

Recursion Basics

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Basics of Recursion

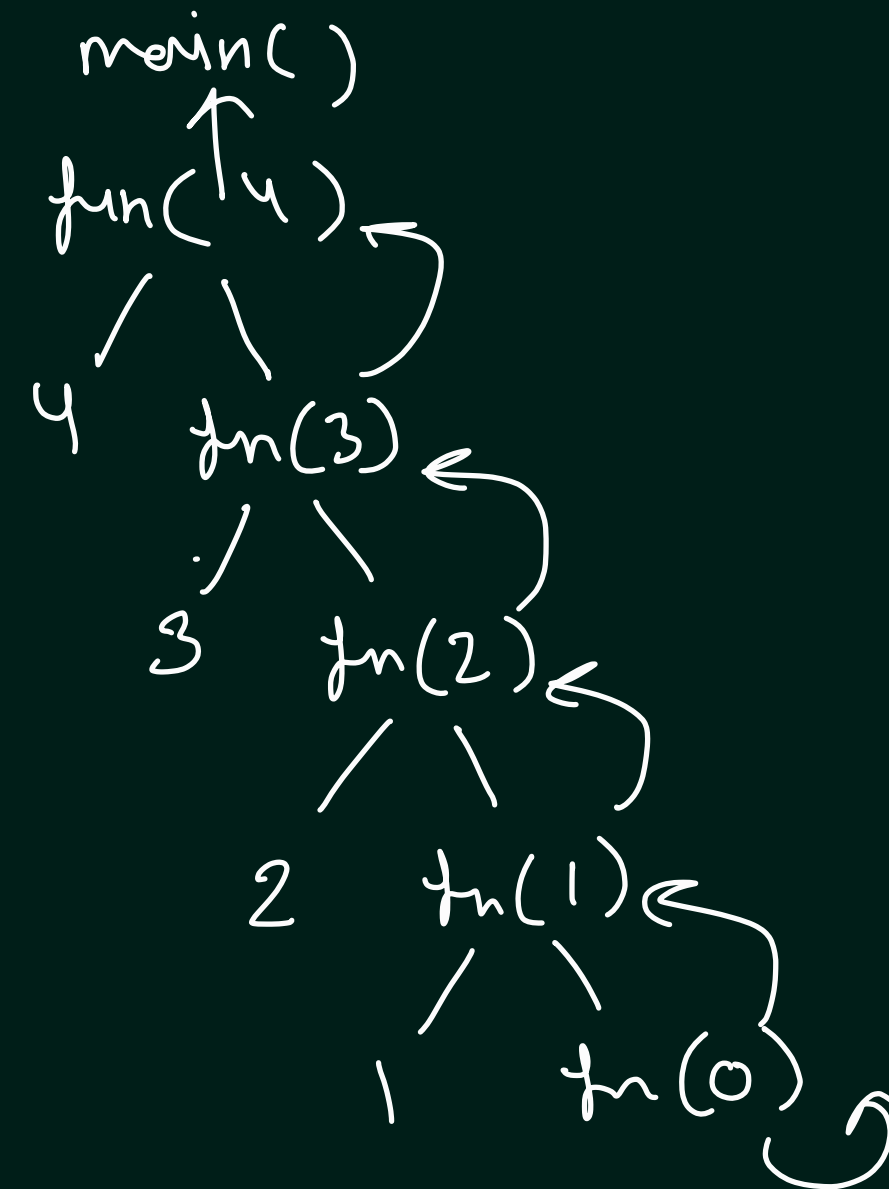
Recursion is the technique of making a function call itself. This technique provides a way to break complicated problems down into simple problems which are easier to solve.

base
case

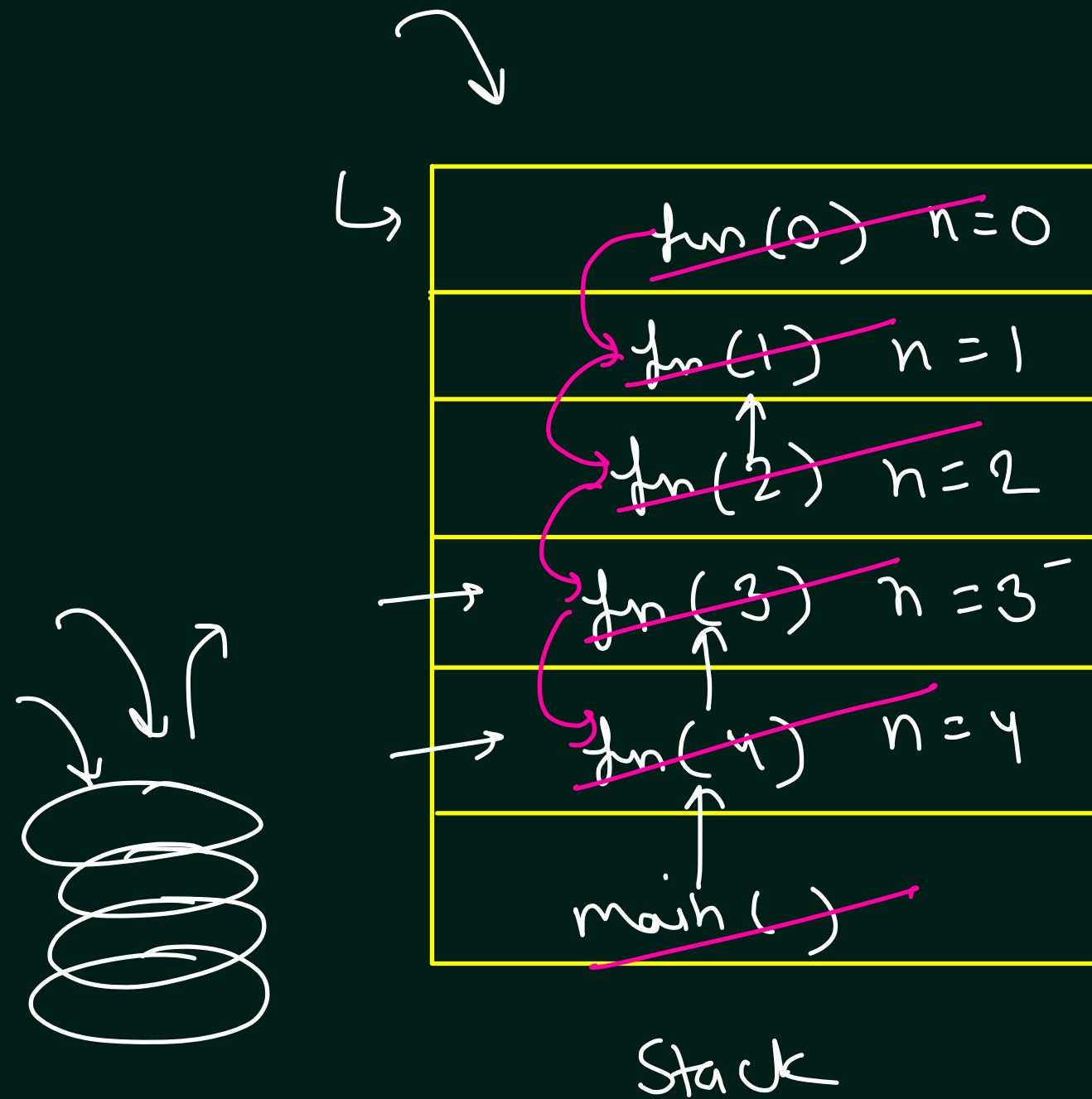
```
2 usages
static void fun(int n) {
1 | if(n > 0) { ←
2 |     System.out.println(n);
3 |     fun(n - 1);
  | }
  }
```

o/p : 4 3 2 1

~~n = 4 3 2 1 0~~



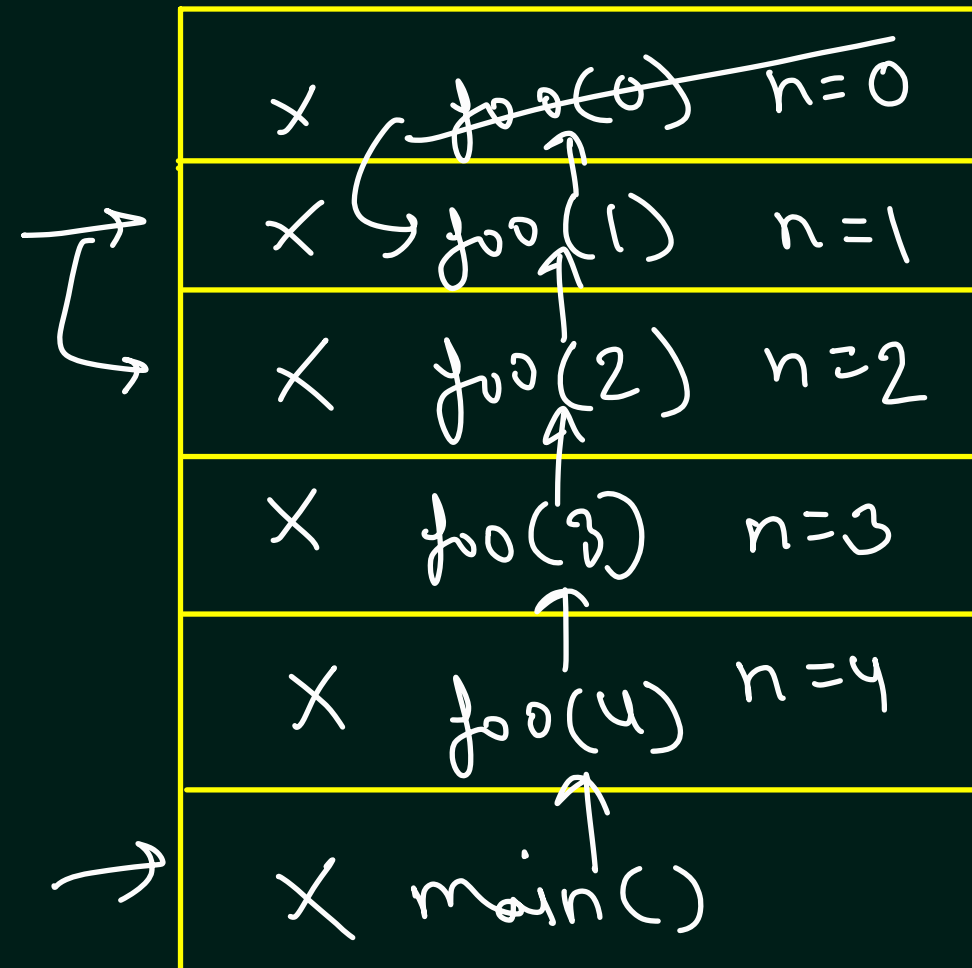
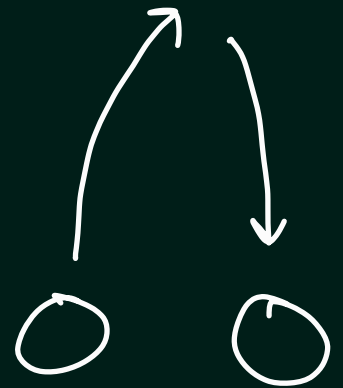
Recursion Stack



op/ : 4 3 2 1

```
2 usages
static void fun(int n) {
    if(n > 0) {
        System.out.println(n);
        fun(n - 1);
    }
}
```

Recursion Stack

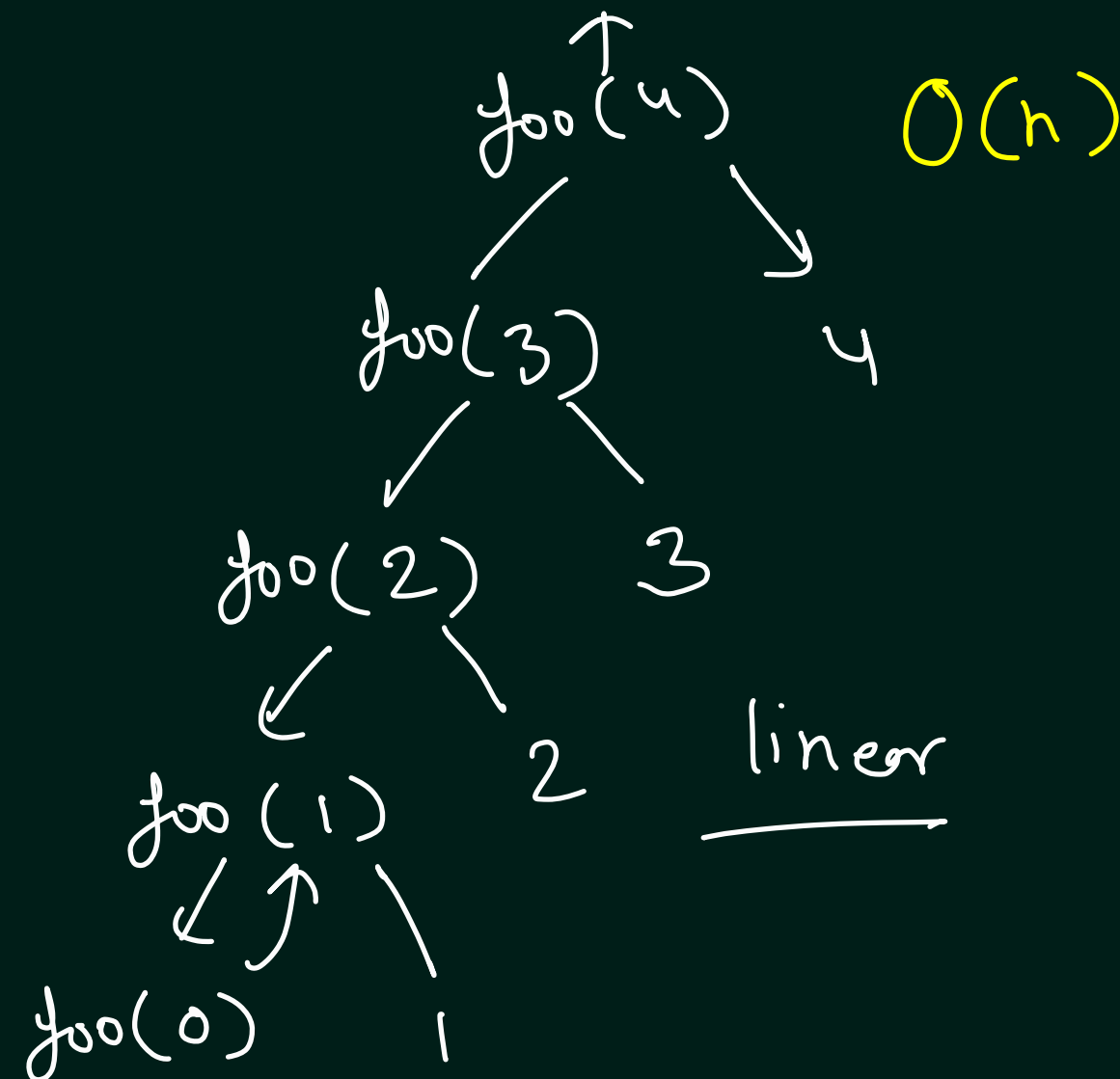


Op: 1 2 3 4

```

main()
  ↓
2 usages
static void foo(int n) {
    if(n > 0) {
        foo(n: n - 1);
        System.out.println(n); ← desc
    }
}
  
```

Arrows show the call flow: main() calls foo(2), which calls foo(1), which calls foo(0). The return path is labeled 'desc'.



Linear Recursion & Tree Recursion

↑

```

fn(n) {
  if ( ) {
    → fn(n-1);
  }
}

```

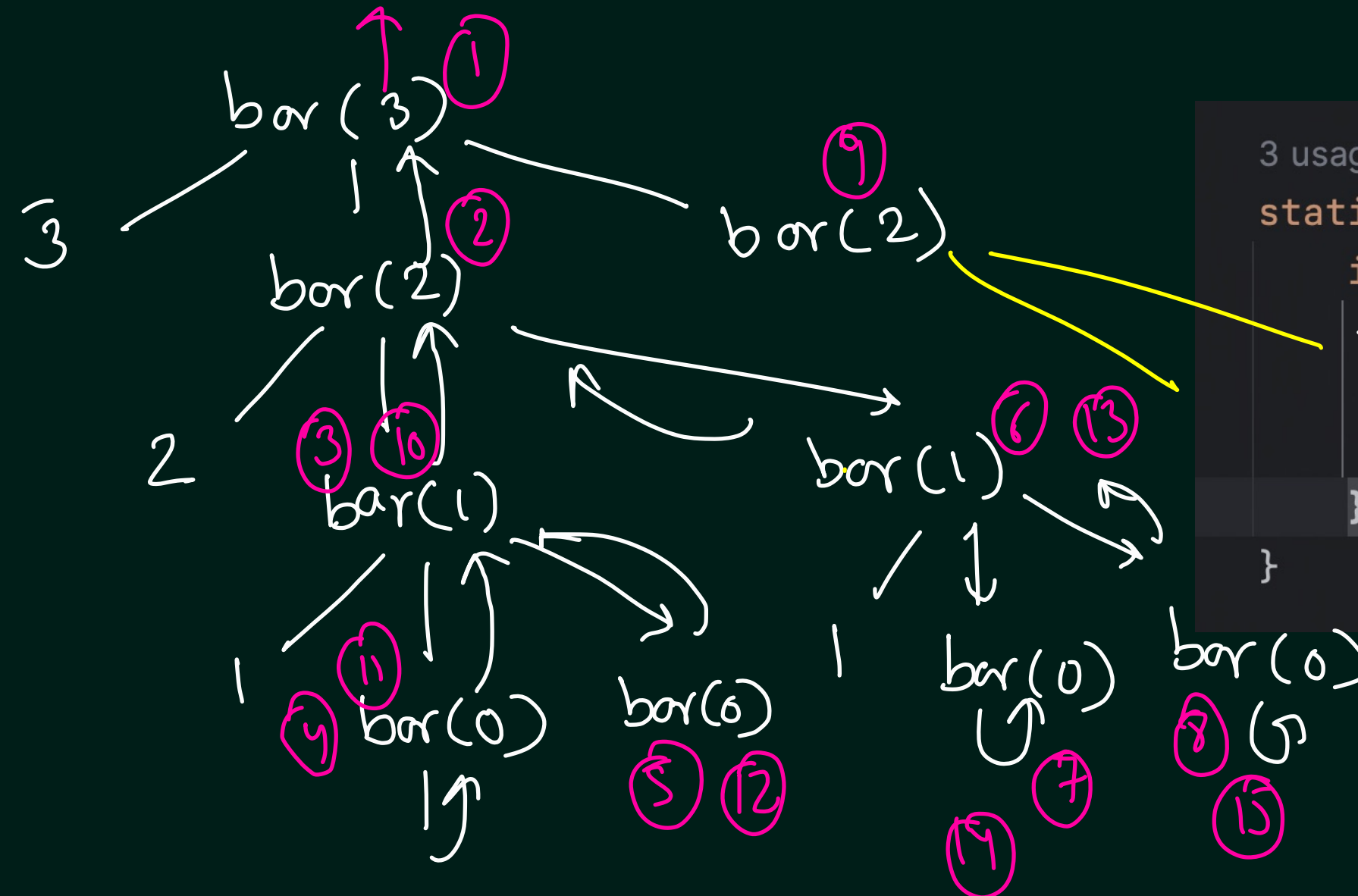
```

fn(n) {
  if ( ) {
    → fn(n-1);
    → fn(n-1);
  }
}

```



8 (15)


$$n=3$$

```
if(n > 0) {
```

→ `System.out.println(n);`

\Rightarrow `bar(n: n-1);`

→ `bar(n: n-1);` `bar(2)`

$$\rightarrow 2^{n+1} - 1$$

Op: 3 2 1 | 2 1 1

$$2^4 - 1 = 15$$

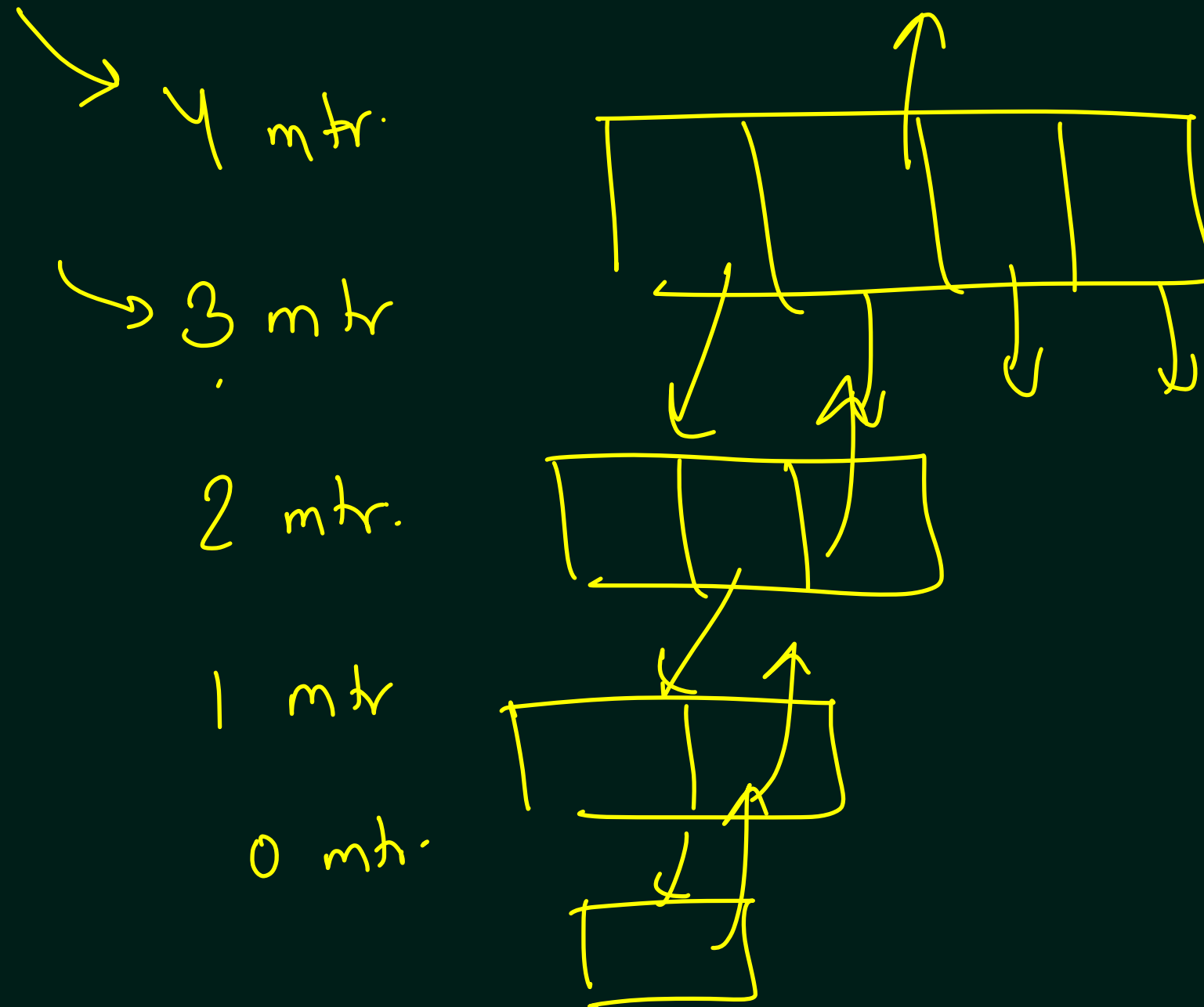
$$\left[2^0 + 2^1 + 2^2 + 2^3 \dots 2^n = 2^{n+1} - 1 \right] \rightarrow \boxed{O(2^n)}$$

3 Steps To Solve Recursion Problem

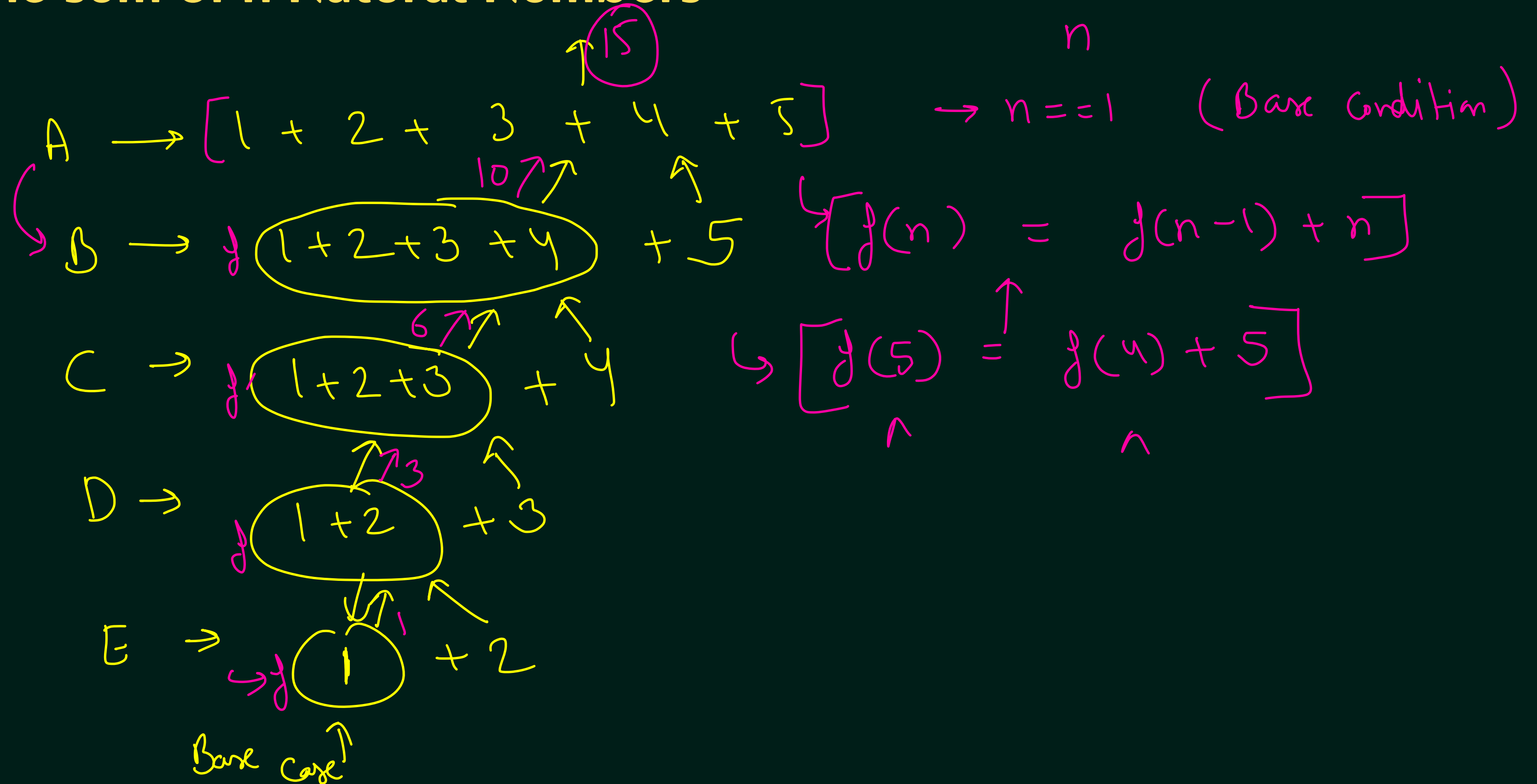
→ Step 1: Find the Base Case

→ Step 2: Find the relation between the Problem and the Sub Problem

Step 3: Generalise the relation found in the step 2



The sum of n Natural Numbers



The sum of n Natural Numbers

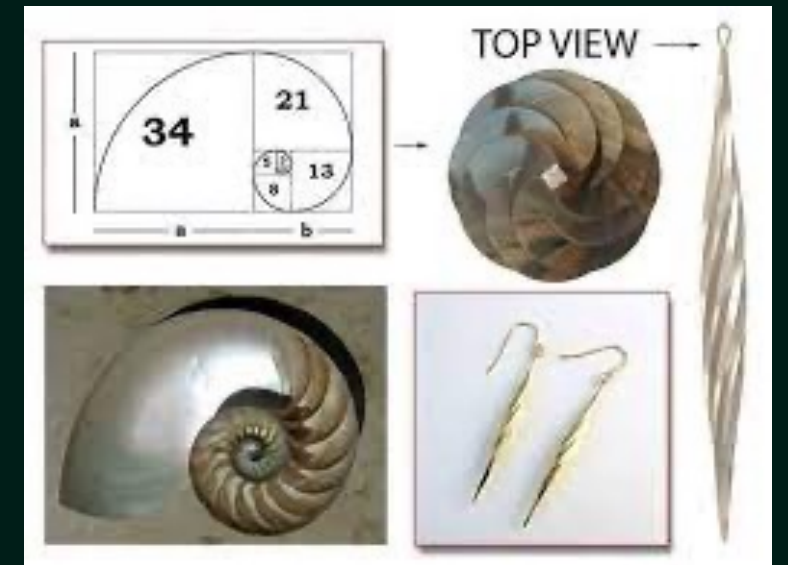
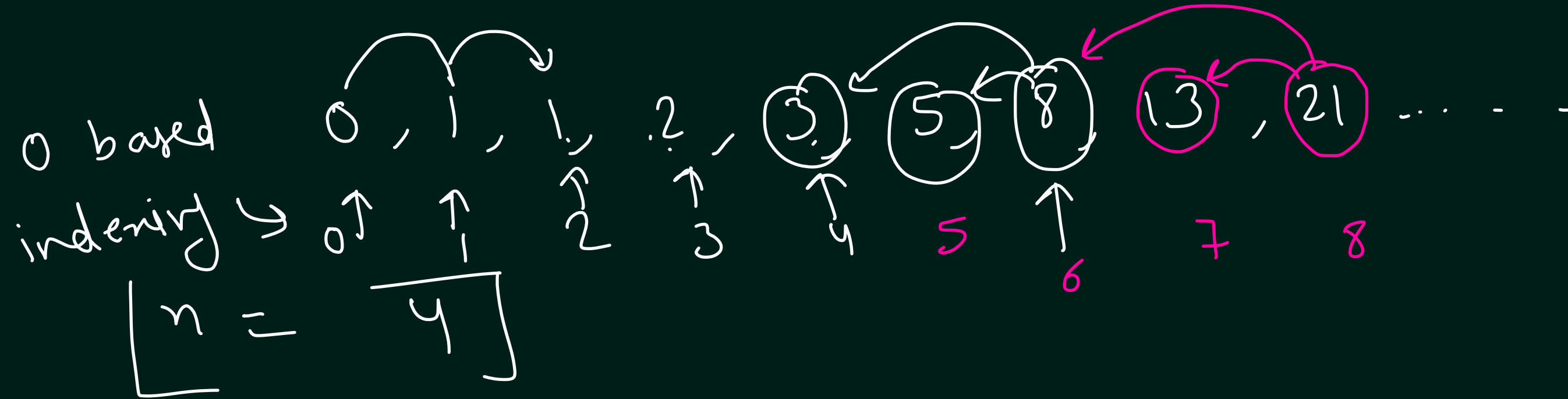
$$\begin{array}{l}
 f(5) + 5 \quad \uparrow 15 \\
 \swarrow \uparrow 10 \\
 f(4) + 4 \\
 \swarrow \uparrow 6 \\
 f(3) + 3 \\
 \swarrow \uparrow 3 \\
 f(2) + 2 \\
 \swarrow \uparrow 1 \\
 f(1)
 \end{array}$$

```

15 → System.out.println(sumOfN(n: 5)); ← main
    }

2 usages          n = 5
static int sumOfN(int n) {
    if(n == 1) return 1;
    return sumOfN(n: n-1) + n;
}
                                10 + 5
  
```

Fibonacci Number

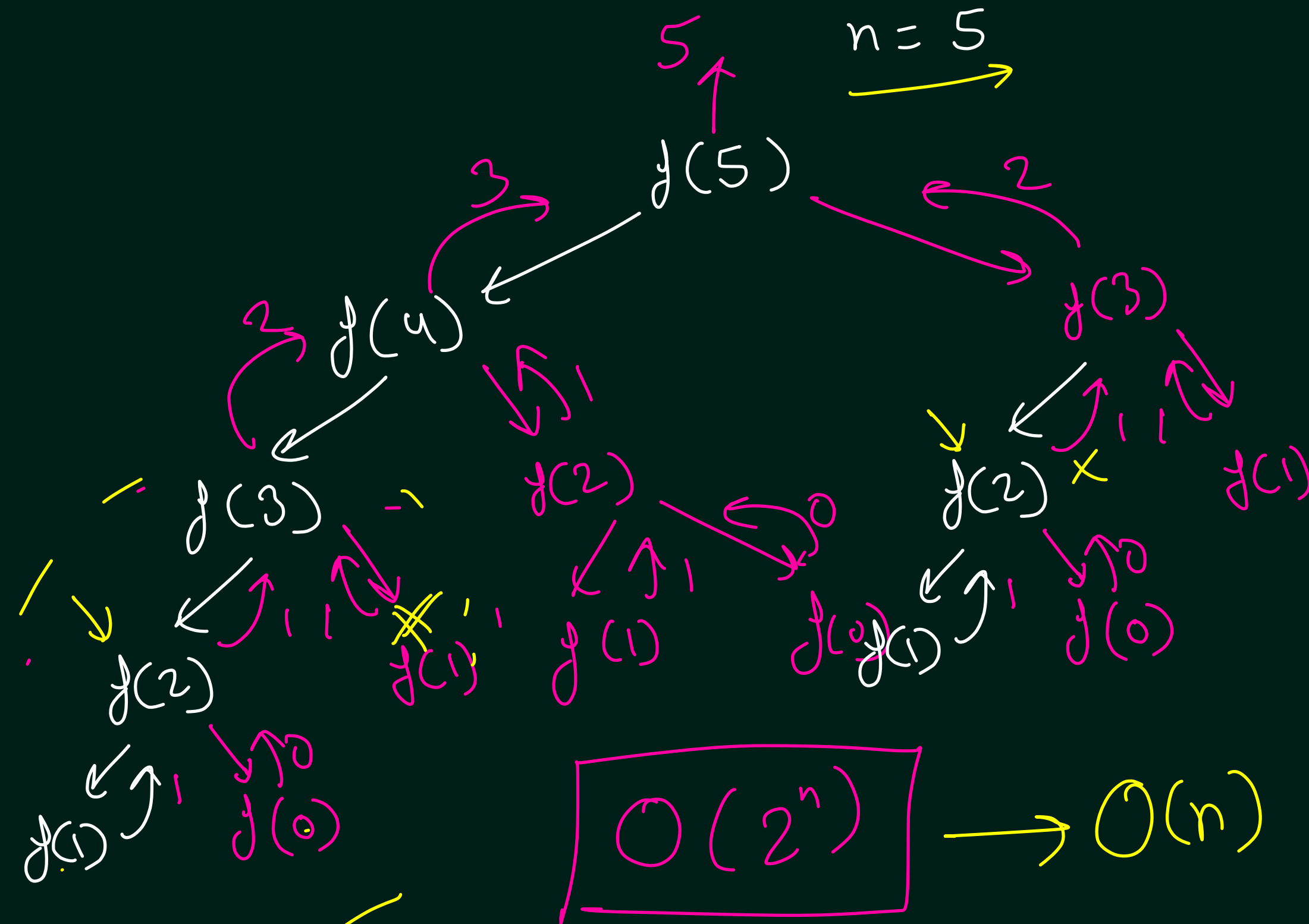


$$\rightarrow f(n) = f(n-1) + f(n-2) \leftarrow$$

$$\begin{aligned} f(8) &= f(7) + f(6) \\ &= 13 + 8 = 21 \end{aligned}$$

$$f(2) = f(1) + f(0) = 1$$





$O(2^n) \rightarrow O(n)$

memoization

0	1	1	2	3	5
0	1	2	3	4	5

3 usages

5

```
static int fibOfN(int n) {
    if(n <= 1) return n;
    return fibOfN(n: n-1) + fibOfN(n: n-2);
}
```

$\text{fib}(4) = 3$ $\text{fib}(3) = 2$