My Notes for AP Calculus BC

Luke Barlow

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9.2.1 Geometric Series Test

Definition 1. A series in the form $\sum ar^n = a + ar + ar^2 + ar^3 + ... + ar^n ...$ is called a geometric series with ratio r.

An infinite geometric series with ratio r diverges if $|r| \geq 1$. If $|r| < 1$, we can say that the series converges by the **geometric series test**. The infinite sum of this series is

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

9.2.2 nth Term Test

9.2.3 Integral Test

If f is positive, continuous, and decreasing for $x \geq m \geq 1$ where m is a positive integer and $a_n = f(x)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\, \mathrm{d}x$ either both converge or diverge. Use implicit integration to determine whether the integral converges or diverges. **Note:** The answer to the limit or the integral is *not* the sum of the infinite series.

9.2.4 p-series and Harmonic Series

A p-series is an infinite series in the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + ... + \frac{1}{n^p} + ...$$

where p is a positive number. The p-series will converge if $p > 1$ and diverge if $1 < p \leq 1$.

9.2.5 Comparison Tests

Direct Comparison Test

Limit Comparison Test

9.2.6 Polynomial Test

9.2.7 Alternating Series

9.2.8 Ratio Test

A series $\sum a_n$ is absolutely convergent if the limit of the ratio of successive terms,

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$$

is less than 1. If the limit is greater than 1 or approaches ∞, then the series diverges. The ratio test is inconclusive if the limit equals 1.

9.2.9 Root Test

9.3 Absolute and Conditional Convergence

9.4 Power Series

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Chapter 10

Parametric, Polar, and Vector-Valued Functions

10.1 Parametric Equations

Parametric equations are functions of a single, independent variable (usually t) called a parameter. Parametric equations represent the coordinates that make up a parametric curve in the form $(x(t), y(t))$.

10.1.1 Derivatives

For a smooth curve C represented by $x = x(t)$ and $y = y(t)$, the slope of the line tangent to C at (x, y) is

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$$

as long as $\frac{\mathrm{d}x}{\mathrm{d}t} \neq 0$.

10.1.2 Arc Length

10.2 Vector-Valued Functions

10.3 Polar Functions