ВАРИАНТ 3

[redacted]

N1

$$\int \frac{\arcsin^3 x}{\sqrt{1-x^2}}\,dx = \int \arcsin^3 x\, d(\arcsin x) =$$

$$= \frac{\arcsin^4 x}{4} + C$$

N2

$$\int \sin^5 2x \cdot \cos 2x\, dx = \frac{1}{2}\int \sin^5 2x\, d(\sin 2x) =$$

$$= \frac{\sin^6 2x}{12} + C$$

N3

$$\int \frac{7x-5}{\sqrt{3+2x-x^2}}\,dx = \int \frac{(7x-5)\,dx}{\sqrt{-(x-1)^2+4}} = \left[\begin{matrix} t = x-1 & dt = dx \\ x = t+1 & \end{matrix}\right] =$$

$$= \int \frac{\overset{7t+2}{7(t+1)-5}}{\sqrt{4-t^2}}\,dt = 7\overbrace{\int \frac{t\,dt}{\sqrt{4-t^2}}}^{A} + 2\overbrace{\int \frac{dt}{\sqrt{4-t^2}}}^{B} =$$

A) $\int \frac{t}{\sqrt{4-t^2}}\,dt = -\frac{1}{2}\int \frac{d(-t^2)}{\sqrt{4-t^2}} = -\frac{1}{2}\,2\sqrt{4-t^2} =$

$= -\sqrt{4-t^2}$

B) $\int \frac{dt}{\sqrt{4-t^2}} = \arcsin\frac{t}{2}$

$= -7\sqrt{3+2x-x^2} + 2\arcsin\frac{x-1}{2} + C.$

№4

№4

$$\int \frac{(x^4-3x^3+x^2-5x+2)\,dx}{(x-1)(x^4-1)} =$$

$$= \int \frac{x^4-3x^3+x^2-5x+2}{(x-1)(x^2+1)(x-1)(x+1)}\,dx = \left[\frac{A}{x+1} + \frac{Bx+C}{x^2+1} +\right.$$

$Bx^2+Bx+Cx+C$

$$+\frac{D}{x-1} + \frac{E}{(x-1)^2} = \frac{A(x^2-2x+1)(x^2+1)+(Bx+C)(x+1)\cdot}{(x-1)(x^4-1)}$$

$(x^4-1)$

$$\frac{\cdot(x^2-2x+1) + D\,\cancel{(\ldots)} + E(x+1)(x^2+1)\cdot}{} =$$

$$Ax^4+Ax^2-2Ax^3-2Ax+Ax^2+A+Bx^4-2Bx^3+Bx^2+$$
$$= +Bx^3-2Bx^2+Bx+Cx^3-2Cx^2+Cx+Cx^2-2Cx+C+$$
$$+Dx^4-D+Ex^3+Ex+Ex^2+E$$

$x^4:\ A+B+D \qquad 1$

$x^3:\ -2A-2B+B+C+E \quad -2C \qquad -3$

$x^2:\ \cancel{A+A+B+B}\ A+A+B-2B+C+E \qquad 1$

$x^1:\ -2A+B-2C+C-2C+E \qquad -5$

$x^0:\ A+C-D+E \qquad 2$

$$\begin{pmatrix} 1 & 0 & 1 & -1 & 1 & 2 \\ -2 & 1 & -3 & 0 & 1 & -5 \\ 2 & -1 & 1 & 0 & 1 & 1 \\ -2 & -1 & 1 & 0 & 1 & -3 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{matrix} -\mathrm{V} \\ +\mathrm{III} \\ \\ +\mathrm{III} \\ \\ \end{matrix} \sim \left(\begin{array}{ccccc|c} 0 & -1 & 1 & -2 & 1 & 1 \\ 0 & 0 & -2 & 0 & 2 & -4 \\ 2 & -1 & 1 & 0 & 1 & 1 \\ 0 & -2 & 2 & 0 & 2 & -2 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{array}\right) \begin{matrix} \\ /:(-2) \\ -2\mathrm{V} \\ /:2 \\ \\ \end{matrix}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 & -1 & -1 \\ 0 & -1 & 1 & 0 & 1 & -1 \\ 0 & -3 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 2 \end{array}\right) \begin{array}{l} \\ \\ +\mathrm{II} \\ +3\mathrm{II} \\ \\ \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 2 & 0 & -2 \\ 0 & 0 & -2 & 4 & -3 & -4 \\ 0 & 0 & 1 & 0 & -1 & 2 \end{array}\right) \begin{array}{l} \\ \\ \\ +2\mathrm{V} \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{cccccc} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 4 & -5 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{array}\right) \begin{array}{l} \\ \\ \\ -4\mathrm{V} \\ \\ \end{array} \sim \left(\begin{array}{cccccc} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & -5 & 4 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{array}\right) \begin{array}{l} \\ \\ \\ /:(-5) \\ \\ \end{array}$$

$$\left(\begin{array}{cccccc} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4/5 \end{array}\right) \qquad \left.\begin{array}{l} A = 3/2 \\ B = 0 \\ C = 1 \\ D = -1/2 \\ E = -1 \end{array}\right] =$$

$$= \int \left( \frac{1}{x^2+1} + \frac{3}{2(x+1)} - \frac{1}{2(x-1)} - \frac{1}{(x-1)^2} \right) dx =$$

$$= \operatorname{arctg} x + \frac{3}{2} \ln|x+1| - \frac{1}{2} \ln|x-1| + \frac{1}{x-1} + C$$

N5

$$\int \frac{5\sqrt{x+1}}{(x+1)^2\sqrt{x}}\,dx = 5\int \frac{1}{\sqrt[2]{(x+1)^3}\,\sqrt{x}}\,dx = 5\int \frac{dx}{\sqrt{x}\,\sqrt{(x+1)^2}\,\sqrt{x+1}} =$$

$/ : \sqrt{x+1}$

$$= 5\int \frac{dx}{\frac{\sqrt{x}\,(x+1)^2}{\sqrt{x+1}}} = \Big[ t = \frac{\sqrt{x}}{\sqrt{x+1}},\ t^2 = \frac{x}{x+1} \longrightarrow$$

$$\longrightarrow x = t^2(x+1) \longrightarrow x = \frac{t^2}{1-t^2} = \frac{-t^2}{t^2-1} = -\left(\frac{1-t^2}{1-t^2}\right) =$$

$$= -\left(-\frac{1}{1-t^2} + 1\right) = -\frac{1}{t^2-1} - 1$$

$$dx = \frac{-(-2t)}{(t^2-1)^2}\,dt = \frac{2t}{(t^2-1)^2}\,dt \Big] = 5\int \frac{2\cancel{t}\,dt}{\cancel{(t^2-1)^2}\cdot \cancel{t}\left(-\frac{1}{\cancel{t^2-1}}\right)^2} =$$

$$= 5\int 2\,dt = 10t = 10\sqrt{\frac{x}{x+1}} + C.$$

N6 $\quad \sqrt{25\left(\frac{1-\cos^2 x}{\cos^2 x}\right)} = 5\,tg\,x$

$$\int \frac{\sqrt{(x^2-25)^3}}{x^6}\,dx = \left[x = \frac{5}{\cos t} \quad dx = \frac{5\sin t}{\cos^2 t}\,dt = 5\,\frac{tg\,t}{\cos t}\,dt\right] =$$

$$= \int \frac{(5\,tg\,t)^3\, 5\,tg\,t}{\frac{5^6}{\cos^6 t}\cdot \cos t}\,dt = \frac{1}{25}\int \cos^5 t\, tg^4 t\,dt =$$

$$= \frac{1}{25}\int \cos t \sin^4 t\,dt = \frac{1}{25}\int \sin^4 t\, d(\sin t) =$$

$$= \frac{\sin^5 t}{125} =$$

$$= \left[x = \frac{5}{\cos t} \to t = \arccos\frac{5}{x},\ \sin t = \sqrt{1-\left(\frac{5}{x}\right)^2} = \frac{\sqrt{x^2-25}}{x},\ \sin^5 t = \frac{\sqrt{x^2-25}\,(x^2-25)^2}{x^5}\right] =$$

$$= \frac{\sqrt{x^2-25}\,(x^4-50x^2+625)}{125x^5} + C.$$

N7

$$\int \frac{1\,dx}{(1+\sin x-\cos x)^2} = \left[ t=\operatorname{tg}\frac{x}{2};\ \frac{x}{2}=\operatorname{arctg} t \quad \cos x=\frac{1-t^2}{1+t^2};\ \sin x=\frac{2t}{1+t^2} \quad dx=\frac{2}{1+t^2}dt \right] =$$

$$= \int \frac{2\,dt}{(1+t^2)\left(1+\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}\right)^2} = \int \frac{2\,dt}{(1+t^2)\left(\frac{\cancel{1}+t^2+2t-\cancel{1}+t^2}{1+t^2}\right)^2}$$

$$= \int \frac{2\,dt}{(\cancel{1+t^2})\left(\frac{2t(t+1)}{\cancel{1+t^2}}\right)^2} = \int \frac{2(1+t^2)\,dt}{4t^2(t+1)^2} =$$

$$= \frac{1}{2}\int \frac{1+t^2}{t^2(t+1)^2} = \left[ \frac{t^2+1}{t^2(t+1)^2} = \frac{A}{t+1} + \frac{B}{(t+1)^2} + \right.$$

$(t^2(t+1))$ $(t^2$ $(t(t+1)^2$ $(t+1)^2$

$$+\frac{C}{t} + \frac{D}{t^2},\quad A(t^3+t^2)+Bt^2+Ct^3+2Ct^2+Ct$$

$$+Dt^2+2Dt+D \Rightarrow \begin{array}{l} t^3:\ A+C=0 \quad A=2 \\ t^2:\ A+B+2C+D=1 \\ t^1:\ C+2D=0 \ \Rightarrow\ C=-2 \\ t^0:\ D=1 \end{array}$$

$2+B-4+1=1$
$-1+B=1$
$B=2$

$$\left.(2;\ 2;\ -2;\ 1)\right]$$

$$=\frac{1}{2}\int\left(\frac{2}{(t+1)}+\frac{2}{(t+1)^2}-\frac{2}{t}+\frac{1}{t^2}\right)dt=$$

$$=\ln|t+1|-\frac{1}{(t+1)}-\ln|t|-\frac{1/2}{t}=$$

$$=\ln|t+1|-\ln|t|+\frac{-3t-1}{2(t^2+t)}=$$

$$=\ln\left|tg\frac{x}{2}+1\right|-\ln\left|tg\frac{x}{2}\right|+\frac{-3tg\frac{x}{2}-1}{2\left(tg^2\frac{x}{2}+tg\frac{x}{2}\right)}+C$$

№ 8

$$\int_{-1}^{0}(x^2+x)e^x\,dx = \left[\begin{array}{l|l} x^2+x=u & du=2x+1\,dx \\ e^x dx = dv & v=e^x \end{array}\right] =$$

$$= \left(e^x(x^2+x) - \int_{-1}^{0} e^x(2x+1)\,dx\right)\Bigg|_{-1}^{0} =$$

$$e^x(x^2+x)\Big|_{-1}^{0} - \int_{-1}^{0} 2e^x x\,dx - \int_{-1}^{0} e^x\,dx = \left[\begin{array}{l|l} 2x=u & du=2dx \\ e^x dx = dV & V=e^x \end{array}\right]$$

$$= e^x(x^2+x)\Big|_{-1}^{0} - e^x\Big|_{-1}^{0} - \left(e^x 2x\Big|_{-1}^{0} - \int_{-1}^{0} 2e^x\,dx\right) =$$

$$= \left(e^x(x^2+x) - e^x - 2e^x x + 2e^x\right)\Bigg|_{-1}^{0} =$$

$$= -\left(\cancel{e^{-1}}\cdot 0 - e^{-1} + 2e^{-1} + 2e^{-1}\right) + \left(1\cdot 0 - 1 - 0 + 2\cdot 1\right) =$$

$$= -3e^{-1} + 1 = 1 - \frac{3}{e}.$$

N9

$$\int_1^5 \frac{1}{x+\sqrt{3x+1}}\,dx = \left[ t=\sqrt{3x+1} \quad t^2=3x+1 \quad dx=\frac{2t}{3}dt \quad x=\frac{t^2-1}{3}; \quad \begin{matrix} x=1 \to t=2 \\ x=5 \to t=4 \end{matrix} \right] = \int_2^4 \frac{2t\,dt}{3\left(\frac{t^2-1}{3}+t\right)} =$$

$$= \frac{2}{\cancel{3}}\int_2^4 \frac{\cancel{3}t\,dt}{t^2-1+3t} = \left[ t=\frac{1}{2}(2t+3)-\frac{3}{2} \right] =$$

$$= 2\left(\frac{1}{2}\int_2^4 \frac{2t+3\,dt}{t^2-1+3t} - \int_2^4 \frac{3\,dt}{2(t^2+3t-1)}\right) =$$

$$= \ln\left|t^2+3t-1\right|\Bigg|_2^4 - \cancel{2}\cdot\frac{3}{\cancel{2}}\int_2^4 \frac{dt}{\left(t+\frac{3}{2}\right)^2-\frac{13}{4}}$$

B) $$-3\int_2^4 \frac{dt}{\left(t+\frac{3}{2}\right)^2-\frac{13}{4}} = \left[ u=t+\frac{3}{2} \quad \begin{matrix} t=2 \to u=3{,}5 \\ t=4 \to u=5{,}5 \end{matrix} \right] =$$

$$= -3\int_{3,5}^{5,5} \frac{du}{\left(u-\frac{\sqrt{13}}{2}\right)\left(u+\frac{\sqrt{13}}{2}\right)} = \cancel{\left[ \frac{1}{\sqrt{13}}\left(u-\frac{\sqrt{13}}{2}\right) \ldots \right]}$$

$$\left[1 = -\frac{1}{\sqrt{13}}\left(\left(u-\frac{\sqrt{13}}{2}\right)-\left(u+\frac{\sqrt{13}}{2}\right)\right)\right] =$$

$$= -3\int_{3,5}^{5,5} \frac{-\frac{1}{u+\frac{\sqrt{13}}{2}} - \frac{1}{u-\frac{\sqrt{13}}{2}}}{\sqrt{13}}\, du =$$

$$= +\frac{3}{\sqrt{13}}\int_{3,5}^{5,5}\left(+\frac{2du}{2u+\sqrt{13}}\right) + \frac{3}{\sqrt{13}}\int_{3,5}^{5,5}\frac{2du}{2u-\sqrt{13}} =$$

$$= \frac{3\cdot\cancel{2}}{\sqrt{13}\cdot\cancel{2}}\ln|2u+\sqrt{13}| - \frac{3\cdot\cancel{2}}{\sqrt{13}\cdot\cancel{2}}\ln|2u-\sqrt{13}|\Bigg|_{3,5}^{5,5}$$

$$A+B = \ln(t^2+3t-1)\Big|_2^4 + \left(\frac{3}{\sqrt{13}}\ln|2u+\sqrt{13}| -\right.$$

$$\left.-\frac{3}{\sqrt{13}}\ln(2u-\sqrt{13})\right)\Bigg|_{3,5}^{5,5} =$$

$$= \ln(16+12-1) - \ln(4+6-1) + \frac{3}{\sqrt{13}}\ln\left|\frac{11+\sqrt{13}}{11-\sqrt{13}}\right| -$$

$$-\frac{3}{\sqrt{13}}\ln\left|\frac{7+\sqrt{13}}{7-\sqrt{13}}\right| = \ln 27 - \ln 9 + \frac{3}{\sqrt{13}}\ln\left|\frac{11+\sqrt{13}}{11-\sqrt{13}}\right| -$$

$$-\frac{3}{\sqrt{13}}\ln\left|\frac{7+\sqrt{13}}{7-\sqrt{13}}\right|$$

№10

$y = x\ln^2 x \quad y = x\ln x \qquad S - ?$

$x\ln^2 x = x\ln x$

$x\ln^2 x - x\ln x = 0$

$x\ln x(\ln x - 1) = 0$

$\left[\begin{array}{l} x\ln x = 0 \\ \ln x = 1 \end{array}\right. \quad \left[\begin{array}{l} \left[\begin{array}{l} x = 0 \\ \ln x = 0 \end{array}\right. \quad x = 1 \\ \quad x = e \end{array}\right.$

$x = 1, \quad x = e.$

$$S = \int_1^e (x\ln x - x\ln^2 x)\,dx =$$

$$= \underbrace{\int_1^e x\ln x\,dx}_{A} - \underbrace{\int_1^e x\ln^2 x\,dx}_{B} =$$

$$A: \int_1^e x\ln x\,dx = \begin{bmatrix} u = \ln x & du = \frac{1}{x} \\ dv = x\,dx & v = \frac{x^2}{2} \end{bmatrix} =$$

$$= \ln x \cdot \frac{x^2}{2}\Big|_1^e - \int_1^e \frac{x^2}{2}\cdot\frac{1}{x} = \frac{\ln x \cdot x^2}{2} - \frac{1}{2}\cdot\frac{x^2}{2} = \frac{x^2 2\ln x - x^2}{4}$$

B) $\int_1^e x\ln^2 x = \left[ t=\ln^2 x \quad t^2=\ln^2 x \quad x^2 = e^t = e^{\ln x} \quad x^2 = e^{2\ln x} \right] =$

$= \int_1^e e^{2t} \cdot t^2\, dt = \left[ \begin{matrix} u=t^2 & du = 2t\,dt \\ dV = e^{2t}dt & V = \frac{1}{2}e^{2t} \end{matrix} \right]$

$= \frac{e^{2t}}{2} \cdot t^2 \Big|_1^e - \int \frac{e^{2t}}{\cancel{2}} \cancel{2} t\,dt = \left[ \begin{matrix} t=u & du = 1\,dt \\ e^{2t}dt = dV & V = \frac{1}{2}e^{2t} \end{matrix} \right]$

$= \frac{t^2 e^{2t}}{2} - \left( \frac{e^{2t}\cancel{t}}{2} - \int_1^e \frac{e^{2t}}{2} dt \right) = \left( \frac{t^2 e^{2t}}{2} - \frac{e^{2t}\cancel{t}}{2} + \frac{e^{2t}}{4} \right) \Big|_1^e =$

$A+B = \left( \frac{2x^2 \ln x - x^2}{4} + \frac{\ln^2 x\, e^{2\ln x}}{2} - \frac{e^{2\ln x} \cdot \ln x}{2} \right.$

$\left. + \frac{e^{2\ln x}}{4} \right) \Big|_1^e = \cancel{\frac{2e^2 \cdot 1 - e^2}{4} + \frac{\ln^2 e \cdot e^2}{2} - \frac{e^2 \cdot 1}{2} +}$

$\cancel{+\frac{e^2}{4} - (0 + 1) \cdot e^2} = \left( \frac{2e^2 \cdot 1 - e^2}{4} + \frac{\ln^2 e \cdot e^2}{2} - \right.$

$- \frac{e^2}{2} + \frac{e^2}{4} - \left( \cancel{0} + \frac{\ln^2 1 \cdot e^0}{2} - \frac{e^0 \cdot \ln 1}{2} + \frac{e^0}{4} \right) =$

$= \frac{e^2}{4} + \frac{e^2}{2} - \frac{e^2}{2} + \frac{e^2}{4} - \frac{1}{4} = \cancel{\ } + \frac{1}{2}$ Ответ: $\frac{1}{2}$

№11

$y = 1 - \ln\cos x \qquad 0 \le x \le \frac{\pi}{3}$

$$L = \int_a^b \sqrt{y'^2 + 1}\,dx$$

$$y' = -\frac{(-\sin x)}{\cos x} = \operatorname{tg} x$$

$$L = \int_0^{\frac{\pi}{3}} \sqrt{\operatorname{tg}^2 x + 1}\,dx = \int_0^{\frac{\pi}{3}} \sqrt{\frac{\sin^2 x + \cos^2 x}{\cos^2 x}}\,dx =$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{\frac{1}{\cos^2 x}}\,dx = \int_0^{\frac{\pi}{3}} \frac{1}{\cos x}\,dx = \int_0^{\frac{\pi}{3}} \sec x\,dx =$$

$$= \ln\left|\sec x + \operatorname{tg} x\right|\Big|_0^{\pi/3} = \ln\left|\frac{1+\sin x}{\cos x}\right|\Big|_0^{\pi/3} =$$

$$= \ln\left|\frac{1+0}{1}\right| + \ln\left|\frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right| = \ln|2+\sqrt{3}| \approx$$

$$\approx 1,31691$$

№12

ⓐ $x = 2(t - \sin t)$ $\quad 2t - 2\sin t$ $\qquad 0 \le t \le 2\pi$

$y = 2(1 - \cos t)$

$$S = \int_0^{2\pi} y(t)\,x'(t)\,dt$$

$$x'(t) = 2 - 2\cos t$$

$$S = \int_0^{2\pi} (2 - 2\cos t)(2 - 2\cos t)\,dt =$$

$$= \int_0^{2\pi} (2 - 2\cos t)^2\,dt = 4\int_0^{2\pi} (1 - \cos t)^2\,dt =$$

$$= 4\left(\int_0^{2\pi} (1 - 2\cos t + \cos^2 t)\,dt =\right.$$

$$= 4\left(\int_0^{2\pi} \left(1 - 2\cos t + \frac{1}{2} + \frac{\cos 2t}{2}\right)dt\right) =$$

$$= 4\left(\frac{3}{2}t - 2\cdot\sin t + \frac{1}{2}\cdot\frac{1}{2}\cdot\sin 2t\right)\Big|_0^{2\pi} =$$

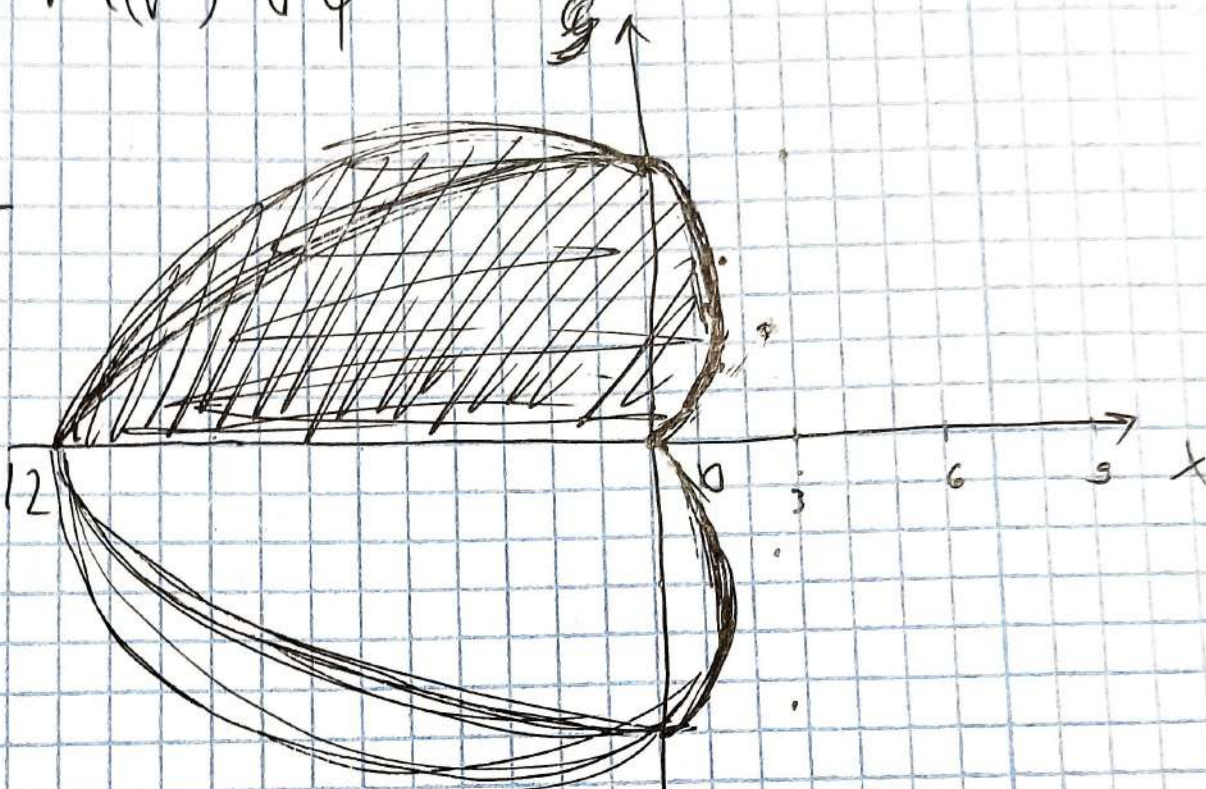
$$= 4\left(\frac{3}{\cancel{2}}\cancel{2}\pi\right) - 4\left(\frac{3}{2}\cdot 0\right) = 12\pi.$$

№12

(8) $r = 6(1 - \cos\varphi)$

$$L = \int_a^b \sqrt{r^2 + (r')^2}\, d\varphi$$

| $\varphi$ | $r$ |
|---|---|
| $0$ | $0$ |
| $\frac{\pi}{3}$ | $3$ |
| $\pi$ | $6$ |
| $\frac{3\pi}{2}$ | $6$ |
| $-\pi$ | $12$ |




$\frac{\pi}{4}$ $\quad 6\left(1 - \frac{\sqrt{2}}{2}\right)$

$6\,\frac{2-\sqrt{2}}{2} =$

$= 6 - 3\sqrt{2} \approx 1{,}8$

$r' = +6\sin\varphi$

$$L = \int_0^{\pi} \sqrt{(6 - 6\cos\varphi)^2 + 36\sin^2\varphi}\, d\varphi =$$

$$=\int_0^{\pi}\sqrt{36-72\cos\varphi+36\cos^2\varphi+36\sin^2\varphi}\,d\varphi$$

$$=\int_0^{\pi}\sqrt{36-72\cos\varphi+36\left(\frac{1+\cos2\varphi}{2}\right)+36\left(\frac{1-\cos2\varphi}{2}\right)}\,d\varphi=$$

$$=\int_0^{\pi}\sqrt{36-72\cos\varphi+18+\cancel{18\cos2\varphi}+18-\cancel{18\cos2\varphi}}\,d\varphi$$

$$=\int_0^{\pi}\sqrt{72(1-\cos\varphi)\cdot2:2}\,d\varphi=$$

$$\left[t=\frac{\varphi}{2}\quad dt=\frac{1}{2}dx\right]$$

$$=6\sqrt{2}\int_0^{\pi}\sqrt{2}\cdot\sin\frac{\varphi}{2}\,d\varphi=12\int_0^{\frac{\pi}{4}}\sin t\,dt=$$

$$=-24\cos t\Big|_0^{\pi}=-24\cos\frac{\varphi}{2}\Big|_0^{\pi}=$$

$$=-24\cdot0+24=24$$

$$L_{\text{полн}}=L\cdot2=24\cdot2=48.$$

Ответ: 48.

N13

$$\int_0^{+\infty} e^{-2x}(4x-3)\,dx - ?$$

$$\lim_{x\to+\infty} \frac{4x-3}{e^{2x}} = \lim_{x\to+\infty} \frac{4x}{e^{2x}} - \frac{3}{e^{2x}} \Rightarrow$$ предел есть

$$\int \frac{4x-3}{e^{2x}}dx \left[\begin{matrix} t=2x & dx=\frac{1}{2}dt \\ x=\frac{1}{2}t & \end{matrix}\right] =$$

$$= \int \frac{4\cdot\frac{1}{2}t-3}{e^t}\cdot\frac{1}{2}dt = \int \frac{2t-3}{2e^t}dt =$$

$$= \int \left(\underbrace{\frac{t}{e^t}}_{I} - \underbrace{\frac{3}{2e^t}}_{II}\right)dt =$$

$$I)\ \int \frac{t}{e^t}dt = \left[\begin{matrix} u=t & ; du=dt \\ v=e^{-t}dt & dv=-e^{-t} \end{matrix}\right] = -e^{-t}\cdot t +$$

$$+ \int e^{-t}dt = -e^{-t}\cdot t - e^{-t} = -\frac{t}{e^t} - \frac{1}{e^t} =$$

$$= -\frac{t+1}{e^t}$$

$$= -\frac{t+1}{e^t} + \frac{3}{2}e^{-t} = \frac{-2t-2+3}{2e^t} = \frac{-2t+1}{2e^t} =$$

$$= \frac{-4x-1}{2e^{2x}}$$

$$\lim_{b \to \infty} \int_0^b \frac{4x-3}{e^{2x}} = \lim_{b \to \infty} \frac{-4x-1}{2e^{2x}} \Big|_0^b =$$

$$= \lim_{b \to \infty} \left( \frac{-4b-1}{2 \cdot e^{2b}} + \frac{1}{2} \right) = \frac{1}{2}$$

Ответ: $\frac{1}{2}$