

My Notes for AP Calculus BC

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Chapter 1

Limits and Continuity

1.1 Computing Limits

1.2 Limits at Infinity

1.3 Continuity

1.4 Intermediate Value Theorem

1.5 Squeeze Theorem

Chapter 2

Differentiation and the Rate of Change

2.1 Tangent Lines and Rates of Change

2.2 The Derivative Function

2.3 Techniques of Differentiation

2.4 Product Rule and Quotient Rule

2.5 Derivatives of Trig Functions

2.6 The Chain Rule

Chapter 3

Topics in Differentiation

3.1 Implicit Differentiation

3.2 Derivatives of Logarithmic Functions

3.3 Derivatives of Exponential Functions

3.4 Derivatives of Inverse Functions

3.5 Related Rates

3.6 Local Linear Approximation

3.7 L'Hôpital's Rule and Indeterminate Forms

Chapter 4

The Derivative in Graphing and Applications

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4.2 Relative Extrema

4.3 Absolute Maxima and Minima

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Chapter 5

Integration

5.1 Overview of Area

5.2 The Indefinite Integral

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5.5 Area as a Limit and Riemann Sums

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5.7 The Definite Integral

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Chapter 6

Applications of the Definite Integral

6.1 Area Between Two Curves

6.2 Volumes by Slicing

6.3 Disks and Washers

6.4 Length of a Plane Curve

Chapter 7

Principles of Integral Evaluation

7.1 Integration by Parts

7.2 Integration of Rational Functions by Partial
Fractions

7.3 Improper Integrals

Chapter 8

Differential Equations

8.1 Logistic Growth

8.2 Separable Equations

8.3 Exponential Growth and Decay

8.4 Euler's Method

Chapter 9

Infinite Series

9.1 Defining Convergent and Divergent Infinite Series

9.2 Geometric Series

Definition 1. A series in the form $\sum ar^n = a + ar + ar^2 + ar^3 + \dots + ar^n \dots$ is called a geometric series with ratio r .

An infinite geometric series with ratio r diverges if $|r| \geq 1$. If $|r| < 1$, we can say that the series converges by the **geometric series test**. The infinite sum of this series is

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

9.3 nth Term Test

9.4 Integral Test

Definition 2. If f is positive, continuous, and decreasing for $x \geq m \geq 1$ where m is a positive integer and $a_n = f(x)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x) dx$ either both converge or diverge.

Use implicit integration to determine whether the integral converges or diverges. **Note:** The answer to the limit or the integral is *not* the sum of the infinite series.

9.5 p-series and Harmonic Series

Definition 3. A p-series is an infinite series in the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$$

where p is a positive number.

The p-series will converge if $p > 1$ and diverge if $1 < p \leq 1$

9.6 Comparison Tests

9.7 Polynomial Test

9.8 Alternating Series

Chapter 10

Parametric, Polar, and Vector-Valued Functions

10.1 Parametric Equations

Parametric equations are functions of a single, independent variable (usually t) called a parameter.

Parametric equations represent the coordinates that make up a parametric curve in the form $(x(t), y(t))$.

Definition 4 (Derivative of a Parametric Equation). For a smooth curve C represented by $x = x(t)$ and $y = y(t)$, the slope of the line tangent to C at (x, y) is

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

as long as $\frac{dx}{dt} \neq 0$.

10.2 Vector-Valued Functions

10.3 Polar Functions