 x+1|$$

$$\int \frac{2\sin x\cos x}{\cos^2 x+1}\,dx = \begin{bmatrix} \cos^2 x+1 = t \\ dt = -2\cos x \sin x\,dx \end{bmatrix} =$$

$$= \int \frac{-dt}{t} = -\ln|\cos^2 x+1| + C$$

N31

$$\int \frac{x^2 dx}{x^3+1} = \frac{1}{2}\int (x^3+1)^{-1} d(x^3+1) = \frac{1}{2}\ln|x^3+1| + C$$

N32

$$\int tg 3x\, dx = -\frac{1}{3} Ln|\cos 3x| + C$$

N33

$$\int \frac{dx}{x \cdot \ln x} = \int (\ln x)^{-1} d(\ln|x|) = Ln|Ln|x|| + C$$

N34

$$\int \frac{1}{1+9x^2} dx = \frac{1}{3}\cdot\frac{1}{1} arctg \frac{3x}{1} = \frac{arctg 3x}{3} + C$$

N35

$$\int \frac{dx}{\sqrt{4-x^2}} = \arcsin\frac{x}{2} + C$$

N36

$$\int \frac{dx}{9x^2+4} = \frac{1}{2}\cdot\frac{1}{3}\cdot \operatorname{arctg}\frac{3x}{2} = \frac{\operatorname{arctg}\frac{3x}{2}}{6} + C$$

N38

$$\int \frac{dx}{4-9x^2} = \frac{1}{3}\cdot\int \frac{d(3x)}{(2-3x)(2+3x)} = \left[\frac{A}{2-3x} + \frac{B}{2+3x} = \right.$$

$$\left. = \frac{2A+3Ax+2B-3Bx}{(2-3x)(2+3x)} \Rightarrow \begin{matrix} 3A-3B=0 \\ 2A+2B=1 \end{matrix} \quad \begin{matrix} A=B \\ A=\frac{1}{4}=B \end{matrix} \right] =$$

$$= \frac{1}{3}\int \frac{1}{4(2-3x)}\, d(3x) + \frac{1}{3}\int \frac{1}{4(2+3x)}\, d(3x) =$$

$$= -\frac{1}{3}\cdot\frac{1}{4}\ln|8-12x| + \frac{1}{3}\cdot\frac{1}{4}\cdot\ln|8+12x| =$$

$$= \frac{\ln|2+3x|}{3} - \frac{\ln|2-3x|}{3} + C$$

N37

$$\int \frac{dx}{\sqrt{9x^2+4}} = \frac{1}{3}\ln\left|3x+\sqrt{9x^2+4}\right| + C$$

N39

$$\int \frac{dx}{\sqrt{4-9x^2}} = \frac{\arcsin \frac{3x}{2}}{3} + C$$

N40

$$\int \frac{x}{x^4+1}\,dx = \frac{1}{2}\int \frac{d(x^2)}{x^4+1} = \frac{1}{2}\,\mathrm{arctg}\, x^2 + C$$

N41

$$\int \frac{e^x\,dx}{e^{2x}+4} = \int \frac{d(e^x)}{(e^x)^2+2^2} = \frac{1}{2}\,\mathrm{arctg}\,\frac{e^x}{2} + C$$

N42

$$\int \frac{1+x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int \frac{-2x\,dx}{\sqrt{1-x^2}} + \int \frac{dx}{\sqrt{1-x^2}} = -\frac{1}{2}\int \frac{d(1-x^2)}{\sqrt{1-x^2}} + \int \frac{dx}{\sqrt{1-x^2}} = -\sqrt{1-x^2} + \arcsin x + C$$

№43

$$\int e^{\sin x}\cos x\,dx = \int e^{\sin x}\,d(\sin x) = e^{\sin x} + C$$

№44

$$\int \frac{3x-1}{x^2+9}\,dx = \frac{3}{2}\int \frac{2x\,dx}{x^2+9} - \int \frac{1\,dx}{x^2+9} =$$

$$= \frac{1}{2}\cdot\frac{3}{2}\,2\ln|x^2+9| - \frac{1}{3}\operatorname{arctg}\frac{x}{3} + C =$$

$$= \frac{3\ln|x^2+9|}{2} - \frac{\operatorname{arctg}\frac{x}{3}}{3} + C$$

№45

$$\int\sqrt{\frac{1-x}{1+x}} = \left[\begin{array}{l} t^2=\frac{1-x}{1+x} \quad t=\sqrt{\frac{1-x}{1+x}} \\ 1-x = t^2+t^2x \Leftrightarrow x(1+t^2)=1-t^2 \end{array}\right.$$

$$\left. x=\frac{1-t^2}{1+t^2} \quad dx=\frac{-4t}{(1+t^2)^2}dt\right] = \int \frac{-4t^2}{(1+t^2)^2}dt =$$

$$= \left[\frac{At+B}{(t^2+1)} + \frac{Ct+D}{(1+t^2)^2} = \frac{At^3+At+B+Bt^2+Ct+D}{(1+t^2)^2}\right.$$

$$\left. A=0 \quad B=4 \quad C=0 \quad D=-4\right] =$$

$$= \left(\int \underbrace{\frac{4}{1+t^2}}_{\mathrm{I}} - \int \underbrace{\frac{4}{(1+t^2)^2}}_{\mathrm{II}}\right)$$

I) $\int \mathrm{I} = 4\,\mathrm{arctg}\, t$

II)

$$4\int \frac{\cancel{4}\,1}{(1+t^2)^2} = 4\int \frac{1}{1+t^2}\, d(\operatorname{arctg} t) =$$

$$= \left[u = \operatorname{arctg} t\right] = \left[t = \operatorname{tg} u \quad 1+t^2 = 1+\operatorname{tg}^2 u\right.$$

$$\left.\operatorname{tg}^2 u = \frac{1}{\cos^2 u} - 1 \Rightarrow 1+t^2 = \frac{1}{\cos^2 u}\right] =$$

$$= 4\int \cancel{1}\cos^2 u\, du = 4\int \frac{\cos 2u + 1}{2}\, du =$$

$$= \cancel{2}\cdot\frac{1}{\cancel{2}}\sin 2u + 2u = 2\sin u \cos u + 2u =$$

$$= 2\operatorname{tg} u \cos^2 u + 2u = 2\operatorname{tg} u \frac{1}{\operatorname{tg}^2 u + 1} + 2u =$$

$$= 2\cancel{t}\,t \cdot \frac{1}{t^2+1} + 2\operatorname{arctg} t$$

итог)

$$-(I - II) = -4\operatorname{arctg} t + 2t\frac{1}{t^2+1} + 2\operatorname{arctg} t =$$

$$= -2\operatorname{arctg} t + \frac{2t}{t^2+1} =$$

$$= \frac{2\,\frac{\sqrt{1-x}}{\sqrt{1+x}}}{\frac{1-x}{1+x}+1} - 2\,\mathrm{arctg}\sqrt{\frac{1-x}{1+x}} = \sqrt{1-x}\,\sqrt{1+x} -$$

$$\cancel{1}-\cancel{x}+\cancel{1}+\cancel{x}$$

$$-2\,\mathrm{arctg}\sqrt{\frac{1-x}{1+x}} + C$$

№46

$$\int \frac{1+x-x^2}{\sqrt{(1-x^2)^3}}\,dx = \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{x}{\sqrt{(1-x^2)^3}}\,dx =$$

$$= \arcsin x + \frac{1}{2}\,\hat{a}\,\frac{1}{\sqrt{1-x}} = \arcsin x + \frac{1}{\sqrt{1-x}} + C$$

№47

$$\int \frac{x}{x-\sqrt{x^2-1}}\,dx = \int \frac{x^2+x\sqrt{x^2-1}}{\cancel{x^2}-\cancel{x^2}+1}\,dx = \frac{x^3}{3} +$$

$$+ \frac{1}{2}\int \sqrt{x^2-1}\,d(x^2-1) =$$

$$= \frac{x^3}{3} + \frac{(x^2-1)\sqrt{x^2-1}}{3} + C$$

№48

$$\int x\sqrt[8]{9+x}\,dx = \left[\begin{array}{l} x = t^8 - 9 \\ t^8 = 9 + x \qquad t = \sqrt[8]{9+x} \\ dx = 8t^7\,dt \end{array}\right] = \int 8t^8\left(t^8 - 9\right)dt = 8\int\left(t^{16} - 9t^8\right)dt$$

$$= 8\cdot\frac{t^{17}}{17} - \cancel{9}\cdot 8\,\frac{t^9}{\cancel{9}} = \frac{8\left(\sqrt[8]{9+x}\right)^{17}}{17} - 8\cdot\left(\sqrt[8]{9+x}\right)^9 + C$$

№50

$$\int \frac{dx}{x-\sqrt{x^2-1}} = \cancel{\int \frac{x+\sqrt{x^2-1}}{x^2-x^2+1}dx = \frac{x^2}{2} + \int \sqrt{x^2-1}\,dx}$$

$$= \left[\begin{array}{ll} t = -x + \sqrt{x^2-1} & (t+x)^2 = x^2-1 \\ 2tx = -1 - t^2 & x = \dfrac{-t^2-1}{2t} \end{array} \right.$$

$$-4t^2 + 2t^2 + 2 = -2t^2 + 2$$

$$\left. dx = \frac{-2t\cdot 2t - 2(-t^2-1)}{4t^2}dt = -\frac{t^2-1}{2t^2}dt \right] =$$

$$= \int \frac{t^2-1}{2t^3}dt = \frac{1}{2}\int\frac{dt}{t} - \frac{1}{2}\int\frac{1}{t^3}dt =$$

$$= \frac{1}{2}\ln|t| + \frac{1}{2\cdot 2t^2} = \frac{1}{2}\ln|t| + \frac{1}{4t^2} =$$

$$= \frac{1}{2}\ln\left|-x+\sqrt{x^2-1}\right| + \frac{1}{4(x^2-1-2x\sqrt{x^2-1}+x^2)} + C$$

№51

$$\int x \cdot \cos x \cdot dx = \begin{bmatrix} u = x & du = dx \\ dV = \cos x\, dx & V = \sin x \end{bmatrix} =$$

$$= x \cdot \sin x - \int \sin x\, dx = x \sin x + \cos x + C$$

№52

$$\int \arccos x\, dx = \begin{bmatrix} u = \arccos x & du = -\frac{1}{\sqrt{1-x^2}} \\ dV = dx & V = x \end{bmatrix} =$$

$$= x \arccos x + \int \frac{x}{\sqrt{1-x^2}} dx = x \arccos x -$$

$$- \sqrt{1-x^2} + C$$

№53

$$\int x \operatorname{arctg} x\, dx = \begin{bmatrix} u = \operatorname{arctg} x & du = \frac{1\, dx}{1+x^2} \\ x\, dx = dv & V = \frac{x^2}{2} \end{bmatrix} =$$

$$= \frac{x^2 \cdot \operatorname{arctg} x}{2} - \int \frac{x^2+1-1\, dx}{2(1+x^2)} =$$

$$= \frac{x^2 \operatorname{arctg} x}{2} - \frac{1}{2}\cdot x + \frac{1}{2}\operatorname{arctg} x = \frac{x^2 \operatorname{arctg} x + \operatorname{arctg} x}{2} - \frac{1}{2}x + C.$$

N54

$$\int x\cdot e^{-x}dx = \begin{bmatrix} u = e^{-x} & du = -e^{-x}dx \\ x\,dx = dV & V = \frac{x^2}{2} \end{bmatrix} = \frac{x^2}{2}\cdot e^{-x} + \int \frac{x^2}{2}\cdot e^{-x}dx =$$

неберущ. инт.

$$= \frac{e^{-x}\cdot x^2}{2}$$

$$\int x e^{-x}dx = \begin{bmatrix} u = x & du = dx \\ dV = e^{-x}dx & V = -e^{-x} \end{bmatrix} = -e^{-x}\cdot x + \int e^{-x}dx =$$

$$= -e^{-x}(x+1) = -\frac{x+1}{e^x} + C$$

N55

$$\int x^2 e^x dx = \begin{bmatrix} u = x^2 & du = 2x\,dx \\ e^x dx = dV & V = e^x \end{bmatrix} = e^x x^2 - \int e^x 2x\,dx =$$

$$\begin{bmatrix} u = x & du = dx \\ dV = e^x dx & V = e^x \end{bmatrix}$$

$$= e^x x^2 - 2\left(e^x x - \int e^x dx\right) = e^x(x^2 - 2x + 2) + C$$

N56

$$\int \ln^2 x\,dx = \begin{bmatrix} u = \ln^2 x & du = 2\ln x \cdot \frac{1}{x}dx \\ dx = dV & V = x \end{bmatrix} =$$

$$= \ln^2 x \cdot x - \int \cancel{x}\frac{2\ln x}{\cancel{x}}\,dx \begin{bmatrix} \ln x = u & du = \frac{1}{x}dx \\ dx = dv & v = x \end{bmatrix} =$$

$$= \ln^2 x \cdot x - 2\ln x \cdot x + 2\int x \cdot \frac{1}{x}dx =$$

$$= \ln^2 x \cdot x - 2\ln x \cdot x + 2x + C$$

N57

$$\int e^x \sin x\,dx = \begin{bmatrix} u = \sin x & du = \cos x\,dx \\ e^x dx = dV & V = e^x \end{bmatrix} =$$

$$= e^x \sin x - \int e^x \cos x\,dx = \begin{bmatrix} \cos x = u & du = -\sin x\,dx \\ e^x dx = dV & V = e^x \end{bmatrix}$$

$$= e^x \sin x - \left(\cos x \cdot e^x + \int \sin x \cdot e^x dx\right)$$

$$\int e^x \sin x\,dx = \frac{e^x \sin x - \cos x \cdot e^x}{2} + C$$

№58

$$\int \sqrt{a^2+x^2}\,dx = \left[\begin{array}{l} t = \operatorname{arctg}\frac{x}{a}; \\ x = a\,\operatorname{tg} t; \\ dx = \frac{a}{\cos^2 t}\,dt \end{array}\right] = \left[a^2 + a^2\operatorname{tg}^2 t = a^2(1+\operatorname{tg}^2 t) = \frac{a^2}{\cos^2 t}\right] = \int \sqrt{\frac{a^2}{\cos^2 t}} \cdot \frac{a}{\cos^2 t}\,dt =$$

$$= \int \frac{a^2}{\cos^3 t}\,dt = a^2 \int \frac{1}{\cos^2 t} \cdot \frac{1}{\cos t}\,dt =$$

$$= a^2 \int \frac{1}{\cos t}\,d(\operatorname{tg} t) = \left[\sqrt{\frac{1}{\cos^2 t}} = \sqrt{1+\operatorname{tg}^2 t}\right] =$$

$$= a^2 \int \sqrt{1+\operatorname{tg}^2 t}\,d(\operatorname{tg} t)$$

N59

$$\int \ln x \, dx = \begin{bmatrix} u = \ln x & du = \frac{1}{x} dx \\ dx = dV & V = x \end{bmatrix} =$$

$$= \ln x \; x - \int x \frac{1}{x} dx = x(\ln x - 1) + C$$

N60

$$\int \ln^2 x \, dx = = N56$$

N61

$$\int \frac{x^2 dx}{(1+x^2)^2} = \left[\begin{matrix} u = x & du = dx \\ dV = \frac{x}{(1+x^2)^2} dx & \underbrace{V = \frac{1}{2}\int \frac{d(x^2)}{(1+x^2)^2}} = \end{matrix} \right.$$

$$\left. = -\frac{1}{2} \frac{1}{(1+x^2)} \right] = \frac{-x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2} \cdot dx =$$

$$= \frac{-x}{2(1+x^2)} + \frac{\operatorname{arctg} x}{2} + C$$

N62

$$\cancel{\int x\cdot e^{\sqrt[3]{x}}\,dx = \begin{bmatrix} u=x & du=dx \\ e^{\sqrt[3]{x}}dx=dv & v=e^{\sqrt[3]{x}}\cdot\frac{1}{3}\cdot x^{-\frac{2}{3}} \end{bmatrix}} =$$

$$\cancel{=\frac{e^{\sqrt[3]{x}}\cdot x}{3\cdot x^{2/3}}-\frac{1}{3}\int e^{\sqrt[3]{x}}\cdot x^{-\frac{2}{3}}\cdot dx}$$

$$\int x\cdot e^{\sqrt[3]{x}}\,dx = \begin{bmatrix} t=\sqrt[3]{x} & x=t^3 \\ dx=3t^2dt & \end{bmatrix} = \int t^3\cdot e^t\cdot 3t^2\,dt =$$

$$=3\int t^5 e^t\,dt = \begin{bmatrix} u=t^5 & du=5t^4dt \\ e^t dt=dv & v=e^t \end{bmatrix} =$$

$$=3\left(e^t t^5-\int e^t\cdot 5t^4\,dt\right)=\begin{bmatrix} t^4=u & du=4t^3dt \\ e^t dt=dv & V=e^t \end{bmatrix}$$

$$=3\left(e^t t^5-5\cdot e^t\cdot t^4+5\int 4t^3 e^t\,dt\right)=\begin{bmatrix} t^3=u & du=3t^2 \\ e^t dt & \end{bmatrix}$$

$$=3\left(e^t t^5-5e^t t^4+5\cdot 4\cdot t^3 e^t-20\cdot\int e^t 3t^2\,dt\right)=$$

$$=3\left(e^t t^5-5e^t t^4+20t^3e^t-60e^t t^2+60\cdot 2\int e^t t\,dt\right)$$

$u=t$, $du=dt$

$$=3\left(e^t t^5-5e^t t^4+20t^3e^t-60e^t t^2+120e^t t-\right.$$

$$\left.-e^t\cdot 120\right)=$$

$= e^t(3t^5 - 15t^4 + 60t^3 - 180t^2 + 360u - 360) =$

$= e^{\sqrt[3]{x}}(3\sqrt[3]{x^5} - 15\sqrt[3]{x^4} + 60\sqrt[3]{x^3} - 180\sqrt[3]{x^2} + 360\sqrt[3]{x^2} - 360)$

$+C$

N63

$$\int \frac{dx}{1+\sqrt{x+1}} = \left[\begin{array}{l} t = 1+\sqrt{x+1} \\ (t-1)^2 = x+1 \end{array} \Rightarrow \begin{array}{l} t^2 - 2t + \cancel{1} = x + \cancel{1} \\ x = t^2 - 2t \end{array}\right] =$$

$$dx = (2t-2)\,dt$$

$$= \int \frac{2(t-1)}{t}\,dt = 2t + 2\ln|t| =$$

$$= 2\left(1+\sqrt{x+1} + \ln\left|1+\sqrt{x+1}\right|\right) + C$$

N64

$$\int \cos^2 x\,dx = \left[\begin{array}{l} t = \operatorname{tg} x \quad x = \operatorname{arctg} t \\ \operatorname{tg}^2 x = \frac{1}{\cos^2 x} - 1 \\ dx = \frac{1}{1+t^2}\,dt \\ \cos^2 x = \frac{1}{t^2+1} \end{array}\right] = \int \frac{1 \pm t^2}{t^2+1}\,dt \cdot \frac{1}{1+t^2} =$$

$$= \int \frac{1+t^2-t^2}{(t^2+1)^2}\,dt = \int \frac{1}{1+t^2} - \int \frac{t^2}{(t^2+1)^2} \text{ (см N 61)} =$$

$$= \cancel{\ldots}\ \mathrm{arctg}\, t + \frac{\cancel{t}}{2(t^2+1)^{\cancel{2}}} - \frac{\mathrm{arctg}\, \cancel{t}}{2} =$$

$$= \frac{\cancel{t}}{2(t^2+1)^{\cancel{2}}} + \frac{\mathrm{arctg}\, t}{2} = \frac{\mathrm{tg}\, x}{2(\mathrm{tg}^2 x+1)^{\cancel{2}}} + \frac{x}{2} + C$$

$$= \frac{2\frac{\sin x}{\cos x}\cdot\cos^{\cancel{2}} x}{2\cdot 2} + \frac{x}{2} = \frac{\sin 2x}{4} + \frac{x}{2} + C$$

можно было проще

$$\int \cos^2 x\, dx = \frac{1}{2}\int (\cos 2x - 1)\, d\cancel{x} = \frac{1}{2}\cdot\frac{1}{2}\sin 2x -$$

$$- \frac{x}{2} + C$$

N65

$$\int \frac{dx}{1-\cos x} = \int \frac{dx}{1 - \frac{1-\mathrm{tg}^2\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}}} = \left[\begin{array}{l} \cancel{\tfrac{x}{2}} = 2\,\mathrm{arctg}\, t \\ d\cancel{x} = 2\frac{1}{t^2+1}\,dt \\ t = \mathrm{tg}\,\frac{\cancel{2}x}{2} \end{array}\right] =$$

$$= \int \frac{1+\cancel{t^2}}{\cancel{2}t^2}\cdot\frac{\cancel{2}}{\cancel{t^2+1}}\, dt = \frac{t^{-1}}{-1} =$$

$$= -\frac{1}{t} = -\frac{1}{\mathrm{tg}\,\frac{x}{2}} + C$$

N 66

$$\int \frac{dx \cdot \cos 2x}{1+\sin x \cos x} = \Big[t = 1+\sin x \cos x$$

$$dt = (\sin x \cos x)' = \cos^2 x - \sin^2 x = \cos 2x \Big] =$$

$$= \int \frac{\cancel{\cos 2x}}{t} \cdot \frac{1}{\cancel{\cos 2x}} = \ln|1+\cos x \sin x|$$

N67

$$\int \frac{dx}{\sin x} = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c$$

N68

$$\int \frac{dx}{\cos x} = \ln\left|\operatorname{tg}\left(\frac{x}{2}+\frac{\pi}{2}\right)\right| + c$$

N69

$$\int \frac{\sin^3 x}{\cos x}\,dx = -\int \frac{\overset{1-\cos^2 x}{\sin^2 x}}{\cos x}\,d(\cos x) =$$

$$= -\left(\int \frac{1}{\cos x}\,d(\cos x) - \int \frac{\cos^2 x}{\cos x}\,d(\cos x)\right) =$$

$$= -\ln|\cos x| + \frac{\cos^2 x}{2} + C$$

N70

$$\int \frac{dx}{\cos^4 x} = \int \frac{\cos^2 x + \sin^2 x}{\cos^4 x}\,dx =$$

$$= \int \frac{1}{\cos^2 x}\,dx + \int \frac{\sin^2 x}{\cos^4 x}\,dx = \operatorname{tg} x +$$

$$+ \int \operatorname{tg}^2 x\,\frac{1}{\cos^2 x}\,dx = \operatorname{tg} x + \int \operatorname{tg}^2 x\,d(\operatorname{tg} x) =$$

$$= \operatorname{tg} x + \frac{\operatorname{tg}^3 x}{3} + C.$$

№71

$$\int \cos^3 x\,dx = \int \cos^2 x\, d(\sin x) =$$

$$= \int 1\, d(\sin x) - \int \sin^2 x\, d(\sin x) = \sin x - \frac{\sin^3 x}{3} + C$$

№72

$$\int \sin^5 x\,dx = -\int \sin^4 x\, d(\cos x) =$$

$$= -\left(\int 1\, d(\cos x) - 2\int \cos^2 x\, d(\cos x) + \int \cos^4 x\, d(\cos x)\right) = -\cos x +$$

$$+ 2\,\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + C.$$

№73

$$\int \cos 2x \sin 3x \, dx = \int \frac{\sin(5x) + \sin x}{2} dx =$$

$$= \frac{1}{2}\frac{1}{5}\int \sin 5x (5x) + \frac{1}{2}\int \sin x \, dx =$$

$$= \frac{-\cos 5x}{10} - \frac{\cos x}{2} + C.$$

№74

$$\int \sin x \cdot \sin 2x \, dx = \int \frac{\cos x - \cos 3x}{2} dx =$$

$$= \frac{1}{2}\int \sin x - \frac{1}{2}\frac{1}{3}\sin 3x = \frac{\sin x}{2} - \frac{\sin 3x}{6}$$

$$+ C$$

№75

$$\int \sin x \cos 3x \, dx = \int \frac{\sin 4x + \sin(-2x)}{2} =$$

$$= \frac{-\cos 4x}{8} + \frac{\cos 2x}{4} + C$$

№76

$$\int \frac{x^3 dx}{\sqrt{x-1}} = \left[\begin{array}{ll} t = \cancel{\sqrt{x-1}} & dt = 1 \\ x = t+1 & \end{array} \right] = \int \frac{(t+1)^3}{\sqrt{t}} dt =$$

$$= \int \frac{t^{\frac{6}{2}} + 3t^{\frac{4}{2}} + 3t^{\frac{2}{2}} + 1}{t^{\frac{1}{2}}} = \int t^{5/2} + 3\int t^{3/2} + 3\int t^{\frac{1}{2}} + \int \frac{1}{\sqrt{t}} =$$

$$= \frac{2 \cdot t^{7/2}}{7} + 3 \cdot \frac{2 t^{5/2}}{5} + \cancel{3}\frac{2 t^{3/2}}{\cancel{3}} + 2\sqrt{t} =$$

$$= \frac{2(x-1)^{7/2}}{7} + \frac{6(x-1)^{5/2}}{5} + 2(x-1)^{3/2} + 2\sqrt{x-1} + C$$

№77

$$\int \frac{dx}{x\sqrt{x+1}} = \left[\begin{array}{lll} t = \sqrt{x+1} & t^2 = x+1 & x = t^2 - 1 \\ dx = 2t\,dt & & \end{array} \right] =$$

$$= \int \frac{2\cancel{t}}{(t^2-1)\cancel{t}} dt = \frac{1}{2} 2 \cdot \ln \left| \frac{t-1}{t+1} \right| + C =$$

$$= \ln\left|\sqrt{x+1} - 1\right| - \ln\left|\sqrt{x+1} + 1\right| + C$$

$$\cancel{-3+6t^2 \quad \int\left(\frac{3}{1-2t^2}+\frac{3}{(1-2t)^2}\right)dt = -3\int\frac{1}{1-(\sqrt{2}t)^2}} +$$

$$\cancel{3\int\frac{1}{(1-2t)^2}} = 3\int\frac{1\,dt}{2t^2-1} + 3\int^{A}\frac{1\,dt}{(2t^2-1)^2}$$

A) $$\cancel{3}\int\frac{-1dt+2t^2}{(2t^2-1)^2} = -\left(\int\frac{2t^2-1}{(2t^2-1)^2}dt - \int\frac{2t^2}{(2t^2-1)^2}\right) =$$

$$= -\left(\int\frac{1}{2t^2-1}dt - 2\int\frac{t^2}{(2t^2-1)^2}dt\right) = \left[\begin{matrix} u = t & du = dt \\ dV = \frac{t}{(2t^2-1)^2}dt & \end{matrix}\right.$$

$$\left. V = \int\frac{t}{(2t^2-1)^2}dt = \frac{1}{4}\int\frac{d(2t^2)}{(2t^2-1)^2} = -\frac{1}{2t^2-1}\cancel{4}\right] =$$

$$= -\left(\int\frac{1}{2t^2-1}dt + 2\cdot t\cdot\frac{1}{2t^2-1} + 2\int -\frac{1}{2t^2-1}\cdot dt\right) =$$

$$= +\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{2}t-1}{\sqrt{2}t+1}\right| - \frac{2t}{2t^2-1}$$

итого: $$3\int\frac{1\,dt}{2t^2-1} + 3\int\frac{dt}{2t^2-1} - \frac{6t}{2t^2-1} =$$

$$= 6\cdot\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}t-1}{\sqrt{2}t+1}\right| - \frac{6t}{2t^2-1} + C =$$

№78

$$\int \frac{dx}{1+\sqrt{x}} =$$

$$= \left[\begin{array}{l} 1+\sqrt{x} = t \\ t^2-2t+1 = x \end{array} \quad dx = (2t-2)\,dt\right] =$$

$$= \int \frac{2(t-1)}{t}\,dt = 2(\sqrt{x}+1) - 2\ln|\sqrt{x}+1| + C$$

№79

$$\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}} = \left[\begin{array}{l} \sqrt[4]{x} = t \quad t^4 = x \quad \sqrt{x} = t^2 \\ dx = 4t^3\,dt \end{array}\right] =$$

$$= \int \frac{4t^3}{t^2+t}\,dt = 4\int \frac{t^2 \pm 1}{t+1} = 4\int \frac{(t-1)\cancel{(t+1)}}{\cancel{t+1}}\,dt -$$

$$-4\int \frac{+1}{t+1}\,dt = 4\int t\,dt\ 4\int 1\,dt -$$

$$-4\ln|t+1| = 2t^2 - 4t - 4\ln|t+1| =$$

$$= 2\sqrt{x} - 4\sqrt[4]{x} - 4\ln|\sqrt[4]{x}+1| + C$$

N80

$$\int \frac{x}{x(x+1)}dx = \ln|x+1| + C$$

N81

$$\int \frac{3-4x}{2x^2-3x+1}dx = -\int \frac{4x-3}{2x^2-3x+1}dx =$$

$$= -\int \frac{d(2x^2-3x+1)}{2x^2-3x+1} = -\ln|2x^2-3x+1| + C$$

N82

$$\int \frac{x-3}{\sqrt{3-2x-x^2}}dx = -\frac{1}{2}\left(\int \frac{-2x-2}{\sqrt{-x^2-2x+3}}dx + \frac{8\,dx}{\sqrt{-x^2-2x+3}}\right) =$$

$-2x-2$

$$= -\frac{1}{2}\left(\int \frac{d(-x^2-2x+3)}{\sqrt{-x^2-2x+3}} + \int \frac{8}{\sqrt{-x^2-2x+3}}dx\right) =$$

$-(x^2+2x-3+4-4)$

$-((x+1)^2-4)$

$$= -\cancel{\frac{1}{2}\cdot 2\sqrt{-x^2-2x+3} + 4\cdot\ln\left|x+1+\sqrt{(x+1)^2-2}\right| + C}$$

$$= -\sqrt{3-2x-x^2} - 4\arcsin\left(\frac{x+1}{2}\right) + C$$

N83

$$\int \frac{dx}{\sqrt{9x^2-6x+2}} = \frac{1}{3}\ln\left|3x+1+\sqrt{(3x+1)^2+1}\right| + C$$

$\cancel{18x - 6}$

$(3x+1)^2+1$

N84

$$\int \frac{\sqrt{x}}{\sqrt{2x+3}}dx = \left[t = \frac{\sqrt{x}}{\sqrt{2x+3}} \Rightarrow t^2 = \frac{x}{2x+3} \right.$$

$$x = 2xt^2 + 3t^2 \Rightarrow x(1-2t^2) = 3t^2$$

$$x = \frac{3t^2}{1-2t^2}$$

$$dx = \frac{6t(1-2t^2) - (-4t)\cdot 3t^2}{(1-2t^2)^2} =$$

$$\left. = \frac{6t - \cancel{12t^3} + \cancel{12t^3}}{(1-2t^2)^2} \right] = \int \frac{6t^2}{(2t^2-1)^2}dt =$$

$$= \left[\frac{At+B}{1-2t^2}^{(1-2t^2)} + \frac{Ct+D}{(1-2t^2)^2} = \frac{-At + 2At^3 - B + 2Bt^2}{(1-2t)^2} \right.$$

$\underline{+Ct+D}$

$A + C = 0 \qquad +2B = 6 \to B = +3$

$+2A = 0$

$-B + D = 0 \to D = 3$

$$= \frac{3 \ln\left|\frac{\sqrt{2}\sqrt{x} - \sqrt{2x+3}}{\sqrt{2}\sqrt{x} + \sqrt{2x+3}}\right|}{\sqrt{2}} + 2\sqrt{x}\sqrt{2x+3} + C$$

N85

$$\int \frac{dx}{x(x^2+1)} =$$

$$\left[\frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)} \right.$$

$$\left. \begin{matrix} A+B=0 \\ C=0 \\ A=1 \end{matrix} \quad B=-1 \right] = \int \frac{1\,dx}{x}$$

$$- \int \frac{1}{x^2+1}\,dx = \ln|x| - \frac{1}{2}\ln|x^2+1| + C$$

N86

$$\int \frac{dx}{1+x^3} = \left[\frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} = \right.$$

$$= \frac{Ax^2 - Ax + A + Bx^2 + Bx + Cx + C}{x^3+1} \quad \left| \begin{matrix} A+B=0 \\ B-A+C=0 \\ A+C=0 \end{matrix} \right.$$

$B=C$ $\quad B=\frac{1}{3}$ $\quad -C$ $\quad 2B+A=1$ $\quad 2B=1$ $\quad 3B=-1$

$\cancel{B = \frac{1}{3} = C}$

$$\begin{cases} A+B=0 \\ B-A+C=0 \\ A+C=1 \end{cases} \quad \begin{array}{l} A=-B \\ 2B+C=0 \\ C=1-A \end{array} \quad 2B+1-A \quad \left.\begin{array}{l} 3B=-1 \\ B=-\frac{1}{3} \\ A=\frac{1}{3} \\ C=\frac{2}{3} \end{array}\right] =$$

$2x-1$

$x+2$ $\quad 2\cdot$

$$= \int \frac{1\,dx}{3(x+1)} - \int \frac{\frac{1}{3}x+\frac{2}{3}}{x^2-x+1}dx = \frac{1}{3}\ln|x+1| - \frac{1}{3}\cdot\left(\frac{1}{2}\int\frac{2x+1}{x^2-x+1}dx\right.$$

$$\left.+\int\frac{5/2}{x^2-x+1}dx\right) = \frac{\ln|x+1|}{3} - \frac{1}{6}\ln|x^2-x+1| - \frac{1}{3}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\cdot\frac{5}{2}$$

$$\left.\begin{array}{l} -2\cdot\frac{1}{2} \quad \frac{1}{4}+\frac{3}{4} \\ \left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2 \\ x^2-x+\frac{1}{4} \end{array}\right] \cdot \operatorname{arctg}\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}} + C =$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln|x^2-x+1| - \frac{\cancel{5}\operatorname{arctg}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

$+C.$

$x^0 \ldots) - \text{II} \; (0\,0\,0 \ldots)$ $\quad\quad$ $B+C=0$

N87

$$\int \frac{dx}{(x^2+1)(x+1)^2} = \left[\frac{Ax+B}{x^2+1} + \frac{C}{x+1} + \frac{D}{(x+1)^2} = \frac{Ax^3+2Ax^2+Ax+Bx^2+2Bx+B+Cx^3+Cx^2+Cx+C+Dx^2+D}{(x^2+1)(x+1)^2} \right.$$

$(x^2+2x+1$ $\quad$ $((x^2+1)(x+1)$ $\quad$ x^3+x^2+x+1

$$\begin{cases} A+C=0 \\ 2A+B+C+D=0 \\ A+2B+C=0 \\ B+C+D=1 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right) \begin{matrix} \\ -2\text{I} \\ -\text{I} \\ \\ \end{matrix} = \left(\begin{array}{ccccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 \end{array}\right) - \text{II}$$

$A+C=0$

$-C+D=0$

$B=0$

$2C=1$

$-\frac{1}{2}+D=0 \Rightarrow D=\frac{1}{2}$

$A=-\frac{1}{2}$

$\left. -\frac{1}{2};\ 0;\ \frac{1}{2};\ \frac{1}{2} \right]$

$$\frac{1}{2}\int -\frac{dx}{x^2+1} + \frac{1}{2}\int\frac{dx}{x+1} + \frac{1}{2}\int\frac{dx}{(x+1)^2} = -\frac{1}{4}\ln|x^2+1| + \frac{1}{2}\ln|x+1| - \frac{1}{2(x+1)} + C$$

N88

$$\int \frac{x^3-6}{x^4+6x^2+8}\,dx = \int \frac{x^3-6}{(x^2+4)(x^2+2)} = \left[\frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{x^2+2} = \right.$$

$$= \frac{Ax^3+2Ax+Bx^2+2B+Cx^3+4Cx+Dx^2+4D}{(x^2+4)(x^2+2)} \Rightarrow$$

$$\Rightarrow \begin{array}{lll} x^3\ A+C=1 & A=1-C & A=2 \\ x\ 2A+4C=0 & 2-2C+4C=0 & 2+2C=0 \\ & & C=-1 \\ x^2\ B+D=0 & B=-D & \\ 2B+4D=-6 & -2D+4D=-6 & \\ & 2D=-6 & D=-3 \\ & & B=3 \end{array} \Bigg] =$$

2; 3; -1; -3

$$= \int \frac{2x+3\,dx}{(x^2+4)} - \int \frac{x+3}{x^2+2}\,dx = 2\frac{1}{2}\int \frac{d(x^2)}{x^2+4} + \int \frac{3}{x^2+4}\,dx -$$

$$-\frac{1}{2}\int \frac{d(x^2)}{x^2+2} - \int \frac{3}{x^2+2}\,d(x) = \ln|x^2+4| - \frac{1}{2}\ln|x^2+2| +$$

$$+ \frac{3\,\mathrm{arctg}\left(\frac{x}{2}\right)}{2} - \frac{3\,\mathrm{arctg}\frac{x}{\sqrt{2}}}{\sqrt{2}} + C$$

№89

$$\int \frac{dx}{(x^2+1)^2} = \cancel{\left[\frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}\right]} =$$

$$\cancel{= \frac{Ax^3+Ax+Bx^2+B+Cx+D}{(x^2+1)^2}} \Rightarrow \cancel{\begin{cases} A=0 \\ B=0 \\ A+C=0 \\ D+B=1 \\ D=1 \end{cases}}$$

$$= \int \frac{1 \pm x^2}{(x^2+1)^2}\,dx = \int \frac{1}{x^2+1}\,dx - \int \frac{x^2}{(x^2+1)^2}\,dx =$$

$u = x$ $\quad du = dx$

$dv = \frac{x}{(x^2+1)^2}\,dx$ $\quad v = \int \frac{x}{(x^2+1)^2}\,dx = \frac{1}{2}\int \frac{d(x^2)}{(x^2+1)^2}\,dx = -\frac{1}{2(x^2+1)}$

$$= \int \frac{1}{x^2+1}\,dx + x \cdot \frac{1}{2(x^2+1)} - \int \frac{1}{2(x^2+1)}\,dx =$$

$$= +\frac{1}{2}\int \frac{1}{x^2+1}\,dx + \frac{x}{2(x^2+1)} = +\frac{\operatorname{arctg} x}{2} +$$

$$+ \frac{x}{2(x^2+1)} + C$$

№90

$$\int \frac{x^5}{\underbrace{(x-1)^2}_{x^2-2x+1}(x^2-1)}dx = \int \frac{x^5}{x^4-2x^3+\cancel{x^2}-\cancel{x^2}+2x-1}dx =$$

$$= \left[\begin{array}{l|l} x^5 & x^4-2x^3+2x-1 \\ -\;x^5-2x^4+2x^2-x & x+2 \\ \quad 2x^4-2x^2+x & \\ -\;2x^4-4x^3+4x-2 & \\ -2x^2+4x^3-3x+2 & \end{array}\right] = \int\left(x+2+\frac{4x^3-2x^2-3x+2}{\underbrace{x^4-2x^3+2x-1}_{(x-1)^3(x+1)}}\right)dx =$$

$$= \left[\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}+\frac{D}{(x-1)^3}\right] = \frac{Ax^3-3Ax^2+3Ax-A+}{(x-1)^3(x+1)}$$

(multipliers: $(x-1)^3$; $(x+1((x-1)^2$; $(x+1(x-1$; $(x+1$)

$$\frac{+Bx^3-Bx^2-Bx+B+Cx^2-C+Dx+D}{} \rightarrow$$

$$\begin{aligned} A+B&=4\\ -3A+B+C&=-2\\ 3A-B+D&=-3\\ -A+B-C+D&=2\end{aligned} \quad \begin{pmatrix} 1 & 1 & 0 & 0 & 4\\ -3 & -1 & 1 & 0 & -2\\ 3 & -1 & 0 & 1 & -3\\ -1 & 1 & -1 & 1 & 2\end{pmatrix}\begin{array}{l} \\ +3\text{I}\\ -3\text{I}\\ +\text{I}\end{array} = \begin{pmatrix} 1 & 1 & 0 & 0 & 4\\ 0 & 2 & 1 & 0 & 10\\ 0 & -4 & 0 & 1 & -15\\ 0 & 2 & -1 & 1 & 6\end{pmatrix}\begin{array}{l} \\ \\ +2\text{II}\\ -\text{II}\end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 4 \\ 0 & 2 & 1 & 0 & 10 \\ 0 & 0 & 2 & 1 & 5 \\ 0 & 0 & -2 & 1 & -4 \end{array}\right) + \mathrm{III} \sim \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 4 \\ 0 & 2 & 1 & 0 & 10 \\ 0 & 0 & 2 & 1 & 5 \\ 0 & 0 & 0 & 2 & 1 \end{array}\right)$$

$$\left.\begin{array}{l} 2D = 1 \Rightarrow D = \frac{1}{2} \quad A = \frac{1}{8} \quad B = \frac{31}{8} \\ \qquad\qquad\qquad\qquad C = \frac{9}{4} \end{array}\right] =$$

$$= \int (x+2)\,dx + \int \frac{dx}{8(x+1)} + \int \frac{31\,dx}{8(x-1)} + \int \frac{9\,dx}{4(x-1)^2} +$$

$$(x-1)^2 \cdot 2$$

$$+ \int \frac{1\,dx}{2(x-1)^3} = \frac{x^2}{2} + 2x + \frac{1}{8}\ln|x+1| + \frac{31}{8}\ln|x-$$

$$(x-1)$$

$$+ \frac{9}{4}\left(-\frac{1}{x-1}\right) - \frac{1}{2}\,\frac{1}{2(x-1)^2} =$$

$$= \cancel{\frac{2x^2 + 2x \cdot 4(x-1)^2 - 9^{(x-1)} - 1}{(x-1)^2}}$$

$$= \frac{2x^2(x-1)^2 + 2x \cdot 4 \cdot (x-1)^2 - 9(x-1) - 1}{4(x-1)^2} +$$

$$+\frac{\ln|x+1|+31\ln|x-1|}{8}=$$

$$=\frac{4x^4+4x^3-14x^2-x+8}{4(x-1)^2}+\frac{\ln|x+1|+31\ln|x-1|}{8}+C$$

№91

$$\int\frac{x^4+1}{x^3-x^2+x-1}dx=\left[\begin{array}{l|l} x^4+1 & x^3-x^2+x-1 \\ -\underline{x^4-x^3+x^2-x} & x+1 \\ \quad x^3-x^2+x+1 & \\ -\underline{x^3-x^2+x-1} & \\ \qquad 2 & \end{array}\right]=$$

$$=\int\left(\frac{2}{\underbrace{x^3-x^2+x-1}_{(x-1)(x^2+1)}}+x+1\right)dx=\left[\frac{Ax+B}{x^2+1}^{(x-1)}+\frac{C}{x-1}^{(x^2+1)}=\right.$$

$$=\frac{(Ax+B)(x-1)+C(x^2+1)}{}=\frac{Ax^2-Ax+Bx-B+Cx^2+C}{(x^2+1)(x-1)}$$

$A+C=0 \quad A=-C$

$-A+B=0 \quad A=B \qquad C=1 \to A=-1=B$

$-B+C=2$

$-A+C=2 \qquad\qquad -1;-1;1 \left.\right]$

$2C=2 \quad C=1$

$$= \int \frac{+x+1}{x^2+1}\,dx + \int \frac{1}{x-1}\,dx + \int (x+1)\,dx =$$

$$= \frac{-1}{2}\int \frac{d(x^2)}{x^2+1} - \int \frac{1}{x^2+1}\,dx + \int \frac{1}{x-1}\,dx +$$

$$+ \int x\,dx + \int dx = -\frac{\ln|x^2+1|}{2} - \operatorname{arctg} x +$$

$$+ \ln|x-1| + \frac{x^2}{2} + x + C$$

N92

$$\int \frac{x^2}{1-x^4}\,dx = -\int \frac{x^2}{\underset{(x^2-1)(x^2+1)}{x^4-1}}\,dx = \Big[$$

$$\frac{\overset{(x-1)\,x^2+1}{A}}{x+1} + \frac{\overset{(x+1)\,x^2+1}{B}}{x-1} + \frac{\overset{(x-1)(x+1)\ (x^2-1)}{Cx+D}}{x^2+1} = (Ax-A)(x^2+1) + (Bx+B)$$

$$\cdot (x^2+1) + (Cx+D)(x^2-1) =$$

$$=$$

$x = 1:\ (Bx+B)(x^2+1) = 1$

$2B = 1 \Rightarrow B = \frac{1}{4}$

$x=-1: A(x-1)(x^2+1) = 1$

$-2A\cancel{B}\cdot 2 = 1 \qquad A = -\frac{1}{4}$

$Ax^3 + Ax - Ax^2 - A + Bx^3 + Bx + Bx^2 + B + Cx^3 - Cx +$
$+ Dx^2 - D$

$$\begin{aligned} A+B+C&=0\\ -A+B+D&=1\\ A+B-C&=0\\ -A+B-D&=0 \end{aligned} \qquad \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0\\ -1 & 1 & 0 & 1 & 1\\ 1 & 1 & -1 & 0 & 0\\ -1 & 1 & 0 & -1 & 0 \end{array}\right)\begin{array}{l} \\ +\mathrm{I}\\ -\mathrm{I}\\ +\mathrm{I}\end{array} \sim$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 0 & 0\\ 0 & 2 & 1 & 1 & 1\\ 0 & 0 & -2 & 0 & 0\\ 0 & 2 & 1 & -1 & 0 \end{pmatrix}\begin{array}{l} \\ \\ \\ -\mathrm{II}\end{array} \sim \begin{pmatrix} 1 & 1 & 1 & 0 & 0\\ 0 & 2 & 1 & 1 & 1\\ 0 & 0 & +2 & 0 & 0\\ 0 & 0 & 0 & -2 & -1 \end{pmatrix}$$

$-2D = -1$

$D = \frac{1}{2}$

$C = 0$

$2B + \frac{1}{2} = 1$

$2B = \frac{1}{2}$

$B = \frac{1}{4} \quad A = -\frac{1}{4}$

$-\frac{1}{4} \quad \frac{1}{4} \quad 0 \quad \frac{1}{2}$

$$= -\int \frac{1\,dx}{4(x+1)} - \frac{1}{4}\int \frac{dx}{x-1} \cancel{+} \frac{1}{2}\int \frac{dx}{x^2+1} =$$

$$= -\frac{1}{4}\ln|x+1| - \frac{1}{4}\ln|x-1| - \frac{1}{2}\operatorname{arctg} x + C$$

N93

$$\int \sqrt{\underbrace{1-4x-x^2}_{-(x^2+4x+4)+5}}\,dx = \int \sqrt{5-(x+2)^2}\,dx =$$

$$= \left[t = x+2 \quad dt = dx\right] = \int \sqrt{5-t^2}\,dt =$$

$$\left[\begin{array}{l} 5\cos^2 u = 5 - 5\sin^2 u \Rightarrow t^2 = 5\sin^2 u \\ t = \sin u\cdot\sqrt{5}\ ,\ u = \arcsin\frac{t}{\sqrt{5}} \\ \cancel{du = d\,\arcsin\frac{t}{\sqrt{5}}}\ dt = \sqrt{5}\cos u\,du \end{array}\right] =$$

$$= \int \sqrt{5}\cos u\cdot\sqrt{5}\cos u\,du = \int 5\cos^2 u\,du =$$

$$= 5\int \frac{\cos 2u + 1}{2}\,du = \frac{5}{4}\sin 2u + \frac{5}{2}u =$$

$$\cancel{= \frac{5}{2}\,[illegible]\,\frac{x+2}{\sqrt{5}} + \frac{5}{2}\arcsin\frac{x+2}{\sqrt{5}}}$$

$\sin 2x = 2\sin x\cos x$

$= \frac{5}{4}\sin 2\cdot\arcsin\frac{t}{\sqrt5} + \frac{5}{2}\arcsin\frac{t}{\sqrt5} =$

$= 5\sin u\cos u \qquad u \qquad \arcsin\frac{t}{\sqrt5} = \sqrt5\sin u$

$= \frac{5\arcsin\frac{x+2}{\sqrt5}}{2} + \frac{5\cdot\sin\left(\arcsin\frac{t}{\sqrt5}\right)\cos\left(\arcsin\frac{t}{\sqrt5}\right)}{4_2} =$

$= \frac{5\arcsin\frac{x+2}{\sqrt5}}{2} + \frac{t\sqrt{5-t^2}}{2} =$

$= \frac{5\arcsin\frac{x+2}{\sqrt5}}{2} + \frac{x\sqrt{-x^2-4x+1}}{2} + \sqrt{-x^2-4x+1}$

№94

$\int\sqrt{x^2-2x-1}\,dx = \int\sqrt{(x-1)^2-2}\,dx = \left[t = x-1 \quad dx = dt\right]$

$(x-1)^2+2$

$= \int\sqrt{t^2-2}\,dt = \left[u = \frac{t}{\sqrt2} \quad t = u\sqrt2 \quad dt = \sqrt2\,du\right] =$

$= 2\int\sqrt{u^2-1}\,du \quad \left[tg^2\omega = u^2-1,\ u^2 = \frac{1}{\cos^2\omega}\right.$

$\left.u = \frac{1}{\cos\omega} \quad du = \frac{\sin\omega}{\cos^2\omega}d\omega = \frac{tg\,\omega}{\cos\omega}d\omega\right] =$

$$= 2\int \sqrt{\underbrace{tg^2\omega}_{\frac{1-\cos^2\omega}{\cos^2\omega}}}\, d\omega = 2\int \frac{tg^2\omega}{\cos\omega}\, d\omega =$$

$$= 2\int \frac{\frac{1}{\cos^2\omega} - 1}{\cos\omega}\, d\omega = -2\int \frac{\cos^2\omega - 1}{\cos^3\omega}\, d\omega =$$

$$= -2\left(\int \frac{1}{\cos\omega}\, d\omega - \int \frac{1}{\cos^3\omega}\, d\omega\right) =$$

$$= -2\ln\left|tg\left(\frac{\omega}{2} + \frac{\pi}{2}\right)\right| +$$

$$+ 2\left(\frac{1}{-2}\right)\frac{1}{\cos^2\omega} = -2\sec\omega - \frac{1}{\cos^2\omega}$$

N95

$$\int \frac{dx}{x-\sqrt{x^2-x+1}} = \left[x-\sqrt{x^2-x+1} = -t \qquad \cancel{x^2}-2xt+t^2 = \cancel{x^2}-x+1 \right.$$

$$x(-2t+1) = -t^2+1 \qquad x = \frac{1-t^2}{-2t+1}$$

$$t = \sqrt{x^2-x+1} - x$$

$$dx = \frac{-2t(2t+1)-2(1-t^2)}{(2t+1)^2} \qquad -4t^2-2t-2+2t^2 \qquad -2t^2-2t-2 = -2(t^2+t+1)$$

$$= \left. \frac{-2(t^2+t+1)}{(2t+1)^2}\,dt \right] =$$

$$4\left(t+\frac{1}{2}\right)^2$$

$$= \int \frac{-2(t^2+t+1)}{(2t+1)^2}\cdot(-t)\,dt = 2\int \frac{t^2+t+1}{4t\left(t+\frac{1}{2}\right)^2}\,dt =$$

$$\left(\left(t+\frac{1}{2}\right)\cdot t \qquad \cdot t \qquad \left(t+\frac{1}{2}\right)^2\right.$$

$$= \left[\frac{A}{t+\frac{1}{2}} + \frac{B}{\left(t+\frac{1}{2}\right)^2} + \frac{C}{t} = \frac{A\left(t^2+\frac{t}{2}\right)+Bt+C\left(t^2+t+\frac{1}{4}\right)}{t\cdot\left(t+\frac{1}{2}\right)^2} \right.$$

$$At^2+\frac{At}{2}+Bt+Ct^2+Ct+\frac{C}{4}$$

$A+C=1$ $\qquad A=-3$ $\quad +5/2-1 \quad \frac{3}{2}$

$\frac{A}{2}+B+C=1$ $\qquad -\frac{3}{2}+8/2 = 1-B$ $\qquad -3/2$

$C/4=1$ $\qquad C=4$

$$\boxed{-3,\ -\frac{3}{2},\ 4}$$

$$-\int \frac{3}{t+\frac{1}{2}}\,dt - \frac{3}{2}\int \frac{dt}{\left(t+\frac{1}{2}\right)^2} + 4\int \frac{dt}{t} =$$

$$= -3\ln\left|t+\frac{1}{2}\right| + \frac{3}{2}\,\frac{1}{t+\frac{1}{2}} + 4\ln|t| =$$

$$= -3\ln\left|\sqrt{x^2-x+1}-x+\frac{1}{2}\right| + 4\ln\left|\sqrt{x^2-x+1}-x\right| + \frac{3}{2\left(\sqrt{x^2-x+1}-x+\frac{1}{2}\right)} + C$$

N96

$$\int \frac{1}{x^2\left(\sqrt{1+x^2}+x\right)}dx = \left[1+x^2 = \frac{1}{\cos^2\omega} \to \frac{1}{\cos^2\omega}-1 = x^2 \quad x = tg\,\omega \to \omega = arctg\,x \quad dx = \frac{1}{\cos^2\omega}d\omega \right] =$$

$$= \int \frac{1}{tg^2\omega\left(\frac{1}{\cos\omega}+tg\,\omega\right)} \cdot \frac{1}{\cos^2\omega}\,d\omega =$$

$$= \int \frac{1}{\cos\omega\, tg^2\omega + tg^3\omega\cos^2\omega}\,d\omega = \int \frac{1}{\frac{\sin^2\omega}{\cos\omega}+\frac{\sin^3\omega}{\cos\omega}}\,d\omega =$$

$$= \int \frac{\cos\omega}{\sin^2\omega+\sin^3\omega}\,d\omega = \left[\sin\omega = t \quad dt = \cos\omega\, d\omega\right]$$

$$= \int \frac{\cancel{\cos\omega}}{t^2+t^3}\,dt \cdot \frac{1}{\cancel{\cos\omega}} = \int \frac{1}{t^2(1+t)}\,dt =$$

$$\left[\frac{A}{t}+\frac{B}{t^2}+\frac{C}{t+1}=\frac{At^2+At+Bt+B+Ct^2}{t^2(t+1)}\Rightarrow\right.$$

$$\Rightarrow \left.\begin{matrix} A+C=0 \to C=1 \\ A+B=0 \to A=-1 \\ B=1 \end{matrix}\right] = -\int\frac{1}{t}dt+\int\frac{1}{t^2}dt+$$

$$+\int\frac{1}{t+1}dt = -\ln|t|-\frac{1}{t}+\ln|t+1| =$$

$$= -\ln(|\sin\omega|)+\ln|\sin\omega+1|-\frac{1}{\sin\omega}=$$

$$\cancel{-\ln|\sin(\operatorname{arctg} x)|+\ln|\sin(\operatorname{arctg} x+1)|}-$$

$$\cancel{\frac{1}{\sin(\operatorname{arctg}\omega)}} = \left[x=\operatorname{tg}\omega=\frac{\sin\omega}{\cos\omega}\right. \to$$

$$\operatorname{tg}^2\omega=\frac{1}{\cos^2\omega}-1 \qquad \cos\omega = \frac{1}{\sqrt{\operatorname{tg}^2\omega+1}}$$

$$\left.\Rightarrow \underset{\omega=\operatorname{arctg}x}{\sin\omega}=\operatorname{tg}\omega\cdot\frac{1}{\sqrt{\operatorname{tg}^2\omega+1}}=x\cdot\frac{1}{\sqrt{x^2+1}}\right]=$$

$$= -\ln\left|\frac{x}{\sqrt{x^2+1}}\right|+\ln\left(\left|\frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}\right|\right)-\frac{\sqrt{x^2+1}}{x}=$$

$$= -\ln|x|+\cancel{\ln|\sqrt{x^2+1}|}+\ln|x+\sqrt{x^2+1}|-$$

$$-\cancel{\ln|\sqrt{x^2+1}|}-\frac{\sqrt{x^2+1}}{x}+C=$$

$$= -\ln|x|+\ln|x+\sqrt{x^2+1}|-\frac{\sqrt{x^2+1}}{x}+C$$

№97

$$\int \frac{dx}{1+\sqrt{\underset{(x^2+2x+1)+1}{x^2+2x+2}}} = \left[\begin{matrix} x+1=t & dt=dx \\ x=t-1 & \end{matrix}\right] = \int \frac{dx}{1+\sqrt{t^2+1}} =$$

$$= \int \frac{1-\sqrt{t^2+1}}{\cancel{1}-t^2-\cancel{1}}\,dt = +\overset{A}{\overset{''}{\int}} \frac{\sqrt{t^2+1}}{t^2}\,dt - \int \frac{1}{t^2}\,dt =$$

A) $$\int \frac{\sqrt{t^2+1}}{t^2}\,dt = \left[\begin{matrix} u=\sqrt{t^2+1} & du=\frac{+1}{\cancel{2}\sqrt{t^2+1}}\cancel{2}t\,dt = \frac{t\,dt}{\sqrt{t^2+1}} \\ dV=\frac{1}{t^2}\,dt & V=-\frac{1}{t} \end{matrix}\right] =$$

$$= -\frac{\sqrt{t^2+1}}{t} - \int -\frac{1}{t}\cdot\frac{t}{\sqrt{t^2+1}}\,dt = -\frac{\sqrt{t^2+1}}{t} + \int \frac{1}{\sqrt{t^2+1}}\,dt$$

$$= -\frac{\sqrt{x^2+2x+2}}{x+1} + \ln\left|x+1+\sqrt{x^2+2x+2}\right| + \frac{1}{x+1} + C.$$

№ 100

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \left[\begin{array}{l} 1+x^2 = \frac{1}{\cos^2\varphi} \to x^2 = tg\,\varphi \\ \varphi = arctg\,x. \quad dx = \frac{1}{\cos^2\varphi}\,d\varphi \end{array}\right] =$$

$$= \int \frac{\frac{1}{\cos\varphi}}{2+tg^2\varphi}\cdot\frac{1}{\cos^2\varphi}\,d\varphi = \int \frac{1}{2\cos^3\varphi + \cos^3\varphi\, tg^2\varphi}\,d\varphi =$$

$$= \int \frac{1}{\underbrace{2\cos^3\varphi + \sin^2\varphi\cdot\cos\varphi}_{\cos\varphi(2\cos^2\varphi + \sin^2\varphi)}}\,d\varphi = \int \frac{\cos\varphi}{\cos^2\varphi\,(2\cos^2\varphi + \sin^2\varphi)}\,d\varphi =$$

$$= \left[\begin{array}{l} \cos^2\varphi = 1 - \sin^2\varphi \\ \sin^2\varphi = t^2. \quad t = \sin\varphi \quad dt = \cos\varphi\,d\varphi \end{array}\right] =$$

$$d\varphi = \frac{1}{\cos\varphi}\,dt$$

$$= \int \frac{\cancel{\cos\varphi}}{\underbrace{(1-t^2)}_{1-t^2}\underbrace{(2(1-t^2)+t^2)}_{2-t^2}}\cdot\frac{1}{\cancel{\cos\varphi}}\,dt =$$

$$= \int \frac{1}{(t-1)(t+1)(t^2-2)}\,dt = \left[\frac{Ax+B}{t^2-2} + \frac{C}{t-1} + \frac{D}{t+1}\right.$$

$$= \frac{(A\cancel{t}+B)(t^2-1) + C\overbrace{(t^2-2)(t+1)}^{t^3+t^2-2t-2} + D\overbrace{(t^2-2)(t-1)}^{t^3-t^2-2t+2}}{(t^2-2)(t^2-1)}$$

$$= \frac{At^3\cancel{t} - A\cancel{t} + Bt^2 - B + Ct^3 + Ct^2 - C2t - 2C + Dt^3 - Dt^2 - 2Dt + 2D}{(t^2-2)(t^2-1)}$$

$$\Longrightarrow \begin{cases} C + D + A = 0 \\ B + C - D = 0 \\ -2C - 2D - A = 0 \\ -B - 2C + 2D = 1 \end{cases} \Longleftrightarrow$$

$$\Longleftrightarrow \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 1 & 0 & 2 & 2 & 0 \\ 0 & 1 & 2 & -2 & -1 \end{pmatrix} - \mathrm{I} \sim \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & -2 & -1 \end{pmatrix} - \mathrm{II} \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right) - \mathrm{III} \sim \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & -1 \end{pmatrix}$$

$$-2D = -1 \qquad D = \frac{1}{2}$$

$$C + \frac{1}{2} = 0 \qquad C = -\frac{1}{2}$$

$$B - \frac{1}{2} - \frac{1}{2} = 0 \qquad B = 1 \qquad A = 0$$

$$\boxed{0,\ 1,\ -\frac{1}{2},\ \frac{1}{2}}$$

$$= \int \frac{1}{t^2-2}\,dt - \frac{1}{2}\int \frac{1}{t-1} + \frac{1}{2}\int \frac{1}{t+1} =$$

$$= \frac{1}{2\sqrt{2}} \ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| - \frac{1}{2}\ln|t-1| + \frac{1}{2}\ln|t+1| =$$

$$= \frac{\ln\left|\frac{\sin\varphi-\sqrt{2}}{\sin\varphi+\sqrt{2}}\right|}{2\sqrt{2}} - \frac{\ln|\sin\varphi-1|}{2} + \frac{\ln|\sin\varphi+1|}{2}$$

$=$

$\frac{1}{\cos^2\varphi} - 1 = tg^2\varphi \quad tg^2\varphi+1 = \frac{1}{\cos^2 1} \quad \frac{1}{tg^2\varphi} \quad \cos^2\varphi = \frac{1}{tg^2\varphi+1}$

$$\left[\sin\varphi = \sqrt{1-\cos^2\varphi} = \sqrt{1-\frac{1}{tg^2\varphi+1}} = [\varphi = arctg\, x] = \right.$$

$$\left. = \sqrt{1-\frac{1}{x^2+1}} = \sqrt{\frac{x^2+1-1}{x^2+1}} = \frac{x}{\sqrt{x^2+1}}\right] =$$

$$= \frac{\ln\left|\frac{x-\sqrt{2}\sqrt{x^2+1}}{x+\sqrt{2}\sqrt{x^2+1}}\right|}{2\sqrt{2}} - \frac{\ln\left|\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right|}{2} +$$

$$+ \frac{\ln\left|\frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}\right|}{2} = \frac{\ln|x-\sqrt{2}\sqrt{x^2+1}|}{2\sqrt{2}} -$$

$$- \frac{\ln|x+\sqrt{2}\sqrt{x^2+1}|}{2\sqrt{2}} - \frac{\ln|x-\sqrt{x^2+1}|}{2} +$$

$$+ \cancel{\ln|\sqrt{x^2+1}|} \quad + \frac{\ln|x+\sqrt{x^2+1}|}{2} -$$

$$-\cancel{\frac{\ln\left|\sqrt{x^2+1}\right|}{2}} = -\frac{\ln\left|x+\sqrt{2}\sqrt{x^2+1}\right|}{2\sqrt{2}} -$$

$$-\frac{\ln\left|x-\sqrt{2}\sqrt{x^2+1}\right|}{2\sqrt{2}} + \frac{\ln\left|\sqrt{x^2+1}-x\right|}{2} +$$

$$+\frac{\ln\left|x+\sqrt{x^2+1}\right|}{2} + C$$