

My Notes for AP Calculus BC

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Chapter 1

Limits and Continuity

1.1 Computing Limits

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1.3 Continuity

1.4 Intermediate Value Theorem

1.5 Squeeze Theorem

Chapter 2

Differentiation and the Rate of Change

2.1 Tangent Lines and Rates of Change

2.2 The Derivative Function

2.3 Techniques of Differentiation

2.4 Product Rule and Quotient Rule

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Chapter 3

Topics in Differentiation

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3.6 Local Linear Approximation

3.7 L'Hôpital's Rule and Indeterminate Forms

Chapter 4

The Derivative in Graphing and Applications

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Integration

5.1 Overview of Area

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Chapter 6

Applications of the Definite Integral

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6.4 Length of a Plane Curve

Chapter 7

Principles of Integral Evaluation

7.1 Integration by Parts

7.2 Integration of Rational Functions by Partial
Fractions

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Chapter 8

Differential Equations

8.1 Logistic Growth

8.2 Separable Equations

8.3 Exponential Growth and Decay

8.4 Euler's Method

Chapter 9

Infinite Series

9.1 Defining Convergent and Divergent Infinite Series

9.2 Geometric Series

Definition 1. A series in the form $\sum ar^n = a + ar + ar^2 + ar^3 + \dots + ar^n \dots$ is called a geometric series with ratio r .

An infinite geometric series with ratio r diverges if $|r| \geq 1$. If $|r| < 1$, we can say that the series converges by the **geometric series test**. The infinite sum of this series is

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

9.3 nth Term Test

9.4 Integral Test

Definition 2. If f is positive, continuous, and decreasing for $x \geq m \geq 1$ where m is a positive integer and $a_n = f(x)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x) dx$ either both converge or diverge.

Use implicit integration to determine whether the integral converges or diverges. **Note:** The answer to the limit or the integral is *not* the sum of the infinite series.

9.5 p-series and Harmonic Series

Definition 3. A p-series is an infinite series in the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$$

where p is a positive number.

The p-series will converge if $p > 1$ and diverge if $1 < p \leq 1$

9.6 Comparison Tests

9.7 Polynomial Test

9.8 Alternating Series

9.9 Power Series

9.10 Error Bounds

9.11 Taylor Series

Chapter 10

Parametric, Polar, and Vector-Valued Functions

10.1 Parametric Equations

Parametric equations are functions of a single, independent variable (usually t) called a parameter. Parametric equations represent the coordinates that make up a parametric curve in the form $(x(t), y(t))$.

10.1.1 Derivatives

Definition 4 (First Derivative). For a smooth curve C represented by $x = x(t)$ and $y = y(t)$, the slope of the line tangent to C at (x, y) is

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

as long as $\frac{dx}{dt} \neq 0$.

10.1.2 Arc Length

10.2 Vector-Valued Functions

10.3 Polar Functions