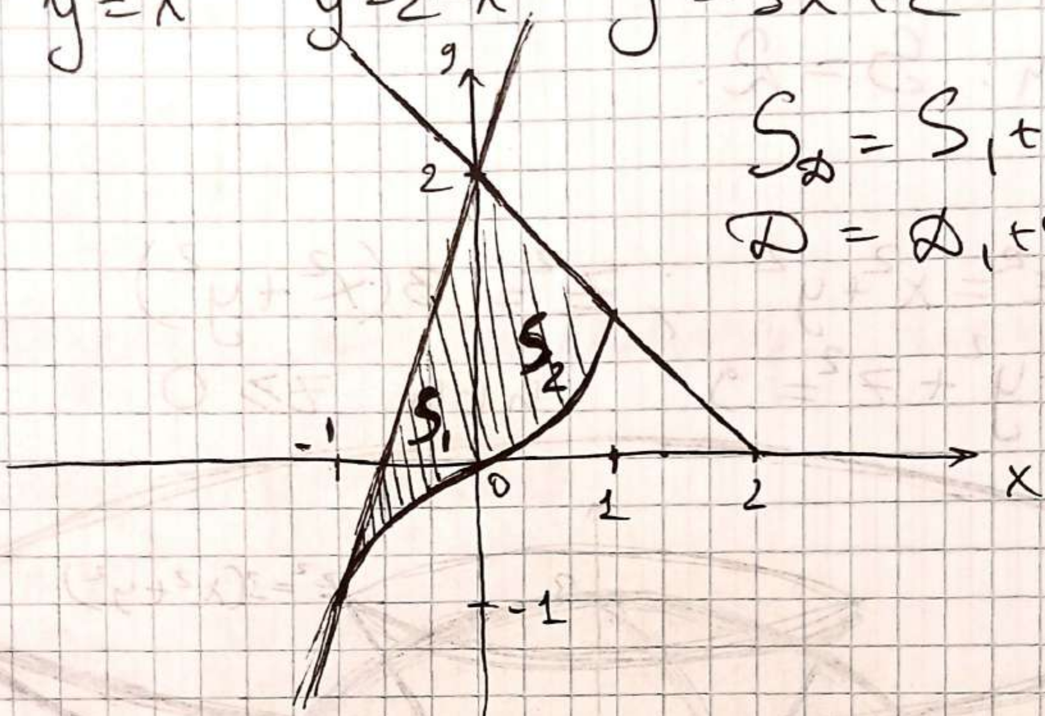
Вариант 3

№1 $y=x^3$ $\quad y=2-x$ $\quad y=3x+2$ $\quad$ при $y \le 2 - x$



$S_D = S_1 + S_2$

$D = D_1 + D_2$

$$S_1 = \iint\limits_{D_1} dx\,dy = \int_{-1}^{0} dx \int_{x^3}^{3x+2} dy = \int_{-1}^{0} (3x+2-x^3)\,dx =$$

$$= \left(3\frac{x^2}{2} + 2x - \frac{x^4}{4}\right)\Big|_{-1}^{0} = 0 - \left(3\frac{1}{2} - 2 - \frac{1}{4}\right) = -\left(\frac{6-8-1}{4}\right) =$$

$$= \frac{3}{4}$$

$$S_2 = \iint\limits_{D_2} dx\,dy = \int_{0}^{1} dx \int_{x^3}^{2-x} dy = \int_{0}^{1} (-x^3+2-x)\,dx =$$

$$= \left(-\frac{x^4}{4} + 2x - \frac{x^2}{2}\right)\Big|_{0}^{1} = -\frac{1}{4} + 2 - \frac{1}{2} = -\frac{1}{4} - \frac{2}{4} + \frac{8}{4} = \frac{5}{4}$$

$$S = S_1 + S_2 = \frac{3}{4} + \frac{5}{4} = \frac{8}{4} = 2.$$

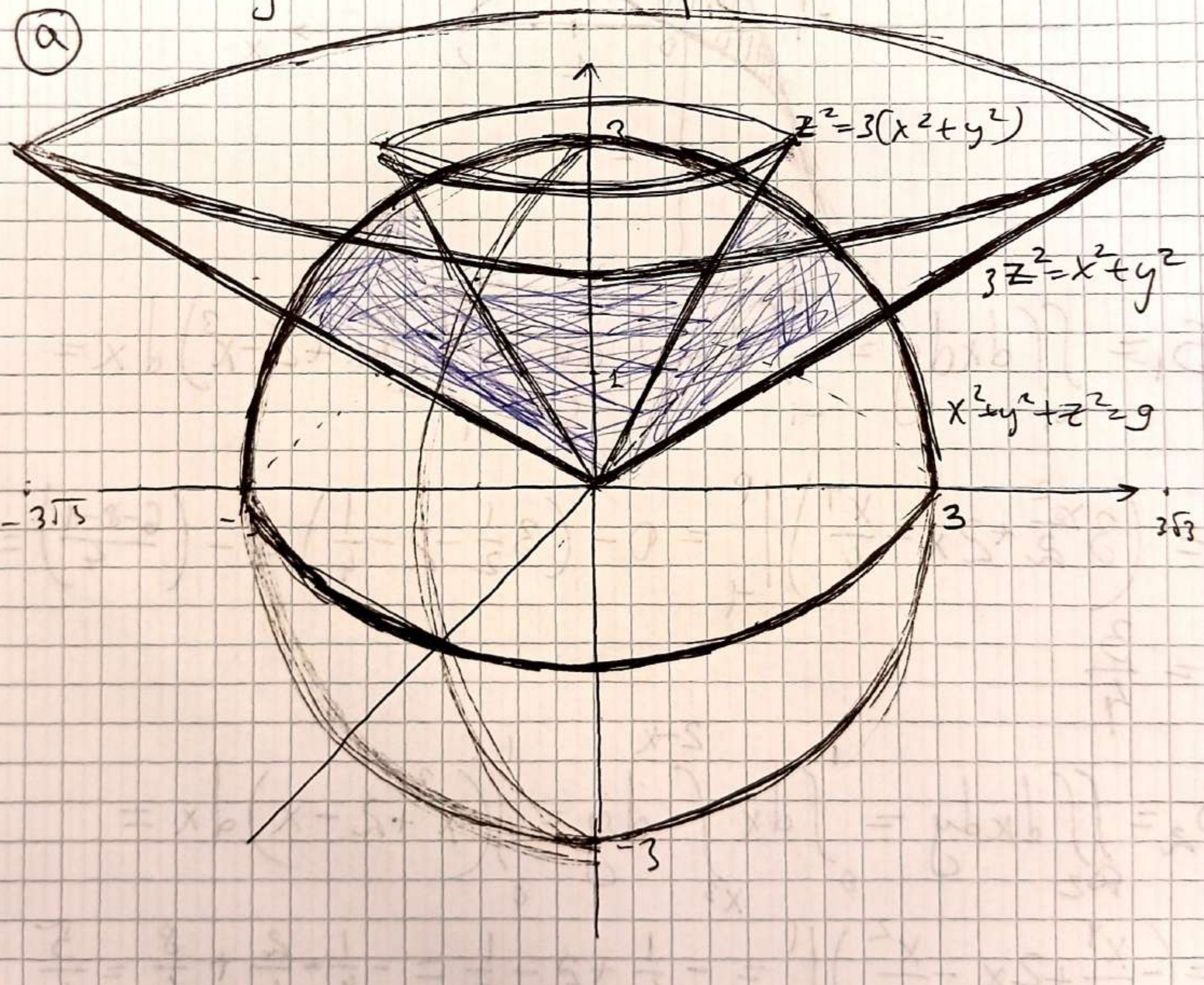
Ответ: $S = 2$.

N2

$3z^2 = x^2 + y^2$; $z^2 = 3(x^2 + y^2)$

$x^2 + y^2 + z^2 = 9$ при $z \geq 0$

а)



Необходимо найти область T сферы, заключенную между 2 конусами

б) $V = \iiint\limits_T dv$

перейдем к цилиндр. координатам

$x = r\cos\varphi \quad y = r\sin\varphi$

$x^2 + y^2 = r^2$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \varphi} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\varphi & -r\sin\varphi & 0 \\ \sin\varphi & r\cos\varphi & 0 \\ 0 & 0 & 1 \end{vmatrix} =$$

$= r$

$V = \iiint\limits_T r\, dr\, d\varphi\, dz$

Пусть $3z^2 = x^2 + y^2$ это конус №1, а $z^2 = 3(x^2 + y^2)$ это конус №2.

1) Сначала найдём линию пересечения ~~полуконуса~~ конуса №1 и сферы

$$z=\sqrt{\frac{x^2+y^2}{3}}=\sqrt{\frac{r^2}{3}} \qquad z=\sqrt{9-x^2+y^2}=-\sqrt{9-r^2}$$

$\varphi\in[0;2\pi]$

Уравнение конуса при $z\geqslant0$ | Уравнение сферы при $z\geqslant0$

$$\sqrt{\frac{r^2}{3}}=\sqrt{9-r^2} \quad)^2$$

$$\frac{r^2}{3}=9-r^2$$

$$r^2-27+3r^2=0$$

$r=\frac{\pm3\sqrt3}{2}$, т.к $z\geqslant0$, то $r=\frac{3\sqrt3}{2}$.

Таким образом конус и сфера пересек. по окружности радиуса $r=\frac{3\sqrt3}{2}$ на высоте $z=\sqrt{9-\frac{9\cdot3}{4}}=\frac{3}{2}$. То есть центр этой окр. находится в точке $O(0;0;\frac{3}{2})$

Вычитаем объем тела, ограниченного сферой и ~~конусом~~ конусом №1.

$$V_1 = \int_0^{2\pi} d\varphi \int_0^{\frac{3\sqrt{3}}{2}} r\,dr \int_{\sqrt{\frac{r^2}{3}}}^{\sqrt{9-r^2}} dz = \underbrace{\int_0^{2\pi} d\varphi \int_0^{\frac{3\sqrt{3}}{2}} \left(r\sqrt{9-r^2}\right) dr}_{A} - \underbrace{\int_0^{2\pi} d\varphi \int_0^{\frac{3\sqrt{3}}{2}} \left(r\sqrt{\frac{r^2}{3}}\right) dr}_{B}$$

A: $$-\frac{1}{2}\int_0^{2\pi} d\varphi \int_0^{\frac{3\sqrt{3}}{2}} \sqrt{9-r^2}\, d(9-r^2) = -\frac{1}{2}\int_0^{2\pi} \frac{2(9-r^2)^{3/2}}{3}\Bigg|_0^{\frac{3\sqrt{3}}{2}} =$$

$$= -\frac{1}{2}\int_0^{2\pi} \left(\frac{2\left(9-\frac{27}{4}\right)^{3/2}}{3} - \frac{2\cdot 9^{3/2}}{3} \right) d\varphi = -\frac{1}{2} \cdot$$

$$\cdot \int_0^{2\pi} \frac{2\cdot\frac{27}{8} - 2\cdot 27}{3} = -\frac{1}{2}\int_0^{2\pi} -\frac{7\cdot 9}{4}\, d\varphi = \frac{63\pi}{4}$$

B: $$\int_0^{2\pi} d\varphi \int_0^{\frac{3\sqrt{3}}{2}} r^2\sqrt{\frac{1}{3}}\, dr = \sqrt{\frac{1}{3}} \int_0^{2\pi} d\varphi\, \frac{r^3}{3}\Bigg|_0^{\frac{3\sqrt{3}}{2}} =$$

$$= \sqrt{\frac{1}{3}} \int_0^{2\pi} \frac{27\sqrt{3}}{8}\, d\varphi = \frac{27\cancel{\sqrt{3}}}{\cancel{8}\,4}\, \cancel{2}\pi\, \frac{1}{\cancel{\sqrt{3}}} = \frac{27}{4}\pi.$$

$$V_1=\frac{63\pi}{4}-\frac{27\pi}{4}=\frac{36}{4}\pi=9\pi.$$

2) Выясним линию пересечения сферы и конуса №2

уравнение конуса №2 при $z\geqslant 0$:

$$z=\sqrt{3x^2+3y^2} \qquad z=\sqrt{3r^2}=r\sqrt{3}$$

$$r\sqrt{3}=\sqrt{9-r^2} \quad \uparrow^2$$

$$3r^2=9-r^2$$

$$4r^2=9 \qquad r=\pm\frac{3}{2}, \text{ при } z\geqslant 0 \quad r=\frac{3}{2}$$

Сфера и конус №2 пересекаются по окружности с радиусом $r=3/2$ на высоте $z=\frac{3\sqrt{3}}{2}$. Центр окружности $O(0;0;\frac{3\sqrt{3}}{2})$

Вычислим объем тела огран сферой и конусом №2

$$V_2=\int_0^{2\pi}d\varphi\int_0^{3/2}r\,dr\int_{r\sqrt{3}}^{\sqrt{9-r^2}}dz=\underbrace{\int_0^{2\pi}d\varphi\int_0^{3/2}r\left(\sqrt{9-r^2}\right)dr}_{C}-$$

$$-\underbrace{\int_0^{2\pi} d\varphi \int_0^{3/2} r^2\sqrt{3}\,dr}_{D}$$

ⓒ $$-\frac{1}{2}\int_0^{2\pi} d\varphi \int_0^{3/2}\sqrt{9-r^2}\,d(9-r^2) = -\frac{1}{2}\int_0^{2\pi} d\varphi \cdot \frac{2(9-r^2)^{3/2}}{3}\Bigg|_0^{3/2} = -\frac{1}{2}\int_0^{2\pi} d\varphi \left(\frac{2\big(\overset{27/4}{9-\frac{9}{4}}\big)^{3/2}}{3} - \frac{2\cdot 9^{3/2}}{3}\right) =$$

$$= -\frac{1}{2}\int_0^{2\pi}\left(\frac{\cancel{2}\cdot\frac{81\sqrt{3}}{\cancel{8}_4} - 2\cdot 27}{3}\right)d\varphi = -\frac{1}{2}\int_0^{2\pi}\left(\frac{81\sqrt{3} - \overset{2\cdot 9}{\cancel{8}}\cdot 27}{12}\right)d\varphi =$$

$$= -\frac{1}{2}\int_0^{2\pi}\left(\frac{27\sqrt{3}}{4} - 18\right)d\varphi = -\frac{1}{\cancel{2}}\left(\frac{27\sqrt{3}}{4} - 18\right)\cdot\cancel{2}\pi =$$

$$= \left(18 - \frac{27\sqrt{3}}{4}\right)\pi \qquad (\approx 15).$$

② $+\int_0^{2\pi} d\varphi \int_0^{3/2} r^2\sqrt{3}\,dr = \sqrt{3}\int_0^{2\pi} d\varphi \left(\frac{r^3}{3}\right)\Big|_0^{3/2} =$

$$= \sqrt{3}\int_0^{2\pi} d\varphi \left(\frac{\frac{27}{8}}{3} - 0\right) = \sqrt{3}\int_0^{2\pi} \frac{9}{8}\,d\varphi = \sqrt{3}\cdot 2\pi\cdot\frac{9}{8} = \frac{9\sqrt{3}}{4}\pi$$

$$V_2 = 18\pi - \frac{27\sqrt{3}}{4}\pi - \frac{9\sqrt{3}}{4}\pi = 18\pi - \frac{36\sqrt{3}}{4}\pi =$$

$$= 18\pi - 9\sqrt{3}\pi = 9\pi(2-\sqrt{3}) \approx 2{,}4\pi.$$

$$V = V_1 - V_2 = 9\pi - 9\pi(2-\sqrt{3}) = -9\pi + 9\pi\sqrt{3} =$$

$$= 9\pi(\sqrt{3}-1)$$

Ответ: $9\pi(\sqrt{3}-1)$.

$A - B = \frac{63\pi}{4} - \frac{27\pi}{4} = \frac{36\pi}{4} = 9\pi.$

Ответ: 9π.

№3

а) $y = \frac{x^2}{2}$; $\rho(x,y) = \frac{8x}{1+2y}$; $x_1 = 0 = A$

$f(x) =$; $x_2 = 3/4 = B$

$$M = \int_{AB} \rho(x,y)\, dL$$

$$dL = \sqrt{1 + (f'(x))^2}\, dx$$

$$f'(x) = 2\frac{x}{2} = x$$

$$dL = \sqrt{1+x^2}\, dx$$

$$M = \int_{AB} \rho\left(x, \frac{x^2}{2}\right)\sqrt{1+x^2}\, dx =$$

$$= \int_{AB} \frac{8x}{1 + \cancel{2}\frac{x^2}{\cancel{2}}}\sqrt{1+x^2}\, dx = \int_{AB} \frac{8}{\sqrt{1+x^2}}\, dx =$$

$$\cancel{= 8 \cdot \ln\left|x + \sqrt{x^2+1}\right|\Big|_{x_1}^{x_2} =}$$

$$= \left[t = \sqrt{1+x^2} \quad dx = \frac{1}{t'} dt \quad t' = \frac{1 \cdot 2x}{2\sqrt{1+x^2}} \right] =$$

$$= 8 \int_{AB} \frac{x}{\cancel{\sqrt{1+x^2}}} \cdot \frac{\cancel{2}\cancel{\sqrt{1+x^2}}}{\cancel{2}x} dt = 8 \int_{AB} 1 \, dt =$$

$$= 8\sqrt{1+x^2} \Big|_0^{3/4} = 8 \cdot \left(\overbrace{\sqrt{1+\frac{9}{16}}}^{\frac{5}{4}} - 1 \right) = 8 \cdot \frac{1}{4} = 2$$

Ответ: $M = 2$.

б) $\begin{cases} x = \cos^3 t = \varphi(t) \\ y = \sin^3 t = \psi(t) \end{cases}$, $\rho(x, y) = 8$ $\quad t_1 = \frac{\pi}{6} \quad t_2 = \frac{\pi}{3}$

$$M = \int_{AB} \rho(x, y) \, dL$$

$$dL = \sqrt{(\varphi'(t))^2 + (\psi'(t))^2} \, dt$$

$$\varphi'(t) = 3\cos^2 t \cdot (-\sin t)$$

$$\psi'(t) = 3\sin^2 t \cdot \cos t$$

$$dL = \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t} =$$

$$= \sqrt{9\cos^2 t \sin^2 t \underbrace{(\cos^2 t + \sin^2 t)}_{=1}} = 3|\cos t||\sin t|\, dt$$

т.к. $t \in [\frac{\pi}{6}; \frac{\pi}{3}] \in I_ч \rightarrow$

$$\Rightarrow dL = 3\cos t \sin t\, dt$$

$$M = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 8 \cdot 3\cos t \sin t\, dt = 12\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\cos t \sin t\, dt =$$

$$= \frac{1}{2} 12 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 2t\, d(2t) = 6 \cdot (-\cos 2t)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} =$$

$$= 6\left(-\cos\frac{2\pi}{3} + \cos\frac{\pi}{3}\right) = 6\left(\frac{1}{2} + \frac{1}{2}\right) = 6.$$

Ответ 6.

N.4

$$\begin{cases} x = 2t-1 = \varphi(t) \\ y = t+1 = \psi(t) \\ z = t-1 = \chi(t) \end{cases}$$

M - ?

$$\rho(x,y,z) = \frac{y^2+z^2}{(x+1)^2} \qquad t_1 = \sqrt{6} \qquad t_2 = 2\sqrt{6}$$

$$AB = [t_1, t_2]$$

$$M = \int_{AB} \rho(x,y,z)\, dL$$

$$dL = \sqrt{(\varphi'(t))^2 + (\psi'(t))^2 + (\chi'(t))^2}\, dt$$

$$\varphi'(t) = 2 \qquad \psi'(t) = 1 \qquad \chi'(t) = 1$$

$$dL = \sqrt{6}\, dt$$

$$M = \int_{\sqrt{6}}^{2\sqrt{6}} \rho(2t-1, t+1, t-1)\sqrt{6}\, dt =$$

$$= \int_{\sqrt{6}}^{2\sqrt{6}} \frac{t^2 + \cancel{2t} + 1 + t^2 - \cancel{2t} + 1}{4t^2} \sqrt{6}\, dt =$$

$$= \sqrt{6} \int_{\sqrt{6}}^{2\sqrt{6}} \frac{2t^2+2}{4t^2}\, dt = \sqrt{6} \int_{\sqrt{6}}^{2\sqrt{6}} \frac{t^2+1}{2t^2}\, dt =$$

$$= \sqrt{6}\int_{\sqrt{6}}^{2\sqrt{6}}\left(\frac{1}{2}+\frac{1}{2t^2}\right)dt = \sqrt{6}\left(\frac{1}{2}t-\frac{1}{2}\frac{1}{t}\right)\Bigg|_{\sqrt{6}}^{2\sqrt{6}} =$$

$$= \sqrt{6}\left(\frac{t^2-1}{2t}\right)\Bigg|_{\sqrt{6}}^{2\sqrt{6}} = \sqrt{6}\left(\frac{4\cdot 6-1}{4\sqrt{6}}-\frac{6-1}{2\sqrt{6}}\right) =$$

$$= \cancel{\sqrt{6}}\left(\frac{23-10}{4\cancel{\sqrt{6}}}\right) = \frac{13}{4} = 3{,}25$$

Ответ: $M = 3{,}25$.

N5 $\bar{a} = (x+y)\vec{i} + (z-3y)\vec{k}$

σ: $x+y-z=-2 \quad /:-2$

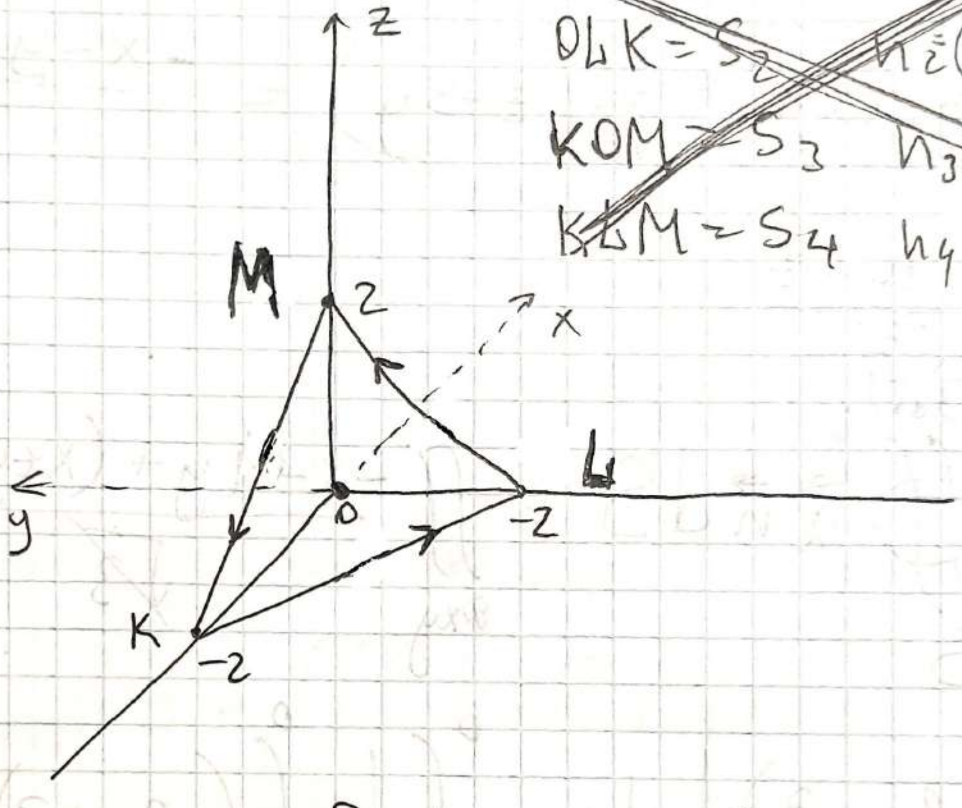
$-\frac{x}{2} - \frac{y}{2} + \frac{z}{2} = 1$

~~$OLM = S_1$ $n_1=(1;0;0)$~~

~~$OLK = S_2$ $n_2(0;1;0)$~~

~~$KOM = S_3$ $n_3=(0;0;-1)$~~

~~$KLM = S_4$ $n_4=(-1;-1;1)$~~



(1) Найдите поток Q век поля $\vec{a}$ через часть пл-ти σ вырезанной координатными пл-ми в сторону нормали $\vec{n}$ напр от начала координат O(0;0;0)

$a_x = x+y$; $a_y = 0$; $a_z = z-3y$

σ: $z = x+y+2$

$\frac{\partial z}{\partial x} = p(x,y) = 1 \qquad \frac{\partial z}{\partial y} = q(x,y) = 1$

Радикал равен $\sqrt{1+p^2(x,y)+q^2(x,y)} = \sqrt{3}$.

$$\cos\alpha = \frac{-1}{\sqrt{3}} \qquad \cos\beta = -\frac{1}{\sqrt{3}} \qquad \cos\gamma = \frac{1}{\sqrt{3}}$$

$$\vec{a}\cdot\vec{n} = (x+y)\cos\alpha + 0\cdot\cos\beta + (z-3y)\cos\gamma =$$

$$= -\frac{1}{\sqrt{3}}(x+y) + \frac{1}{\sqrt{3}}(z-3y) = \frac{-x-y+z-3y}{\sqrt{3}} =$$

$$= \frac{-x-4y+z}{\sqrt{3}}$$

$$dS = \sqrt{3}\,dx\,dy$$

$$z = 4$$

$$Q = \oiint\limits_S \vec{a}\vec{n}\,dS = \iint\limits_{D_{xy}} \frac{-x-4y+(x+y+2)}{\cancel{\sqrt{3}}}\cancel{\sqrt{3}}\,dx\,dy =$$

$$= \iint\limits_{D_{xy}} (-3y+2)\,dx\,dy = \int_{-2}^{0} dx \int_{-2-x}^{0} (-3y+2)\,dy =$$

$$= \int_{-2}^{0} dx \left(\frac{-3y^2}{2} + 2y\right)\Bigg|_{-2-x}^{0} = \int_{-2}^{0} \left(\frac{-3(-2-x)^2}{2} + 2(-2-x)\right) dx =$$

$-12 + 12x + 3x^2 - 8 - 4x$

$$= \int_{-2}^{0} \left(\frac{-3(4+4x+x^2)+4(-2-x)}{2}\right) dx = \int_{-2}^{0} \frac{3x^2+16x+20}{2}\, dx =$$

$$= \frac{1}{2}\left(3\frac{x^3}{3} + 16\frac{x^2}{2} + 20x\right)\Bigg|_{-2}^{0} = -\frac{1}{2}\left(-8 + 8\cdot\underset{-8}{4} - 40\right) = 8.$$

② Th Остроградского – Гаусса

$$Q = \oiint_S a_n\, dS = \iiint_V \operatorname{div} \vec{a}\, dx\,dy\,dz$$

$$\operatorname{div} \vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} = 1 + 0 + 1 = 2$$

$$Q = 2 \cdot \iiint dV = 2 \cdot V_{OLMK}.$$

$$V_{OLMK} = \overset{V}{\frac{1}{3}}\, OL \cdot \frac{1}{2}\, OK \cdot OM = \frac{1}{6}\, 8 = \frac{4}{3}$$

$$Q = 2 \cdot \frac{4}{3} = \frac{8}{3}$$

③ Найти циркуляцию C.

$$C = \int\limits_{KLMK} a_x dx + a_y dy + a_z dz = \int\limits_{KLMK} (x+y)dx +$$

$$+ (z-3y)dz$$

KL: $x+y=-2$ $\quad y=-x-2$

LM: $z=2+y$ $\quad z-y=2$

MK: $z=2+x$ $\quad z-x=2$

$$C = \int\limits_{KL} + \int\limits_{LM} + \int\limits_{MK}$$

1) $\int\limits_{KL} - ?$ $\quad \begin{cases} z=0 \ ; dz=0 \\ x+y=-2 \end{cases}$

$$\int\limits_{KL} = \int_{-2}^{0} (x+y)dx = \int_{-2}^{0} -2dx = -2x \Big|_{-2}^{0} = -4$$

$$\cancel{\int_{-2}^{0} (\overset{2-2y}{2+y-3y})dy = 2y - y^2 \Big|_{-2}^{0} = -(-4-4) = 8}$$

2) $\int\limits_{LM} - ?$ $\quad \begin{cases} x=0 & dx=0 \\ z-y=2 & dz=dy \end{cases}$

$y=z-2 \quad z=2+y$

$$\int\limits_{LM} = \int_{0}^{2} (\overset{z-3z+6}{z-3(z-2)})dz = \int_{0}^{2} (-2z+6)dz = -2\frac{z^2}{2} + 6z \Big|_{0}^{2} =$$

$$= \left(-z^2+6z\right)\Big|_0^2 = -4+12 = 8$$

3) $\int\limits_{MK} \begin{cases} y = 0 & dy = 0 \\ x = z-2 & dx = dz \end{cases}$

$$\int\limits_{MK} = \int\limits_2^0 (z-2)\,dx + z\,dz = \int\limits_2^0 (z-2+z)\,dz =$$

$$= \left(2\frac{z^2}{2} - 2z\right)\Big|_2^0 = -(4-4) = 0$$

$C = -4+8+0 = 4$

Ответ: 1) $Q=8$

2) $Q = 8/3$

3) $C = 4$

Проверка циркуляции:

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y & 0 & z-3y \end{vmatrix} = \left(\frac{\partial}{\partial y}(z-3y) - 0\right)\vec{i} + \left(x+y\frac{\partial}{\partial z} - (z-3y)\frac{\partial}{\partial x}\right)\vec{j} +$$

$n = (-1;-1;1)$

$$+ \left(\frac{\partial}{\partial x}\cdot 0 - (x+y)\frac{\partial}{\partial y}\right)\vec{k} = -3\vec{i} - \vec{k} \quad C = 3S_{OLM} - S_{OKL} = 6-2=4.$$

№6

$$\vec{a} = (2xy - 6x)\vec{i} + (x^2 - 2yz)\vec{j} - y^2\vec{k}$$

1

$$\operatorname{rot}\vec{a} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy-6x & x^2-2yz & -y^2 \end{vmatrix} = \left(-y^2\frac{\partial}{\partial y} - (x^2-2yz)\frac{\partial}{\partial z}\right) i +$$

$$+\left((2xy-6x)\frac{\partial}{\partial z} + y^2\frac{\partial}{\partial x}\right) j + \left((x^2-2yz)\frac{\partial}{\partial x} - (2xy-6x)\frac{\partial}{\partial y}\right) \bar{k}$$

$$= (-2y+2y)\bar{i} + 0\bar{j} + (2x-2x)\bar{k} = 0$$

$\operatorname{rot}\vec{a} = 0 \rightarrow$ поле потенциальное

$$\operatorname{div}\bar{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} = (2y-6) + (-2z) - 0 \neq 0$$

поле не соленоидально

2)

$$\vec{a}(M) = \operatorname{grad} U(M)$$

$$U(M) = \int_{M_0}^{M} dU + C = \int_{M_0}^{M} a_x dx + a_y dy + a_z dz + C$$

начальная точка $M_0 = O(0;0;0)$, текущая $M(x,y,z)$

$$dU = (2xy - 6x)dx + (x^2 - 2yz)dy + (-y^2)dz$$

$$U(x,y,z) = \int_{(0;0;0)}^{(x,y,z)} (2\tilde{x}\tilde{y} - 6\tilde{x})d\tilde{x} + (\tilde{x}^2 - 2\tilde{y}\tilde{z})d\tilde{y} - \tilde{y}^2 d\tilde{z} + C$$

Пусть путь интегр. это ломаная $OABM$

$O(0;0;0)$ $A(x;0;0)$ $B(x;y;0)$ $M(x,y,z)$

$$\int_{OA} = \int_0^x (2\tilde{x}\tilde{y} - 6\tilde{x})d\tilde{x} = \left(2\tilde{y}\frac{\tilde{x}^2}{2} - 3\cancel{6}\frac{\tilde{x}^2}{2}\right)\Big|_0^x = 2x^2 0 - 3x^2 = -3x^2$$

$dy = 0$
$dz = 0$

$$\int_{AB} = \int_0^y \tilde{x}^2 d\tilde{y} = \tilde{x}^2\tilde{y}\Big|_0^y = x^2 y \; ; \int_{BM} = -\int_0^z \tilde{y}^2 d\tilde{z} = -\tilde{y}^2\tilde{z}\Big|_0^z =$$

$$= -y^2 z$$

$$\boxed{U(x,y,z) = -3x^2 + x^2 y - y^2 z + C}$$

Ответ: поле не соленоид., поле потенц.
$U(x,y,z) = -3x^2 + x^2 y - y^2 z + C$