Bilkent University
Department of Computer Engineering

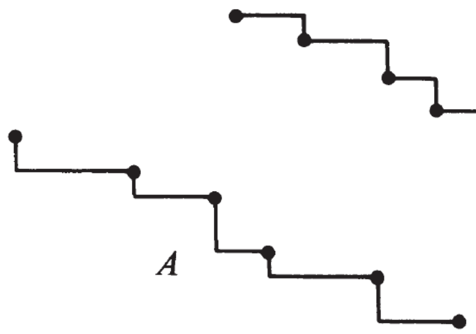
CS 478/564 COMPUTATIONAL GEOMETRY
HOMEWORK 3

Instructor: Uğur Güdükbay
Due date: 24^{th} April 2025, Thursday, 23:55.

In doing the assignments, you are on the honor system, meaning that any submitted assignment is solely the work of the submitter. You are free to discuss the questions with your friends, but you should submit your own work.

You must use a Word Processor to prepare your answers. Handwritten homework will not be graded.

1) Given two sets A and B of points in the plane, each arranged as a staircase (that is, each set coincides with its set of maxima in the dominance relation – Section 4.1.3) Find the pair (p_i, p_j), $p_i \in A$ and $p_j \in B$, closest in the L_l-metric. Is linear time achievable? *(Preparata 5.8.2)*



2) You are given two arbitrary convex polygons P and Q with N and M vertices respectively. The polygons can be disjoint, one inside the other or the boundaries might intersect a number of times; in other words, they can be in any configuration. What are the smallest and the largest number of vertices on the boundary of Convex Hull of P ∪ Q in terms of N and M? The result can be a constant number as well. Draw one example for the smallest and one example for the largest case.

3) Consider the following definition for the Delaunay triangulation of a set of N points in the plane, no four of which are cocircular. Two points p_i and p_j determine an edge of the Delaunay triangulation if and only if there exists a circle passing by these two points that does not contain any other point in its interior. Show that this definition leads to a triangulation that satisfies the circumcircle property, i.e., the circumcircle of each triangle does not contain any other point in its interior. Therefore, it is the same as the dual graph of the Voronoi diagram of the set of points. *(Preparata 5.8.8)* Also show that the above definition for Delaunay triangulation gives a unique triangulation (assuming no four points are cocircular). *(Preparata 5.8.9).*

4) For a set of N points S in the plane, propose a Delaunay Triangulation algorithm (DELAUNAY_TRIANGULATION) that is optimal (**O(*NlogN*)**). Hint: Transform DELAUNAY_TRIANGULATION into VORONOI_DIAGRAM in **O*(N)***. The VORONOI_DIAGRAM algorithm in the textbook uses a DCEL representation for storing the Voronoi Diagram.

5) M is a 2D matrix that represents a discrete 2D plane. M[i][j] = k, if there is a point k located in coordinates (i, j); otherwise, M[i][j] = null. We want to mark each empty cell with a marker of the point that is closest to that cell, assuming Manhattan distance. For example, cells adjacent to (i, j) will be marked with k if M[i][j] = k, and so on. If the 2D matrix includes only one point, then all the cells will be marked with that point. In case there are multiple points with the same closest distance to a specific cell, it is enough to mark it with one of the points. For example, if M[i-1][j] = k and M[i+1][j] = n, (i, j) can be k or n. Give an O(XY) algorithm to mark all the cells, where X and Y are the dimensions of the 2D Matrix. The resulting matrix would look like a discretized Voronoi Diagram.