

Week 3 LIVE 

Recursion Problems and Doubts Session

In This Lecture

1. Matrix Path Problem
2. Power with Modulo
3. Explain the Josephus Problem Again
4. Explain the Count occurrences Problem Again

Power with Modulo

`Math.pow(3.0, 4.0)`

$\hookrightarrow \equiv 3^4 \rightarrow 81$

$$3^4 = 3^3 \times 3$$

$$5^4 = 5^3 \times 5$$

$\hookrightarrow \text{power}(a, b, m)$

$$\equiv a^b$$

$$\leftarrow \left(a^{b-1} \right) \times a^1$$

$$\hookrightarrow a^{b-1+1} \rightarrow a^b$$

`power(a, b)`

`power(a, b-1) * a`

$$\hookrightarrow \boxed{\begin{array}{l} b = 0 \\ a^0 = 1 \end{array}}$$

Power with Modulo

$$f(a, b)$$



$$f(a, b-1)$$



$$f(a, b-2)$$

⋮

$$f(a, 0)$$



$$O(b)$$



$$O(\log b)$$

$$\text{fastPower}(a, b)$$

$$b \rightarrow \frac{b}{2} \rightarrow \frac{b}{2^2} \rightarrow \frac{b}{2^4} \dots 1$$

↑
k steps

$$\left(\frac{b}{2^k} \right) = 1$$

$$b = 2^k$$

$$\log_2 b = \cancel{k \log_2 2}$$

$$k = \log_2 b$$

Power with Modulo

$$\rightarrow a^b * a^c = a^{b+c} \quad \checkmark$$

$$\rightarrow (a^b)^c = a^{b*c}$$

$$\rightarrow 5^4 = (5^2)^2$$

$$\rightarrow 3^5 = (3) * 3^4$$

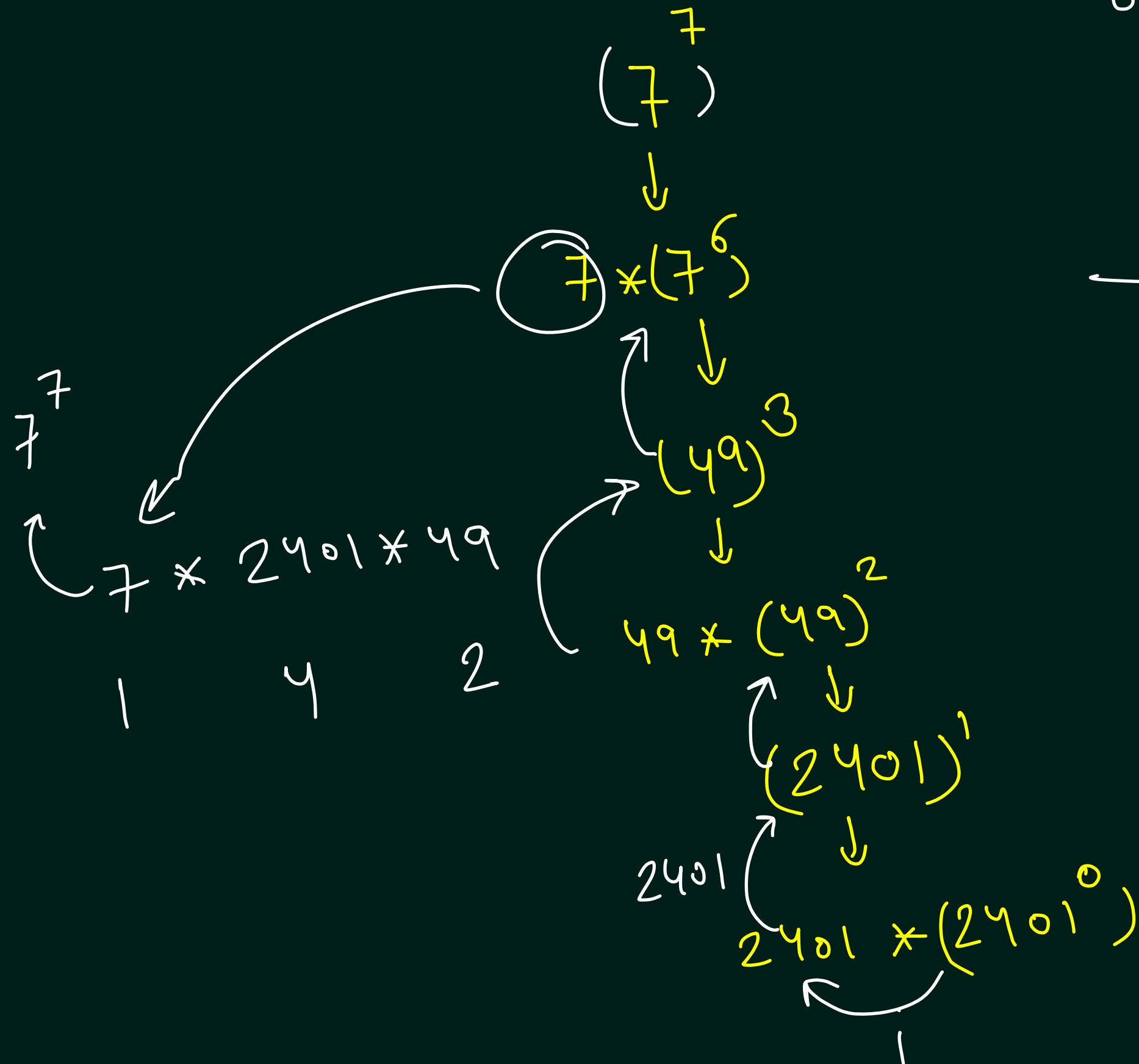
$$7^3 = 7 * (7^2)$$

$$5^{10} = (5^2)^5$$

$$\Rightarrow [a^b = (a^2)^{b/2} \quad \text{if } b \text{ is even}]$$

$$\Rightarrow [a^b = a * a^{b-1} \quad \text{if } b \text{ is odd/even}]$$

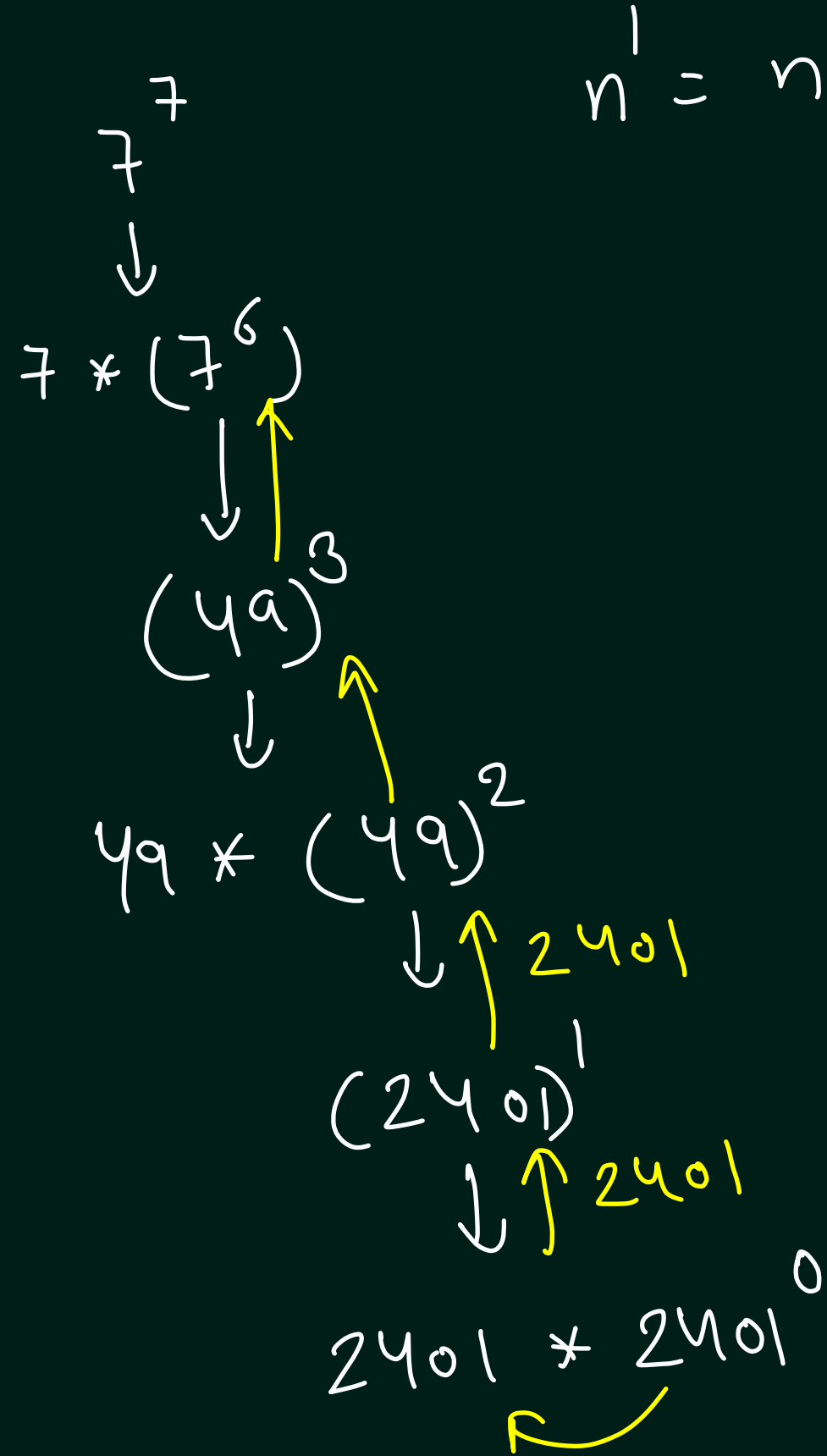
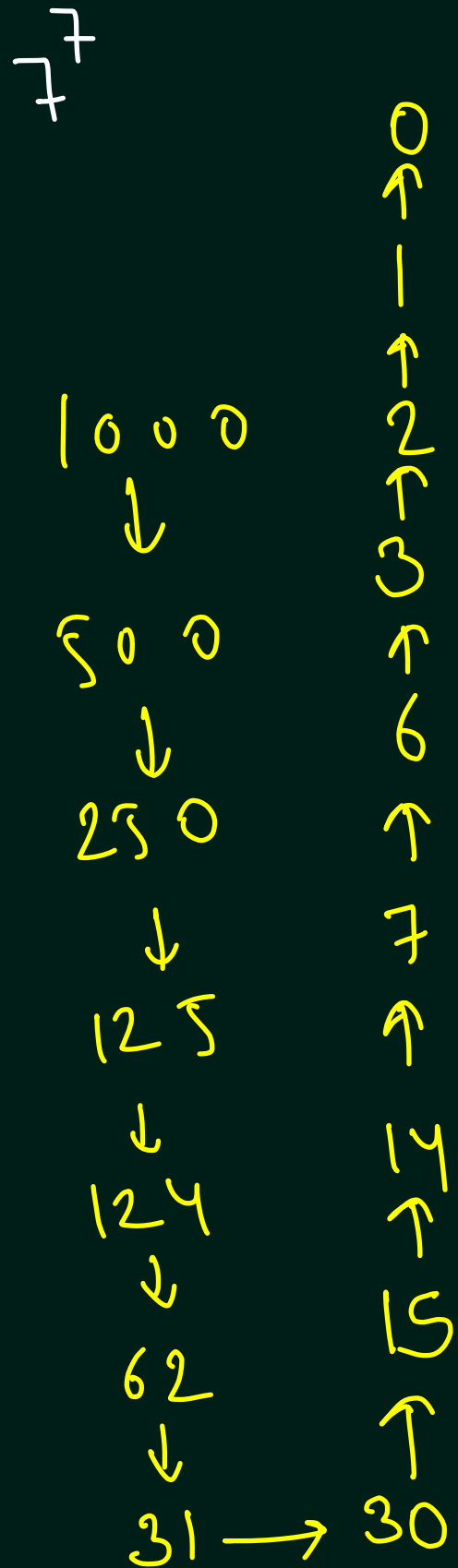
Power with Modulo



Even $\rightarrow n/2 = 0$ ✓
 $\hookrightarrow n \& 1 = 0 \therefore \text{LSB is zero}$

$$\begin{array}{r} \rightarrow (110)_2 \\ \underline{0001} \\ (000)_2 \end{array}$$

Power with Modulo



3 usages

```
static double fastPower(double a, int b) {
    if(b == 0) return 1;

    if(b % 2 == 0) {
        return fastPower(a: a*a, b: b/2);
    } else {
        return a * fastPower(a, b: b-1);
    }
}
```

$$\log_2 1000 = \log_2 10^3$$

$$= 3 \log_2 10$$

$$= 9.9 \dots \rightarrow \boxed{10}$$

Power with Modulo

$$\rightarrow (a+b) \% m = (a \% m + b \% m) \% m$$

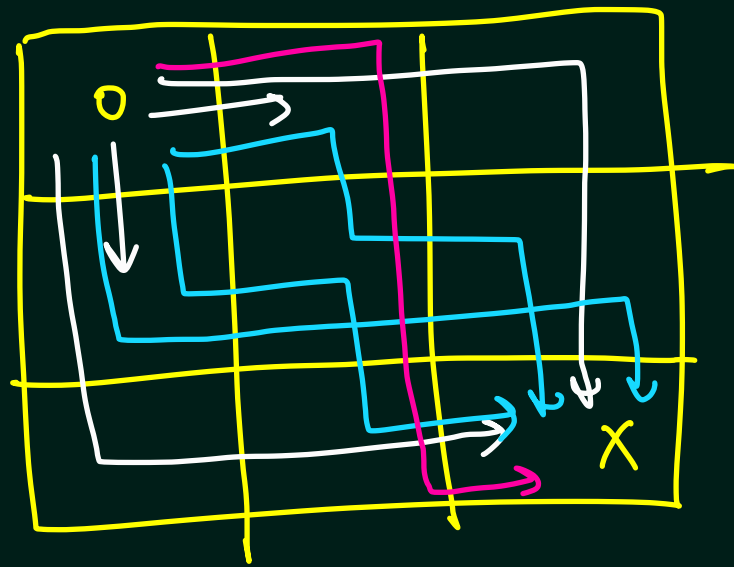
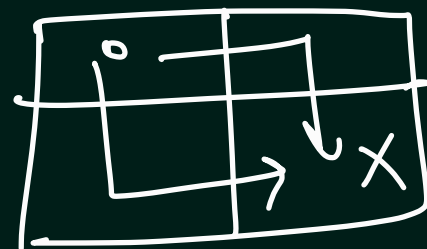
$$\boxed{\rightarrow (a * b) \% m = (a \% m * b \% m) \% m} \rightarrow \text{int} \rightarrow (0 \dots m-1)$$

if negative

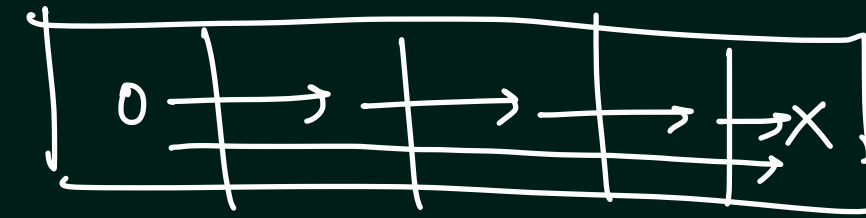
$$(32^{48}) \% 30 \rightarrow [0 \dots 29]$$

$$a^{-b} = \left(\frac{1}{a}\right)^b$$

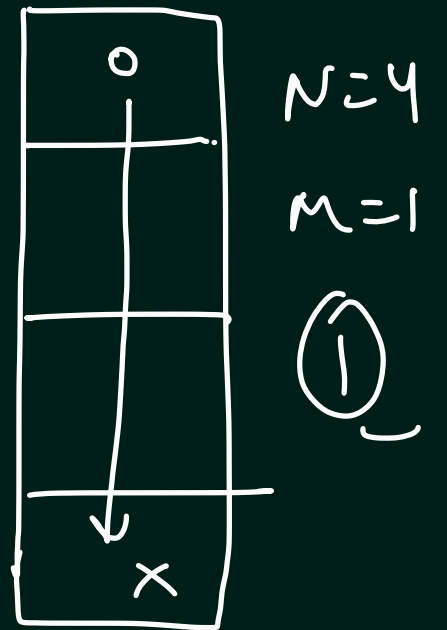
Matrix Path Problem


$$(N \times M)$$

$$(N-1)$$

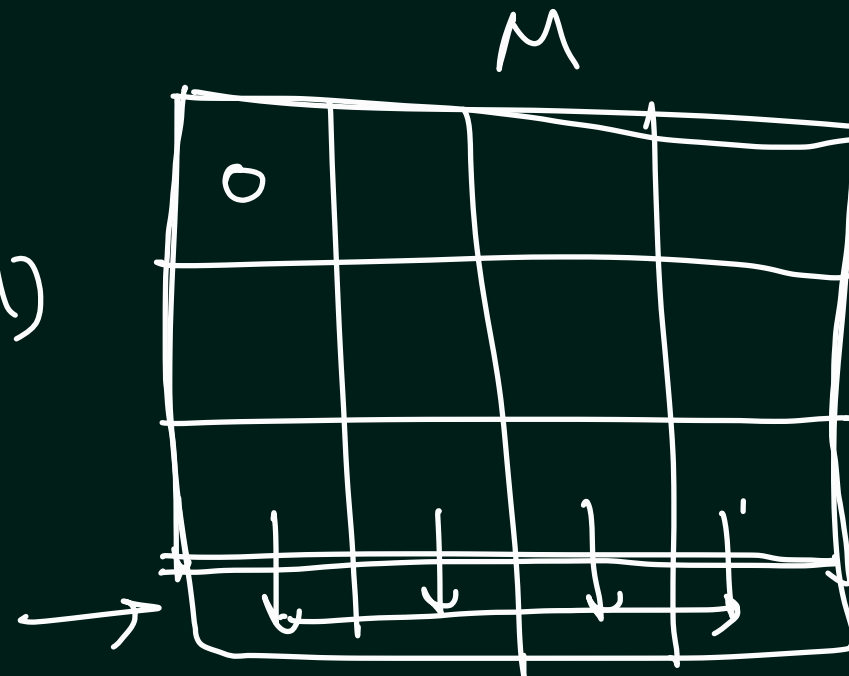
Right
Bottom


$$N = 1$$
$$M = 5$$

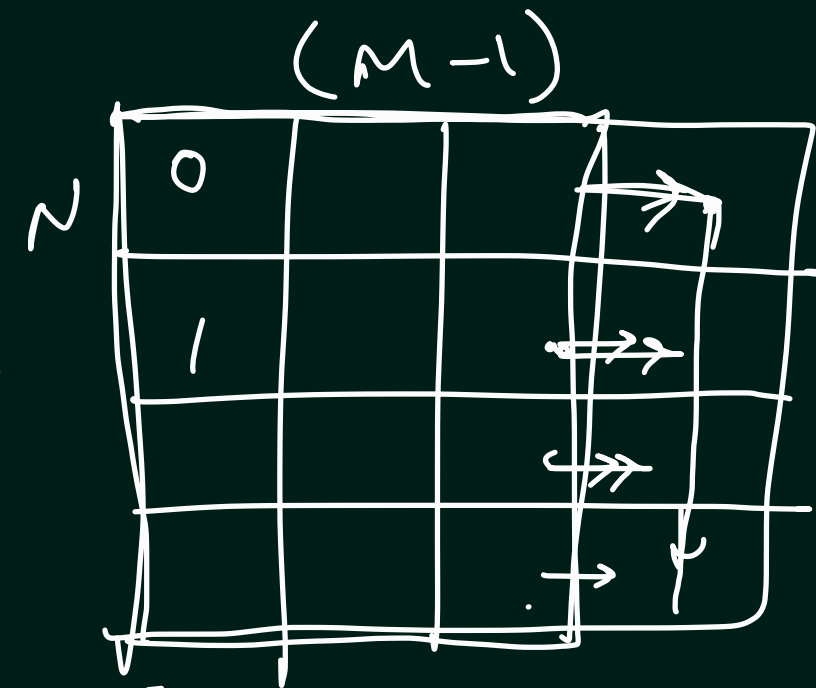
Path = ① ✓


$$N=4$$
$$n=1$$

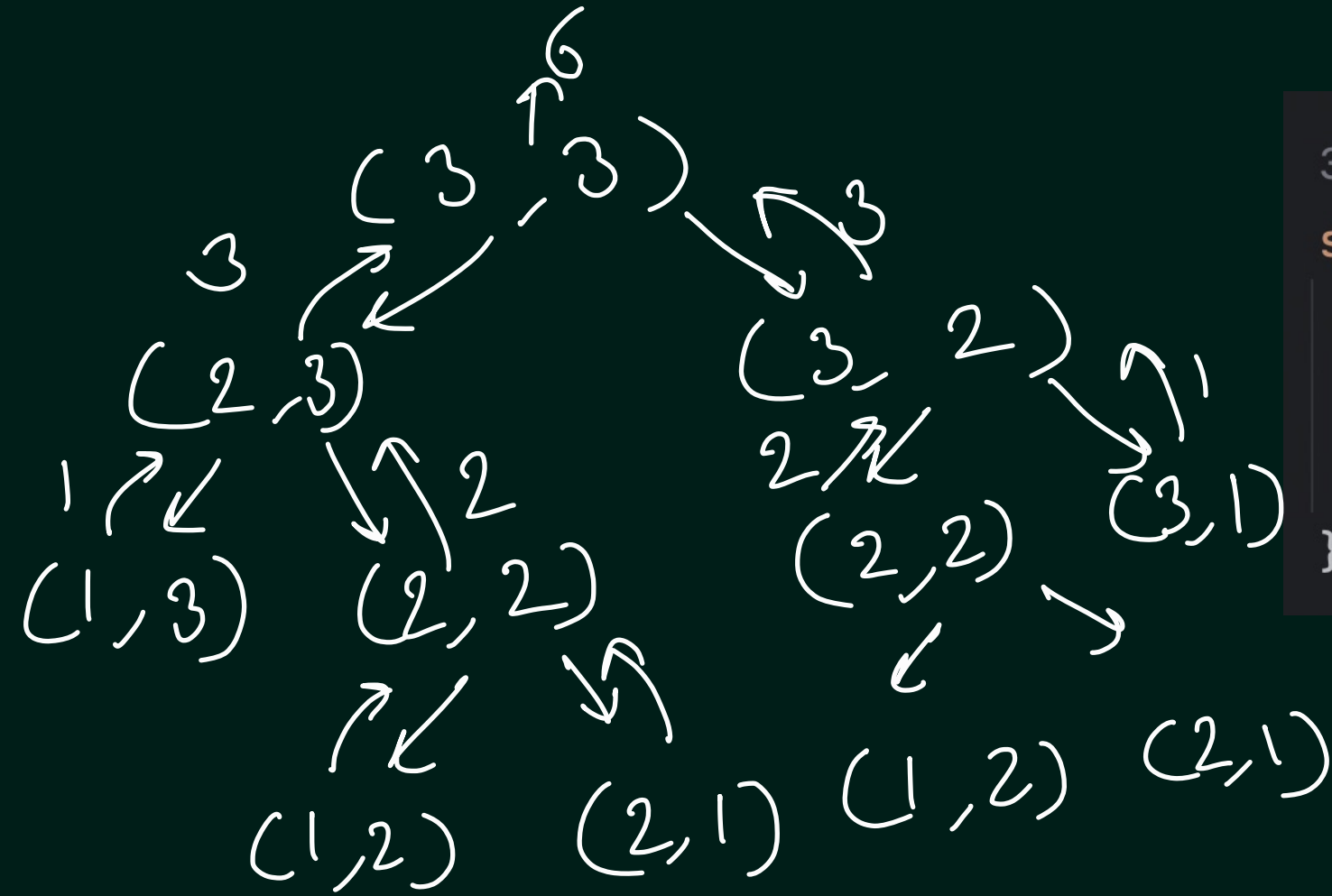
①



+


$$(M-1)$$

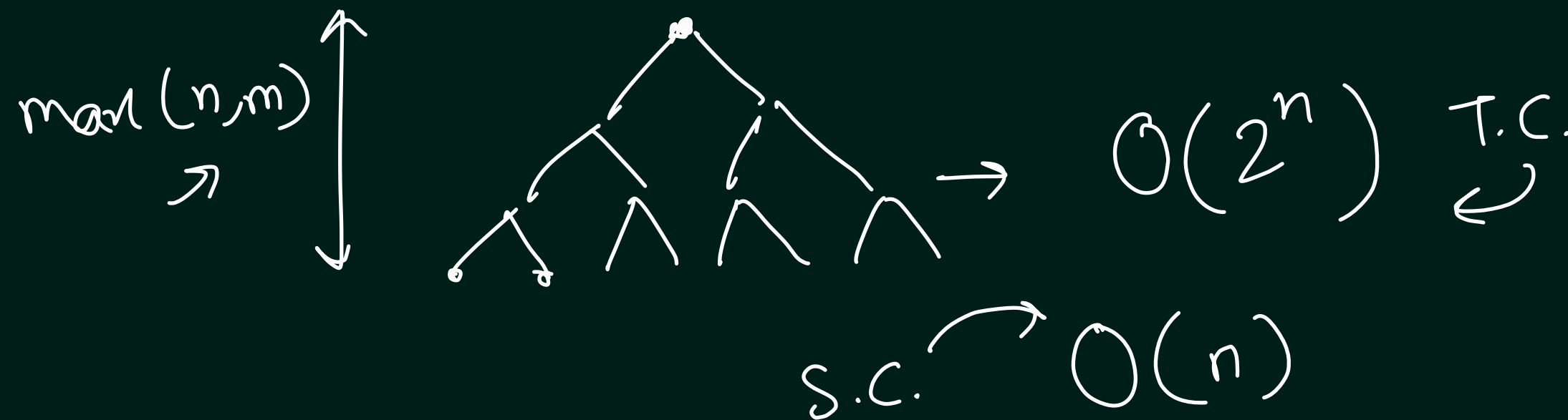
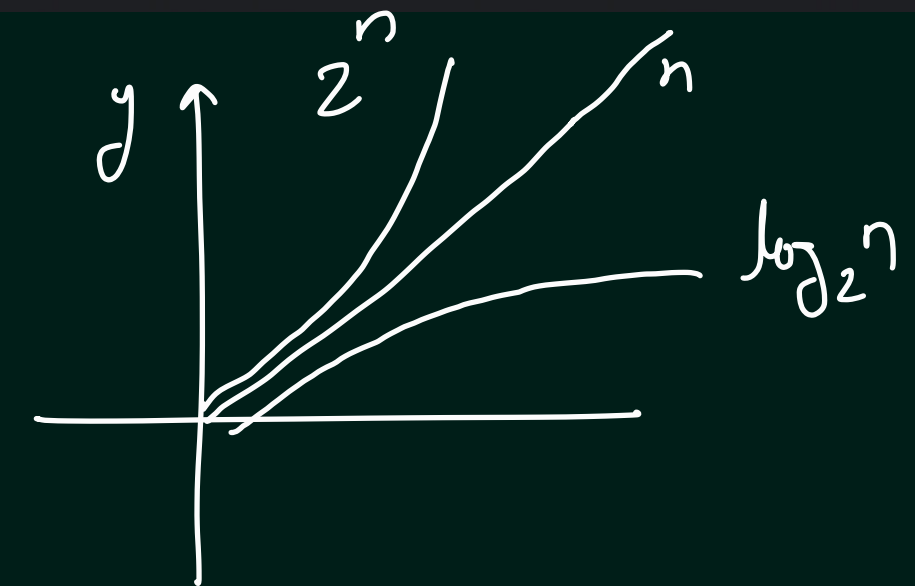
Matrix Path Problem



3 usages

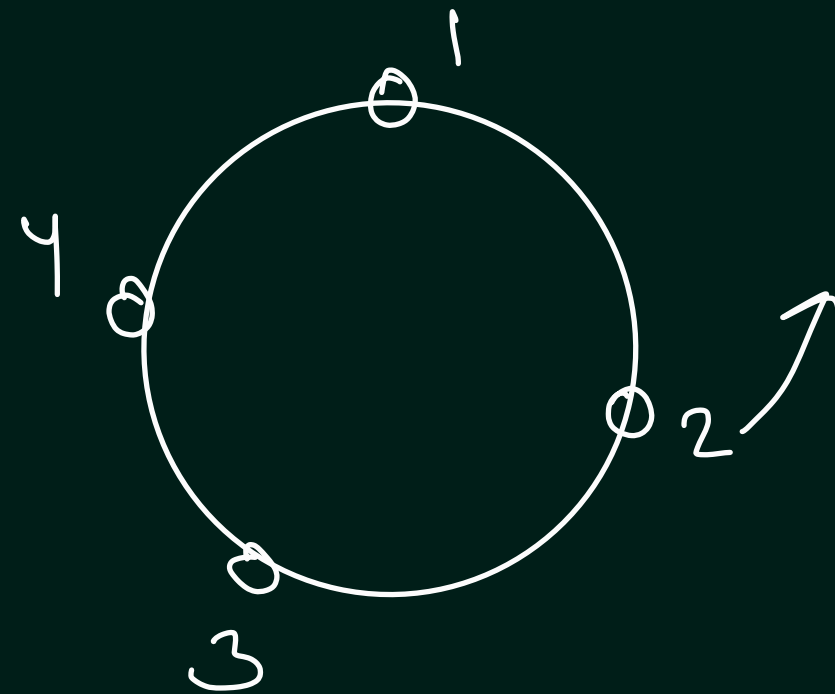
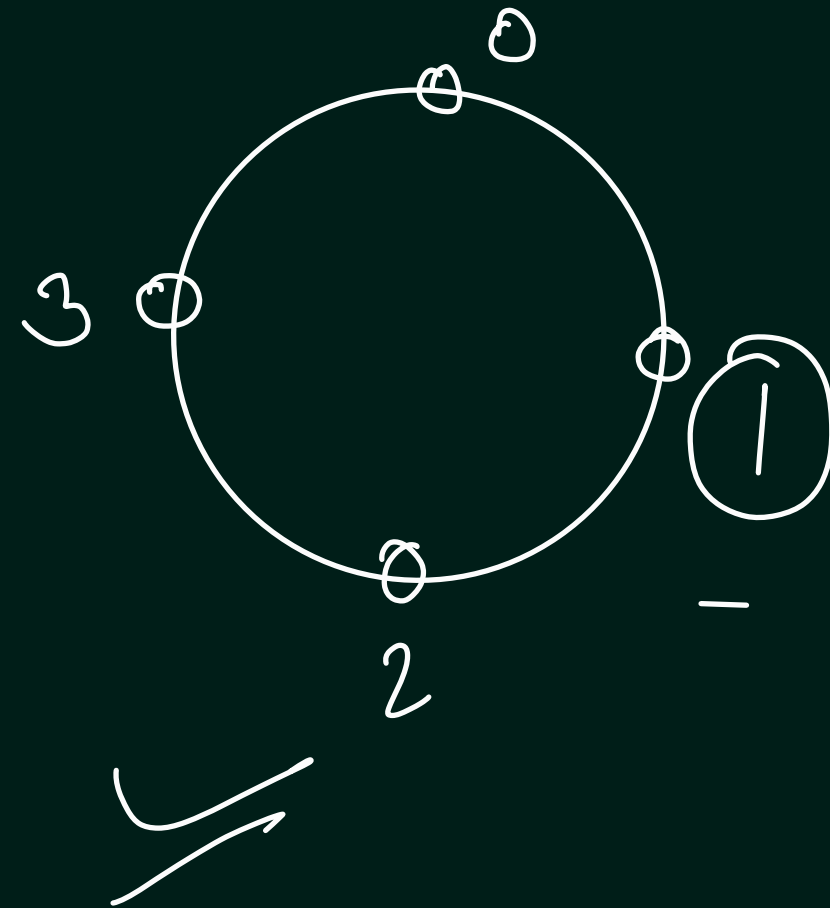
```
static int matrixPaths(int n, int m) {
    if(n == 1 || m == 1) return 1;

    return matrixPaths(n-1, m) + matrixPaths(n, m-1);
}
```



Doubts Session

Josephus Problem



Solution {

josephus(n, k) {

→ return helperJos(n, k) + 1;

helperJosephus(n, k) {

.

}

}

Doubts Session

Count occurrences

$a(b(bab\bar{a}d)) \rightarrow s$

$\rightarrow O(n \times m)$

t
 (ba)
 \downarrow
 $\boxed{\text{Count} = 2}$

$f(s, t, i)$
 \downarrow

$f(s, t, i+1)$

