



1.

Consider Two level H atom

$$\text{--- } 10.2 \text{ eV} \quad \approx 1.6 \times 10^{-18} \text{ J}$$

$$\text{--- } 0 \text{ eV} = 0 \text{ J}$$

5000°

now	$T =$	10	100	1000	10000	↓	100000 K
	$e^{-0/kT}$	1	1	1	1	1	1
	$e^{-1.6 \times 10^{-18}/kT}$	~0	~0	4×10^{-51}	9×10^{-6}	0.01	0.31
	$kT \text{ (eV)}$	9×10^{-4}	9×10^{-3}	9×10^{-2}	0.9	4.3	8.6
	$g = 2n^2$						

so for $n=1$

$$g e^{-0/kT} = 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2$$

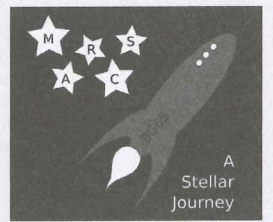
for $n=2$

$$g e^{-1.6 \times 10^{-18}/kT} = \sim 0 \quad \sim 0 \quad 32 \times 10^{-51} \quad 72 \times 10^{-6} \quad 0.08 \quad 2.5$$

$$\text{part func } u = \sum_i g_i e^{-\epsilon_i/kT} = 2 \quad 2 \quad 2 \quad 2 \quad 2.08 \quad 4.5$$

$$\frac{N_0(n=1)}{N_H} \sim 1 \quad 1 \quad 1 \quad 1 \quad 0.96 \quad 0.44$$

$$\frac{N_{n=2}(n=2)}{N_H} \sim 0 \quad 0 \quad \sim 10^{-51} \quad \sim 10^{-6} \quad 0.04 \quad 0.58$$



if have for excited level, in two level case²
ignoring g ,

$$\frac{N_j}{N} = \frac{e^{-E_j/kT}}{1 + e^{-E_j/kT}}$$

we see $N_j/N \rightarrow \frac{1}{2}$ when $e^{-E_j/kT} \rightarrow 1$
ie. when $E_j/kT \rightarrow 0$

So in such a case, $\frac{1}{2}$ is not achievable
except as $T \rightarrow \infty$

Including g

$$\Rightarrow \frac{g_u e^{-E_u/kT}}{g_l + g_u e^{-E_j/kT}}$$

as $T \rightarrow \infty$ $\frac{N_u}{N} = \frac{g_u}{g_l + g_u}$ eg. $\frac{8}{10} = \frac{4}{5}$

ie. even distribution depending on statistical
weight. (ie. for each degenerate state)



3

This is because if one has equilibrium (coll. dominated)
with rate coeff:

C_{lu}

Then

$$C_{ul} = \frac{g_l}{g_u} C_{lu} e^{W/kT}$$

here
 $W = E_u$

$$= \frac{g_l}{g_u} C_{lu} e^{E_u/kT}$$

$$\therefore \frac{C_{\downarrow}}{C_{\uparrow}} = \frac{g_l}{g_u} e^{E_u/kT}$$

$$\Rightarrow \frac{C_{\downarrow}}{C_{\uparrow}} = \frac{g_l}{g_u} e^{E_u/kT}$$

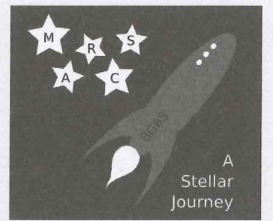
i.e. for $kT \ll E_u \rightarrow \infty$

$kT \gg E_u \rightarrow \frac{g_l}{g_u} \Rightarrow \sim 1$

$E_u = kT \rightarrow \frac{g_l}{g_u} e \Rightarrow \sim e = 2.7$

↑
ignoring g.

So even if $E_u = kT$, then $C_{\downarrow} > C_{\uparrow}$, as expected,
leading to still $\frac{N_u}{N} \ll \frac{1}{2}$



4.

i.e. we need

$$- N_e C_{\uparrow} + N_u C_{\downarrow} = 0$$

steady state
for lower level.

$$\Rightarrow \frac{N_u}{N_e} = \frac{C_{\uparrow}}{C_{\downarrow}} = \frac{g_u}{g_e} e^{-E_u/kT} \quad \text{as expected.}$$

$$\text{But at } E_u = kT \quad \frac{C_{\uparrow}}{C_{\downarrow}} = \frac{g_u}{g_e} \frac{1}{e} \sim \frac{1}{e} \ll 1$$

i.e. downward transitions still much more efficient,
leads to "lack of balance"

i.e. the ease of deexcitation compared to
excitation leads to low levels remaining
dominant until $kT \gg E_u$

and even then only leads to a "even"
distribution among states.