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M R S A C
A Stellar Journey

Consider Two level H atom

—— 10.2eV. $\simeq 1.6 \times 10^{-18}$ J

—— 0eV = 0 J

					5000° ↓	
now if T =	10	100	1000	10000		100000 K
$e^{-0/kT}$ =	1	1	1	1	1	1
$e^{-1.6\times10^{-18}/kT}$ =	~ 0	~ 0	4×10^{-51}	9×10^{-6}	0.01	0.31
kT (eV)	9×10^{-4}	9×10^{-3}	9×10^{-2}	0.9	4.3	8.6

$g = 2n^2$

so for $n=1$

$g\,e^{-0/kT}$ =	2	2	2	2	2	2

for $n=2$

$g\,e^{-\ldots/kT}$ =	~ 0	~ 0	32×10^{-51}	72×10^{-6}	0.08	2.5
part func $u = \sum_i g\,e^{-E_i/kT}$ =	2	2	2	2	2.08	4.5.
$\frac{N_{0}(n=1)}{N_H} \sim$	1	1	1	1	0.96	0.44
$\frac{N_{10.2}(n=2)}{N_H} \sim$	0	0	$\sim 10^{-51}$	$\sim 10^{-6}$	0.04	0.56

if have for excited level, in two level case ignoring g,

$$\frac{N_j}{N} = \frac{e^{-E_j/kT}}{1 + e^{-E_j/kT}}$$

we see $N_j/N \rightarrow 1/2$ when $e^{-E_j/kT} \rightarrow 1$

i.e. when $E_j/kT \rightarrow 0$

So in such a case, $\frac{1}{2}$ is not achievable except as $T \rightarrow \infty$

Including g

$$\Rightarrow \quad \frac{g_u e^{-E_u/kT}}{g_l + g_u e^{-E_j/kT}}$$

as $T \rightarrow \infty$ $\frac{N_u}{N} = \frac{g_u}{g_l + g_u}$ eg. $\frac{8}{10} = \frac{4}{5}$

i.e. even distribution depending on statistical weight. (i.e. for each degenerate state)

This is because if one has equilibrium (coll. dominated) with rate coeff: C_{lu}

Then

$$C_{ul} = \frac{g_l}{g_u} C_{lu} e^{W/kT}$$ here $W = E_u$

$$= \frac{g_l}{g_u} C_{lu} e^{E_u/kT}$$

so

$$\frac{C\downarrow}{C\uparrow} = \frac{g_l}{g_u} e^{E_u/kT}$$

$$\Rightarrow \quad \frac{C\downarrow}{C\uparrow} = \frac{g_l}{g_u} e^{E_u/kT}$$

i.e. for
$kT \ll E_u \rightarrow \infty$
$kT \gg E_u \rightarrow \frac{g_l}{g_u} \Rightarrow \sim 1$
$E_u = kT \rightarrow \frac{g_l}{g_u} e \Rightarrow \sim e = 2.7$

ignoring g.

So even if $E_u = kT$, then $C\downarrow > C\uparrow$, as expected, leading to still $\frac{N_u}{N} \ll \frac{1}{2}$

i.e. we need

$$- N_\ell C_\uparrow + N_u C_\downarrow = 0$$

steady state for lower level.

$$\Rightarrow \frac{N_u}{N_\ell} = \frac{C_\uparrow}{C_\downarrow} = \frac{g_u}{g_\ell} e^{-E_u/kT}$$ as expected.

But at $E_u = kT$ $\frac{C_\uparrow}{C_\downarrow} = \frac{g_u}{g_\ell} \frac{1}{e} \sim \frac{1}{e} \ll 1$

i.e. downward transitions still much more efficient, leads to "lack of balance"

i.e the ease of de-excitation compared to excitation leads to low levels remaining dominant until $kT \gg E_u$

and even then only leads to a "even" distribution among states.