

# My Notes for AP Calculus BC

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## Chapter 1

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## Chapter 2

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## Chapter 9

# Infinite Series

### 9.1 Defining Convergent and Divergent Infinite Series

### 9.2 Convergence Tests

#### 9.2.1 Geometric Series Test

**Definition 1.** A series in the form  $\sum ar^n = a + ar + ar^2 + ar^3 + \dots + ar^n \dots$  is called a geometric series with ratio  $r$ .

An infinite geometric series with ratio  $r$  diverges if  $|r| \geq 1$ . If  $|r| < 1$ , we can say that the series converges by the **geometric series test**. The infinite sum of this series is

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

#### 9.2.2 nth Term Test

#### 9.2.3 Integral Test

**Definition 2.** If  $f$  is positive, continuous, and decreasing for  $x \geq m \geq 1$  where  $m$  is a positive integer and  $a_n = f(x)$ , then  $\sum_{n=1}^{\infty} a_n$  and  $\int_1^{\infty} f(x) dx$  either both converge or diverge.

Use implicit integration to determine whether the integral converges or diverges. **Note:** The answer to the limit or the integral is *not* the sum of the infinite series.

### 9.2.4 p-series and Harmonic Series

**Definition 3.** A p-series is an infinite series in the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$$

where  $p$  is a positive number.

The p-series will converge if  $p > 1$  and diverge if  $1 < p \leq 1$

### 9.2.5 Comparison Tests

**Direct Comparison Test**

**Limit Comparison Test**

### 9.2.6 Polynomial Test

### 9.2.7 Alternating Series

### 9.2.8 Ratio Test

A series  $\sum a_n$  is absolutely convergent if the limit of the ratio of successive terms,

$$\lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|}$$

is less than 1. If the limit is greater than 1 or approaches  $\infty$ , then the series diverges. The ratio test is inconclusive if the limit equals 1.

### 9.2.9 Root Test

## 9.3 Absolute and Conditional Convergence

## 9.4 Power Series

## 9.5 Error Bounds

## 9.6 Taylor Series

## Chapter 10

# Parametric, Polar, and Vector-Valued Functions

### 10.1 Parametric Equations

Parametric equations are functions of a single, independent variable (usually  $t$ ) called a parameter. Parametric equations represent the coordinates that make up a parametric curve in the form  $(x(t), y(t))$ .

#### 10.1.1 Derivatives

**Definition 4** (First Derviative). For a smooth curve  $C$  represented by  $x = x(t)$  and  $y = y(t)$ , the slope of the line tangent to  $C$  at  $(x, y)$  is

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

as long as  $\frac{dx}{dt} \neq 0$ .

#### 10.1.2 Arc Length

### 10.2 Vector-Valued Functions

### 10.3 Polar Functions