Bilkent University
Department of Computer Engineering

CS 478/564 COMPUTATIONAL GEOMETRY
HOMEWORK 1

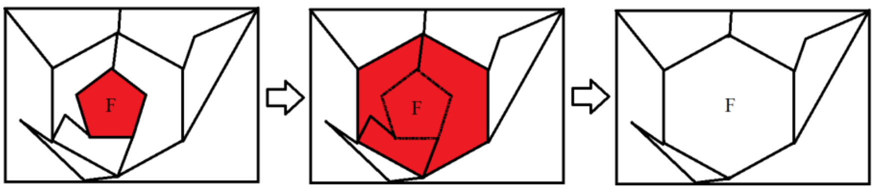
Instructor: Uğur Güdükbay
Due date: Wednesday, February 26th, 2024 (23.55)

In doing the assignments, you are on the honour system, meaning that any submitted assignment is solely the work of the submitter. You are free to discuss the questions with your friends, but you should submit your own work.

You must use a Word Processor to prepare your answers. Handwritten homeworks will not be graded.

1. Prove that for any segment tree **T (l, r)** the maximum number of standard intervals that can get allocated for an arbitrary interval **[b, e]** is equal to $\lceil \log_2 n \rceil + \lfloor \log_2 n \rfloor - 2$ where $n = r - l$, the total number of elementary intervals. Give an example interval for the segment tree **T (1, 16)** for which this maximum is achieved.

2. Given **Doubly Connected Edge List** representation of a **Planar Straight-Line Graph** and a face **F**, give an algorithm to expand F such that it covers all the neighbors of F (**you are going to calculate union of F with all the neighbor faces of F**). You need to modify the graph, you can use *DeleteEdge(int i)* function to remove an edge. You can assume that F is not adjacent to the outer face.



3. Given a **Doubly-Connected-Edge-List** (DCEL) representation of a planar graph, and two faces $\mathbf{F_a}$ and $\mathbf{F_b}$, write a Pidgin-Algol procedure to determine whether the polygon formed by the union of $\mathbf{F_a}$ and $\mathbf{F_b}$ is convex or concave. $\mathbf{F_a}$ and $\mathbf{F_b}$ are adjacent faces and the common edge is not included in the polygon that is formed by their union.

4. Let **S** be an arbitrary set of points in the plane. Is there only one unique simple polygon whose vertices are the set **S**? Either prove the uniqueness, or show an example of two different simple polygons with the same set of vertices.

5. Using Euler's formula as a starting point, prove the following inequalities for a **Planar Straight Line Graph** (PSLG) with the additional property that each vertex has degree greater than or equal to 3.

v <= 2/3 e	e <= 3v – 6
e <= 3f – 6	f <= 2/3 e
v <= 2f – 4	f <= 2v – 4

Also prove that if a Planar Straight Line Graph (PSLG) is triangulated (that is every face is a triangle) then e = 3v – 6. Note that in this case some vertices may have degree less than 3.