

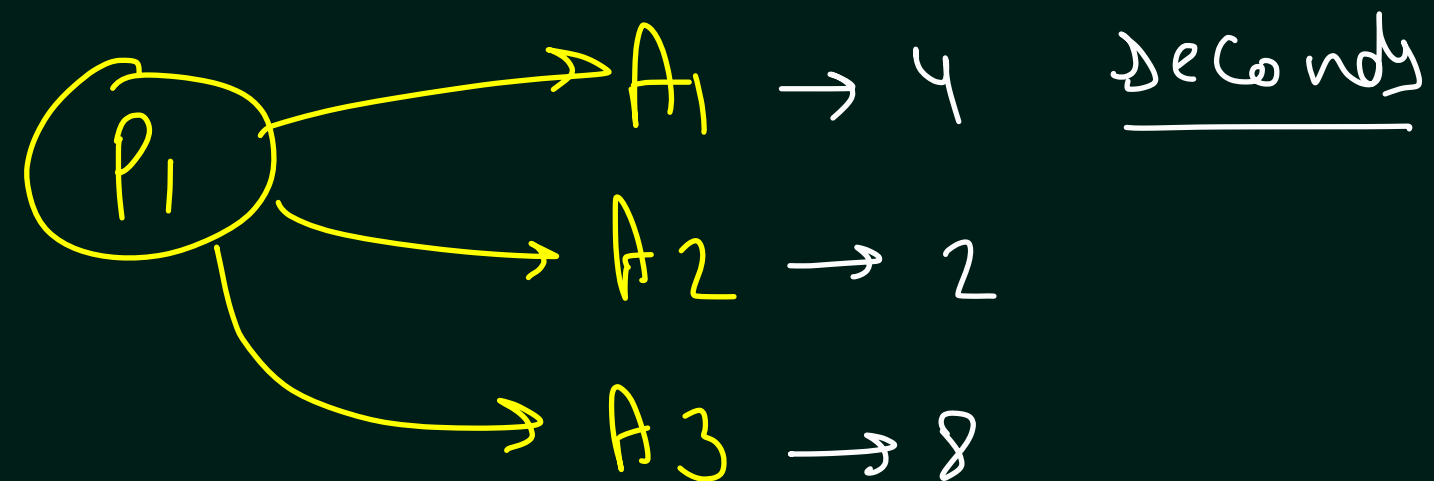
# Time & Space Complexity

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# What is Time Complexity

The time complexity of an algorithm quantifies the amount of time taken by an algorithm to run as a function of the length of the input.

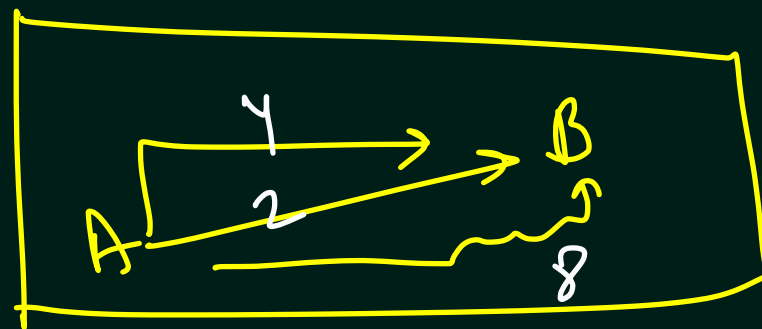


$A^O$  slow

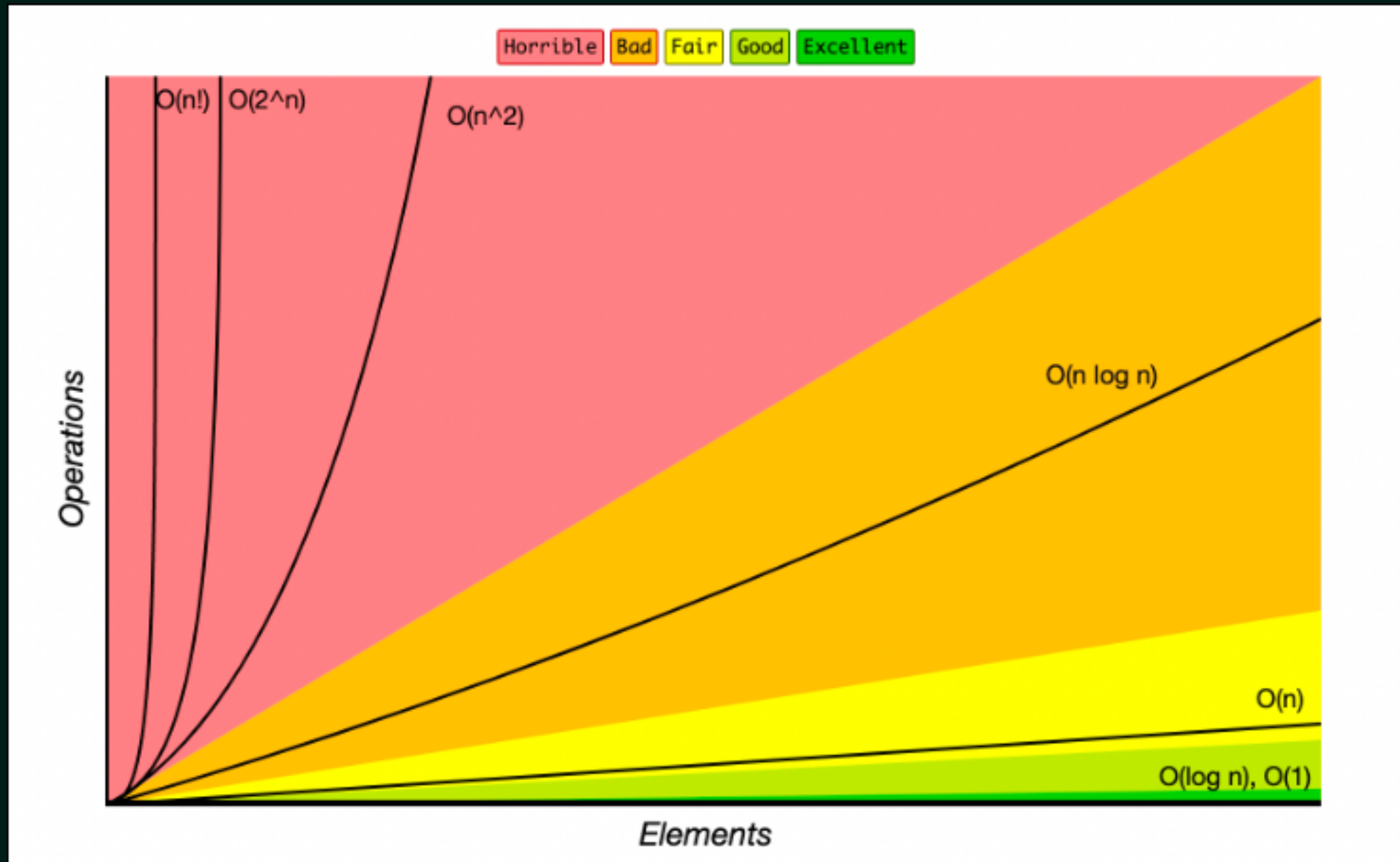
$B^O$  fast

Standard

$C^O$  super fast

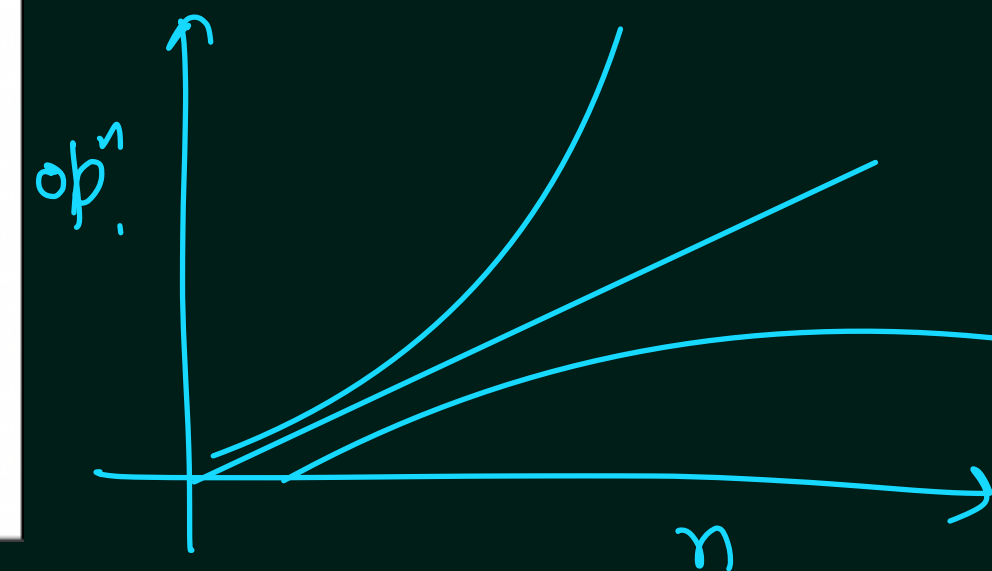


# The Big O Notation



## Asymptotic Notations

$\checkmark \rightarrow O()$  Worst Case  
 $\times \rightarrow \Theta()$  Average Case  
 $\times \rightarrow \Omega()$  Best Case



1, 2, 3 ... N

① for (1, 2, 3 ... N)

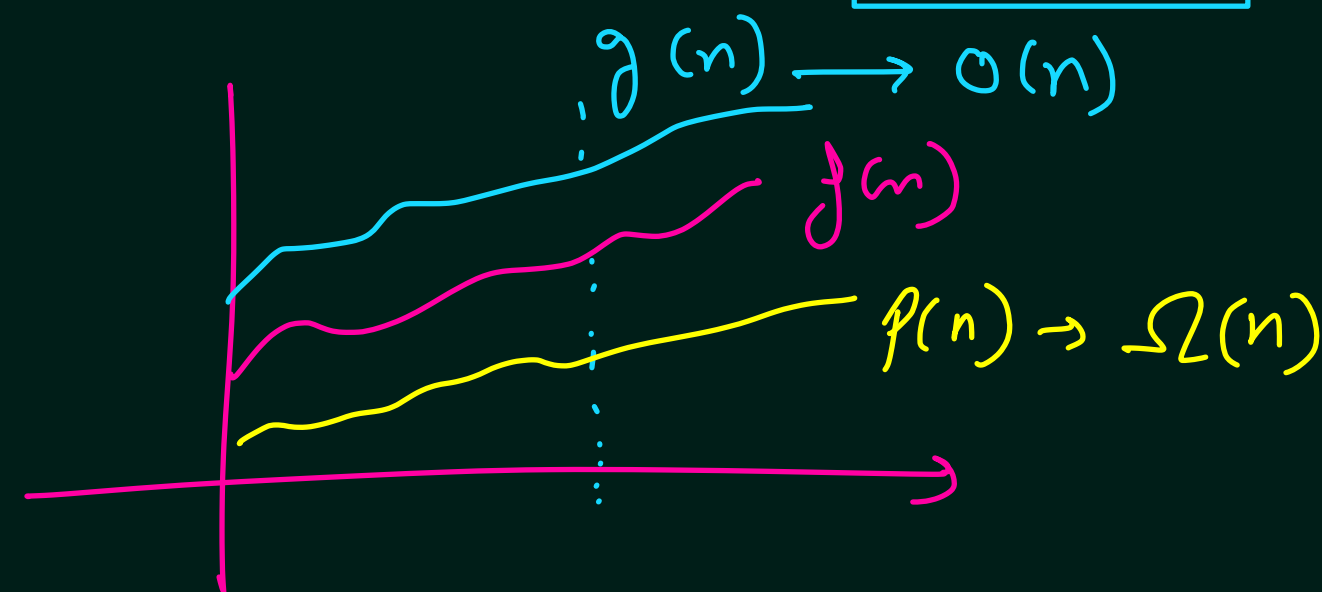
sum += i

$O(N)$

②  $sum(n) = \frac{n(n+1)}{2}$

$O(1)$

1 q/b → 1 ms  
 ↳ 10<sup>3</sup> ms  
 ↳ 10<sup>6</sup> ms



$O(n) = g(n) > f(n)$

# Time Complexity of Various Functions

Q1.  $T(n) = n^2 + \boxed{4n^3} + 1000$  ① Find the largest growing factor

Q2.  $T(n) = 40n + \boxed{n^2} + 12 \rightarrow O(n^2)$  ② Remove the constant

Q3.  $T(n) = n^2 + \boxed{2^n} \rightarrow O(2^n)$

$O(n^3)$

Q4.  $T(n) = 20\log(n) + 4n + \boxed{3n^2} \rightarrow O(n^2)$



# Time Complexity of Recursive Functions

Q1.  $T(n) = T(n-1) + C$

Q2.  $T(n) = 2T(n-1) + C$

Q3.  $T(n) = 2T(n-1) + nC$

Q4.  $T(n) = 2T(n/2) + C$

```
function(n) {  
    sum = sum + n;  
    → function(n-1);  
}
```

$$T(n) = T(n-1) + C$$

# Time Complexity of Recursive Functions

$$\begin{aligned}
 \textcircled{1} \quad T(n) &= (T(n-1)) + C \\
 &= ((T(n-2)) + C) + C \\
 &= ((T(n-3)) + C) + C + C
 \end{aligned}$$

$$\begin{aligned}
 n - k &= 0 \\
 \Rightarrow k &= n
 \end{aligned}$$

$$\begin{aligned}
 T(0) &= 1 \\
 T(1) &= 1
 \end{aligned}$$

$$T(n) = T(n-k) + kC$$

$$T(n) = T(0) + nC$$

$$[T(n) = 1 + \cancel{nC}] \rightarrow \boxed{O(n)}$$



# Time Complexity of Recursive Functions

$$(2) \quad T(n) = 2T(n-1) + C$$

$$= 2[2T(n-2) + C] + C$$

$$= 2[2[2T(n-3) + C] + C] + C$$

$$T(0) = n - k = 0$$

$$k = n$$

$$T(0) = 1000$$

(let's assume)

$$T(n) = 2^k T(n-k) + kC$$

$$= 2^n T(0) + nC$$

$$T(n) = \boxed{2^n \times 1000} + nC \rightarrow O(2^n)$$

# Time Complexity of Recursive Functions

$$\begin{aligned}
 \textcircled{3} \quad T(n) &= 2T(n/2) + C \\
 &= 2[2T(n/4) + C] + C \\
 &= 2[2[2T(n/8) + C] + C] + C
 \end{aligned}$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kC$$

$$= 2^{\log_2 n} T(1) + \log_2 n C$$

$$T(n) = \boxed{n * 1} + \log_2 n * C$$

$\rightarrow O(n)$

$$x = 2^{\log_2 n}$$

$$\log_2 x = \log_2 2^{\log_2 n}$$

$$= \log_2 n \log_2 2$$

$$\log_2 x = \log_2 n$$

$$\boxed{x = n} = 2^{\log_2 n}$$

$$T(1) = \frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = \log_2 2^k$$

$$= k \log_2 2$$

$$\boxed{k = \log_2 n}$$

# Time Constraints and Time Complexity

Let  $n$  be the main variable in the problem.

- If  $n \leq 12$ , the time complexity can be  $O(n!)$ . → Hardest
- If  $n \leq 25$ , the time complexity can be  $O(2^n)$ .
- If  $n \leq 100$ , the time complexity can be  $O(n^4)$ .
- If  $n \leq 500$ , the time complexity can be  $O(n^3)$ .
- If  $n \leq 10^4$ , the time complexity can be  $O(n^2)$ .
- If  $n \leq 10^6$ , the time complexity can be  $O(n \log n)$ .
- If  $n \leq 10^8$ , the time complexity can be  $O(n)$ .
- If  $n > 10^8$ , the time complexity can be  $O(\log n)$  or  $O(1)$ . → smallest

$10^8$  Operations Rule

# Time Constraints and Time Complexity

# Space Complexity

The space Complexity of an algorithm is the total space taken by the algorithm with respect to the input size. Space complexity includes both Auxiliary space and space used by input.

$O(n)$

↳  $N$

$O(10) \rightarrow \boxed{O(1)}$

$\boxed{O(n^2)}$

function  $(n)$  {

// int a[] = new int[n];

// int b[] = new int[n];

int c[] = new int[10];

int d[][] = new int[n][n];

$O(2n)$

↳  $O(n)$

←

