```
Integrate[Sqrt (q r) BesselJ[l + 1 / 2, q r] Exp[- r / ns] r^ (ns - t),
 {r, 0, Infinity}, Assumptions →
  ns > 0 && Element[ns, Reals] && Element[t, Integers] && ns + l + 2 > t >= 0 &&
   Element[q, Reals] && q > 0 && Element[l, Integers] && l ≥ 0 ]
```

$$2^{-\frac{1}{2}-l}\, ns^{\frac{5}{2}+l+ns-t}\, q^{\frac{3}{2}+l}\, \text{Sqrt}\, \text{Gamma}\left[\frac{5}{2}+l+ns-t\right]\, \text{Hypergeometric2F1Regularized}\left[\frac{1}{4}(5+2\,l+2\,ns-2\,t),\ \frac{1}{4}(7+2\,l+2\,ns-2\,t),\ \frac{3}{2}+l,\ -ns^2\,q^2\right]$$

```
Simplify[%]
```

$$2^{-\frac{1}{2}-l}\, ns^{\frac{5}{2}+l+ns-t}\, q^{\frac{3}{2}+l}\, \text{Sqrt}\, \text{Gamma}\left[\frac{5}{2}+l+ns-t\right]\, \text{Hypergeometric2F1Regularized}\left[\frac{1}{4}(5+2\,l+2\,ns-2\,t),\ \frac{1}{4}(7+2\,l+2\,ns-2\,t),\ \frac{3}{2}+l,\ -ns^2\,q^2\right]$$

```
Integrate[BesselJ[nu, b r] Exp[- a r] r^ (mu - 1), {r, 0, Infinity},
 Assumptions → a > 0 && Element[a, Reals] && Element[b, Reals] &&
   b > 0 && Element[nu, Reals] && nu > 0 && Element[mu, Reals] ]
```

$$\text{If}\left[mu+nu>0,\ 2^{-nu}\,a^{-mu-nu}\,b^{nu}\,\text{Gamma}[mu+nu]\ \text{Hypergeometric2F1Regularized}\left[\frac{mu+nu}{2},\ \frac{1}{2}(1+mu+nu),\ 1+nu,\ -\frac{b^2}{a^2}\right],\ \text{Integrate}\left[e^{-a\,r}\,r^{-1+mu}\,\text{BesselJ}[nu,\,b\,r],\ \{r,\,0,\,\infty\},\ \text{Assumptions}\to mu<0\ \&\&\ nu>0\ \&\&\ mu+nu\le 0\ \&\&\ b>0\ \&\&\ a>0\right]\right]$$

```
Integrate[Sqrt[0.05] BesselJ[0.5, 0.05 r] Exp[- r / 5] r^ (5 + 0.5), {r, 0, Infinity}]
```

$$1.24091\times 10^6$$

```
Integrate[Sqrt[2 / Pi] Sin[0.05 r] Exp[- r / 5] r^ (5), {r, 0, Infinity}]
```

$$1.24091\times 10^6$$

```
Integrate[Sqrt[2 / Pi] Sin[q r] Exp[- r / nu] r^ (nu - t),
 {r, 0, Infinity}, Assumptions → q > 0 && Element[q, Reals] &&
   nu > 0 && Element[nu, Reals] && t >= 0 && Element[t, Integers] ]
```

$$\sqrt{\frac{2}{\pi}}\ \text{If}\left[t<2+nu,\ \left(\frac{1}{nu^2}+q^2\right)^{\frac{1}{2}(-1-nu+t)}\ \text{Gamma}[1+nu-t]\ \text{Sin}[(1+nu-t)\ \text{ArcTan}[nu\,q]],\ \text{Integrate}\left[e^{-\frac{r}{nu}}\,r^{nu-t}\,\text{Sin}[q\,r],\ \{r,\,0,\,\infty\},\ \text{Assumptions}\to nu\in\text{Reals}\ \&\&\ q\in\text{Reals}\ \&\&\ t\in\text{Integers}\ \&\&\ t\ge 0\ \&\&\ t\ge 2+nu\ \&\&\ nu>0\ \&\&\ q>0\right]\right]$$

```
Integrate[BesselJ[l + 1 / 2, q r] Exp[- r / nstar] r^ (nstar - t), {r, 0, Infinity},
 Assumptions → t >= 0 && Element[t, Integers] && Element[q, Reals] &&
   q >= 0 && Element[nstar, Reals] && nstar > 0 && nstar - t + l + 2 > 0]
```

$$2^{-\frac{1}{2}-l}\, nstar^{\frac{3}{2}+l+nstar-t}\, q^{\frac{1}{2}+l}\, \text{Gamma}\left[\frac{3}{2}+l+nstar-t\right]\, \text{Hypergeometric2F1Regularized}\left[\frac{1}{4}(3+2\,l+2\,nstar-2\,t),\ \frac{1}{4}(5+2\,l+2\,nstar-2\,t),\ \frac{3}{2}+l,\ -nstar^2\,q^2\right]$$

Calc

```
l = 0
```

0

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\sqrt{\frac{2}{\pi}}\ \nu^{1-t+\nu}\ \left(1+q^2\,\nu^2\right)^{\frac{1}{2}\,(-1+t-\nu)}\ \text{Gamma}[1-t+\nu]\ \text{Sin}[(1-t+\nu)\ \text{ArcTan}[q\,\nu]]$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1)
```

$$\sqrt{\frac{2}{\pi}}\ \left(q^2+\frac{1}{\nu^2}\right)^{\frac{1}{2}\,(-1+t-\nu)}\ \text{Gamma}[1-t+\nu]\ \text{Sin}[(1-t+\nu)\ \text{ArcTan}[q\,\nu]]$$

```
FortranForm[% /. ν → nu]
```

```
      Sqrt(2/Pi)*(nu**(-2) + q**2)**((-1 - nu + t)/2.)*
     -  Gamma(1 + nu - t)*Sin((1 + nu - t)*ArcTan(nu*q))
```

```
l = 1
```

```
1
```

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\frac{1}{q}\sqrt{\frac{2}{\pi}}\ \nu^{-t+\nu}\ \left(1+q^2\,\nu^2\right)^{\frac{1}{2}\,(-1+t-\nu)}\ \text{Gamma}[-t+\nu]$$
$$\left(q\,(t-\nu)\,\nu\,\text{Cos}[(1-t+\nu)\ \text{ArcTan}[q\,\nu]]-\sqrt{1+q^2\,\nu^2}\ \text{Sin}[(t-\nu)\ \text{ArcTan}[q\,\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q}\sqrt{\frac{2}{\pi}}\ \nu^{-t+\nu}\ \left(1+q^2\,\nu^2\right)^{\frac{1}{2}\,(-1+t-\nu)}\ \text{Gamma}[-t+\nu]$$
$$\left(q\,(t-\nu)\,\nu\,\text{Cos}[(1-t+\nu)\ \text{ArcTan}[q\,\nu]]-\sqrt{1+q^2\,\nu^2}\ \text{Sin}[(t-\nu)\ \text{ArcTan}[q\,\nu]]\right)$$

```
FortranForm[% /. ν → nu]
```

```
      (nu**(nu - t)*Sqrt(2/Pi)*
     -    (1 + nu**2*q**2)**((-1 - nu + t)/2.)*Gamma(nu - t)*
     -    (nu*q*(-nu + t)*Cos((1 + nu - t)*ArcTan(nu*q)) -
     -      Sqrt(1 + nu**2*q**2)*Sin((-nu + t)*ArcTan(nu*q))))/q
```

```
l = 2
```

```
2
```

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$-\frac{1}{q^2}\sqrt{\frac{2}{\pi}}\ \nu^{-1-t+\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}\ \text{Gamma}[-1-t+\nu]$$
$$\left(3q\nu(-1-t+\nu)\sqrt{1+q^2\nu^2}\ \text{Cos}[(t-\nu)\ \text{ArcTan}[q\nu]]+\right.$$
$$3\left(1+q^2\nu^2\right)\text{Sin}[(1+t-\nu)\ \text{ArcTan}[q\nu]]+$$
$$\left.q^2\nu^2\left(t+t^2-2t\nu+(-1+\nu)\nu\right)\text{Sin}[(1-t+\nu)\ \text{ArcTan}[q\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1)
```

$$-\frac{1}{q^2}\sqrt{\frac{2}{\pi}}\ \nu^{-1-t+\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}\ \text{Gamma}[-1-t+\nu]$$
$$\left(3q\nu(-1-t+\nu)\sqrt{1+q^2\nu^2}\ \text{Cos}[(t-\nu)\ \text{ArcTan}[q\nu]]+\right.$$
$$3\left(1+q^2\nu^2\right)\text{Sin}[(1+t-\nu)\ \text{ArcTan}[q\nu]]+$$
$$\left.q^2\nu^2\left(t+t^2-2t\nu+(-1+\nu)\nu\right)\text{Sin}[(1-t+\nu)\ \text{ArcTan}[q\nu]]\right)$$

```
FortranForm[% /. ν → nu]
```

```
      -((nu**(-1 + nu - t)*Sqrt(2/Pi)*
     -      (1 + nu**2*q**2)**((-1 - nu + t)/2.)*
     -      Gamma(-1 + nu - t)*
     -      (3*nu*q*Sqrt(1 + nu**2*q**2)*(-1 + nu - t)*
     -         Cos((-nu + t)*ArcTan(nu*q)) +
     -        nu**2*q**2*((-1 + nu)*nu + t - 2*nu*t + t**2)*
     -         Sin((1 + nu - t)*ArcTan(nu*q)) +
     -        3*(1 + nu**2*q**2)*Sin((1 - nu + t)*ArcTan(nu*q))))/
     -    q**2)
```

```
l = 3
```

3

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\frac{1}{q^3}\sqrt{\frac{2}{\pi}}\ \nu^{-2-t+\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}$$
$$\left(-15q\nu\left(1+q^2\nu^2\right)\text{Cos}[(1+t-\nu)\ \text{ArcTan}[q\nu]]\ \text{Gamma}[-1-t+\nu]+\right.$$
$$q^3\nu^3\,\text{Cos}[(1-t+\nu)\ \text{ArcTan}[q\nu]]\ \text{Gamma}[1-t+\nu]+$$
$$3\sqrt{1+q^2\nu^2}\ \left(2q^2\nu^2\,\text{Gamma}[-t+\nu]\ \text{Sin}[(t-\nu)\ \text{ArcTan}[q\nu]]-\right.$$
$$\left.\left.5\left(1+q^2\nu^2\right)\text{Gamma}[-2-t+\nu]\ \text{Sin}[(2+t-\nu)\ \text{ArcTan}[q\nu]]\right)\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2) ^ (1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q^3}\sqrt{\frac{2}{\pi}}\;\nu^{-2-t+\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}$$
$$\left(-15\,q\,\nu\left(1+q^2\nu^2\right)\text{Cos}[(1+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-1-t+\nu]+\right.$$
$$q^3\nu^3\,\text{Cos}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[1-t+\nu]+$$
$$3\sqrt{1+q^2\nu^2}\,\left(2\,q^2\nu^2\,\text{Gamma}[-t+\nu]\,\text{Sin}[(t-\nu)\,\text{ArcTan}[q\nu]]-\right.$$
$$\left.\left.5\left(1+q^2\nu^2\right)\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\nu]]\right)\right)$$

```
FortranForm[% /. ν → nu]
```

```
      (nu**(-2 + nu - t)*Sqrt(2/Pi)*
     -    (1 + nu**2*q**2)**((-1 - nu + t)/2.)*
     -    (-15*nu*q*(1 + nu**2*q**2)*
     -       Cos((1 - nu + t)*ArcTan(nu*q))*Gamma(-1 + nu - t) +
     -      nu**3*q**3*Cos((1 + nu - t)*ArcTan(nu*q))*
     -       Gamma(1 + nu - t) +
     -      3*Sqrt(1 + nu**2*q**2)*
     -       (2*nu**2*q**2*Gamma(nu - t)*
     -          Sin((-nu + t)*ArcTan(nu*q)) -
     -         5*(1 + nu**2*q**2)*Gamma(-2 + nu - t)*
     -          Sin((2 - nu + t)*ArcTan(nu*q)))))/q**3
```

```
l = 4
```

4

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\frac{1}{q^4}\sqrt{\frac{2}{\pi}}\;\nu^{-3-t+\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}$$
$$\left(-105\,q\,\nu\left(1+q^2\nu^2\right)^{3/2}\text{Cos}[(2+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-2-t+\nu]+\right.$$
$$10\,q^3\nu^3\sqrt{1+q^2\nu^2}\,\text{Cos}[(t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-t+\nu]+$$
$$45\,q^2\nu^2\,\text{Gamma}[-1-t+\nu]\,\text{Sin}[(1+t-\nu)\,\text{ArcTan}[q\nu]]+45\,q^4\nu^4\,\text{Gamma}[-1-t+\nu]$$
$$\text{Sin}[(1+t-\nu)\,\text{ArcTan}[q\nu]]-105\,\text{Gamma}[-3-t+\nu]\,\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$210\,q^2\nu^2\,\text{Gamma}[-3-t+\nu]\,\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$105\,q^4\nu^4\,\text{Gamma}[-3-t+\nu]\,\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]+$$
$$\left.q^4\nu^4\,\text{Gamma}[1-t+\nu]\,\text{Sin}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q^4}\sqrt{\frac{2}{\pi}}\,\nu^{-3-t+\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}$$
$$\Big(-105\,q\,\nu\left(1+q^2\nu^2\right)^{3/2}\text{Cos}[(2+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-2-t+\nu]+$$
$$10\,q^3\nu^3\sqrt{1+q^2\nu^2}\,\text{Cos}[(t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-t+\nu]+$$
$$45\,q^2\nu^2\,\text{Gamma}[-1-t+\nu]\,\text{Sin}[(1+t-\nu)\,\text{ArcTan}[q\nu]]+45\,q^4\nu^4\,\text{Gamma}[-1-t+\nu]$$
$$\text{Sin}[(1+t-\nu)\,\text{ArcTan}[q\nu]]-105\,\text{Gamma}[-3-t+\nu]\,\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$210\,q^2\nu^2\,\text{Gamma}[-3-t+\nu]\,\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$105\,q^4\nu^4\,\text{Gamma}[-3-t+\nu]\,\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]+$$
$$q^4\nu^4\,\text{Gamma}[1-t+\nu]\,\text{Sin}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\Big)$$

```
FortranForm[% /. ν → nu]
```

```
      (nu**(-3 + nu - t)*Sqrt(2/Pi)*
     -   (1 + nu**2*q**2)**((-1 - nu + t)/2.)*
     -   (-105*nu*q*(1 + nu**2*q**2)**1.5*
     -      Cos((2 - nu + t)*ArcTan(nu*q))*Gamma(-2 + nu - t) +
     -     10*nu**3*q**3*Sqrt(1 + nu**2*q**2)*
     -      Cos((-nu + t)*ArcTan(nu*q))*Gamma(nu - t) +
     -     nu**4*q**4*Gamma(1 + nu - t)*
     -      Sin((1 + nu - t)*ArcTan(nu*q)) +
     -     45*nu**2*q**2*Gamma(-1 + nu - t)*
     -      Sin((1 - nu + t)*ArcTan(nu*q)) +
     -     45*nu**4*q**4*Gamma(-1 + nu - t)*
     -      Sin((1 - nu + t)*ArcTan(nu*q)) -
     -     105*Gamma(-3 + nu - t)*
     -      Sin((3 - nu + t)*ArcTan(nu*q)) -
     -     210*nu**2*q**2*Gamma(-3 + nu - t)*
     -      Sin((3 - nu + t)*ArcTan(nu*q)) -
     -     105*nu**4*q**4*Gamma(-3 + nu - t)*
     -      Sin((3 - nu + t)*ArcTan(nu*q))))/q**4
```

```
l = 5
```

5

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$-\frac{1}{q^5}\sqrt{\frac{2}{\pi}}\,\nu^{-4-t+\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}$$
$$\Big(945\,q\,\nu\left(1+q^2\nu^2\right)^2\text{Cos}[(3+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-3-t+\nu]-$$
$$105\,q^3\nu^3\left(1+q^2\nu^2\right)\text{Cos}[(1+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-1-t+\nu]+$$
$$q^5\nu^5\,\text{Cos}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[1-t+\nu]+$$
$$15\,q^4\nu^4\sqrt{1+q^2\nu^2}\,\text{Gamma}[-t+\nu]\,\text{Sin}[(t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$420\,q^2\nu^2\sqrt{1+q^2\nu^2}\,\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$420\,q^4\nu^4\sqrt{1+q^2\nu^2}\,\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\nu]]+$$
$$945\sqrt{1+q^2\nu^2}\,\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\nu]]+$$
$$1890\,q^2\nu^2\sqrt{1+q^2\nu^2}\,\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\nu]]+$$
$$945\,q^4\nu^4\sqrt{1+q^2\nu^2}\,\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\nu]]\Big)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2) ^ (1/2 (1 - t + ν)))^(-1)
```

$$-\frac{1}{q^5}\sqrt{\frac{2}{\pi}}\ \nu^{-4-t+\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}$$
$$\Big(945\,q\,\nu\left(1+q^2\,\nu^2\right)^2\,\text{Cos}[(3+t-\nu)\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-3-t+\nu]-$$
$$105\,q^3\,\nu^3\left(1+q^2\,\nu^2\right)\text{Cos}[(1+t-\nu)\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-1-t+\nu]+$$
$$q^5\,\nu^5\,\text{Cos}[(1-t+\nu)\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[1-t+\nu]+$$
$$15\,q^4\,\nu^4\sqrt{1+q^2\,\nu^2}\,\text{Gamma}[-t+\nu]\,\text{Sin}[(t-\nu)\,\text{ArcTan}[q\,\nu]]-$$
$$420\,q^2\,\nu^2\sqrt{1+q^2\,\nu^2}\,\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\,\nu]]-$$
$$420\,q^4\,\nu^4\sqrt{1+q^2\,\nu^2}\,\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\,\nu]]+$$
$$945\sqrt{1+q^2\,\nu^2}\,\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\,\nu]]+$$
$$1890\,q^2\,\nu^2\sqrt{1+q^2\,\nu^2}\,\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\,\nu]]+$$
$$945\,q^4\,\nu^4\sqrt{1+q^2\,\nu^2}\,\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\,\nu]]\Big)$$

```
FortranForm[% /. ν → nu]
```

```
      -((nu**(-4 + nu - t)*Sqrt(2/Pi)*
     -     (1 + nu**2*q**2)**((-1 - nu + t)/2.)*
     -     (945*nu*q*(1 + nu**2*q**2)**2*
     -        Cos((3 - nu + t)*ArcTan(nu*q))*Gamma(-3 + nu - t) -
     -       105*nu**3*q**3*(1 + nu**2*q**2)*
     -        Cos((1 - nu + t)*ArcTan(nu*q))*Gamma(-1 + nu - t) +
     -       nu**5*q**5*Cos((1 + nu - t)*ArcTan(nu*q))*
     -        Gamma(1 + nu - t) +
     -       15*nu**4*q**4*Sqrt(1 + nu**2*q**2)*Gamma(nu - t)*
     -        Sin((-nu + t)*ArcTan(nu*q)) -
     -       420*nu**2*q**2*Sqrt(1 + nu**2*q**2)*
     -        Gamma(-2 + nu - t)*Sin((2 - nu + t)*ArcTan(nu*q)) -
     -       420*nu**4*q**4*Sqrt(1 + nu**2*q**2)*
     -        Gamma(-2 + nu - t)*Sin((2 - nu + t)*ArcTan(nu*q)) +
     -       945*Sqrt(1 + nu**2*q**2)*Gamma(-4 + nu - t)*
     -        Sin((4 - nu + t)*ArcTan(nu*q)) +
     -       1890*nu**2*q**2*Sqrt(1 + nu**2*q**2)*
     -        Gamma(-4 + nu - t)*Sin((4 - nu + t)*ArcTan(nu*q)) +
     -       945*nu**4*q**4*Sqrt(1 + nu**2*q**2)*
     -        Gamma(-4 + nu - t)*Sin((4 - nu + t)*ArcTan(nu*q))))/
     -    q**5)
```

```
l = 6
```

```
6
```

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$-\frac{1}{q^6}\sqrt{\frac{2}{\pi}}\ \nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\ \text{Gamma}[-5-t+\nu]$$

$$\left(-21\,q^5\,(1+t-\nu)\,(2+t-\nu)\,(3+t-\nu)\,(4+t-\nu)\,(5+t-\nu)\,\text{Cos}[(t-\nu)\,\text{ArcTan}[q\nu]]+\right.$$

$$\frac{1}{\nu^2}1260\,q^3\,(3+t-\nu)\,(4+t-\nu)\,(5+t-\nu)\left(1+q^2\nu^2\right)\text{Cos}[(2+t-\nu)\,\text{ArcTan}[q\nu]]+$$

$$\frac{1}{\nu^4}10\,395\,q\,(-5-t+\nu)\left(1+q^2\nu^2\right)^2\text{Cos}[(4+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu}210\,q^4\,(-5-t+\nu)$$

$$(-4-t+\nu)\,(-3-t+\nu)\,(-2-t+\nu)\sqrt{1+q^2\nu^2}\ \text{Sin}[(1+t-\nu)\,\text{ArcTan}[q\nu]]-$$

$$\frac{1}{\nu^3}4725\,q^2\,(-5-t+\nu)\,(-4-t+\nu)\left(1+q^2\nu^2\right)^{3/2}\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]+$$

$$\frac{1}{\nu^5}10\,395\left(1+q^2\nu^2\right)^{5/2}\text{Sin}[(5+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\sqrt{1+q^2\nu^2}}q^6\nu\,(-5-t+\nu)$$

$$\left.(-4-t+\nu)\,(-3-t+\nu)\,(-2-t+\nu)\,(-1-t+\nu)\,(-t+\nu)\,\text{Sin}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1)
```

$$-\frac{1}{q^6}\sqrt{\frac{2}{\pi}}\ \nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\ \text{Gamma}[-5-t+\nu]$$

$$\left(-21\,q^5\,(1+t-\nu)\,(2+t-\nu)\,(3+t-\nu)\,(4+t-\nu)\,(5+t-\nu)\,\text{Cos}[(t-\nu)\,\text{ArcTan}[q\nu]]+\right.$$

$$\frac{1}{\nu^2}1260\,q^3\,(3+t-\nu)\,(4+t-\nu)\,(5+t-\nu)\left(1+q^2\nu^2\right)\text{Cos}[(2+t-\nu)\,\text{ArcTan}[q\nu]]+$$

$$\frac{1}{\nu^4}10\,395\,q\,(-5-t+\nu)\left(1+q^2\nu^2\right)^2\text{Cos}[(4+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu}210\,q^4\,(-5-t+\nu)$$

$$(-4-t+\nu)\,(-3-t+\nu)\,(-2-t+\nu)\sqrt{1+q^2\nu^2}\ \text{Sin}[(1+t-\nu)\,\text{ArcTan}[q\nu]]-$$

$$\frac{1}{\nu^3}4725\,q^2\,(-5-t+\nu)\,(-4-t+\nu)\left(1+q^2\nu^2\right)^{3/2}\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]+$$

$$\frac{1}{\nu^5}10\,395\left(1+q^2\nu^2\right)^{5/2}\text{Sin}[(5+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\sqrt{1+q^2\nu^2}}q^6\nu\,(-5-t+\nu)$$

$$\left.(-4-t+\nu)\,(-3-t+\nu)\,(-2-t+\nu)\,(-1-t+\nu)\,(-t+\nu)\,\text{Sin}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\right)$$

```
FortranForm[% /. ν → nu]
```

```
      -((nu**(nu - t)*Sqrt(2/Pi)*
     -       (1 + nu**2*q**2)**((-nu + t)/2.)*Gamma(-5 + nu - t)*
     -       (-21*q**5*(1 - nu + t)*(2 - nu + t)*(3 - nu + t)*
     -          (4 - nu + t)*(5 - nu + t)*
     -          Cos((-nu + t)*ArcTan(nu*q)) +
     -         (1260*q**3*(1 + nu**2*q**2)*(3 - nu + t)*
     -            (4 - nu + t)*(5 - nu + t)*
     -            Cos((2 - nu + t)*ArcTan(nu*q)))/nu**2 +
     -         (10395*q*(1 + nu**2*q**2)**2*(-5 + nu - t)*
     -            Cos((4 - nu + t)*ArcTan(nu*q)))/nu**4 +
     -         (nu*q**6*(-5 + nu - t)*(-4 + nu - t)*(-3 + nu - t)*
     -            (-2 + nu - t)*(-1 + nu - t)*(nu - t)*
     -            Sin((1 + nu - t)*ArcTan(nu*q)))/
     -          Sqrt(1 + nu**2*q**2) +
     -         (210*q**4*Sqrt(1 + nu**2*q**2)*(-5 + nu - t)*
     -            (-4 + nu - t)*(-3 + nu - t)*(-2 + nu - t)*
     -            Sin((1 - nu + t)*ArcTan(nu*q)))/nu -
     -         (4725*q**2*(1 + nu**2*q**2)**1.5*(-5 + nu - t)*
     -            (-4 + nu - t)*Sin((3 - nu + t)*ArcTan(nu*q)))/
     -          nu**3 + (10395*(1 + nu**2*q**2)**2.5*
     -            Sin((5 - nu + t)*ArcTan(nu*q)))/nu**5))/q**6)
```

```
l = 7
```

7

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2]
```

$$\frac{1}{q^7}\sqrt{\frac{2}{\pi}}\,\nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\left(-\frac{1}{\nu^5}135\,135\,q\left(1+q^2\nu^2\right)^{5/2}\text{Cos}[(5+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-5-t+\nu]+\right.$$
$$\frac{1}{\nu^3}17\,325\,q^3\left(1+q^2\nu^2\right)^{3/2}\text{Cos}[(3+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-3-t+\nu]-$$
$$\frac{1}{\nu}378\,q^5\sqrt{1+q^2\nu^2}\,\text{Cos}[(1+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-1-t+\nu]+$$
$$\left(q^7\nu\,\text{Cos}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[1-t+\nu]\right)\Big/\left(\sqrt{1+q^2\nu^2}\right)+$$
$$28\,q^6\,\text{Gamma}[-t+\nu]\,\text{Sin}[(t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^2}$$
$$3150\,q^4\left(1+q^2\nu^2\right)\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^4}$$
$$62\,370\left(q+q^3\nu^2\right)^2\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^6}$$
$$\left.135\,135\left(1+q^2\nu^2\right)^3\text{Gamma}[-6-t+\nu]\,\text{Sin}[(6+t-\nu)\,\text{ArcTan}[q\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2) ^ (1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q^7}\sqrt{\frac{2}{\pi}}\ \nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\left(-\frac{1}{\nu^5}135\,135\,q\left(1+q^2\nu^2\right)^{5/2}\text{Cos}[(5+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-5-t+\nu]+\right.$$
$$\frac{1}{\nu^3}17\,325\,q^3\left(1+q^2\nu^2\right)^{3/2}\text{Cos}[(3+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-3-t+\nu]-$$
$$\frac{1}{\nu}378\,q^5\sqrt{1+q^2\nu^2}\ \text{Cos}[(1+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-1-t+\nu]+$$
$$\left(q^7\nu\,\text{Cos}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[1-t+\nu]\right)\Big/\left(\sqrt{1+q^2\nu^2}\right)+$$
$$28\,q^6\,\text{Gamma}[-t+\nu]\,\text{Sin}[(t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^2}$$
$$3150\,q^4\left(1+q^2\nu^2\right)\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^4}$$
$$62\,370\left(q+q^3\nu^2\right)^2\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^6}$$
$$\left.135\,135\left(1+q^2\nu^2\right)^3\text{Gamma}[-6-t+\nu]\,\text{Sin}[(6+t-\nu)\,\text{ArcTan}[q\nu]]\right)$$

```
FortranForm[% /. ν → nu]
```

```
      (nu**(nu - t)*Sqrt(2/Pi)*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -    ((-135135*q*(1 + nu**2*q**2)**2.5*
     -         Cos((5 - nu + t)*ArcTan(nu*q))*Gamma(-5 + nu - t))/
     -       nu**5 + (17325*q**3*(1 + nu**2*q**2)**1.5*
     -         Cos((3 - nu + t)*ArcTan(nu*q))*Gamma(-3 + nu - t))/
     -       nu**3 - (378*q**5*Sqrt(1 + nu**2*q**2)*
     -         Cos((1 - nu + t)*ArcTan(nu*q))*Gamma(-1 + nu - t))/
     -       nu + (nu*q**7*Cos((1 + nu - t)*ArcTan(nu*q))*
     -         Gamma(1 + nu - t))/Sqrt(1 + nu**2*q**2) +
     -      28*q**6*Gamma(nu - t)*Sin((-nu + t)*ArcTan(nu*q)) -
     -      (3150*q**4*(1 + nu**2*q**2)*Gamma(-2 + nu - t)*
     -         Sin((2 - nu + t)*ArcTan(nu*q)))/nu**2 +
     -      (62370*(q + nu**2*q**3)**2*Gamma(-4 + nu - t)*
     -         Sin((4 - nu + t)*ArcTan(nu*q)))/nu**4 -
     -      (135135*(1 + nu**2*q**2)**3*Gamma(-6 + nu - t)*
     -         Sin((6 - nu + t)*ArcTan(nu*q)))/nu**6))/q**7
```

```
l = 8
```

```
8
```

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\frac{1}{q^8}\sqrt{\frac{2}{\pi}}\ \nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\left(-\frac{1}{\nu^6}2\,027\,025\,q\left(1+q^2\nu^2\right)^3\text{Cos}[(6+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-6-t+\nu]+\right.$$
$$\frac{1}{\nu^4}270\,270\,q^3\left(1+q^2\nu^2\right)^2\text{Cos}[(4+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-4-t+\nu]-$$
$$\frac{1}{\nu^2}6930\,q^5\left(1+q^2\nu^2\right)\text{Cos}[(2+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-2-t+\nu]+$$
$$36\,q^7\,\text{Cos}[(t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-t+\nu]+\frac{1}{\nu}$$
$$630\,q^6\sqrt{1+q^2\nu^2}\,\text{Gamma}[-1-t+\nu]\,\text{Sin}[(1+t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^3}$$
$$51\,975\,q^4\left(1+q^2\nu^2\right)^{3/2}\text{Gamma}[-3-t+\nu]\,\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^5}$$
$$945\,945\,q^2\left(1+q^2\nu^2\right)^{5/2}\text{Gamma}[-5-t+\nu]\,\text{Sin}[(5+t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$\frac{1}{\nu^7}2\,027\,025\left(1+q^2\nu^2\right)^{7/2}\text{Gamma}[-7-t+\nu]\,\text{Sin}[(7+t-\nu)\,\text{ArcTan}[q\nu]]+$$
$$\left.\left(q^8\,\nu\,\text{Gamma}[1-t+\nu]\,\text{Sin}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\right)\Big/\left(\sqrt{1+q^2\nu^2}\right)\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q^8}\sqrt{\frac{2}{\pi}}\ \nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\left(-\frac{1}{\nu^6}2\,027\,025\,q\left(1+q^2\nu^2\right)^3\text{Cos}[(6+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-6-t+\nu]+\right.$$
$$\frac{1}{\nu^4}270\,270\,q^3\left(1+q^2\nu^2\right)^2\text{Cos}[(4+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-4-t+\nu]-$$
$$\frac{1}{\nu^2}6930\,q^5\left(1+q^2\nu^2\right)\text{Cos}[(2+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-2-t+\nu]+$$
$$36\,q^7\,\text{Cos}[(t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-t+\nu]+\frac{1}{\nu}$$
$$630\,q^6\sqrt{1+q^2\nu^2}\,\text{Gamma}[-1-t+\nu]\,\text{Sin}[(1+t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^3}$$
$$51\,975\,q^4\left(1+q^2\nu^2\right)^{3/2}\text{Gamma}[-3-t+\nu]\,\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^5}$$
$$945\,945\,q^2\left(1+q^2\nu^2\right)^{5/2}\text{Gamma}[-5-t+\nu]\,\text{Sin}[(5+t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$\frac{1}{\nu^7}2\,027\,025\left(1+q^2\nu^2\right)^{7/2}\text{Gamma}[-7-t+\nu]\,\text{Sin}[(7+t-\nu)\,\text{ArcTan}[q\nu]]+$$
$$\left.\left(q^8\,\nu\,\text{Gamma}[1-t+\nu]\,\text{Sin}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\right)\Big/\left(\sqrt{1+q^2\nu^2}\right)\right)$$

```
FortranForm[% /. ν → nu]
```

```
      (nu**(nu - t)*Sqrt(2/Pi)*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -    ((-2027025*q*(1 + nu**2*q**2)**3*
     -         Cos((6 - nu + t)*ArcTan(nu*q))*Gamma(-6 + nu - t))/
     -       nu**6 + (270270*q**3*(1 + nu**2*q**2)**2*
     -         Cos((4 - nu + t)*ArcTan(nu*q))*Gamma(-4 + nu - t))/
     -       nu**4 - (6930*q**5*(1 + nu**2*q**2)*
     -         Cos((2 - nu + t)*ArcTan(nu*q))*Gamma(-2 + nu - t))/
     -       nu**2 + 36*q**7*Cos((-nu + t)*ArcTan(nu*q))*
     -       Gamma(nu - t) +
     -      (nu*q**8*Gamma(1 + nu - t)*
     -         Sin((1 + nu - t)*ArcTan(nu*q)))/Sqrt(1 + nu**2*q**2)
     -        + (630*q**6*Sqrt(1 + nu**2*q**2)*Gamma(-1 + nu - t)*
     -         Sin((1 - nu + t)*ArcTan(nu*q)))/nu -
     -      (51975*q**4*(1 + nu**2*q**2)**1.5*Gamma(-3 + nu - t)*
     -         Sin((3 - nu + t)*ArcTan(nu*q)))/nu**3 +
     -      (945945*q**2*(1 + nu**2*q**2)**2.5*Gamma(-5 + nu - t)*
     -         Sin((5 - nu + t)*ArcTan(nu*q)))/nu**5 -
     -      (2027025*(1 + nu**2*q**2)**3.5*Gamma(-7 + nu - t)*
     -         Sin((7 - nu + t)*ArcTan(nu*q)))/nu**7))/q**8
```

```
l = 9
```

9

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2]
```

$$\frac{1}{q^9}\sqrt{\frac{2}{\pi}}\,\nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\left(-\frac{1}{\nu^7}34\,459\,425\,q\left(1+q^2\nu^2\right)^{7/2}\text{Cos}[(7+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-7-t+\nu]+\right.$$
$$\frac{1}{\nu^5}4\,729\,725\,q^3\left(1+q^2\nu^2\right)^{5/2}\text{Cos}[(5+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-5-t+\nu]-$$
$$\frac{1}{\nu^3}135\,135\,q^5\left(1+q^2\nu^2\right)^{3/2}\text{Cos}[(3+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-3-t+\nu]+$$
$$\frac{1}{\nu}990\,q^7\sqrt{1+q^2\nu^2}\,\text{Cos}[(1+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-1-t+\nu]-$$
$$\left(q^9\nu\,\text{Cos}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[1-t+\nu]\right)\Big/\left(\sqrt{1+q^2\nu^2}\right)-$$
$$45\,q^8\,\text{Gamma}[-t+\nu]\,\text{Sin}[(t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^2}$$
$$13\,860\,q^6\left(1+q^2\nu^2\right)\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^4}$$
$$945\,945\,q^4\left(1+q^2\nu^2\right)^2\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^6}$$
$$16\,216\,200\,q^2\left(1+q^2\nu^2\right)^3\text{Gamma}[-6-t+\nu]\,\text{Sin}[(6+t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$\left.\frac{1}{\nu^8}34\,459\,425\left(1+q^2\nu^2\right)^4\text{Gamma}[-8-t+\nu]\,\text{Sin}[(8+t-\nu)\,\text{ArcTan}[q\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q^9}\sqrt{\frac{2}{\pi}}\,\nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\left(-\frac{1}{\nu^7}34\,459\,425\,q\left(1+q^2\nu^2\right)^{7/2}\text{Cos}[(7+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-7-t+\nu]+\right.$$
$$\frac{1}{\nu^5}4\,729\,725\,q^3\left(1+q^2\nu^2\right)^{5/2}\text{Cos}[(5+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-5-t+\nu]-$$
$$\frac{1}{\nu^3}135\,135\,q^5\left(1+q^2\nu^2\right)^{3/2}\text{Cos}[(3+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-3-t+\nu]+$$
$$\frac{1}{\nu}990\,q^7\sqrt{1+q^2\nu^2}\,\text{Cos}[(1+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-1-t+\nu]-$$
$$\left(q^9\nu\,\text{Cos}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[1-t+\nu]\right)\Big/\left(\sqrt{1+q^2\nu^2}\right)-$$
$$45\,q^8\,\text{Gamma}[-t+\nu]\,\text{Sin}[(t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^2}$$
$$13\,860\,q^6\left(1+q^2\nu^2\right)\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^4}$$
$$945\,945\,q^4\left(1+q^2\nu^2\right)^2\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^6}$$
$$16\,216\,200\,q^2\left(1+q^2\nu^2\right)^3\text{Gamma}[-6-t+\nu]\,\text{Sin}[(6+t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$\left.\frac{1}{\nu^8}34\,459\,425\left(1+q^2\nu^2\right)^4\text{Gamma}[-8-t+\nu]\,\text{Sin}[(8+t-\nu)\,\text{ArcTan}[q\nu]]\right)$$

```
FortranForm[% /. ν → nu]
```

```
      (nu**(nu - t)*Sqrt(2/Pi)*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -    ((-34459425*q*(1 + nu**2*q**2)**3.5*
     -         Cos((7 - nu + t)*ArcTan(nu*q))*Gamma(-7 + nu - t))/
     -       nu**7 + (4729725*q**3*(1 + nu**2*q**2)**2.5*
     -         Cos((5 - nu + t)*ArcTan(nu*q))*Gamma(-5 + nu - t))/
     -       nu**5 - (135135*q**5*(1 + nu**2*q**2)**1.5*
     -         Cos((3 - nu + t)*ArcTan(nu*q))*Gamma(-3 + nu - t))/
     -       nu**3 + (990*q**7*Sqrt(1 + nu**2*q**2)*
     -         Cos((1 - nu + t)*ArcTan(nu*q))*Gamma(-1 + nu - t))/
     -       nu - (nu*q**9*Cos((1 + nu - t)*ArcTan(nu*q))*
     -         Gamma(1 + nu - t))/Sqrt(1 + nu**2*q**2) -
     -      45*q**8*Gamma(nu - t)*Sin((-nu + t)*ArcTan(nu*q)) +
     -      (13860*q**6*(1 + nu**2*q**2)*Gamma(-2 + nu - t)*
     -         Sin((2 - nu + t)*ArcTan(nu*q)))/nu**2 -
     -      (945945*q**4*(1 + nu**2*q**2)**2*Gamma(-4 + nu - t)*
     -         Sin((4 - nu + t)*ArcTan(nu*q)))/nu**4 +
     -      (16216200*q**2*(1 + nu**2*q**2)**3*Gamma(-6 + nu - t)*
     -         Sin((6 - nu + t)*ArcTan(nu*q)))/nu**6 -
     -      (34459425*(1 + nu**2*q**2)**4*Gamma(-8 + nu - t)*
     -         Sin((8 - nu + t)*ArcTan(nu*q)))/nu**8))/q**9
```

```
l = 10
```

```
10
```

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$
\begin{aligned}
&-\frac{1}{q^{10}}\sqrt{\frac{2}{\pi}}\;\nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\\
&\left(\frac{1}{\nu^8}654\,729\,075\,q\left(1+q^2\nu^2\right)^4\text{Cos}[(8+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-8-t+\nu]-\right.\\
&\quad\frac{1}{\nu^6}91\,891\,800\left(q+q^3\nu^2\right)^3\text{Cos}[(6+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-6-t+\nu]+\\
&\quad\frac{1}{\nu^4}2\,837\,835\,q^5\left(1+q^2\nu^2\right)^2\text{Cos}[(4+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-4-t+\nu]-\\
&\quad\frac{1}{\nu^2}25\,740\,q^7\left(1+q^2\nu^2\right)\text{Cos}[(2+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-2-t+\nu]+\\
&\quad55\,q^9\,\text{Cos}[(t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-t+\nu]+\frac{1}{\nu}\\
&\quad1485\,q^8\sqrt{1+q^2\nu^2}\,\text{Gamma}[-1-t+\nu]\,\text{Sin}[(1+t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^3}\\
&\quad315\,315\,q^6\left(1+q^2\nu^2\right)^{3/2}\text{Gamma}[-3-t+\nu]\,\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^5}\\
&\quad18\,918\,900\,q^4\left(1+q^2\nu^2\right)^{5/2}\text{Gamma}[-5-t+\nu]\,\text{Sin}[(5+t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^7}\\
&\quad310\,134\,825\,q^2\left(1+q^2\nu^2\right)^{7/2}\text{Gamma}[-7-t+\nu]\,\text{Sin}[(7+t-\nu)\,\text{ArcTan}[q\nu]]+\\
&\quad\frac{1}{\nu^9}654\,729\,075\left(1+q^2\nu^2\right)^{9/2}\text{Gamma}[-9-t+\nu]\,\text{Sin}[(9+t-\nu)\,\text{ArcTan}[q\nu]]+\\
&\quad\left.\left(q^{10}\,\nu\,\text{Gamma}[1-t+\nu]\,\text{Sin}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\right)\Big/\left(\sqrt{1+q^2\nu^2}\right)\right)
\end{aligned}
$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1)
```

$$
\begin{aligned}
&-\frac{1}{q^{10}}\sqrt{\frac{2}{\pi}}\;\nu^{-t+\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\\
&\Bigg(\frac{1}{\nu^8}654\,729\,075\,q\left(1+q^2\,\nu^2\right)^4\text{Cos}[(8+t-\nu)\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-8-t+\nu]-\\
&\quad\frac{1}{\nu^6}91\,891\,800\left(q+q^3\,\nu^2\right)^3\text{Cos}[(6+t-\nu)\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-6-t+\nu]+\\
&\quad\frac{1}{\nu^4}2\,837\,835\,q^5\left(1+q^2\,\nu^2\right)^2\text{Cos}[(4+t-\nu)\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-4-t+\nu]-\\
&\quad\frac{1}{\nu^2}25\,740\,q^7\left(1+q^2\,\nu^2\right)\text{Cos}[(2+t-\nu)\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-2-t+\nu]+\\
&\quad55\,q^9\,\text{Cos}[(t-\nu)\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-t+\nu]+\frac{1}{\nu}\\
&\quad1485\,q^8\sqrt{1+q^2\,\nu^2}\,\text{Gamma}[-1-t+\nu]\,\text{Sin}[(1+t-\nu)\,\text{ArcTan}[q\,\nu]]-\frac{1}{\nu^3}\\
&\quad315\,315\,q^6\left(1+q^2\,\nu^2\right)^{3/2}\text{Gamma}[-3-t+\nu]\,\text{Sin}[(3+t-\nu)\,\text{ArcTan}[q\,\nu]]+\frac{1}{\nu^5}\\
&\quad18\,918\,900\,q^4\left(1+q^2\,\nu^2\right)^{5/2}\text{Gamma}[-5-t+\nu]\,\text{Sin}[(5+t-\nu)\,\text{ArcTan}[q\,\nu]]-\frac{1}{\nu^7}\\
&\quad310\,134\,825\,q^2\left(1+q^2\,\nu^2\right)^{7/2}\text{Gamma}[-7-t+\nu]\,\text{Sin}[(7+t-\nu)\,\text{ArcTan}[q\,\nu]]+\\
&\quad\frac{1}{\nu^9}654\,729\,075\left(1+q^2\,\nu^2\right)^{9/2}\text{Gamma}[-9-t+\nu]\,\text{Sin}[(9+t-\nu)\,\text{ArcTan}[q\,\nu]]+\\
&\quad\left(q^{10}\,\nu\,\text{Gamma}[1-t+\nu]\,\text{Sin}[(1-t+\nu)\,\text{ArcTan}[q\,\nu]]\right)\Big/\left(\sqrt{1+q^2\,\nu^2}\right)\Bigg)
\end{aligned}
$$

```
FortranForm[% /. ν → nu]
```

```
      -((nu**(nu - t)*Sqrt(2/Pi)*
     -     (1 + nu**2*q**2)**((-nu + t)/2.)*
     -     ((654729075*q*(1 + nu**2*q**2)**4*
     -          Cos((8 - nu + t)*ArcTan(nu*q))*Gamma(-8 + nu - t))
     -        /nu**8 - (91891800*(q + nu**2*q**3)**3*
     -          Cos((6 - nu + t)*ArcTan(nu*q))*Gamma(-6 + nu - t))
     -        /nu**6 + (2837835*q**5*(1 + nu**2*q**2)**2*
     -          Cos((4 - nu + t)*ArcTan(nu*q))*Gamma(-4 + nu - t))
     -        /nu**4 - (25740*q**7*(1 + nu**2*q**2)*
     -          Cos((2 - nu + t)*ArcTan(nu*q))*Gamma(-2 + nu - t))
     -        /nu**2 + 55*q**9*Cos((-nu + t)*ArcTan(nu*q))*
     -       Gamma(nu - t) +
     -      (nu*q**10*Gamma(1 + nu - t)*
     -         Sin((1 + nu - t)*ArcTan(nu*q)))/
     -       Sqrt(1 + nu**2*q**2) +
     -      (1485*q**8*Sqrt(1 + nu**2*q**2)*Gamma(-1 + nu - t)*
     -         Sin((1 - nu + t)*ArcTan(nu*q)))/nu -
     -      (315315*q**6*(1 + nu**2*q**2)**1.5*
     -         Gamma(-3 + nu - t)*Sin((3 - nu + t)*ArcTan(nu*q)))
     -        /nu**3 + (18918900*q**4*(1 + nu**2*q**2)**2.5*
     -         Gamma(-5 + nu - t)*Sin((5 - nu + t)*ArcTan(nu*q)))
     -        /nu**5 - (310134825*q**2*(1 + nu**2*q**2)**3.5*
     -         Gamma(-7 + nu - t)*Sin((7 - nu + t)*ArcTan(nu*q)))
     -        /nu**7 + (654729075*(1 + nu**2*q**2)**4.5*
     -         Gamma(-9 + nu - t)*Sin((9 - nu + t)*ArcTan(nu*q)))
     -        /nu**9))/q**10)
```

```
l = 11
```

```
11
```

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\frac{1}{q^{11}}\sqrt{\frac{2}{\pi}}\,\nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$

$$\left(-\frac{1}{\nu^9}13\,749\,310\,575\,q\left(1+q^2\nu^2\right)^{9/2}\text{Cos}[(9+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-9-t+\nu]+\right.$$

$$\frac{1}{\nu^7}1\,964\,187\,225\,q^3\left(1+q^2\nu^2\right)^{7/2}\text{Cos}[(7+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-7-t+\nu]-$$

$$\frac{1}{\nu^5}64\,324\,260\,q^5\left(1+q^2\nu^2\right)^{5/2}\text{Cos}[(5+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-5-t+\nu]+$$

$$\frac{1}{\nu^3}675\,675\,q^7\left(1+q^2\nu^2\right)^{3/2}\text{Cos}[(3+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-3-t+\nu]-$$

$$\frac{1}{\nu}2145\,q^9\sqrt{1+q^2\nu^2}\,\text{Cos}[(1+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-1-t+\nu]+$$

$$\left(q^{11}\nu\,\text{Cos}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[1-t+\nu]\right)\Big/\left(\sqrt{1+q^2\nu^2}\right)+$$

$$66\,q^{10}\,\text{Gamma}[-t+\nu]\,\text{Sin}[(t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^2}$$

$$45\,045\,q^8\left(1+q^2\nu^2\right)\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^4}$$

$$7\,567\,560\,q^6\left(1+q^2\nu^2\right)^2\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^6}$$

$$413\,513\,100\,q^4\left(1+q^2\nu^2\right)^3\text{Gamma}[-6-t+\nu]\,\text{Sin}[(6+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^8}$$

$$6\,547\,290\,750\,q^2\left(1+q^2\nu^2\right)^4\text{Gamma}[-8-t+\nu]\,\text{Sin}[(8+t-\nu)\,\text{ArcTan}[q\nu]]-$$

$$\left.\frac{1}{\nu^{10}}13\,749\,310\,575\left(1+q^2\nu^2\right)^5\text{Gamma}[-10-t+\nu]\,\text{Sin}[(10+t-\nu)\,\text{ArcTan}[q\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q^{11}}\sqrt{\frac{2}{\pi}}\,\nu^{-t+\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\left(-\frac{1}{\nu^9}13\,749\,310\,575\,q\left(1+q^2\nu^2\right)^{9/2}\text{Cos}[(9+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-9-t+\nu]+\right.$$
$$\frac{1}{\nu^7}1\,964\,187\,225\,q^3\left(1+q^2\nu^2\right)^{7/2}\text{Cos}[(7+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-7-t+\nu]-$$
$$\frac{1}{\nu^5}64\,324\,260\,q^5\left(1+q^2\nu^2\right)^{5/2}\text{Cos}[(5+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-5-t+\nu]+$$
$$\frac{1}{\nu^3}675\,675\,q^7\left(1+q^2\nu^2\right)^{3/2}\text{Cos}[(3+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-3-t+\nu]-$$
$$\frac{1}{\nu}2145\,q^9\sqrt{1+q^2\nu^2}\,\text{Cos}[(1+t-\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[-1-t+\nu]+$$
$$\left(q^{11}\nu\,\text{Cos}[(1-t+\nu)\,\text{ArcTan}[q\nu]]\,\text{Gamma}[1-t+\nu]\right)\Big/\left(\sqrt{1+q^2\nu^2}\right)+$$
$$66\,q^{10}\,\text{Gamma}[-t+\nu]\,\text{Sin}[(t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^2}$$
$$45\,045\,q^8\left(1+q^2\nu^2\right)\text{Gamma}[-2-t+\nu]\,\text{Sin}[(2+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^4}$$
$$7\,567\,560\,q^6\left(1+q^2\nu^2\right)^2\text{Gamma}[-4-t+\nu]\,\text{Sin}[(4+t-\nu)\,\text{ArcTan}[q\nu]]-\frac{1}{\nu^6}$$
$$413\,513\,100\,q^4\left(1+q^2\nu^2\right)^3\text{Gamma}[-6-t+\nu]\,\text{Sin}[(6+t-\nu)\,\text{ArcTan}[q\nu]]+\frac{1}{\nu^8}$$
$$6\,547\,290\,750\,q^2\left(1+q^2\nu^2\right)^4\text{Gamma}[-8-t+\nu]\,\text{Sin}[(8+t-\nu)\,\text{ArcTan}[q\nu]]-$$
$$\left.\frac{1}{\nu^{10}}13\,749\,310\,575\left(1+q^2\nu^2\right)^5\text{Gamma}[-10-t+\nu]\,\text{Sin}[(10+t-\nu)\,\text{ArcTan}[q\nu]]\right)$$

```
FortranForm[% /. ν → nu]
```

```
      (nu**(nu - t)*Sqrt(2/Pi)*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -    ((-13749310575*q*(1 + nu**2*q**2)**4.5*
     -         Cos((9 - nu + t)*ArcTan(nu*q))*Gamma(-9 + nu - t))/
     -       nu**9 + (1964187225*q**3*(1 + nu**2*q**2)**3.5*
     -         Cos((7 - nu + t)*ArcTan(nu*q))*Gamma(-7 + nu - t))/
     -       nu**7 - (64324260*q**5*(1 + nu**2*q**2)**2.5*
     -         Cos((5 - nu + t)*ArcTan(nu*q))*Gamma(-5 + nu - t))/
     -       nu**5 + (675675*q**7*(1 + nu**2*q**2)**1.5*
     -         Cos((3 - nu + t)*ArcTan(nu*q))*Gamma(-3 + nu - t))/
     -       nu**3 - (2145*q**9*Sqrt(1 + nu**2*q**2)*
     -         Cos((1 - nu + t)*ArcTan(nu*q))*Gamma(-1 + nu - t))/
     -       nu + (nu*q**11*Cos((1 + nu - t)*ArcTan(nu*q))*
     -         Gamma(1 + nu - t))/Sqrt(1 + nu**2*q**2) +
     -      66*q**10*Gamma(nu - t)*Sin((-nu + t)*ArcTan(nu*q)) -
     -      (45045*q**8*(1 + nu**2*q**2)*Gamma(-2 + nu - t)*
     -         Sin((2 - nu + t)*ArcTan(nu*q)))/nu**2 +
     -      (7567560*q**6*(1 + nu**2*q**2)**2*Gamma(-4 + nu - t)*
     -         Sin((4 - nu + t)*ArcTan(nu*q)))/nu**4 -
     -      (413513100*q**4*(1 + nu**2*q**2)**3*Gamma(-6 + nu - t)*
     -         Sin((6 - nu + t)*ArcTan(nu*q)))/nu**6 +
     -      (6547290750*q**2*(1 + nu**2*q**2)**4*
     -         Gamma(-8 + nu - t)*Sin((8 - nu + t)*ArcTan(nu*q)))/
     -       nu**8 - (13749310575*(1 + nu**2*q**2)**5*
     -         Gamma(-10 + nu - t)*Sin((10 - nu + t)*ArcTan(nu*q)))
     -        /nu**10))/q**11
```

```
l = 12
```

```
12
```

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[- r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\frac{1}{q^{12}}\sqrt{\frac{2}{\pi}}\left(-316\,234\,143\,225\,q\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\right.$$
$$\cos[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-10-t+\nu]+$$
$$45\,831\,035\,250\,q^3\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-8-t+\nu]-$$
$$1\,571\,349\,780\,q^5\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-6-t+\nu]+18\,378\,360\,q^7$$
$$\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\cos[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\Gamma[-4-t+\nu]-75\,075\,q^9\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-2-t+\nu]+$$
$$78\,q^{11}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\cos[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-t+\nu]+$$
$$3003\,q^{10}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-1-t+\nu]$$
$$\sin[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$1\,351\,350\,q^8\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-3-t+\nu]$$
$$\sin[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$192\,972\,780\,q^6\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-5-t+\nu]$$
$$\sin[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$9\,820\,936\,125\,q^4\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-7-t+\nu]$$
$$\sin[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$151\,242\,416\,325\,q^2\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-9-t+\nu]$$
$$\sin[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$316\,234\,143\,225\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-11-t+\nu]$$
$$\sin[11\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+q^{12}\left(\frac{1}{\nu}\right)^{-1+t-\nu}$$
$$\left.\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}\Gamma[1-t+\nu]\,\sin[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2) ^ (1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q^{12}}\sqrt{\frac{2}{\pi}}\left(-316\,234\,143\,225\,q\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\right.$$
$$\cos[10\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\,\Gamma[-10-t+\nu]+$$
$$45\,831\,035\,250\,q^3\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[8\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\,\Gamma[-8-t+\nu]-$$
$$1\,571\,349\,780\,q^5\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[6\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\,\Gamma[-6-t+\nu]+18\,378\,360\,q^7$$
$$\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\cos[4\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]$$
$$\Gamma[-4-t+\nu]-75\,075\,q^9\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[2\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\,\Gamma[-2-t+\nu]+$$
$$78\,q^{11}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\cos[t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\,\Gamma[-t+\nu]+$$
$$3003\,q^{10}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-1-t+\nu]$$
$$\sin[\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]-$$
$$1\,351\,350\,q^8\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-3-t+\nu]$$
$$\sin[3\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]+$$
$$192\,972\,780\,q^6\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-5-t+\nu]$$
$$\sin[5\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]-$$
$$9\,820\,936\,125\,q^4\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-7-t+\nu]$$
$$\sin[7\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]+$$
$$151\,242\,416\,325\,q^2\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-9-t+\nu]$$
$$\sin[9\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]-$$
$$316\,234\,143\,225\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-11-t+\nu]$$
$$\sin[11\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]+q^{12}\left(\frac{1}{\nu}\right)^{-1+t-\nu}$$
$$\left.\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}\Gamma[1-t+\nu]\sin[\arctan[q\,\nu]-t\arctan[q\,\nu]+\nu\arctan[q\,\nu]]\right)$$

FortranForm[% /. ν → nu]

```
      (Sqrt(2/Pi)*(-316234143225*(1/nu)**(-nu + t)*q*
     -      (nu**(-2) + q**2)**5*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(10*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-10 + nu - t) +
     -     45831035250*(1/nu)**(-nu + t)*q**3*
     -      (nu**(-2) + q**2)**4*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(8*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-8 + nu - t) -
     -     1571349780*(1/nu)**(-nu + t)*q**5*(nu**(-2) + q**2)**3*
     -      (1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(6*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-6 + nu - t) +
     -     18378360*(1/nu)**(-nu + t)*q**7*(nu**(-2) + q**2)**2*
     -      (1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(4*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-4 + nu - t) -
     -     75075*(1/nu)**(-nu + t)*q**9*(nu**(-2) + q**2)*
     -      (1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(2*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-2 + nu - t) +
     -     78*(1/nu)**(-nu + t)*q**11*
     -      (1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(nu*ArcTan(nu*q) - t*ArcTan(nu*q))*Gamma(nu - t) +
     -     (1/nu)**(-1 - nu + t)*q**12*
     -      (1 + nu**2*q**2)**((-1 - nu + t)/2.)*
     -      Gamma(1 + nu - t)*
     -      Sin(ArcTan(nu*q) + nu*ArcTan(nu*q) - t*ArcTan(nu*q))\
     -      + 3003*(1/nu)**(1 - nu + t)*q**10*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-1 + nu - t)*
     -      Sin(ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))\
     -      - 1351350*(1/nu)**(1 - nu + t)*q**8*(nu**(-2) + q**2)*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-3 + nu - t)*
     -      Sin(3*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       + 192972780*(1/nu)**(1 - nu + t)*q**6*
     -      (nu**(-2) + q**2)**2*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-5 + nu - t)*
     -      Sin(5*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       - 9820936125*(1/nu)**(1 - nu + t)*q**4*
     -      (nu**(-2) + q**2)**3*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-7 + nu - t)*
     -      Sin(7*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       + 151242416325*(1/nu)**(1 - nu + t)*q**2*
     -      (nu**(-2) + q**2)**4*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-9 + nu - t)*
     -      Sin(9*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       - 316234143225*(1/nu)**(1 - nu + t)*
     -      (nu**(-2) + q**2)**5*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-11 + nu - t)*
     -      Sin(11*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))))/q**12
```

l = 13

13

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[- r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\frac{1}{q^{13}}\sqrt{\frac{2}{\pi}}\left(-7\,905\,853\,580\,625\,q\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\right.$$
$$\cos[11\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-11-t+\nu]+$$
$$1\,159\,525\,191\,825\,q^3\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[9\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-9-t+\nu]-$$
$$41\,247\,931\,725\,q^5\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[7\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-7-t+\nu]+$$
$$523\,783\,260\,q^7\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[5\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-5-t+\nu]-$$
$$2\,552\,550\,q^9\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[3\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-3-t+\nu]+$$
$$4095\,q^{11}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\cos[\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]$$
$$\Gamma[-1-t+\nu]-q^{13}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}$$
$$\cos[\arctan[q\nu]-t\arctan[q\nu]+\nu\arctan[q\nu]]\,\Gamma[1-t+\nu]-$$
$$91\,q^{12}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-t+\nu]\sin[t\arctan[q\nu]-\nu\arctan[q\nu]]+$$
$$120\,120\,q^{10}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-2-t+\nu]$$
$$\sin[2\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]-41\,351\,310\,q^8\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{t-\nu}$$
$$\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-4-t+\nu]\sin[4\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]+$$
$$5\,237\,832\,600\,q^6\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-6-t+\nu]$$
$$\sin[6\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]-$$
$$252\,070\,693\,875\,q^4\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-8-t+\nu]$$
$$\sin[8\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]+$$
$$3\,794\,809\,718\,700\,q^2\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-10-t+\nu]$$
$$\sin[10\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]-$$
$$7\,905\,853\,580\,625\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-12-t+\nu]$$
$$\left.\sin[12\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2) ^ (1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q^{13}}\sqrt{\frac{2}{\pi}}\left(-7\,905\,853\,580\,625\,q\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\right.$$
$$\cos[11\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\,\Gamma[-11-t+\nu]+$$
$$1\,159\,525\,191\,825\,q^3\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[9\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\,\Gamma[-9-t+\nu]-$$
$$41\,247\,931\,725\,q^5\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[7\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\,\Gamma[-7-t+\nu]+$$
$$523\,783\,260\,q^7\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[5\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\,\Gamma[-5-t+\nu]-$$
$$2\,552\,550\,q^9\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[3\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\,\Gamma[-3-t+\nu]+$$
$$4095\,q^{11}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\cos[\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]$$
$$\Gamma[-1-t+\nu]-q^{13}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}$$
$$\cos[\arctan[q\,\nu]-t\arctan[q\,\nu]+\nu\arctan[q\,\nu]]\,\Gamma[1-t+\nu]-$$
$$91\,q^{12}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-t+\nu]\sin[t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]+$$
$$120\,120\,q^{10}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-2-t+\nu]$$
$$\sin[2\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]-41\,351\,310\,q^8\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{t-\nu}$$
$$\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-4-t+\nu]\sin[4\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]+$$
$$5\,237\,832\,600\,q^6\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-6-t+\nu]$$
$$\sin[6\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]-$$
$$252\,070\,693\,875\,q^4\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-8-t+\nu]$$
$$\sin[8\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]+$$
$$3\,794\,809\,718\,700\,q^2\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-10-t+\nu]$$
$$\sin[10\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]-$$
$$7\,905\,853\,580\,625\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-12-t+\nu]$$
$$\left.\sin[12\arctan[q\,\nu]+t\arctan[q\,\nu]-\nu\arctan[q\,\nu]]\right)$$

FortranForm[% /. ν → nu]

```
      (Sqrt(2/Pi)*(-7905853580625*(1/nu)**(1 - nu + t)*q*
     -      (nu**(-2) + q**2)**5*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Cos(11*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-11 + nu - t) +
     -      1159525191825*(1/nu)**(1 - nu + t)*q**3*
     -      (nu**(-2) + q**2)**4*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Cos(9*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-9 + nu - t) -
     -      41247931725*(1/nu)**(1 - nu + t)*q**5*
     -      (nu**(-2) + q**2)**3*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Cos(7*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-7 + nu - t) +
     -      523783260*(1/nu)**(1 - nu + t)*q**7*
     -      (nu**(-2) + q**2)**2*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Cos(5*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-5 + nu - t) -
     -      2552550*(1/nu)**(1 - nu + t)*q**9*(nu**(-2) + q**2)*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Cos(3*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-3 + nu - t) +
     -      4095*(1/nu)**(1 - nu + t)*q**11*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Cos(ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))*
     -      Gamma(-1 + nu - t) -
     -      (1/nu)**(-1 - nu + t)*q**13*
     -      (1 + nu**2*q**2)**((-1 - nu + t)/2.)*
     -      Cos(ArcTan(nu*q) + nu*ArcTan(nu*q) - t*ArcTan(nu*q))*
     -      Gamma(1 + nu - t) +
     -      91*(1/nu)**(-nu + t)*q**12*
     -      (1 + nu**2*q**2)**((-nu + t)/2.)*Gamma(nu - t)*
     -      Sin(nu*ArcTan(nu*q) - t*ArcTan(nu*q)) +
     -      120120*(1/nu)**(-nu + t)*q**10*(nu**(-2) + q**2)*
     -      (1 + nu**2*q**2)**((-nu + t)/2.)*Gamma(-2 + nu - t)*
     -      Sin(2*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       - 41351310*(1/nu)**(-nu + t)*q**8*
     -      (nu**(-2) + q**2)**2*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Gamma(-4 + nu - t)*
     -      Sin(4*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       + 5237832600*(1/nu)**(-nu + t)*q**6*
     -      (nu**(-2) + q**2)**3*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Gamma(-6 + nu - t)*
     -      Sin(6*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       - 252070693875*(1/nu)**(-nu + t)*q**4*
     -      (nu**(-2) + q**2)**4*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Gamma(-8 + nu - t)*
     -      Sin(8*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       + 3794809718700*(1/nu)**(-nu + t)*q**2*
     -      (nu**(-2) + q**2)**5*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Gamma(-10 + nu - t)*
     -      Sin(10*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q)) -
     -      7905853580625*(1/nu)**(-nu + t)*(nu**(-2) + q**2)**6*
     -      (1 + nu**2*q**2)**((-nu + t)/2.)*Gamma(-12 + nu - t)*
     -      Sin(12*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))))/q**13
```

l = 14

14

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$
\begin{aligned}
&-\frac{1}{q^{14}}\sqrt{\frac{2}{\pi}}\left(213\,458\,046\,676\,875\,q\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\right.\\
&\quad\cos[12\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-12-t+\nu]-\\
&\quad31\,623\,414\,322\,500\,q^3\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\\
&\quad\cos[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-10-t+\nu]+\\
&\quad1\,159\,525\,191\,825\,q^5\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\\
&\quad\cos[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-8-t+\nu]-\\
&\quad15\,713\,497\,800\,q^7\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\\
&\quad\cos[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-6-t+\nu]+87\,297\,210\,q^9\\
&\quad\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\cos[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\\
&\quad\Gamma[-4-t+\nu]-185\,640\,q^{11}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\\
&\quad\cos[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-2-t+\nu]+\\
&\quad105\,q^{13}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\cos[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-t+\nu]+\\
&\quad5460\,q^{12}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-1-t+\nu]\\
&\quad\sin[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-\\
&\quad4\,594\,590\,q^{10}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-3-t+\nu]\\
&\quad\sin[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+\\
&\quad1\,309\,458\,150\,q^8\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-5-t+\nu]\\
&\quad\sin[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-\\
&\quad151\,242\,416\,325\,q^6\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-7-t+\nu]\\
&\quad\sin[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+\\
&\quad6\,957\,151\,150\,950\,q^4\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-9-t+\nu]\\
&\quad\sin[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-\\
&\quad102\,776\,096\,548\,125\,q^2\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\\
&\quad\Gamma[-11-t+\nu]\sin[11\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+\\
&\quad213\,458\,046\,676\,875\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-13-t+\nu]\\
&\quad\sin[13\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+\\
&\quad q^{14}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}\Gamma[1-t+\nu]\\
&\quad\left.\sin[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\right)
\end{aligned}
$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2) ^ (1/2 (1 - t + ν)))^(-1)
```

$$
\begin{aligned}
&-\frac{1}{q^{14}}\sqrt{\frac{2}{\pi}}\left(213\,458\,046\,676\,875\,q\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\right.\\
&\quad\text{Cos}[12\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-12-t+\nu]-\\
&\quad31\,623\,414\,322\,500\,q^3\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\\
&\quad\text{Cos}[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-10-t+\nu]+\\
&\quad1\,159\,525\,191\,825\,q^5\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\\
&\quad\text{Cos}[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-8-t+\nu]-\\
&\quad15\,713\,497\,800\,q^7\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\\
&\quad\text{Cos}[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-6-t+\nu]+87\,297\,210\,q^9\\
&\quad\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\text{Cos}[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\\
&\quad\text{Gamma}[-4-t+\nu]-185\,640\,q^{11}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\\
&\quad\text{Cos}[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-2-t+\nu]+\\
&\quad105\,q^{13}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\text{Cos}[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-t+\nu]+\\
&\quad5460\,q^{12}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-1-t+\nu]\\
&\quad\text{Sin}[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-\\
&\quad4\,594\,590\,q^{10}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-3-t+\nu]\\
&\quad\text{Sin}[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+\\
&\quad1\,309\,458\,150\,q^8\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-5-t+\nu]\\
&\quad\text{Sin}[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-\\
&\quad151\,242\,416\,325\,q^6\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-7-t+\nu]\\
&\quad\text{Sin}[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+\\
&\quad6\,957\,151\,150\,950\,q^4\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-9-t+\nu]\\
&\quad\text{Sin}[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-\\
&\quad102\,776\,096\,548\,125\,q^2\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\\
&\quad\text{Gamma}[-11-t+\nu]\,\text{Sin}[11\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+\\
&\quad213\,458\,046\,676\,875\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-13-t+\nu]\\
&\quad\text{Sin}[13\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+\\
&\quad q^{14}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}\text{Gamma}[1-t+\nu]\\
&\quad\left.\text{Sin}[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\right)
\end{aligned}
$$

```
FortranForm[% /. ν → nu]
```

```
      -((Sqrt(2/Pi)*(213458046676875*(1/nu)**(-nu + t)*q*
     -        (nu**(-2) + q**2)**6*
     -        (1 + nu**2*q**2)**((-nu + t)/2.)*
     -        Cos(12*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -          t*ArcTan(nu*q))*Gamma(-12 + nu - t) -
```

```
-         31623414322500*(1/nu)**(-nu + t)*q**3*
-          (nu**(-2) + q**2)**5*
-          (1 + nu**2*q**2)**((-nu + t)/2.)*
-          Cos(10*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q))*Gamma(-10 + nu - t) +
-         1159525191825*(1/nu)**(-nu + t)*q**5*
-          (nu**(-2) + q**2)**4*
-          (1 + nu**2*q**2)**((-nu + t)/2.)*
-          Cos(8*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q))*Gamma(-8 + nu - t) -
-         15713497800*(1/nu)**(-nu + t)*q**7*
-          (nu**(-2) + q**2)**3*
-          (1 + nu**2*q**2)**((-nu + t)/2.)*
-          Cos(6*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q))*Gamma(-6 + nu - t) +
-         87297210*(1/nu)**(-nu + t)*q**9*(nu**(-2) + q**2)**2*
-          (1 + nu**2*q**2)**((-nu + t)/2.)*
-          Cos(4*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q))*Gamma(-4 + nu - t) -
-         185640*(1/nu)**(-nu + t)*q**11*(nu**(-2) + q**2)*
-          (1 + nu**2*q**2)**((-nu + t)/2.)*
-          Cos(2*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q))*Gamma(-2 + nu - t) +
-         105*(1/nu)**(-nu + t)*q**13*
-          (1 + nu**2*q**2)**((-nu + t)/2.)*
-          Cos(nu*ArcTan(nu*q) - t*ArcTan(nu*q))*Gamma(nu - t)\
-         + (1/nu)**(-1 - nu + t)*q**14*
-          (1 + nu**2*q**2)**((-1 - nu + t)/2.)*
-          Gamma(1 + nu - t)*
-          Sin(ArcTan(nu*q) + nu*ArcTan(nu*q) - t*ArcTan(nu*q))
-           + 5460*(1/nu)**(1 - nu + t)*q**12*
-          (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-          Gamma(-1 + nu - t)*
-          Sin(ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
-           - 4594590*(1/nu)**(1 - nu + t)*q**10*
-          (nu**(-2) + q**2)*
-          (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-          Gamma(-3 + nu - t)*
-          Sin(3*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) +
-         1309458150*(1/nu)**(1 - nu + t)*q**8*
-          (nu**(-2) + q**2)**2*
-          (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-          Gamma(-5 + nu - t)*
-          Sin(5*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) -
-         151242416325*(1/nu)**(1 - nu + t)*q**6*
-          (nu**(-2) + q**2)**3*
-          (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-          Gamma(-7 + nu - t)*
-          Sin(7*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) +
-         6957151150950*(1/nu)**(1 - nu + t)*q**4*
-          (nu**(-2) + q**2)**4*
-          (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-          Gamma(-9 + nu - t)*
-          Sin(9*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) -
-         102776096548125*(1/nu)**(1 - nu + t)*q**2*
-          (nu**(-2) + q**2)**5*
-          (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-          Gamma(-11 + nu - t)*
-          Sin(11*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) +
-         213458046676875*(1/nu)**(1 - nu + t)*
-          (nu**(-2) + q**2)**6*
-          (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-          Gamma(-13 + nu - t)*
-          Sin(13*ArcTan(nu*q) - nu*ArcTan(nu*q) +
```

l = 15

15

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\frac{1}{q^{15}}\sqrt{\frac{2}{\pi}}\left(-6\,190\,283\,353\,629\,375\,q\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\right.$$
$$\cos[13\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-13-t+\nu]+$$
$$924\,984\,868\,933\,125\,q^3\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[11\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-11-t+\nu]-$$
$$34\,785\,755\,754\,750\,q^5\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[9\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-9-t+\nu]+$$
$$496\,939\,367\,925\,q^7\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[7\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-7-t+\nu]-$$
$$3\,055\,402\,350\,q^9\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[5\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-5-t+\nu]+$$
$$7\,936\,110\,q^{11}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[3\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-3-t+\nu]-$$
$$7140\,q^{13}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\cos[\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]$$
$$\Gamma[-1-t+\nu]+q^{15}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}$$
$$\cos[\arctan[q\nu]-t\arctan[q\nu]+\nu\arctan[q\nu]]\,\Gamma[1-t+\nu]+$$
$$120\,q^{14}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-t+\nu]\sin[t\arctan[q\nu]-\nu\arctan[q\nu]]-$$
$$278\,460\,q^{12}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-2-t+\nu]$$
$$\sin[2\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]+$$
$$174\,594\,420\,q^{10}\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-4-t+\nu]$$
$$\sin[4\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]-$$
$$43\,212\,118\,950\,q^8\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-6-t+\nu]$$
$$\sin[6\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]+$$
$$4\,638\,100\,767\,300\,q^6\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-8-t+\nu]$$
$$\sin[8\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]-$$
$$205\,552\,193\,096\,250\,q^4\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-10-t+\nu]$$
$$\sin[10\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]+$$
$$2\,988\,412\,653\,476\,250\,q^2\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-12-t+\nu]$$
$$\sin[12\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]-$$
$$6\,190\,283\,353\,629\,375\left(q^2+\frac{1}{\nu^2}\right)^7\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-14-t+\nu]$$
$$\left.\sin[14\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2) ^ (1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q^{15}}\sqrt{\frac{2}{\pi}}\left(-6\,190\,283\,353\,629\,375\,q\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\right.$$
$$\cos[13\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]\,\Gamma[-13-t+\nu]+$$
$$924\,984\,868\,933\,125\,q^3\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[11\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]\,\Gamma[-11-t+\nu]-$$
$$34\,785\,755\,754\,750\,q^5\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[9\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]\,\Gamma[-9-t+\nu]+$$
$$496\,939\,367\,925\,q^7\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[7\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]\,\Gamma[-7-t+\nu]-$$
$$3\,055\,402\,350\,q^9\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[5\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]\,\Gamma[-5-t+\nu]+$$
$$7\,936\,110\,q^{11}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\cos[3\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]\,\Gamma[-3-t+\nu]-$$
$$7140\,q^{13}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\cos[\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]$$
$$\Gamma[-1-t+\nu]+q^{15}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}$$
$$\cos[\arctan(q\nu)-t\arctan(q\nu)+\nu\arctan(q\nu)]\,\Gamma[1-t+\nu]+$$
$$120\,q^{14}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-t+\nu]\sin[t\arctan(q\nu)-\nu\arctan(q\nu)]-$$
$$278\,460\,q^{12}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-2-t+\nu]$$
$$\sin[2\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]+$$
$$174\,594\,420\,q^{10}\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-4-t+\nu]$$
$$\sin[4\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]-$$
$$43\,212\,118\,950\,q^8\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-6-t+\nu]$$
$$\sin[6\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]+$$
$$4\,638\,100\,767\,300\,q^6\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-8-t+\nu]$$
$$\sin[8\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]-$$
$$205\,552\,193\,096\,250\,q^4\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-10-t+\nu]$$
$$\sin[10\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]+$$
$$2\,988\,412\,653\,476\,250\,q^2\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-12-t+\nu]$$
$$\sin[12\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]-$$
$$6\,190\,283\,353\,629\,375\left(q^2+\frac{1}{\nu^2}\right)^7\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\Gamma[-14-t+\nu]$$
$$\left.\sin[14\arctan(q\nu)+t\arctan(q\nu)-\nu\arctan(q\nu)]\right)$$

```
FortranForm[% /. ν → nu]
```

```
      (Sqrt(2/Pi)*(-6190283353629375*(1/nu)**(1 - nu + t)*q*
     -        (nu**(-2) + q**2)**6*
     -        (1 + nu**2*q**2)**((1 - nu + t)/2.)*
```

```
-        Cos(13*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-13 + nu - t) +
-       924984868933125*(1/nu)**(1 - nu + t)*q**3*
-        (nu**(-2) + q**2)**5*
-        (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-        Cos(11*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-11 + nu - t) -
-       34785755754750*(1/nu)**(1 - nu + t)*q**5*
-        (nu**(-2) + q**2)**4*
-        (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-        Cos(9*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-9 + nu - t) +
-       496939367925*(1/nu)**(1 - nu + t)*q**7*
-        (nu**(-2) + q**2)**3*
-        (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-        Cos(7*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-7 + nu - t) -
-       3055402350*(1/nu)**(1 - nu + t)*q**9*
-        (nu**(-2) + q**2)**2*
-        (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-        Cos(5*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-5 + nu - t) +
-       7936110*(1/nu)**(1 - nu + t)*q**11*(nu**(-2) + q**2)*
-        (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-        Cos(3*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-3 + nu - t) -
-       7140*(1/nu)**(1 - nu + t)*q**13*
-        (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-        Cos(ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))*
-        Gamma(-1 + nu - t) +
-       (1/nu)**(-1 - nu + t)*q**15*
-        (1 + nu**2*q**2)**((-1 - nu + t)/2.)*
-        Cos(ArcTan(nu*q) + nu*ArcTan(nu*q) - t*ArcTan(nu*q))*
-        Gamma(1 + nu - t) -
-       120*(1/nu)**(-nu + t)*q**14*
-        (1 + nu**2*q**2)**((-nu + t)/2.)*Gamma(nu - t)*
-        Sin(nu*ArcTan(nu*q) - t*ArcTan(nu*q)) -
-       278460*(1/nu)**(-nu + t)*q**12*(nu**(-2) + q**2)*
-        (1 + nu**2*q**2)**((-nu + t)/2.)*Gamma(-2 + nu - t)*
-        Sin(2*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
-         + 174594420*(1/nu)**(-nu + t)*q**10*
-        (nu**(-2) + q**2)**2*(1 + nu**2*q**2)**((-nu + t)/2.)*
-        Gamma(-4 + nu - t)*
-        Sin(4*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
-         - 43212118950*(1/nu)**(-nu + t)*q**8*
-        (nu**(-2) + q**2)**3*(1 + nu**2*q**2)**((-nu + t)/2.)*
-        Gamma(-6 + nu - t)*
-        Sin(6*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
-         + 4638100767300*(1/nu)**(-nu + t)*q**6*
-        (nu**(-2) + q**2)**4*(1 + nu**2*q**2)**((-nu + t)/2.)*
-        Gamma(-8 + nu - t)*
-        Sin(8*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
-         - 205552193096250*(1/nu)**(-nu + t)*q**4*
-        (nu**(-2) + q**2)**5*(1 + nu**2*q**2)**((-nu + t)/2.)*
-        Gamma(-10 + nu - t)*
-        Sin(10*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)) +
-       2988412653476250*(1/nu)**(-nu + t)*q**2*
-        (nu**(-2) + q**2)**6*(1 + nu**2*q**2)**((-nu + t)/2.)*
-        Gamma(-12 + nu - t)*
-        Sin(12*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)) -
-       6190283353629375*(1/nu)**(-nu + t)*
-        (nu**(-2) + q**2)**7*(1 + nu**2*q**2)**((-nu + t)/2.)*
-        Gamma(-14 + nu - t)*
-        Sin(14*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))))/q**15
```

```
l = 16
```

```
16
```

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\frac{1}{q^{16}}\sqrt{\frac{2}{\pi}}\left(-191\,898\,783\,962\,510\,625\,q\left(q^2+\frac{1}{\nu^2}\right)^7\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\right.$$
$$\text{Cos}[14\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-14-t+\nu]+$$
$$28\,887\,988\,983\,603\,750\,q^3\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\text{Cos}[12\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-12-t+\nu]-$$
$$1\,109\,981\,842\,719\,750\,q^5\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\text{Cos}[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-10-t+\nu]+$$
$$16\,564\,645\,597\,500\,q^7\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\text{Cos}[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-8-t+\nu]-$$
$$110\,430\,970\,650\,q^9\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\text{Cos}[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-6-t+\nu]+333\,316\,620\,q^{11}$$
$$\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\text{Cos}[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-4-t+\nu]-406\,980\,q^{13}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\text{Cos}[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-2-t+\nu]+$$
$$136\,q^{15}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\text{Cos}[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-t+\nu]+$$
$$9180\,q^{14}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-1-t+\nu]$$
$$\text{Sin}[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$13\,226\,850\,q^{12}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-3-t+\nu]$$
$$\text{Sin}[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$6\,721\,885\,170\,q^{10}\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-5-t+\nu]$$
$$\text{Sin}[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$1\,490\,818\,103\,775\,q^8\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-7-t+\nu]$$
$$\text{Sin}[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$150\,738\,274\,937\,250\,q^6\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\text{Gamma}[-9-t+\nu]\,\text{Sin}[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$6\,474\,894\,082\,531\,875\,q^4\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\text{Gamma}[-11-t+\nu]\,\text{Sin}[11\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$92\,854\,250\,304\,440\,625\,q^2\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\text{Gamma}[-13-t+\nu]\,\text{Sin}[13\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$191\,898\,783\,962\,510\,625\left(q^2+\frac{1}{\nu^2}\right)^7\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\text{Gamma}[-15-t+\nu]$$
$$\text{Sin}[15\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+q^{16}\left(\frac{1}{\nu}\right)^{-1+t-\nu}$$
$$\left.\left(1+q^2\,\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}\text{Gamma}[1-t+\nu]\,\text{Sin}[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2) ^ (1/2 (1 - t + ν)))^(-1)
```

$$\frac{1}{q^{16}}\sqrt{\frac{2}{\pi}}\left(-191\,898\,783\,962\,510\,625\,q\left(q^2+\frac{1}{\nu^2}\right)^7\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\right.$$
$$\cos[14\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-14-t+\nu]+$$
$$28\,887\,988\,983\,603\,750\,q^3\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[12\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-12-t+\nu]-$$
$$1\,109\,981\,842\,719\,750\,q^5\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[10\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-10-t+\nu]+$$
$$16\,564\,645\,597\,500\,q^7\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[8\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-8-t+\nu]-$$
$$110\,430\,970\,650\,q^9\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[6\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-6-t+\nu]+333\,316\,620\,q^{11}$$
$$\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\cos[4\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]$$
$$\Gamma[-4-t+\nu]-406\,980\,q^{13}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}$$
$$\cos[2\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-2-t+\nu]+$$
$$136\,q^{15}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\nu^2\right)^{\frac{t-\nu}{2}}\cos[t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-t+\nu]+$$
$$9180\,q^{14}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-1-t+\nu]$$
$$\sin[\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]-$$
$$13\,226\,850\,q^{12}\left(q^2+\frac{1}{\nu^2}\right)\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-3-t+\nu]$$
$$\sin[3\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]+$$
$$6\,721\,885\,170\,q^{10}\left(q^2+\frac{1}{\nu^2}\right)^2\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-5-t+\nu]$$
$$\sin[5\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]-$$
$$1\,490\,818\,103\,775\,q^8\left(q^2+\frac{1}{\nu^2}\right)^3\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-7-t+\nu]$$
$$\sin[7\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]+$$
$$150\,738\,274\,937\,250\,q^6\left(q^2+\frac{1}{\nu^2}\right)^4\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\Gamma[-9-t+\nu]\sin[9\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]-$$
$$6\,474\,894\,082\,531\,875\,q^4\left(q^2+\frac{1}{\nu^2}\right)^5\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\Gamma[-11-t+\nu]\sin[11\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]+$$
$$92\,854\,250\,304\,440\,625\,q^2\left(q^2+\frac{1}{\nu^2}\right)^6\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}$$
$$\Gamma[-13-t+\nu]\sin[13\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]-$$
$$191\,898\,783\,962\,510\,625\left(q^2+\frac{1}{\nu^2}\right)^7\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\nu^2\right)^{\frac{1}{2}(1+t-\nu)}\Gamma[-15-t+\nu]$$
$$\sin[15\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]+q^{16}\left(\frac{1}{\nu}\right)^{-1+t-\nu}$$
$$\left.\left(1+q^2\nu^2\right)^{\frac{1}{2}(-1+t-\nu)}\Gamma[1-t+\nu]\sin[\arctan[q\nu]-t\arctan[q\nu]+\nu\arctan[q\nu]]\right)$$

FortranForm[% /. ν → nu]

```
      (Sqrt(2/Pi)*(-191898783962510625*(1/nu)**(-nu + t)*q*
     -      (nu**(-2) + q**2)**7*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(14*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-14 + nu - t) +
     -      28887988983603750*(1/nu)**(-nu + t)*q**3*
     -      (nu**(-2) + q**2)**6*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(12*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-12 + nu - t) -
     -      1109981842719750*(1/nu)**(-nu + t)*q**5*
     -      (nu**(-2) + q**2)**5*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(10*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-10 + nu - t) +
     -      16564645597500*(1/nu)**(-nu + t)*q**7*
     -      (nu**(-2) + q**2)**4*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(8*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-8 + nu - t) -
     -      110430970650*(1/nu)**(-nu + t)*q**9*
     -      (nu**(-2) + q**2)**3*(1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(6*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-6 + nu - t) +
     -      333316620*(1/nu)**(-nu + t)*q**11*(nu**(-2) + q**2)**2*
     -      (1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(4*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-4 + nu - t) -
     -      406980*(1/nu)**(-nu + t)*q**13*(nu**(-2) + q**2)*
     -      (1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(2*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-2 + nu - t) +
     -      136*(1/nu)**(-nu + t)*q**15*
     -      (1 + nu**2*q**2)**((-nu + t)/2.)*
     -      Cos(nu*ArcTan(nu*q) - t*ArcTan(nu*q))*Gamma(nu - t) +
     -      (1/nu)**(-1 - nu + t)*q**16*
     -      (1 + nu**2*q**2)**((-1 - nu + t)/2.)*
     -      Gamma(1 + nu - t)*
     -      Sin(ArcTan(nu*q) + nu*ArcTan(nu*q) - t*ArcTan(nu*q))\
     -      + 9180*(1/nu)**(1 - nu + t)*q**14*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-1 + nu - t)*
     -      Sin(ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))\
     -      - 13226850*(1/nu)**(1 - nu + t)*q**12*
     -      (nu**(-2) + q**2)*(1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-3 + nu - t)*
     -      Sin(3*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       + 6721885170*(1/nu)**(1 - nu + t)*q**10*
     -      (nu**(-2) + q**2)**2*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-5 + nu - t)*
     -      Sin(5*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       - 1490818103775*(1/nu)**(1 - nu + t)*q**8*
     -      (nu**(-2) + q**2)**3*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-7 + nu - t)*
     -      Sin(7*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       + 150738274937250*(1/nu)**(1 - nu + t)*q**6*
     -      (nu**(-2) + q**2)**4*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-9 + nu - t)*
     -      Sin(9*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       - 6474894082531875*(1/nu)**(1 - nu + t)*q**4*
     -      (nu**(-2) + q**2)**5*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
     -      Gamma(-11 + nu - t)*
     -      Sin(11*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q)) +
     -      92854250304440625*(1/nu)**(1 - nu + t)*q**2*
     -      (nu**(-2) + q**2)**6*
     -      (1 + nu**2*q**2)**((1 - nu + t)/2.)*
```

```
-       Gamma(-13 + nu - t)*
-       Sin(13*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-         t*ArcTan(nu*q)) -
-      191898783962510625*(1/nu)**(1 - nu + t)*
-       (nu**(-2) + q**2)**7*
-       (1 + nu**2*q**2)**((1 - nu + t)/2.)*
-       Gamma(-15 + nu - t)*
-       Sin(15*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-         t*ArcTan(nu*q)))/q**16
```

l = 17

17

```
Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[- r / ν] r ^ (ν - t + 1 / 2),
 {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
   Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2 ]
```

$$\frac{1}{q^{17}}\sqrt{\frac{2}{\pi}}\left(-6\,332\,659\,870\,762\,850\,625\,q\left(\frac{1}{\nu}\right)^{15+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{15}{2}+\frac{t}{2}-\frac{\nu}{2}}\right.$$
$$\text{Cos}[15\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-15-t+\nu]+$$
$$959\,493\,919\,812\,553\,125\,q^3\left(\frac{1}{\nu}\right)^{13+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{13}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[13\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-13-t+\nu]-$$
$$37\,554\,385\,678\,684\,875\,q^5\left(\frac{1}{\nu}\right)^{11+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{11}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[11\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-11-t+\nu]+$$
$$581\,419\,060\,472\,250\,q^7\left(\frac{1}{\nu}\right)^{9+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{9}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-9-t+\nu]-$$
$$4\,141\,161\,399\,375\,q^9\left(\frac{1}{\nu}\right)^{7+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{7}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-7-t+\nu]+14\,054\,850\,810$$
$$q^{11}\left(\frac{1}{\nu}\right)^{5+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{5}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-5-t+\nu]-21\,366\,450\,q^{13}\left(\frac{1}{\nu}\right)^{3+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{3}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-3-t+\nu]+$$
$$11\,628\,q^{15}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-1-t+\nu]-q^{17}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\,\nu^2\right)^{-\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[1-t+\nu]-$$
$$153\,q^{16}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-t+\nu]\,\text{Sin}[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$581\,400\,q^{14}\left(\frac{1}{\nu}\right)^{2+t-\nu}\left(1+q^2\,\nu^2\right)^{1+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-2-t+\nu]$$
$$\text{Sin}[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-611\,080\,470\,q^{12}\left(\frac{1}{\nu}\right)^{4+t-\nu}$$
$$\left(1+q^2\,\nu^2\right)^{2+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-4-t+\nu]\,\text{Sin}[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$265\,034\,329\,560\,q^{10}\left(\frac{1}{\nu}\right)^{6+t-\nu}\left(1+q^2\,\nu^2\right)^{3+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-6-t+\nu]$$
$$\text{Sin}[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$53\,835\,098\,191\,875\,q^8\left(\frac{1}{\nu}\right)^{8+t-\nu}\left(1+q^2\,\nu^2\right)^{4+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-8-t+\nu]$$
$$\text{Sin}[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$5\,179\,915\,266\,025\,500\,q^6\left(\frac{1}{\nu}\right)^{10+t-\nu}\left(1+q^2\,\nu^2\right)^{5+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-10-t+\nu]$$
$$\text{Sin}[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$216\,659\,917\,377\,028\,125\,q^4\left(\frac{1}{\nu}\right)^{12+t-\nu}\left(1+q^2\,\nu^2\right)^{6+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-12-t+\nu]$$
$$\text{Sin}[12\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$3\,070\,380\,543\,400\,170\,000\,q^2\left(\frac{1}{\nu}\right)^{14+t-\nu}\left(1+q^2\,\nu^2\right)^{7+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-14-t+\nu]$$
$$\text{Sin}[14\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$6\,332\,659\,870\,762\,850\,625\left(\frac{1}{\nu}\right)^{16+t-\nu}\left(1+q^2\,\nu^2\right)^{8+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-16-t+\nu]$$
$$\left.\text{Sin}[16\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\right)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2) ^ (1/2 (1 - t + ν))) ^ (-1)
```

$$\frac{1}{q^{17}}\sqrt{\frac{2}{\pi}}\left(-6\,332\,659\,870\,762\,850\,625\,q\left(\frac{1}{\nu}\right)^{15+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{15}{2}+\frac{t}{2}-\frac{\nu}{2}}\right.$$
$$\text{Cos}[15\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-15-t+\nu]+$$
$$959\,493\,919\,812\,553\,125\,q^3\left(\frac{1}{\nu}\right)^{13+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{13}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[13\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-13-t+\nu]-$$
$$37\,554\,385\,678\,684\,875\,q^5\left(\frac{1}{\nu}\right)^{11+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{11}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[11\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-11-t+\nu]+$$
$$581\,419\,060\,472\,250\,q^7\left(\frac{1}{\nu}\right)^{9+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{9}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-9-t+\nu]-$$
$$4\,141\,161\,399\,375\,q^9\left(\frac{1}{\nu}\right)^{7+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{7}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-7-t+\nu]+14\,054\,850\,810$$
$$q^{11}\left(\frac{1}{\nu}\right)^{5+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{5}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-5-t+\nu]-21\,366\,450\,q^{13}\left(\frac{1}{\nu}\right)^{3+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{3}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-3-t+\nu]+$$
$$11\,628\,q^{15}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-1-t+\nu]-q^{17}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\,\nu^2\right)^{-\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[1-t+\nu]-$$
$$153\,q^{16}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-t+\nu]\,\text{Sin}[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$581\,400\,q^{14}\left(\frac{1}{\nu}\right)^{2+t-\nu}\left(1+q^2\,\nu^2\right)^{1+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-2-t+\nu]$$
$$\text{Sin}[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-611\,080\,470\,q^{12}\left(\frac{1}{\nu}\right)^{4+t-\nu}$$
$$\left(1+q^2\,\nu^2\right)^{2+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-4-t+\nu]\,\text{Sin}[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$265\,034\,329\,560\,q^{10}\left(\frac{1}{\nu}\right)^{6+t-\nu}\left(1+q^2\,\nu^2\right)^{3+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-6-t+\nu]$$
$$\text{Sin}[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$53\,835\,098\,191\,875\,q^8\left(\frac{1}{\nu}\right)^{8+t-\nu}\left(1+q^2\,\nu^2\right)^{4+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-8-t+\nu]$$
$$\text{Sin}[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$5\,179\,915\,266\,025\,500\,q^6\left(\frac{1}{\nu}\right)^{10+t-\nu}\left(1+q^2\,\nu^2\right)^{5+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-10-t+\nu]$$
$$\text{Sin}[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$216\,659\,917\,377\,028\,125\,q^4\left(\frac{1}{\nu}\right)^{12+t-\nu}\left(1+q^2\,\nu^2\right)^{6+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-12-t+\nu]$$
$$\text{Sin}[12\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$3\,070\,380\,543\,400\,170\,000\,q^2\left(\frac{1}{\nu}\right)^{14+t-\nu}\left(1+q^2\,\nu^2\right)^{7+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-14-t+\nu]$$
$$\text{Sin}[14\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$6\,332\,659\,870\,762\,850\,625\left(\frac{1}{\nu}\right)^{16+t-\nu}\left(1+q^2\,\nu^2\right)^{8+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-16-t+\nu]$$
$$\left.\text{Sin}[16\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\right)$$

FortranForm[% /. ν → nu]

```
      (Sqrt(2/Pi)*(-6332659870762850625*(1/nu)**(15 - nu + t)*q*
     -      (1 + nu**2*q**2)**(7.5 - nu/2. + t/2.)*
     -      Cos(15*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-15 + nu - t) +
     -     959493919812553125*(1/nu)**(13 - nu + t)*q**3*
     -      (1 + nu**2*q**2)**(6.5 - nu/2. + t/2.)*
     -      Cos(13*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-13 + nu - t) -
     -     37554385678684875*(1/nu)**(11 - nu + t)*q**5*
     -      (1 + nu**2*q**2)**(5.5 - nu/2. + t/2.)*
     -      Cos(11*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-11 + nu - t) +
     -     581419060472250*(1/nu)**(9 - nu + t)*q**7*
     -      (1 + nu**2*q**2)**(4.5 - nu/2. + t/2.)*
     -      Cos(9*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-9 + nu - t) -
     -     4141161399375*(1/nu)**(7 - nu + t)*q**9*
     -      (1 + nu**2*q**2)**(3.5 - nu/2. + t/2.)*
     -      Cos(7*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-7 + nu - t) +
     -     14054850810*(1/nu)**(5 - nu + t)*q**11*
     -      (1 + nu**2*q**2)**(2.5 - nu/2. + t/2.)*
     -      Cos(5*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-5 + nu - t) -
     -     21366450*(1/nu)**(3 - nu + t)*q**13*
     -      (1 + nu**2*q**2)**(1.5 - nu/2. + t/2.)*
     -      Cos(3*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q))*Gamma(-3 + nu - t) +
     -     11628*(1/nu)**(1 - nu + t)*q**15*
     -      (1 + nu**2*q**2)**(0.5 - nu/2. + t/2.)*
     -      Cos(ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))*
     -      Gamma(-1 + nu - t) -
     -     (1/nu)**(-1 - nu + t)*q**17*
     -      (1 + nu**2*q**2)**(-0.5 - nu/2. + t/2.)*
     -      Cos(ArcTan(nu*q) + nu*ArcTan(nu*q) - t*ArcTan(nu*q))*
     -      Gamma(1 + nu - t) +
     -     153*(1/nu)**(-nu + t)*q**16*
     -      (1 + nu**2*q**2)**(-nu/2. + t/2.)*Gamma(nu - t)*
     -      Sin(nu*ArcTan(nu*q) - t*ArcTan(nu*q)) +
     -     581400*(1/nu)**(2 - nu + t)*q**14*
     -      (1 + nu**2*q**2)**(1 - nu/2. + t/2.)*
     -      Gamma(-2 + nu - t)*
     -      Sin(2*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       - 611080470*(1/nu)**(4 - nu + t)*q**12*
     -      (1 + nu**2*q**2)**(2 - nu/2. + t/2.)*
     -      Gamma(-4 + nu - t)*
     -      Sin(4*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       + 265034329560*(1/nu)**(6 - nu + t)*q**10*
     -      (1 + nu**2*q**2)**(3 - nu/2. + t/2.)*
     -      Gamma(-6 + nu - t)*
     -      Sin(6*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       - 53835098191875*(1/nu)**(8 - nu + t)*q**8*
     -      (1 + nu**2*q**2)**(4 - nu/2. + t/2.)*
     -      Gamma(-8 + nu - t)*
     -      Sin(8*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -       + 5179915266025500*(1/nu)**(10 - nu + t)*q**6*
     -      (1 + nu**2*q**2)**(5 - nu/2. + t/2.)*
     -      Gamma(-10 + nu - t)*
     -      Sin(10*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q)) -
     -     216659917377028125*(1/nu)**(12 - nu + t)*q**4*
     -      (1 + nu**2*q**2)**(6 - nu/2. + t/2.)*
     -      Gamma(-12 + nu - t)*
     -      Sin(12*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -        t*ArcTan(nu*q)) +
     -     3070380543400170000*(1/nu)**(14 - nu + t)*q**2*
     -      (1 + nu**2*q**2)**(7 - nu/2. + t/2.)*
```

```
-        Gamma(-14 + nu - t)*
-        Sin(14*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)) -
-       6332659870762850625*(1/nu)**(16 - nu + t)*
-        (1 + nu**2*q**2)**(8 - nu/2. + t/2.)*
-        Gamma(-16 + nu - t)*
-        Sin(16*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)))/q**17
```

l = 18

18

Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[- r / ν] r^(ν - t + 1 / 2), {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] && Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2]

$$
\begin{aligned}
&-\frac{1}{q^{18}}\sqrt{\frac{2}{\pi}}\left(221\,643\,095\,476\,699\,771\,875\,q\left(\frac{1}{\nu}\right)^{16+t-\nu}\left(1+q^2\,\nu^2\right)^{8+\frac{t}{2}-\frac{\nu}{2}}\right.\\
&\quad\text{Cos}[16\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-16-t+\nu]-\\
&\quad 33\,774\,185\,977\,401\,870\,000\,q^3\left(\frac{1}{\nu}\right)^{14+t-\nu}\left(1+q^2\,\nu^2\right)^{7+\frac{t}{2}-\frac{\nu}{2}}\\
&\quad\text{Cos}[14\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-14-t+\nu]+\\
&\quad 1\,343\,291\,487\,737\,574\,375\,q^5\left(\frac{1}{\nu}\right)^{12+t-\nu}\left(1+q^2\,\nu^2\right)^{6+\frac{t}{2}-\frac{\nu}{2}}\\
&\quad\text{Cos}[12\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-12-t+\nu]-\\
&\quad 21\,459\,648\,959\,248\,500\,q^7\left(\frac{1}{\nu}\right)^{10+t-\nu}\left(1+q^2\,\nu^2\right)^{5+\frac{t}{2}-\frac{\nu}{2}}\\
&\quad\text{Cos}[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-10-t+\nu]+\\
&\quad 161\,505\,294\,575\,625\,q^9\left(\frac{1}{\nu}\right)^{8+t-\nu}\left(1+q^2\,\nu^2\right)^{4+\frac{t}{2}-\frac{\nu}{2}}\\
&\quad\text{Cos}[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-8-t+\nu]-\\
&\quad 602\,350\,749\,000\,q^{11}\left(\frac{1}{\nu}\right)^{6+t-\nu}\left(1+q^2\,\nu^2\right)^{3+\frac{t}{2}-\frac{\nu}{2}}\\
&\quad\text{Cos}[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-6-t+\nu]+1\,081\,142\,370\\
&\quad q^{13}\left(\frac{1}{\nu}\right)^{4+t-\nu}\left(1+q^2\,\nu^2\right)^{2+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\\
&\quad\text{Gamma}[-4-t+\nu]-813\,960\,q^{15}\left(\frac{1}{\nu}\right)^{2+t-\nu}\left(1+q^2\,\nu^2\right)^{1+\frac{t}{2}-\frac{\nu}{2}}\\
&\quad\text{Cos}[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-2-t+\nu]+\\
&\quad 171\,q^{17}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-t+\nu]+\\
&\quad 14\,535\,q^{16}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-1-t+\nu]\\
&\quad\text{Sin}[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-\\
&\quad 33\,575\,850\,q^{14}\left(\frac{1}{\nu}\right)^{3+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{3}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-3-t+\nu]\\
&\quad\text{Sin}[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+\\
&\quad 28\,109\,701\,620\,q^{12}\left(\frac{1}{\nu}\right)^{5+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{5}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-5-t+\nu]\\
&\quad\text{Sin}[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-\\
&\quad 10\,767\,019\,638\,375\,q^{10}\left(\frac{1}{\nu}\right)^{7+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{7}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-7-t+\nu]\\
&\quad\text{Sin}[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+\\
&\quad 2\,034\,966\,711\,652\,875\,q^8\left(\frac{1}{\nu}\right)^{9+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{9}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-9-t+\nu]\\
&\quad\text{Sin}[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-
\end{aligned}
$$

$$187\,771\,928\,393\,424\,375\,q^6\left(\frac{1}{\nu}\right)^{11+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{11}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-11-t+\nu]$$
$$\text{Sin}[11\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$7\,675\,951\,358\,500\,425\,000\,q^4\left(\frac{1}{\nu}\right)^{13+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{13}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-13-t+\nu]$$
$$\text{Sin}[13\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$107\,655\,217\,802\,968\,460\,625\,q^2\left(\frac{1}{\nu}\right)^{15+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{15}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-15-t+\nu]$$
$$\text{Sin}[15\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$221\,643\,095\,476\,699\,771\,875\left(\frac{1}{\nu}\right)^{17+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{17}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-17-t+\nu]$$
$$\text{Sin}[17\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+q^{18}\left(\frac{1}{\nu}\right)^{-1+t-\nu}$$
$$\left(1+q^2\,\nu^2\right)^{-\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[1-t+\nu]\,\text{Sin}[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\Big)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1)
```

$$-\frac{1}{q^{18}}\sqrt{\frac{2}{\pi}}\Big(221\,643\,095\,476\,699\,771\,875\,q\left(\frac{1}{\nu}\right)^{16+t-\nu}\left(1+q^2\,\nu^2\right)^{8+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[16\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-16-t+\nu]-$$
$$33\,774\,185\,977\,401\,870\,000\,q^3\left(\frac{1}{\nu}\right)^{14+t-\nu}\left(1+q^2\,\nu^2\right)^{7+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[14\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-14-t+\nu]+$$
$$1\,343\,291\,487\,737\,574\,375\,q^5\left(\frac{1}{\nu}\right)^{12+t-\nu}\left(1+q^2\,\nu^2\right)^{6+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[12\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-12-t+\nu]-$$
$$21\,459\,648\,959\,248\,500\,q^7\left(\frac{1}{\nu}\right)^{10+t-\nu}\left(1+q^2\,\nu^2\right)^{5+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-10-t+\nu]+$$
$$161\,505\,294\,575\,625\,q^9\left(\frac{1}{\nu}\right)^{8+t-\nu}\left(1+q^2\,\nu^2\right)^{4+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-8-t+\nu]-$$
$$602\,350\,749\,000\,q^{11}\left(\frac{1}{\nu}\right)^{6+t-\nu}\left(1+q^2\,\nu^2\right)^{3+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-6-t+\nu]+1\,081\,142\,370$$
$$q^{13}\left(\frac{1}{\nu}\right)^{4+t-\nu}\left(1+q^2\,\nu^2\right)^{2+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-4-t+\nu]-813\,960\,q^{15}\left(\frac{1}{\nu}\right)^{2+t-\nu}\left(1+q^2\,\nu^2\right)^{1+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-2-t+\nu]+$$
$$171\,q^{17}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-t+\nu]+$$
$$14\,535\,q^{16}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-1-t+\nu]$$
$$\text{Sin}[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$33\,575\,850\,q^{14}\left(\frac{1}{\nu}\right)^{3+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{3}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-3-t+\nu]$$
$$\text{Sin}[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$28\,109\,701\,620\,q^{12}\left(\frac{1}{\nu}\right)^{5+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{5}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-5-t+\nu]$$
$$\text{Sin}[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$

$$10\,767\,019\,638\,375\,q^{10}\left(\frac{1}{\nu}\right)^{7+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{7}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-7-t+\nu]$$
$$\text{Sin}[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$2\,034\,966\,711\,652\,875\,q^8\left(\frac{1}{\nu}\right)^{9+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{9}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-9-t+\nu]$$
$$\text{Sin}[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$187\,771\,928\,393\,424\,375\,q^6\left(\frac{1}{\nu}\right)^{11+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{11}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-11-t+\nu]$$
$$\text{Sin}[11\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$7\,675\,951\,358\,500\,425\,000\,q^4\left(\frac{1}{\nu}\right)^{13+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{13}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-13-t+\nu]$$
$$\text{Sin}[13\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$107\,655\,217\,802\,968\,460\,625\,q^2\left(\frac{1}{\nu}\right)^{15+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{15}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-15-t+\nu]$$
$$\text{Sin}[15\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$221\,643\,095\,476\,699\,771\,875\left(\frac{1}{\nu}\right)^{17+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{17}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-17-t+\nu]$$
$$\text{Sin}[17\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+q^{18}\left(\frac{1}{\nu}\right)^{-1+t-\nu}$$
$$\left(1+q^2\,\nu^2\right)^{-\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[1-t+\nu]\,\text{Sin}[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\Big)$$

FortranForm[% /. ν → nu]

```
      -((Sqrt(2/Pi)*(22164309547669977187 5*(1/nu)**(16 - nu + t)*
     -         q*(1 + nu**2*q**2)**(8 - nu/2. + t/2.)*
     -         Cos(16*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -           t*ArcTan(nu*q))*Gamma(-16 + nu - t) -
     -        33774185977401870000*(1/nu)**(14 - nu + t)*q**3*
     -         (1 + nu**2*q**2)**(7 - nu/2. + t/2.)*
     -         Cos(14*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -           t*ArcTan(nu*q))*Gamma(-14 + nu - t) +
     -        1343291487737574375*(1/nu)**(12 - nu + t)*q**5*
     -         (1 + nu**2*q**2)**(6 - nu/2. + t/2.)*
     -         Cos(12*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -           t*ArcTan(nu*q))*Gamma(-12 + nu - t) -
     -        21459648959248500*(1/nu)**(10 - nu + t)*q**7*
     -         (1 + nu**2*q**2)**(5 - nu/2. + t/2.)*
     -         Cos(10*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -           t*ArcTan(nu*q))*Gamma(-10 + nu - t) +
     -        161505294575625*(1/nu)**(8 - nu + t)*q**9*
     -         (1 + nu**2*q**2)**(4 - nu/2. + t/2.)*
     -         Cos(8*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -           t*ArcTan(nu*q))*Gamma(-8 + nu - t) -
     -        602350749000*(1/nu)**(6 - nu + t)*q**11*
     -         (1 + nu**2*q**2)**(3 - nu/2. + t/2.)*
     -         Cos(6*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -           t*ArcTan(nu*q))*Gamma(-6 + nu - t) +
     -        1081142370*(1/nu)**(4 - nu + t)*q**13*
     -         (1 + nu**2*q**2)**(2 - nu/2. + t/2.)*
     -         Cos(4*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -           t*ArcTan(nu*q))*Gamma(-4 + nu - t) -
     -        813960*(1/nu)**(2 - nu + t)*q**15*
     -         (1 + nu**2*q**2)**(1 - nu/2. + t/2.)*
     -         Cos(2*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -           t*ArcTan(nu*q))*Gamma(-2 + nu - t) +
     -        171*(1/nu)**(-nu + t)*q**17*
     -         (1 + nu**2*q**2)**(-nu/2. + t/2.)*
     -         Cos(nu*ArcTan(nu*q) - t*ArcTan(nu*q))*Gamma(nu - t)\
     -         + (1/nu)**(-1 - nu + t)*q**18*
     -         (1 + nu**2*q**2)**(-0.5 - nu/2. + t/2.)*
     -         Gamma(1 + nu - t)*
     -         Sin(ArcTan(nu*q) + nu*ArcTan(nu*q) - t*ArcTan(nu*q))
```

```
-           + 14535*(1/nu)**(1 - nu + t)*q**16*
-          (1 + nu**2*q**2)**(0.5 - nu/2. + t/2.)*
-          Gamma(-1 + nu - t)*
-          Sin(ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
-           - 33575850*(1/nu)**(3 - nu + t)*q**14*
-          (1 + nu**2*q**2)**(1.5 - nu/2. + t/2.)*
-          Gamma(-3 + nu - t)*
-          Sin(3*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) +
-         28109701620*(1/nu)**(5 - nu + t)*q**12*
-          (1 + nu**2*q**2)**(2.5 - nu/2. + t/2.)*
-          Gamma(-5 + nu - t)*
-          Sin(5*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) -
-         10767019638375*(1/nu)**(7 - nu + t)*q**10*
-          (1 + nu**2*q**2)**(3.5 - nu/2. + t/2.)*
-          Gamma(-7 + nu - t)*
-          Sin(7*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) +
-         2034966711652875*(1/nu)**(9 - nu + t)*q**8*
-          (1 + nu**2*q**2)**(4.5 - nu/2. + t/2.)*
-          Gamma(-9 + nu - t)*
-          Sin(9*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) -
-         187771928393424375*(1/nu)**(11 - nu + t)*q**6*
-          (1 + nu**2*q**2)**(5.5 - nu/2. + t/2.)*
-          Gamma(-11 + nu - t)*
-          Sin(11*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) +
-         7675951358500425000*(1/nu)**(13 - nu + t)*q**4*
-          (1 + nu**2*q**2)**(6.5 - nu/2. + t/2.)*
-          Gamma(-13 + nu - t)*
-          Sin(13*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) -
-         107655217802968460625*(1/nu)**(15 - nu + t)*q**2*
-          (1 + nu**2*q**2)**(7.5 - nu/2. + t/2.)*
-          Gamma(-15 + nu - t)*
-          Sin(15*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q)) +
-         2216430954766999771875*(1/nu)**(17 - nu + t)*
-          (1 + nu**2*q**2)**(8.5 - nu/2. + t/2.)*
-          Gamma(-17 + nu - t)*
-          Sin(17*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-            t*ArcTan(nu*q))))/q**18)
```

l = 19

19

Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[-r / ν] r^(ν - t + 1 / 2), {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] && Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2]

$$\frac{1}{q^{19}}\sqrt{\frac{2}{\pi}}\left(-8\,200\,794\,532\,637\,891\,559\,375\,q\left(\frac{1}{\nu}\right)^{17+t-\nu}\left(1+q^2\nu^2\right)^{\frac{17}{2}+\frac{t}{2}-\frac{\nu}{2}}\right.$$
$$\cos[17\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-17-t+\nu]+$$
$$1\,255\,977\,541\,034\,632\,040\,625\,q^3\left(\frac{1}{\nu}\right)^{15+t-\nu}\left(1+q^2\nu^2\right)^{\frac{15}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\cos[15\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-15-t+\nu]-$$
$$50\,661\,278\,966\,102\,805\,000\,q^5\left(\frac{1}{\nu}\right)^{13+t-\nu}\left(1+q^2\nu^2\right)^{\frac{13}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\cos[13\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-13-t+\nu]+$$
$$831\,561\,397\,170\,879\,375\,q^7\left(\frac{1}{\nu}\right)^{11+t-\nu}\left(1+q^2\nu^2\right)^{\frac{11}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\cos[11\arctan[q\nu]+t\arctan[q\nu]-\nu\arctan[q\nu]]\,\Gamma[-11-t+\nu]-$$

$$6\,557\,114\,959\,770\,375\,q^9\left(\frac{1}{\nu}\right)^{9+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{9}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-9-t+\nu]+$$
$$26\,428\,139\,112\,375\,q^{11}\left(\frac{1}{\nu}\right)^{7+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{7}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-7-t+\nu]-54\,057\,118\,500$$
$$q^{13}\left(\frac{1}{\nu}\right)^{5+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{5}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-5-t+\nu]+51\,482\,970\,q^{15}\left(\frac{1}{\nu}\right)^{3+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{3}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-3-t+\nu]-$$
$$17\,955\,q^{17}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-1-t+\nu]+q^{19}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\,\nu^2\right)^{-\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[1-t+\nu]+$$
$$190\,q^{18}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-t+\nu]\,\text{Sin}[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$1\,119\,195\,q^{16}\left(\frac{1}{\nu}\right)^{2+t-\nu}\left(1+q^2\,\nu^2\right)^{1+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-2-t+\nu]$$
$$\text{Sin}[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+1\,853\,386\,920\,q^{14}\left(\frac{1}{\nu}\right)^{4+t-\nu}$$
$$\left(1+q^2\,\nu^2\right)^{2+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-4-t+\nu]\,\text{Sin}[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$1\,305\,093\,289\,500\,q^{12}\left(\frac{1}{\nu}\right)^{6+t-\nu}\left(1+q^2\,\nu^2\right)^{3+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-6-t+\nu]$$
$$\text{Sin}[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$452\,214\,824\,811\,750\,q^{10}\left(\frac{1}{\nu}\right)^{8+t-\nu}\left(1+q^2\,\nu^2\right)^{4+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-8-t+\nu]$$
$$\text{Sin}[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$80\,473\,683\,597\,181\,875\,q^8\left(\frac{1}{\nu}\right)^{10+t-\nu}\left(1+q^2\,\nu^2\right)^{5+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-10-t+\nu]$$
$$\text{Sin}[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$7\,164\,221\,267\,933\,730\,000\,q^6\left(\frac{1}{\nu}\right)^{12+t-\nu}\left(1+q^2\,\nu^2\right)^{6+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-12-t+\nu]$$
$$\text{Sin}[12\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$287\,080\,580\,807\,915\,895\,000\,q^4\left(\frac{1}{\nu}\right)^{14+t-\nu}\left(1+q^2\,\nu^2\right)^{7+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-14-t+\nu]$$
$$\text{Sin}[14\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$3\,989\,575\,718\,580\,595\,893\,750\,q^2\left(\frac{1}{\nu}\right)^{16+t-\nu}\left(1+q^2\,\nu^2\right)^{8+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-16-t+\nu]$$
$$\text{Sin}[16\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$8\,200\,794\,532\,637\,891\,559\,375\left(\frac{1}{\nu}\right)^{18+t-\nu}\left(1+q^2\,\nu^2\right)^{9+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-18-t+\nu]$$
$$\text{Sin}[18\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\Big)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → ((ν^-2 + q^2) ^ (1/2 (1 - t + ν))) ^ (-1)
```

$$\frac{1}{q^{19}}\sqrt{\frac{2}{\pi}}\left(-8\,200\,794\,532\,637\,891\,559\,375\,q\left(\frac{1}{\nu}\right)^{17+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{17}{2}+\frac{t}{2}-\frac{\nu}{2}}\right.$$
$$\text{Cos}[17\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-17-t+\nu]+$$

$$1\,255\,977\,541\,034\,632\,040\,625\,q^3\left(\frac{1}{\nu}\right)^{15+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{15}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[15\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-15-t+\nu]-$$
$$50\,661\,278\,966\,102\,805\,000\,q^5\left(\frac{1}{\nu}\right)^{13+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{13}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[13\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-13-t+\nu]+$$
$$831\,561\,397\,170\,879\,375\,q^7\left(\frac{1}{\nu}\right)^{11+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{11}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[11\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-11-t+\nu]-$$
$$6\,557\,114\,959\,770\,375\,q^9\left(\frac{1}{\nu}\right)^{9+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{9}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-9-t+\nu]+$$
$$26\,428\,139\,112\,375\,q^{11}\left(\frac{1}{\nu}\right)^{7+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{7}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-7-t+\nu]-54\,057\,118\,500$$
$$q^{13}\left(\frac{1}{\nu}\right)^{5+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{5}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-5-t+\nu]+51\,482\,970\,q^{15}\left(\frac{1}{\nu}\right)^{3+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{3}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-3-t+\nu]-$$
$$17\,955\,q^{17}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-1-t+\nu]+q^{19}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\,\nu^2\right)^{-\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[1-t+\nu]+$$
$$190\,q^{18}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-t+\nu]\,\text{Sin}[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$1\,119\,195\,q^{16}\left(\frac{1}{\nu}\right)^{2+t-\nu}\left(1+q^2\,\nu^2\right)^{1+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-2-t+\nu]$$
$$\text{Sin}[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+1\,853\,386\,920\,q^{14}\left(\frac{1}{\nu}\right)^{4+t-\nu}$$
$$\left(1+q^2\,\nu^2\right)^{2+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-4-t+\nu]\,\text{Sin}[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$1\,305\,093\,289\,500\,q^{12}\left(\frac{1}{\nu}\right)^{6+t-\nu}\left(1+q^2\,\nu^2\right)^{3+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-6-t+\nu]$$
$$\text{Sin}[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$452\,214\,824\,811\,750\,q^{10}\left(\frac{1}{\nu}\right)^{8+t-\nu}\left(1+q^2\,\nu^2\right)^{4+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-8-t+\nu]$$
$$\text{Sin}[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$80\,473\,683\,597\,181\,875\,q^8\left(\frac{1}{\nu}\right)^{10+t-\nu}\left(1+q^2\,\nu^2\right)^{5+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-10-t+\nu]$$
$$\text{Sin}[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$7\,164\,221\,267\,933\,730\,000\,q^6\left(\frac{1}{\nu}\right)^{12+t-\nu}\left(1+q^2\,\nu^2\right)^{6+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-12-t+\nu]$$
$$\text{Sin}[12\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$287\,080\,580\,807\,915\,895\,000\,q^4\left(\frac{1}{\nu}\right)^{14+t-\nu}\left(1+q^2\,\nu^2\right)^{7+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-14-t+\nu]$$
$$\text{Sin}[14\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$3\,989\,575\,718\,580\,595\,893\,750\,q^2\left(\frac{1}{\nu}\right)^{16+t-\nu}\left(1+q^2\,\nu^2\right)^{8+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-16-t+\nu]$$
$$\text{Sin}[16\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$8\,200\,794\,532\,637\,891\,559\,375\left(\frac{1}{\nu}\right)^{18+t-\nu}\left(1+q^2\,\nu^2\right)^{9+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-18-t+\nu]$$

```
Sin[18 ArcTan[q ν] + t ArcTan[q ν] - ν ArcTan[q ν]]
```

FortranForm[% /. ν → nu]

```
      (Sqrt(2/Pi)*(-8200794532637891559375*(1/nu)**(17 - nu + t)*
     -       q*(1 + nu**2*q**2)**(8.5 - nu/2. + t/2.)*
     -       Cos(17*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -         t*ArcTan(nu*q))*Gamma(-17 + nu - t) +
     -       1255977541034632040625*(1/nu)**(15 - nu + t)*q**3*
     -        (1 + nu**2*q**2)**(7.5 - nu/2. + t/2.)*
     -        Cos(15*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -         t*ArcTan(nu*q))*Gamma(-15 + nu - t) -
     -       506612789661028050000*(1/nu)**(13 - nu + t)*q**5*
     -        (1 + nu**2*q**2)**(6.5 - nu/2. + t/2.)*
     -        Cos(13*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -         t*ArcTan(nu*q))*Gamma(-13 + nu - t) +
     -       831561397170879375*(1/nu)**(11 - nu + t)*q**7*
     -        (1 + nu**2*q**2)**(5.5 - nu/2. + t/2.)*
     -        Cos(11*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -         t*ArcTan(nu*q))*Gamma(-11 + nu - t) -
     -       6557114959770375*(1/nu)**(9 - nu + t)*q**9*
     -        (1 + nu**2*q**2)**(4.5 - nu/2. + t/2.)*
     -        Cos(9*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -         t*ArcTan(nu*q))*Gamma(-9 + nu - t) +
     -       26428139112375*(1/nu)**(7 - nu + t)*q**11*
     -        (1 + nu**2*q**2)**(3.5 - nu/2. + t/2.)*
     -        Cos(7*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -         t*ArcTan(nu*q))*Gamma(-7 + nu - t) -
     -       54057118500*(1/nu)**(5 - nu + t)*q**13*
     -        (1 + nu**2*q**2)**(2.5 - nu/2. + t/2.)*
     -        Cos(5*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -         t*ArcTan(nu*q))*Gamma(-5 + nu - t) +
     -       51482970*(1/nu)**(3 - nu + t)*q**15*
     -        (1 + nu**2*q**2)**(1.5 - nu/2. + t/2.)*
     -        Cos(3*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -         t*ArcTan(nu*q))*Gamma(-3 + nu - t) -
     -       17955*(1/nu)**(1 - nu + t)*q**17*
     -        (1 + nu**2*q**2)**(0.5 - nu/2. + t/2.)*
     -        Cos(ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))*
     -        Gamma(-1 + nu - t) +
     -       (1/nu)**(-1 - nu + t)*q**19*
     -        (1 + nu**2*q**2)**(-0.5 - nu/2. + t/2.)*
     -        Cos(ArcTan(nu*q) + nu*ArcTan(nu*q) - t*ArcTan(nu*q))*
     -        Gamma(1 + nu - t) -
     -       190*(1/nu)**(-nu + t)*q**18*
     -        (1 + nu**2*q**2)**(-nu/2. + t/2.)*Gamma(nu - t)*
     -        Sin(nu*ArcTan(nu*q) - t*ArcTan(nu*q)) -
     -       1119195*(1/nu)**(2 - nu + t)*q**16*
     -        (1 + nu**2*q**2)**(1 - nu/2. + t/2.)*
     -        Gamma(-2 + nu - t)*
     -        Sin(2*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -         + 1853386920*(1/nu)**(4 - nu + t)*q**14*
     -        (1 + nu**2*q**2)**(2 - nu/2. + t/2.)*
     -        Gamma(-4 + nu - t)*
     -        Sin(4*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -         - 1305093289500*(1/nu)**(6 - nu + t)*q**12*
     -        (1 + nu**2*q**2)**(3 - nu/2. + t/2.)*
     -        Gamma(-6 + nu - t)*
     -        Sin(6*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -         + 452214824811750*(1/nu)**(8 - nu + t)*q**10*
     -        (1 + nu**2*q**2)**(4 - nu/2. + t/2.)*
     -        Gamma(-8 + nu - t)*
     -        Sin(8*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
     -         - 80473683597181875*(1/nu)**(10 - nu + t)*q**8*
     -        (1 + nu**2*q**2)**(5 - nu/2. + t/2.)*
     -        Gamma(-10 + nu - t)*
     -        Sin(10*ArcTan(nu*q) - nu*ArcTan(nu*q) +
```

```
-          t*ArcTan(nu*q)) +
-       71642212679337300000*(1/nu)**(12 - nu + t)*q**6*
-        (1 + nu**2*q**2)**(6 - nu/2. + t/2.)*
-        Gamma(-12 + nu - t)*
-        Sin(12*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)) -
-       28708058080791589500*(1/nu)**(14 - nu + t)*q**4*
-        (1 + nu**2*q**2)**(7 - nu/2. + t/2.)*
-        Gamma(-14 + nu - t)*
-        Sin(14*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)) +
-       3989575718580595893750*(1/nu)**(16 - nu + t)*q**2*
-        (1 + nu**2*q**2)**(8 - nu/2. + t/2.)*
-        Gamma(-16 + nu - t)*
-        Sin(16*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)) -
-       8200794532637891559375*(1/nu)**(18 - nu + t)*
-        (1 + nu**2*q**2)**(9 - nu/2. + t/2.)*
-        Gamma(-18 + nu - t)*
-        Sin(18*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)))/q**19
```

l = 20

20

Integrate[Sqrt[q] BesselJ[l + 1 / 2, q r] Exp[- r / ν] r^(ν - t + 1 / 2), {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] && Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2]

$$\frac{1}{q^{20}}\sqrt{\frac{2}{\pi}}\left(-319\,830\,986\,772\,877\,770\,815\,625\,q\left(\frac{1}{\nu}\right)^{18+t-\nu}\left(1+q^2\,\nu^2\right)^{9+\frac{t}{2}-\frac{\nu}{2}}\right.$$
$$\cos[18\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-18-t+\nu]+$$
$$49\,204\,767\,195\,827\,349\,356\,250\,q^3\left(\frac{1}{\nu}\right)^{16+t-\nu}\left(1+q^2\,\nu^2\right)^{8+\frac{t}{2}-\frac{\nu}{2}}$$
$$\cos[16\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-16-t+\nu]-$$
$$2\,009\,564\,065\,655\,411\,265\,000\,q^5\left(\frac{1}{\nu}\right)^{14+t-\nu}\left(1+q^2\,\nu^2\right)^{7+\frac{t}{2}-\frac{\nu}{2}}$$
$$\cos[14\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-14-t+\nu]+$$
$$33\,774\,185\,977\,401\,870\,000\,q^7\left(\frac{1}{\nu}\right)^{12+t-\nu}\left(1+q^2\,\nu^2\right)^{6+\frac{t}{2}-\frac{\nu}{2}}$$
$$\cos[12\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-12-t+\nu]-$$
$$277\,187\,132\,390\,293\,125\,q^9\left(\frac{1}{\nu}\right)^{10+t-\nu}\left(1+q^2\,\nu^2\right)^{5+\frac{t}{2}-\frac{\nu}{2}}$$
$$\cos[10\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-10-t+\nu]+$$
$$1\,192\,202\,719\,958\,250\,q^{11}\left(\frac{1}{\nu}\right)^{8+t-\nu}\left(1+q^2\,\nu^2\right)^{4+\frac{t}{2}-\frac{\nu}{2}}$$
$$\cos[8\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-8-t+\nu]-$$
$$2\,710\,578\,370\,500\,q^{13}\left(\frac{1}{\nu}\right)^{6+t-\nu}\left(1+q^2\,\nu^2\right)^{3+\frac{t}{2}-\frac{\nu}{2}}$$
$$\cos[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-6-t+\nu]+3\,088\,978\,200$$
$$q^{15}\left(\frac{1}{\nu}\right)^{4+t-\nu}\left(1+q^2\,\nu^2\right)^{2+\frac{t}{2}-\frac{\nu}{2}}\cos[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\Gamma[-4-t+\nu]-1\,514\,205\,q^{17}\left(\frac{1}{\nu}\right)^{2+t-\nu}\left(1+q^2\,\nu^2\right)^{1+\frac{t}{2}-\frac{\nu}{2}}$$
$$\cos[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-2-t+\nu]+$$
$$210\,q^{19}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t}{2}-\frac{\nu}{2}}\cos[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\Gamma[-t+\nu]+$$
$$21\,945\,q^{18}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\Gamma[-1-t+\nu]$$

$$\text{Sin}[\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]] - 77\,224\,455\,q^{16}\left(\frac{1}{\nu}\right)^{3+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{3}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-3-t+\nu]\,\text{Sin}[3\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]] + 100\,391\,791\,500\,q^{14}\left(\frac{1}{\nu}\right)^{5+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{5}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-5-t+\nu]\,\text{Sin}[5\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]] - 61\,665\,657\,928\,875\,q^{12}\left(\frac{1}{\nu}\right)^{7+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{7}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-7-t+\nu]\,\text{Sin}[7\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]] + 19\,671\,344\,879\,311\,125\,q^{10}\left(\frac{1}{\nu}\right)^{9+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{9}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-9-t+\nu]\,\text{Sin}[9\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]] - 3\,326\,245\,588\,683\,517\,500\,q^{8}\left(\frac{1}{\nu}\right)^{11+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{11}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-11-t+\nu]\,\text{Sin}[11\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]] + 287\,080\,580\,807\,915\,895\,000\,q^{6}\left(\frac{1}{\nu}\right)^{13+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{13}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-13-t+\nu]\,\text{Sin}[13\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]] - 11\,303\,797\,869\,311\,688\,365\,625\,q^{4}\left(\frac{1}{\nu}\right)^{15+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{15}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-15-t+\nu]\,\text{Sin}[15\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]] + 155\,815\,096\,120\,119\,939\,628\,125\,q^{2}\left(\frac{1}{\nu}\right)^{17+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{17}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-17-t+\nu]\,\text{Sin}[17\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]] - 319\,830\,986\,772\,877\,770\,815\,625\left(\frac{1}{\nu}\right)^{19+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{19}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-19-t+\nu]\,\text{Sin}[19\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]] + q^{20}\left(\frac{1}{\nu}\right)^{-1+t-\nu}\left(1+q^2\,\nu^2\right)^{-\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[1-t+\nu]\,\text{Sin}[\text{ArcTan}[q\,\nu] - t\,\text{ArcTan}[q\,\nu] + \nu\,\text{ArcTan}[q\,\nu]]\Bigg)$$

```
% /. ν^(1-t+ν) (1 + q^2 ν^2)^(1/2 (-1+t-ν)) → (((ν^-2 + q^2)^(1/2 (1 - t + ν)))^(-1))
```

$$\frac{1}{q^{20}}\sqrt{\frac{2}{\pi}}\Bigg(-319\,830\,986\,772\,877\,770\,815\,625\,q\left(\frac{1}{\nu}\right)^{18+t-\nu}\left(1+q^2\,\nu^2\right)^{9+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[18\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-18-t+\nu] + 49\,204\,767\,195\,827\,349\,356\,250\,q^{3}\left(\frac{1}{\nu}\right)^{16+t-\nu}\left(1+q^2\,\nu^2\right)^{8+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[16\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-16-t+\nu] - 2\,009\,564\,065\,655\,411\,265\,000\,q^{5}\left(\frac{1}{\nu}\right)^{14+t-\nu}\left(1+q^2\,\nu^2\right)^{7+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[14\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-14-t+\nu] + 33\,774\,185\,977\,401\,870\,000\,q^{7}\left(\frac{1}{\nu}\right)^{12+t-\nu}\left(1+q^2\,\nu^2\right)^{6+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[12\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-12-t+\nu] - 277\,187\,132\,390\,293\,125\,q^{9}\left(\frac{1}{\nu}\right)^{10+t-\nu}\left(1+q^2\,\nu^2\right)^{5+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[10\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-10-t+\nu] + 1\,192\,202\,719\,958\,250\,q^{11}\left(\frac{1}{\nu}\right)^{8+t-\nu}\left(1+q^2\,\nu^2\right)^{4+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[8\,\text{ArcTan}[q\,\nu] + t\,\text{ArcTan}[q\,\nu] - \nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-8-t+\nu] -$$

$$2\,710\,578\,370\,500\,q^{13}\left(\frac{1}{\nu}\right)^{6+t-\nu}\left(1+q^2\,\nu^2\right)^{3+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[6\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-6-t+\nu]+3\,088\,978\,200$$
$$q^{15}\left(\frac{1}{\nu}\right)^{4+t-\nu}\left(1+q^2\,\nu^2\right)^{2+\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[4\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]$$
$$\text{Gamma}[-4-t+\nu]-1\,514\,205\,q^{17}\left(\frac{1}{\nu}\right)^{2+t-\nu}\left(1+q^2\,\nu^2\right)^{1+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Cos}[2\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-2-t+\nu]+$$
$$210\,q^{19}\left(\frac{1}{\nu}\right)^{t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{t}{2}-\frac{\nu}{2}}\text{Cos}[t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]\,\text{Gamma}[-t+\nu]+$$
$$21\,945\,q^{18}\left(\frac{1}{\nu}\right)^{1+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-1-t+\nu]$$
$$\text{Sin}[\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-77\,224\,455\,q^{16}\left(\frac{1}{\nu}\right)^{3+t-\nu}$$
$$\left(1+q^2\,\nu^2\right)^{\frac{3}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-3-t+\nu]\,\text{Sin}[3\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$100\,391\,791\,500\,q^{14}\left(\frac{1}{\nu}\right)^{5+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{5}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-5-t+\nu]$$
$$\text{Sin}[5\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$61\,665\,657\,928\,875\,q^{12}\left(\frac{1}{\nu}\right)^{7+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{7}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-7-t+\nu]$$
$$\text{Sin}[7\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$19\,671\,344\,879\,311\,125\,q^{10}\left(\frac{1}{\nu}\right)^{9+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{9}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-9-t+\nu]$$
$$\text{Sin}[9\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$3\,326\,245\,588\,683\,517\,500\,q^{8}\left(\frac{1}{\nu}\right)^{11+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{11}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-11-t+\nu]$$
$$\text{Sin}[11\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$287\,080\,580\,807\,915\,895\,000\,q^{6}\left(\frac{1}{\nu}\right)^{13+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{13}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-13-t+\nu]$$
$$\text{Sin}[13\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$11\,303\,797\,869\,311\,688\,365\,625\,q^{4}\left(\frac{1}{\nu}\right)^{15+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{15}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-15-t+\nu]$$
$$\text{Sin}[15\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+$$
$$155\,815\,096\,120\,119\,939\,628\,125\,q^{2}\left(\frac{1}{\nu}\right)^{17+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{17}{2}+\frac{t}{2}-\frac{\nu}{2}}$$
$$\text{Gamma}[-17-t+\nu]\,\text{Sin}[17\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]-$$
$$319\,830\,986\,772\,877\,770\,815\,625\left(\frac{1}{\nu}\right)^{19+t-\nu}\left(1+q^2\,\nu^2\right)^{\frac{19}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[-19-t+\nu]$$
$$\text{Sin}[19\,\text{ArcTan}[q\,\nu]+t\,\text{ArcTan}[q\,\nu]-\nu\,\text{ArcTan}[q\,\nu]]+q^{20}\left(\frac{1}{\nu}\right)^{-1+t-\nu}$$
$$\left.\left(1+q^2\,\nu^2\right)^{-\frac{1}{2}+\frac{t}{2}-\frac{\nu}{2}}\text{Gamma}[1-t+\nu]\,\text{Sin}[\text{ArcTan}[q\,\nu]-t\,\text{ArcTan}[q\,\nu]+\nu\,\text{ArcTan}[q\,\nu]]\right)$$

FortranForm[% /. ν → nu]

```
      (Sqrt(2/Pi)*(-319830986772877770815625*
     -       (1/nu)**(18 - nu + t)*q*
     -       (1 + nu**2*q**2)**(9 - nu/2. + t/2.)*
     -       Cos(18*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -         t*ArcTan(nu*q))*Gamma(-18 + nu - t) +
     -      4920476719582734935625*(1/nu)**(16 - nu + t)*q**3*
     -       (1 + nu**2*q**2)**(8 - nu/2. + t/2.)*
     -       Cos(16*ArcTan(nu*q) - nu*ArcTan(nu*q) +
     -         t*ArcTan(nu*q))*Gamma(-16 + nu - t) -
     -      2009564065655411265000*(1/nu)**(14 - nu + t)*q**5*
     -       (1 + nu**2*q**2)**(7 - nu/2. + t/2.)*
```

```
-        Cos(14*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-14 + nu - t) +
-       337741859774018700000*(1/nu)**(12 - nu + t)*q**7*
-        (1 + nu**2*q**2)**(6 - nu/2. + t/2.)*
-        Cos(12*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-12 + nu - t) -
-       277187132390293125*(1/nu)**(10 - nu + t)*q**9*
-        (1 + nu**2*q**2)**(5 - nu/2. + t/2.)*
-        Cos(10*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-10 + nu - t) +
-       1192202719958250*(1/nu)**(8 - nu + t)*q**11*
-        (1 + nu**2*q**2)**(4 - nu/2. + t/2.)*
-        Cos(8*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-8 + nu - t) -
-       2710578370500*(1/nu)**(6 - nu + t)*q**13*
-        (1 + nu**2*q**2)**(3 - nu/2. + t/2.)*
-        Cos(6*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-6 + nu - t) +
-       3088978200*(1/nu)**(4 - nu + t)*q**15*
-        (1 + nu**2*q**2)**(2 - nu/2. + t/2.)*
-        Cos(4*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-4 + nu - t) -
-       1514205*(1/nu)**(2 - nu + t)*q**17*
-        (1 + nu**2*q**2)**(1 - nu/2. + t/2.)*
-        Cos(2*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q))*Gamma(-2 + nu - t) +
-       210*(1/nu)**(-nu + t)*q**19*
-        (1 + nu**2*q**2)**(-nu/2. + t/2.)*
-        Cos(nu*ArcTan(nu*q) - t*ArcTan(nu*q))*Gamma(nu - t) +
-       (1/nu)**(-1 - nu + t)*q**20*
-        (1 + nu**2*q**2)**(-0.5 - nu/2. + t/2.)*
-        Gamma(1 + nu - t)*
-        Sin(ArcTan(nu*q) + nu*ArcTan(nu*q) - t*ArcTan(nu*q))\
-        + 21945*(1/nu)**(1 - nu + t)*q**18*
-        (1 + nu**2*q**2)**(0.5 - nu/2. + t/2.)*
-        Gamma(-1 + nu - t)*
-        Sin(ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))\
-        - 77224455*(1/nu)**(3 - nu + t)*q**16*
-        (1 + nu**2*q**2)**(1.5 - nu/2. + t/2.)*
-        Gamma(-3 + nu - t)*
-        Sin(3*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
-         + 100391791500*(1/nu)**(5 - nu + t)*q**14*
-        (1 + nu**2*q**2)**(2.5 - nu/2. + t/2.)*
-        Gamma(-5 + nu - t)*
-        Sin(5*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
-         - 61665657928875*(1/nu)**(7 - nu + t)*q**12*
-        (1 + nu**2*q**2)**(3.5 - nu/2. + t/2.)*
-        Gamma(-7 + nu - t)*
-        Sin(7*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
-         + 19671344879311125*(1/nu)**(9 - nu + t)*q**10*
-        (1 + nu**2*q**2)**(4.5 - nu/2. + t/2.)*
-        Gamma(-9 + nu - t)*
-        Sin(9*ArcTan(nu*q) - nu*ArcTan(nu*q) + t*ArcTan(nu*q))
-         - 33262455886835175000*(1/nu)**(11 - nu + t)*q**8*
-        (1 + nu**2*q**2)**(5.5 - nu/2. + t/2.)*
-        Gamma(-11 + nu - t)*
-        Sin(11*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)) +
-       287080580807915895000*(1/nu)**(13 - nu + t)*q**6*
-        (1 + nu**2*q**2)**(6.5 - nu/2. + t/2.)*
-        Gamma(-13 + nu - t)*
-        Sin(13*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)) -
-       113037978693116883656250*(1/nu)**(15 - nu + t)*q**4*
-        (1 + nu**2*q**2)**(7.5 - nu/2. + t/2.)*
-        Gamma(-15 + nu - t)*
-        Sin(15*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)) +
-       155815096120119939628125*(1/nu)**(17 - nu + t)*q**2*
```

```
-        (1 + nu**2*q**2)**(8.5 - nu/2. + t/2.)*
-        Gamma(-17 + nu - t)*
-        Sin(17*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)) -
-       3198309867728777708156 25*(1/nu)**(19 - nu + t)*
-        (1 + nu**2*q**2)**(9.5 - nu/2. + t/2.)*
-        Gamma(-19 + nu - t)*
-        Sin(19*ArcTan(nu*q) - nu*ArcTan(nu*q) +
-          t*ArcTan(nu*q)))))/q**20
```

New Calc with b term

```
l = 10
```

```
10
```

```
(-1)^t ((Gamma[-ν + l + 1 + t] Gamma[-ν - l + t]) / (Gamma[-ν + l + 1] Gamma[-ν - l] t!))
 (ν/2)^t Integrate[Sqrt[q] BesselJ[l + 1/2, q r] Exp[-r/ν] r^(ν - t + 1/2),
   {r, 0, Infinity}, Assumptions → t >= 0 && Element[t, Integers] &&
     Element[q, Reals] && q >= 0 && Element[ν, Reals] && ν >= 1 && t < ν + l + 2]
```

$$-\left((-1)^t\, 2^{\frac{1}{2}-t}\, \nu^{\nu}\, \left(1+q^2\,\nu^2\right)^{\frac{t-\nu}{2}}\, \text{Gamma}[-10+t-\nu]\, \text{Gamma}[11+t-\nu]\right.$$
$$\left(\frac{1}{\nu^8}654\,729\,075\, q\, \left(1+q^2\,\nu^2\right)^4\, \text{Cos}[(8+t-\nu)\, \text{ArcTan}[q\,\nu]]\, \text{Gamma}[-8-t+\nu] -\right.$$
$$\frac{1}{\nu^6}91\,891\,800\, \left(q+q^3\,\nu^2\right)^3\, \text{Cos}[(6+t-\nu)\, \text{ArcTan}[q\,\nu]]\, \text{Gamma}[-6-t+\nu] +$$
$$\frac{1}{\nu^4}2\,837\,835\, q^5\, \left(1+q^2\,\nu^2\right)^2\, \text{Cos}[(4+t-\nu)\, \text{ArcTan}[q\,\nu]]\, \text{Gamma}[-4-t+\nu] -$$
$$\frac{1}{\nu^2}25\,740\, q^7\, \left(1+q^2\,\nu^2\right)\, \text{Cos}[(2+t-\nu)\, \text{ArcTan}[q\,\nu]]\, \text{Gamma}[-2-t+\nu] +$$
$$55\, q^9\, \text{Cos}[(t-\nu)\, \text{ArcTan}[q\,\nu]]\, \text{Gamma}[-t+\nu] + \frac{1}{\nu}$$
$$1485\, q^8\, \sqrt{1+q^2\,\nu^2}\, \text{Gamma}[-1-t+\nu]\, \text{Sin}[(1+t-\nu)\, \text{ArcTan}[q\,\nu]] - \frac{1}{\nu^3}$$
$$315\,315\, q^6\, \left(1+q^2\,\nu^2\right)^{3/2}\, \text{Gamma}[-3-t+\nu]\, \text{Sin}[(3+t-\nu)\, \text{ArcTan}[q\,\nu]] + \frac{1}{\nu^5}$$
$$18\,918\,900\, q^4\, \left(1+q^2\,\nu^2\right)^{5/2}\, \text{Gamma}[-5-t+\nu]\, \text{Sin}[(5+t-\nu)\, \text{ArcTan}[q\,\nu]] - \frac{1}{\nu^7}$$
$$310\,134\,825\, q^2\, \left(1+q^2\,\nu^2\right)^{7/2}\, \text{Gamma}[-7-t+\nu]\, \text{Sin}[(7+t-\nu)\, \text{ArcTan}[q\,\nu]] +$$
$$\frac{1}{\nu^9}654\,729\,075\, \left(1+q^2\,\nu^2\right)^{9/2}\, \text{Gamma}[-9-t+\nu]\, \text{Sin}[(9+t-\nu)\, \text{ArcTan}[q\,\nu]] +$$
$$\left.\left.\left(q^{10}\,\nu\, \text{Gamma}[1-t+\nu]\, \text{Sin}[(1-t+\nu)\, \text{ArcTan}[q\,\nu]]\right) \Big/ \left(\sqrt{1+q^2\,\nu^2}\right)\right)\right) \Big/$$
$$\left(\sqrt{\pi}\, q^{10}\, t!\, \text{Gamma}[-10-\nu]\, \text{Gamma}[11-\nu]\right)$$

```
FortranForm[% /. ν → nu]
```

```
      -(((-1)**t*2**(0.5 - t)*nu**nu*
     -       (1 + nu**2*q**2)**((-nu + t)/2.)*Gamma(-10 - nu + t)*
     -       Gamma(11 - nu + t)*
     -       ((654729075*q*(1 + nu**2*q**2)**4*
     -            Cos((8 - nu + t)*ArcTan(nu*q))*Gamma(-8 + nu - t))
     -          /nu**8 - (91891800*(q + nu**2*q**3)**3*
     -            Cos((6 - nu + t)*ArcTan(nu*q))*Gamma(-6 + nu - t))
     -          /nu**6 + (2837835*q**5*(1 + nu**2*q**2)**2*
     -            Cos((4 - nu + t)*ArcTan(nu*q))*Gamma(-4 + nu - t))
     -          /nu**4 - (25740*q**7*(1 + nu**2*q**2)*
     -            Cos((2 - nu + t)*ArcTan(nu*q))*Gamma(-2 + nu - t))
     -          /nu**2 + 55*q**9*Cos((-nu + t)*ArcTan(nu*q))*
     -         Gamma(nu - t) +
     -        (nu*q**10*Gamma(1 + nu - t)*
     -           Sin((1 + nu - t)*ArcTan(nu*q)))/
     -         Sqrt(1 + nu**2*q**2) +
     -        (1485*q**8*Sqrt(1 + nu**2*q**2)*Gamma(-1 + nu - t)*
     -           Sin((1 - nu + t)*ArcTan(nu*q)))/nu -
     -        (315315*q**6*(1 + nu**2*q**2)**1.5*
     -           Gamma(-3 + nu - t)*Sin((3 - nu + t)*ArcTan(nu*q)))
     -          /nu**3 + (18918900*q**4*(1 + nu**2*q**2)**2.5*
     -           Gamma(-5 + nu - t)*Sin((5 - nu + t)*ArcTan(nu*q)))
     -          /nu**5 - (310134825*q**2*(1 + nu**2*q**2)**3.5*
     -           Gamma(-7 + nu - t)*Sin((7 - nu + t)*ArcTan(nu*q)))
     -          /nu**7 + (654729075*(1 + nu**2*q**2)**4.5*
     -           Gamma(-9 + nu - t)*Sin((9 - nu + t)*ArcTan(nu*q)))
     -          /nu**9))/
     -    (Sqrt(Pi)*q**10*Factorial(t)*Gamma(-10 - nu)*
     -      Gamma(11 - nu)))
```