

Recursion Problems - I

In This Lecture

1. Calculate the value of nCr ✓
2. Josephus Problem ✓

Calculate the value of nCr

$${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$$

$${}^nC_r = \frac{n!}{(n-r)!r!}$$

A B C D E

ABC
ABD
ABE
→ BCD
CDE

Factorial

$$[n! = n \times (n-1)!]$$

$$[0! = 1]$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 \times 1$$

$$= 120$$

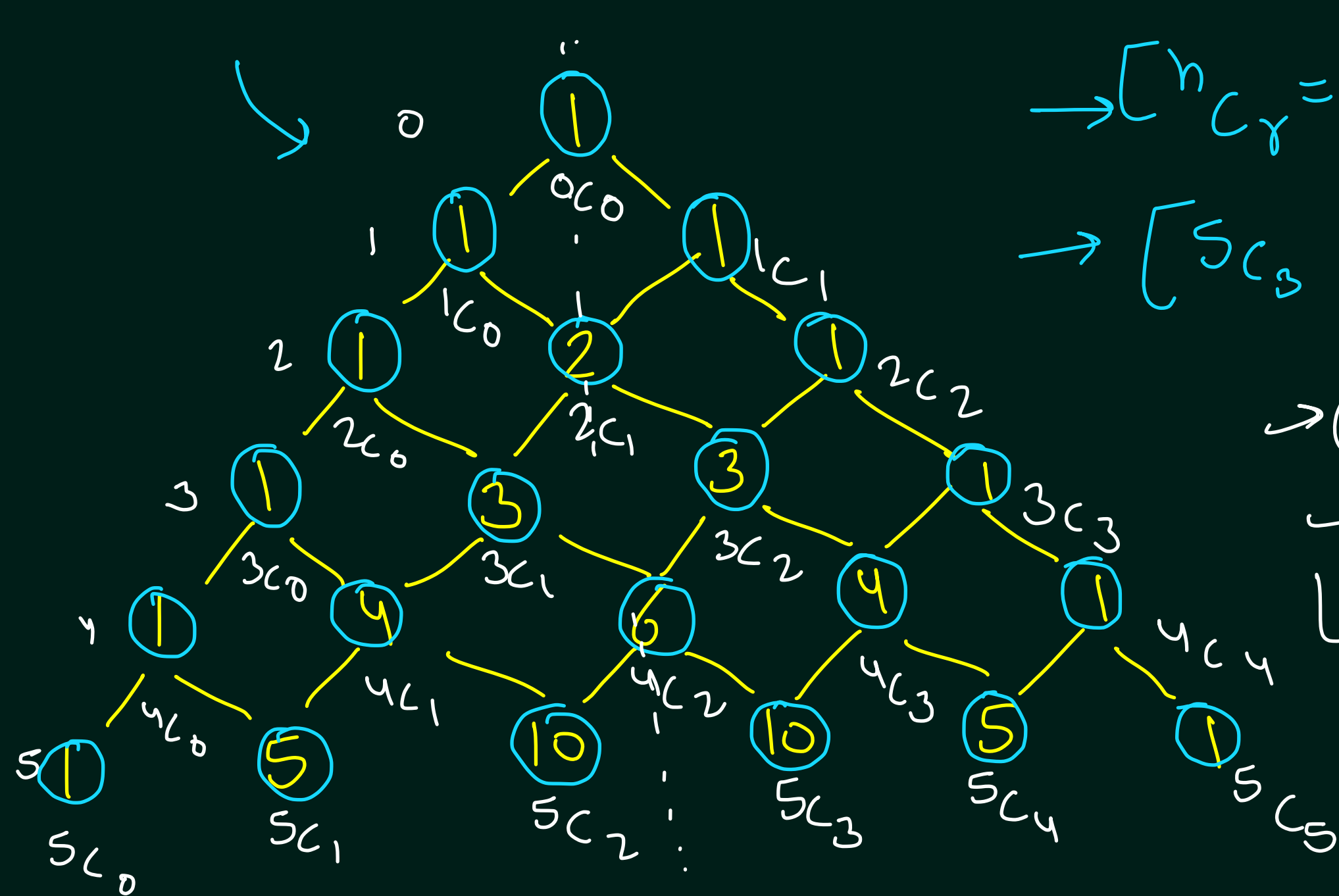
$${}^5C_3 = \frac{5!}{2!3!}$$

$$= \frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3}!}{\cancel{2} \times 1 \times \cancel{3}!}$$

$$= \boxed{10}$$

Calculate the value of nCr

→ Pascal's Triangle



$$\rightarrow [{}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r]$$

$$\rightarrow [5C_3 = 4C_2 + 4C_3]$$

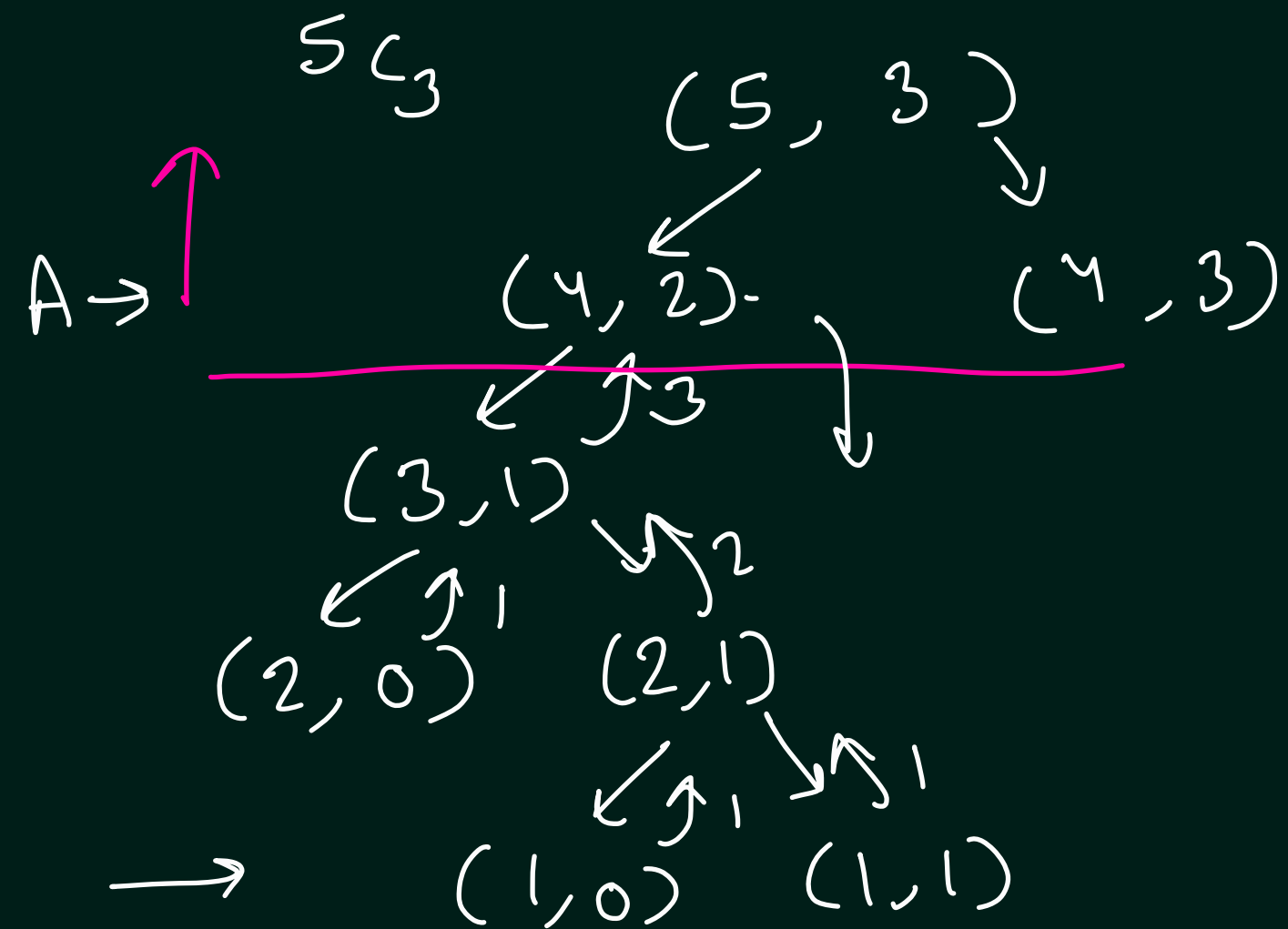
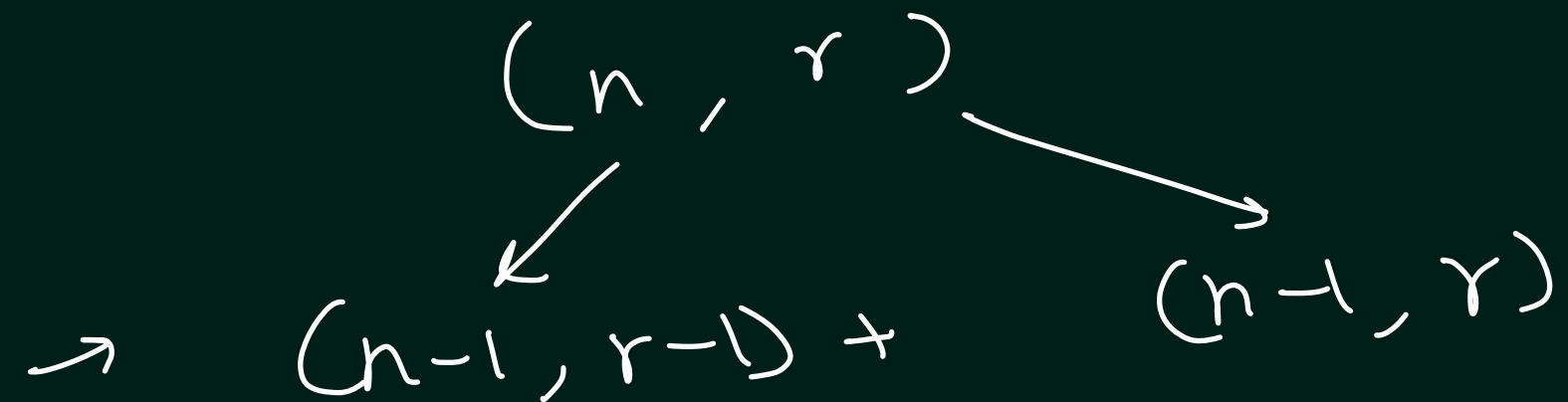
$$\rightarrow (n=r) \quad {}^nC_r = 1$$

$$\rightarrow r=0 \quad {}^nC_r = 1$$

$$\rightarrow [{}^nC_r = {}^nC_{n-r}]$$

$$0 \leq r \leq n$$

Calculate the value of nCr



3 usages

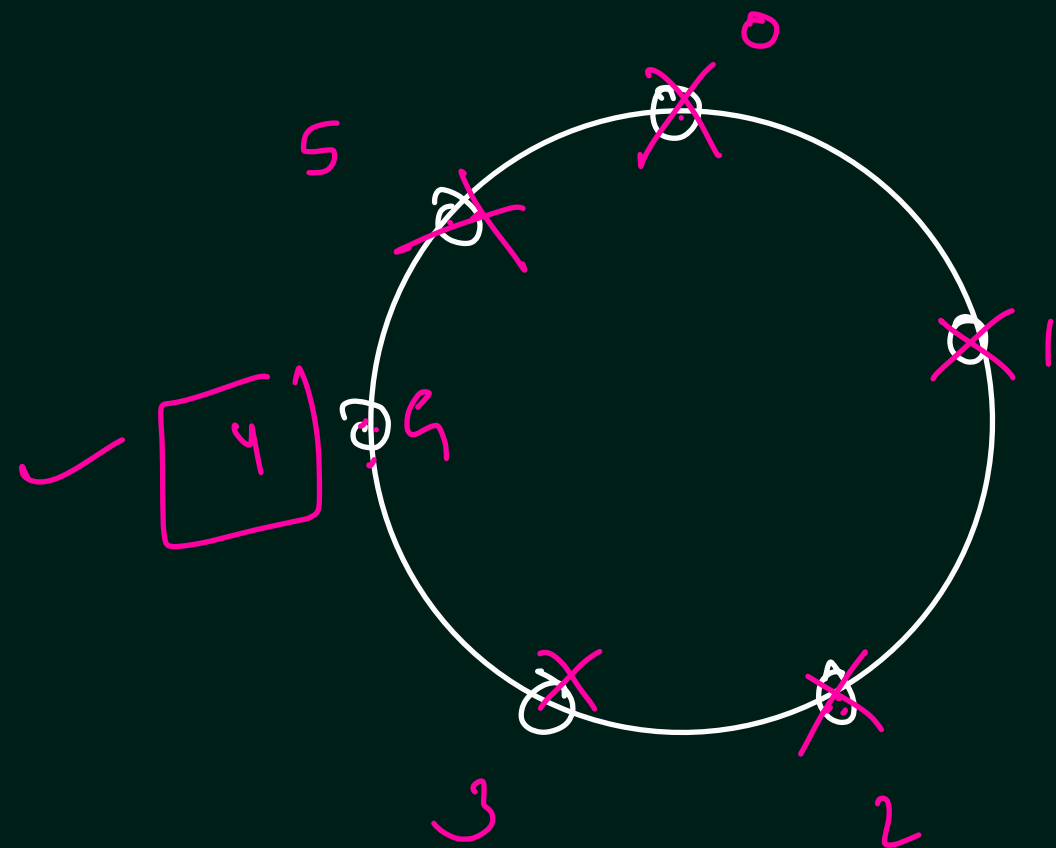
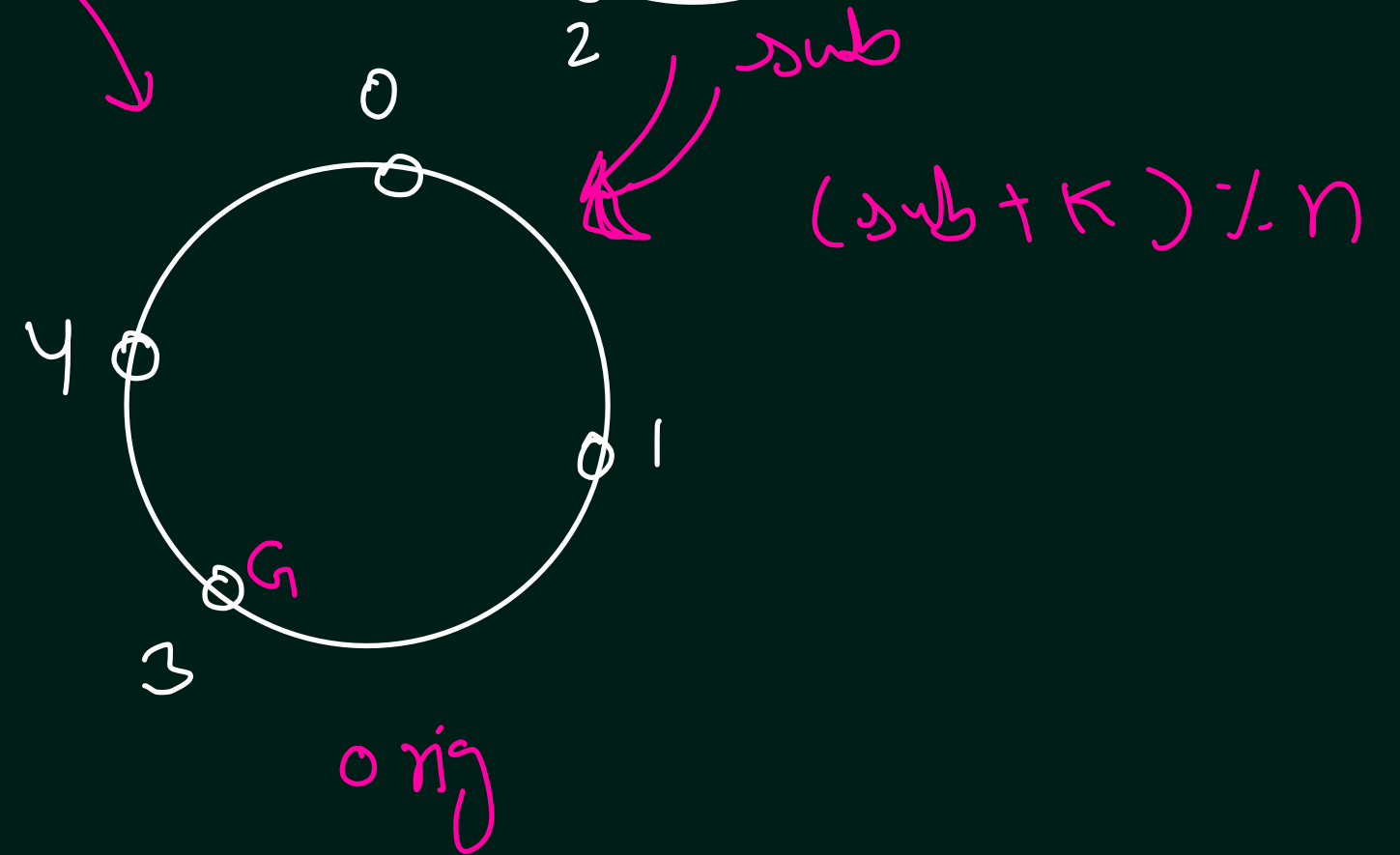
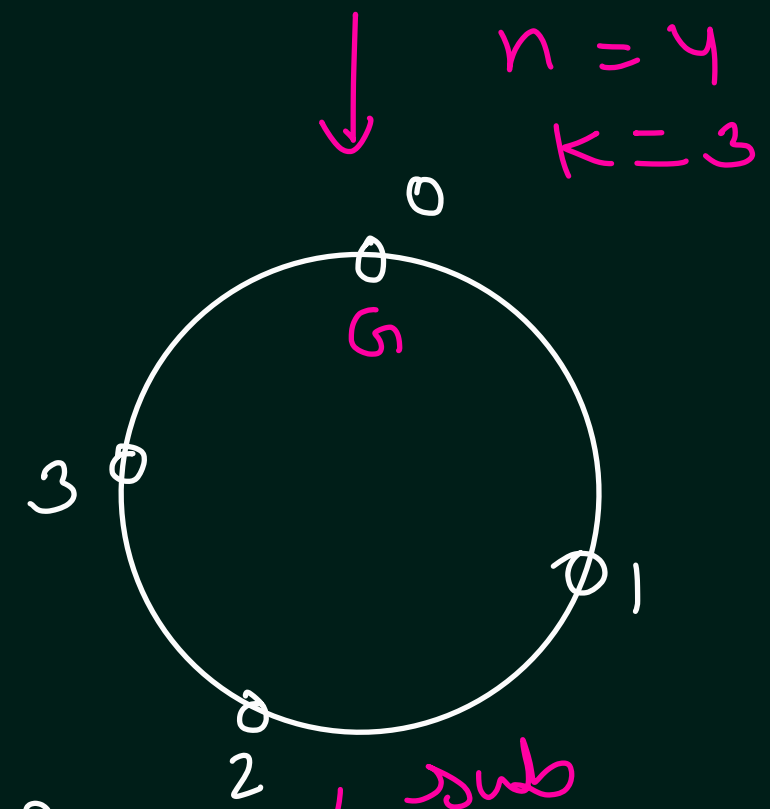
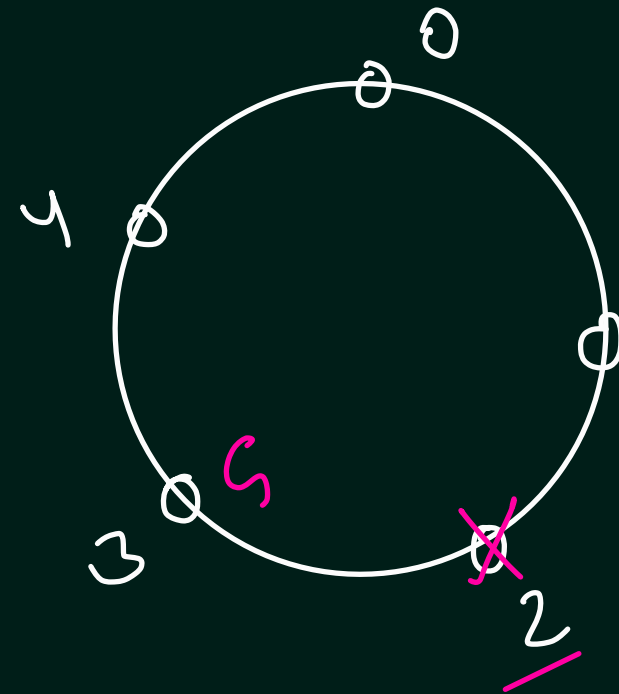
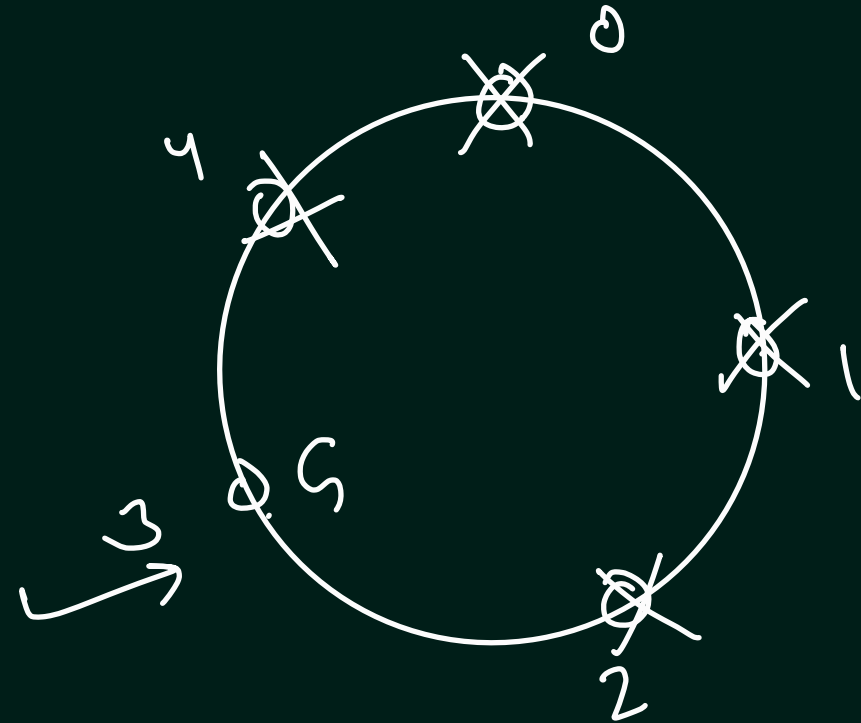
```

static int nCr(int n, int r) {
    if(n==r || r==0) return 1;
    return nCr(n-1, r-1) + nCr(n-1, r);
}
    
```

3 1
1 2

Josephus Problem

$n = 5$
 $k = 3$

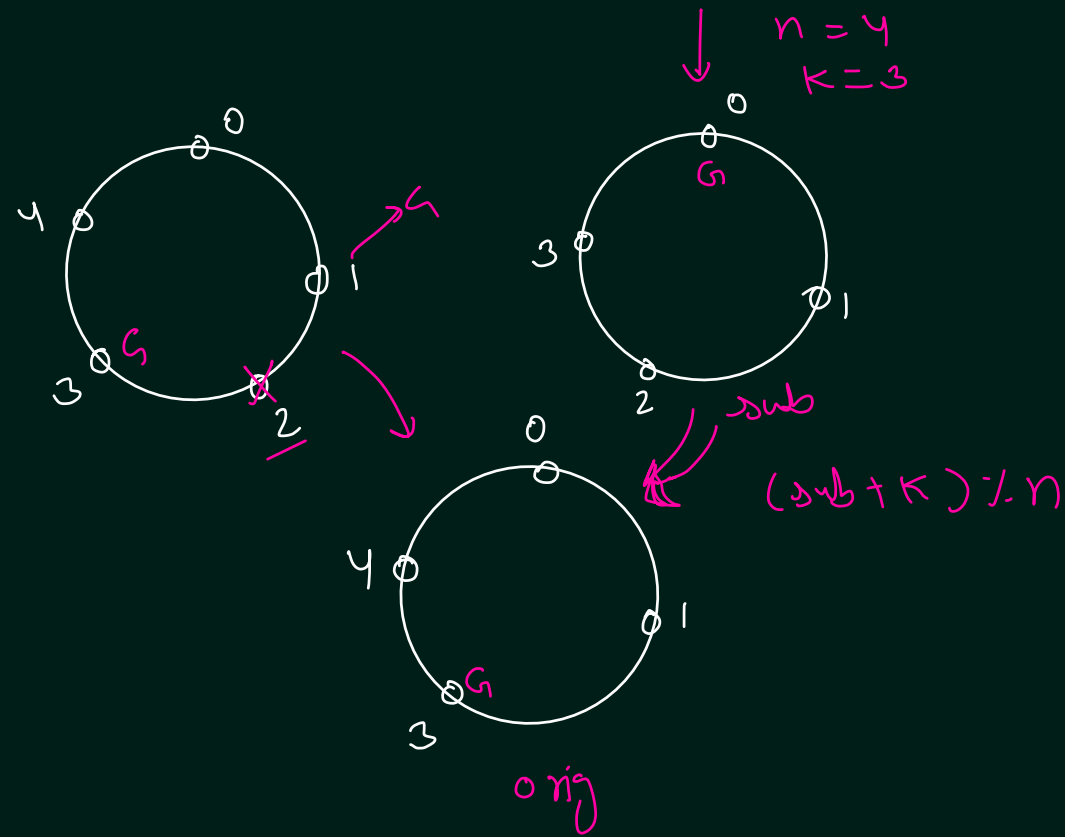


$n = 6$
 $k = 4$

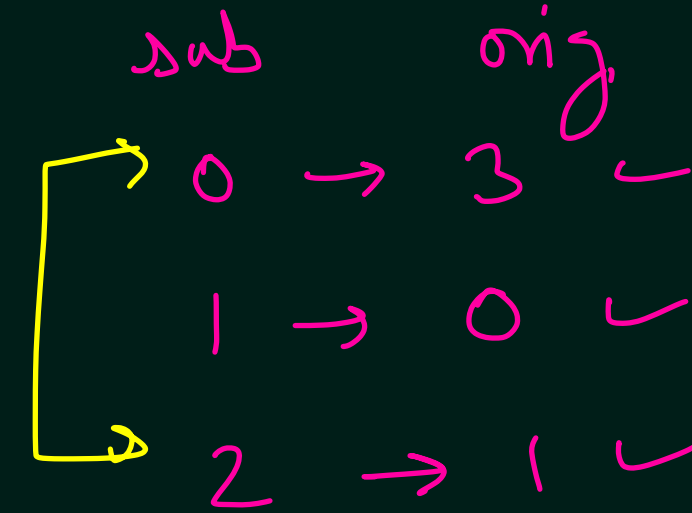
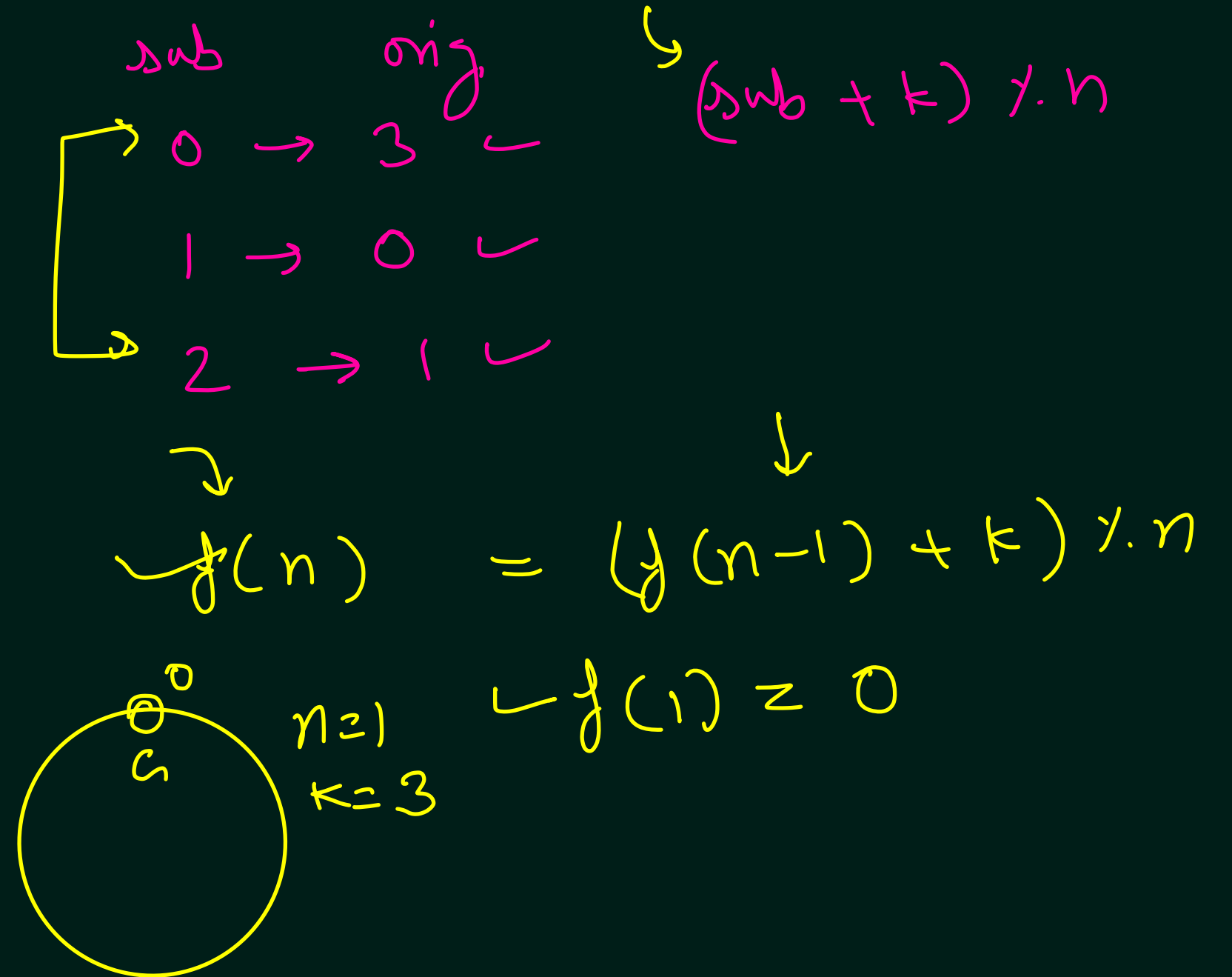
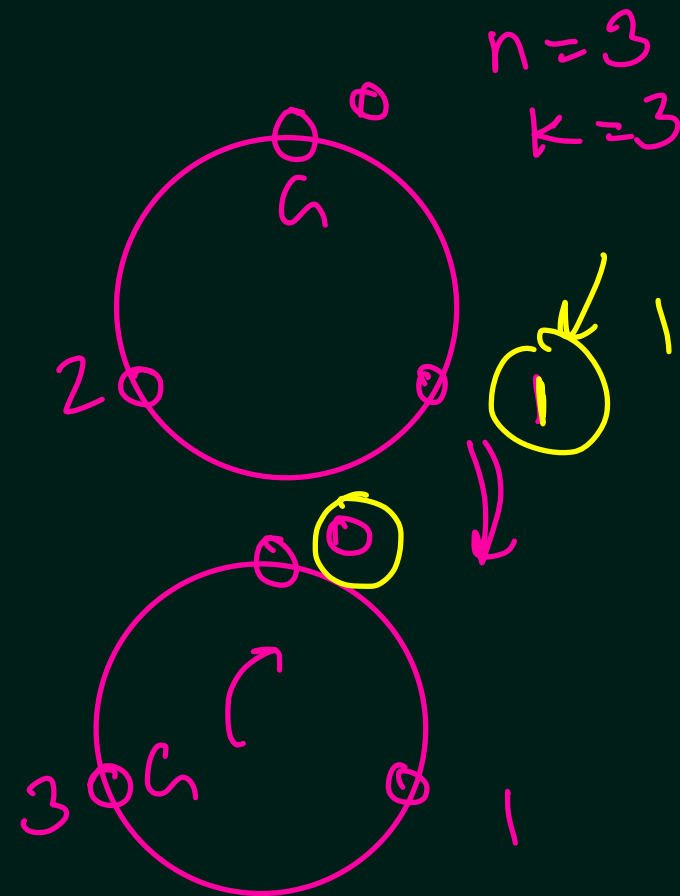
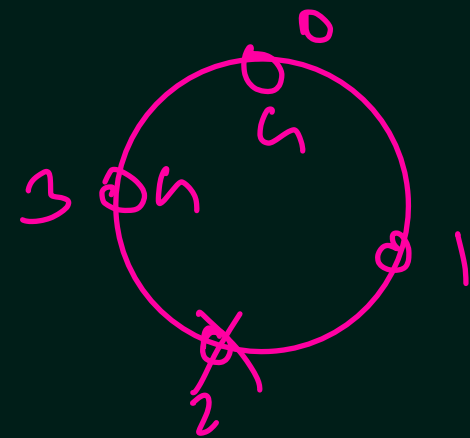
%

sub orig
0 → 3 ✓
1 → 4 ✓
2 → 0 ✓
3 → 1 ✓

Josephus Problem



$n=4$
 $k=3$

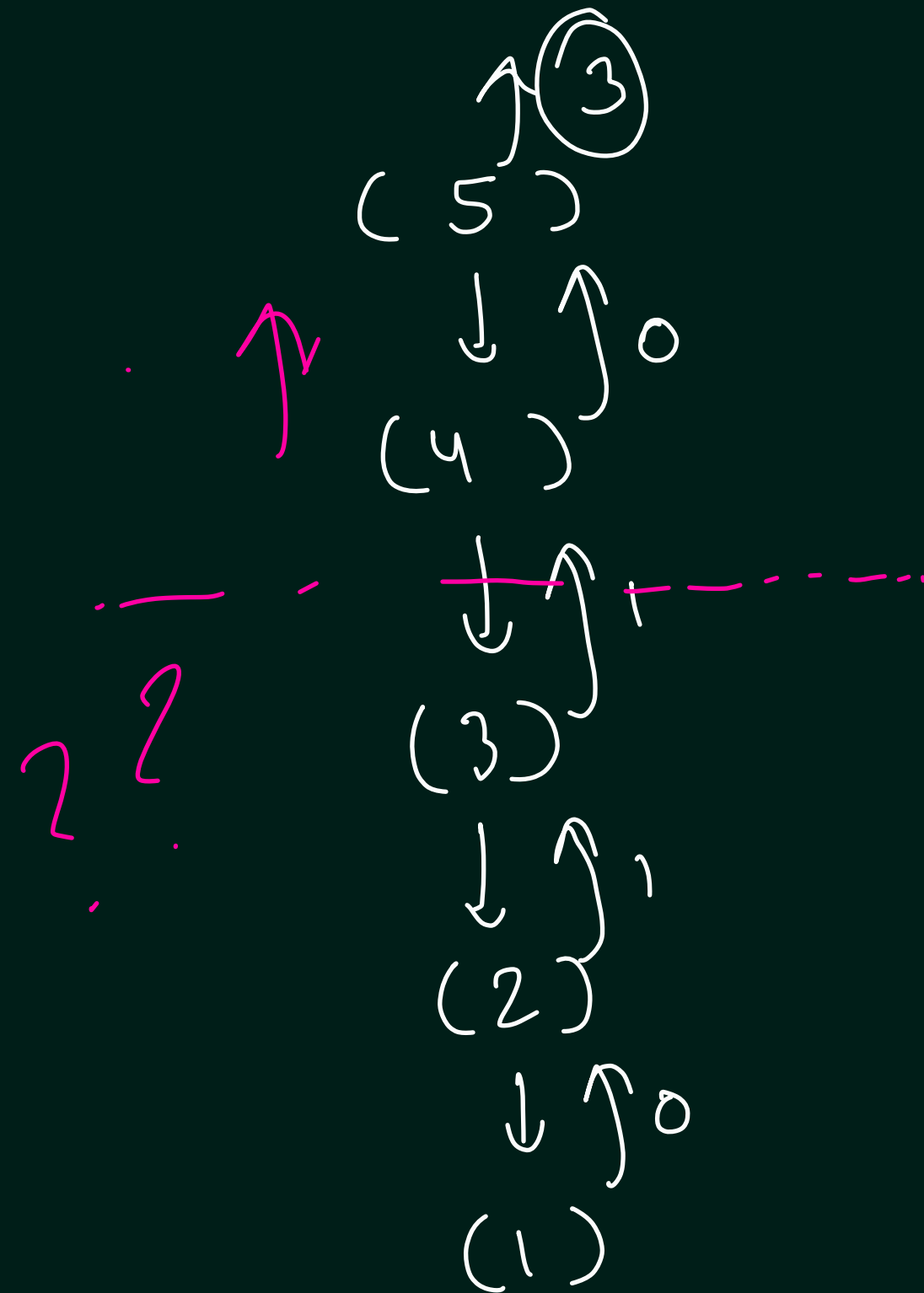


$$f(n) = (f(n-1) + k) \% n$$

$$f(1) = 0$$

Josephus Problem

$n = 5$
 $k = 3$



2 usages

$n=5$

```
static int josephus(int n, int k) {
    if(n==1) return 0;
    return (josephus(n-1, k) + k) % n;
}
```

$(4) \rightarrow (0 + 3) \% 5 = 3$

