

My Notes for AP Calculus BC

Luke Barlow

2023-2024

Contents

1	Limits and Continuity	4
1.1	Computing Limits	4
1.2	Limits at Infinity	4
1.3	Continuity	4
1.4	Intermediate Value Theorem	4
1.5	Squeeze Theorem	4
2	Differentiation and the Rate of Change	5
2.1	Tangent Lines and Rates of Change	5
2.2	The Derivative Function	5
2.3	Techniques of Differentiation	5
2.4	Product Rule and Quotient Rule	5
2.5	Derivatives of Trig Functions	5
2.6	The Chain Rule	5
3	Topics in Differentiation	6
3.1	Implicit Differentiation	6
3.2	Derivatives of Logarithmic Functions	6
3.3	Derivatives of Exponential Functions	6
3.4	Derivatives of Inverse Functions	6
3.5	Related Rates	6
3.6	Local Linear Approximation	6
3.7	L'Hôpital's Rule and Indeterminate Forms	6
4	The Derivative in Graphing and Applications	7
4.1	Increase, Decrease, and Concavity	7
4.2	Relative Extrema	7
4.3	Absolute Maxima and Minima	7
4.4	Applied Max and Min Problems	7
4.5	Rectilinear Motion	7
4.6	Mean Value Theorem	7

5	Integration	8
5.1	Overview of Area	8
5.2	The Indefinite Integral	8
5.3	Slope Fields	8
5.4	Integration By Substitution	8
5.5	Area as a Limit and Riemann Sums	8
5.6	Exact Area Under a Curve (Trapezoid Rule)	8
5.7	The Definite Integral	8
5.8	The Accumulation Function	8
5.9	The Fundamental Theorem of Calculus	8
5.10	Total Change Theorem	8
5.11	Average Value	8
5.12	Definite Integrals by Substitution	8
6	Applications of the Definite Integral	9
6.1	Area Between Two Curves	9
6.2	Volumes by Slicing	9
6.3	Disks and Washers	9
6.4	Length of a Plane Curve	9
7	Principles of Integral Evaluation	10
7.1	Integration by Parts	10
7.2	Integration of Rational Functions by Partial Fractions	10
7.3	Improper Integrals	10
8	Differential Equations	11
8.1	Logistic Growth	11
8.2	Separable Equations	11
8.3	Exponential Growth and Decay	11
8.4	Euler's Method	11
9	Infinite Series	12
9.1	Defining Convergent and Divergent Infinite Series	12
9.2	Convergence Tests	12
9.2.1	Geometric Series Test	12
9.2.2	nth Term Test	12
9.2.3	Integral Test	12
9.2.4	p-series and Harmonic Series	13
9.2.5	Comparison Tests	13
	Direct Comparison Test	13
	Limit Comparison Test	13
9.2.6	Polynomial Test	13
9.2.7	Alternating Series	13
9.2.8	Ratio Test	13
9.2.9	Root Test	13
9.3	Absolute and Conditional Convergence	13

9.4	Power Series	13
9.5	Error Bounds	13
9.6	Taylor Series	13
10	Parametric, Polar, and Vector-Valued Functions	14
10.1	Parametric Equations	14
10.1.1	Derivatives	14
10.1.2	Arc Length	14
10.2	Vector-Valued Functions	14
10.3	Polar Functions	14

Chapter 1

Limits and Continuity

1.1 Computing Limits

1.2 Limits at Infinity

1.3 Continuity

1.4 Intermediate Value Theorem

1.5 Squeeze Theorem

Chapter 2

Differentiation and the Rate of Change

2.1 Tangent Lines and Rates of Change

2.2 The Derivative Function

2.3 Techniques of Differentiation

2.4 Product Rule and Quotient Rule

2.5 Derivatives of Trig Functions

2.6 The Chain Rule

Chapter 3

Topics in Differentiation

3.1 Implicit Differentiation

3.2 Derivatives of Logarithmic Functions

3.3 Derivatives of Exponential Functions

3.4 Derivatives of Inverse Functions

3.5 Related Rates

3.6 Local Linear Approximation

3.7 L'Hôpital's Rule and Indeterminate Forms

Chapter 4

The Derivative in Graphing and Applications

4.1 Increase, Decrease, and Concavity

4.2 Relative Extrema

4.3 Absolute Maxima and Minima

4.4 Applied Max and Min Problems

4.5 Rectilinear Motion

4.6 Mean Value Theorem

Chapter 5

Integration

5.1 Overview of Area

5.2 The Indefinite Integral

5.3 Slope Fields

5.4 Integration By Substitution

5.5 Area as a Limit and Riemann Sums

5.6 Exact Area Under a Curve (Trapezoid Rule)

5.7 The Definite Integral

5.8 The Accumulation Function

5.9 The Fundamental Theorem of Calculus

5.10 Total Change Theorem

5.11 Average Value

5.12 Definite Integrals by Substitution

Chapter 6

Applications of the Definite Integral

6.1 Area Between Two Curves

6.2 Volumes by Slicing

6.3 Disks and Washers

6.4 Length of a Plane Curve

Chapter 7

Principles of Integral Evaluation

7.1 Integration by Parts

7.2 Integration of Rational Functions by Partial
Fractions

7.3 Improper Integrals

Chapter 8

Differential Equations

8.1 Logistic Growth

8.2 Separable Equations

8.3 Exponential Growth and Decay

8.4 Euler's Method

Chapter 9

Infinite Series

9.1 Defining Convergent and Divergent Infinite Series

9.2 Convergence Tests

9.2.1 Geometric Series Test

Definition 1. A series in the form $\sum ar^n = a + ar + ar^2 + ar^3 + \dots + ar^n \dots$ is called a geometric series with ratio r .

An infinite geometric series with ratio r diverges if $|r| \geq 1$. If $|r| < 1$, we can say that the series converges by the **geometric series test**. The infinite sum of this series is

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

9.2.2 nth Term Test

9.2.3 Integral Test

Definition 2. If f is positive, continuous, and decreasing for $x \geq m \geq 1$ where m is a positive integer and $a_n = f(x)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x) dx$ either both converge or diverge.

Use implicit integration to determine whether the integral converges or diverges. **Note:** The answer to the limit or the integral is *not* the sum of the infinite series.

9.2.4 p-series and Harmonic Series

Definition 3. A p-series is an infinite series in the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$$

where p is a positive number.

The p-series will converge if $p > 1$ and diverge if $1 < p \leq 1$

9.2.5 Comparison Tests

Direct Comparison Test

Limit Comparison Test

9.2.6 Polynomial Test

9.2.7 Alternating Series

9.2.8 Ratio Test

9.2.9 Root Test

9.3 Absolute and Conditional Convergence

9.4 Power Series

9.5 Error Bounds

9.6 Taylor Series

Chapter 10

Parametric, Polar, and Vector-Valued Functions

10.1 Parametric Equations

Parametric equations are functions of a single, independent variable (usually t) called a parameter. Parametric equations represent the coordinates that make up a parametric curve in the form $(x(t), y(t))$.

10.1.1 Derivatives

Definition 4 (First Derivative). For a smooth curve C represented by $x = x(t)$ and $y = y(t)$, the slope of the line tangent to C at (x, y) is

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

as long as $\frac{dx}{dt} \neq 0$.

10.1.2 Arc Length

10.2 Vector-Valued Functions

10.3 Polar Functions