Recursion Problems - I

In This Lecture

1. Calculate the value of nCr
2. Josephus Problem

Calculate the value of nCr

$$^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$$

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

A B C D E

ABC
ABD
ABE
→ BCD
COE

$$^5C_3 = \frac{5!}{2!\,3!}$$

$$= \frac{5 * 4 * 3!}{2 \times 1 * 3!}$$

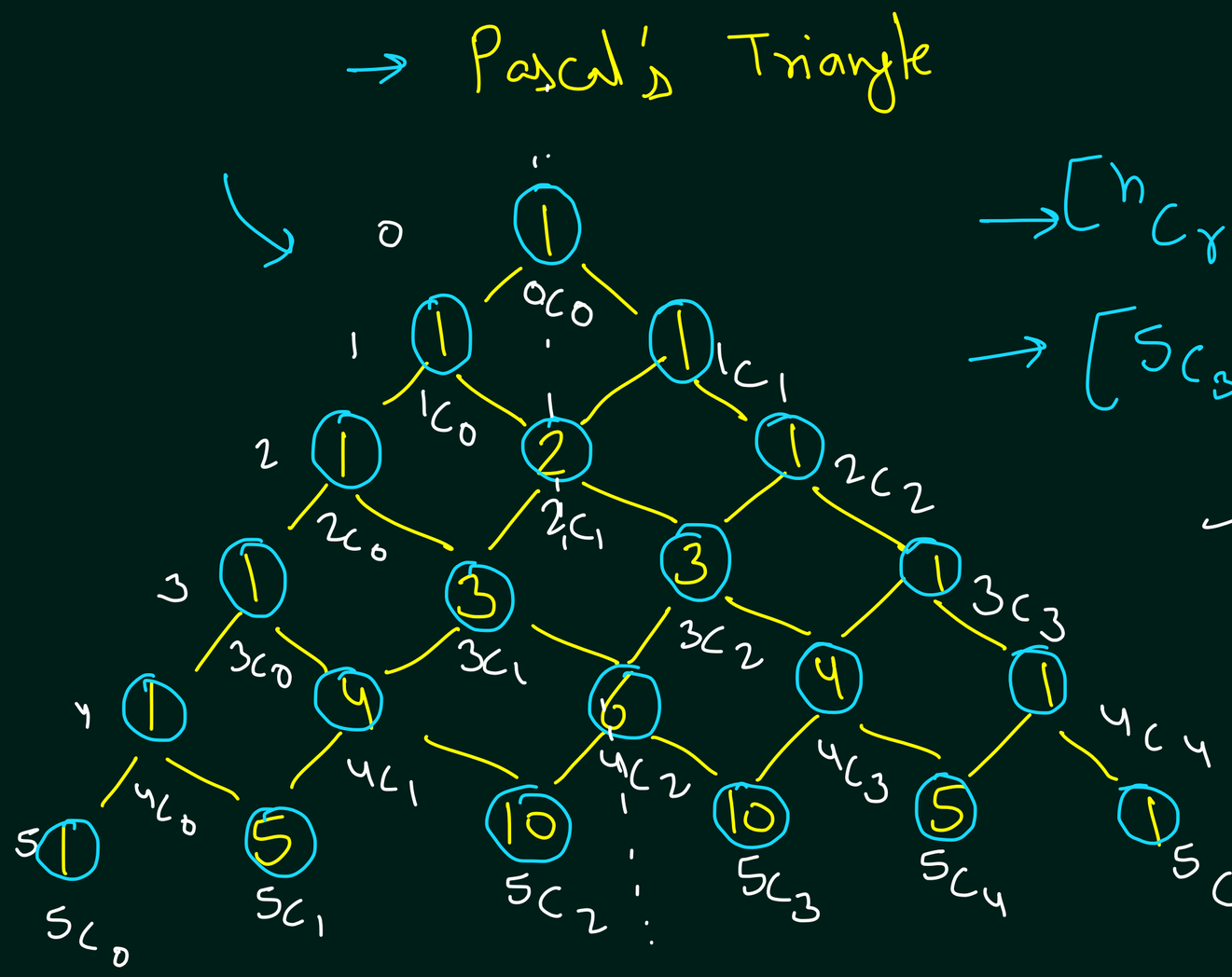
$$= \boxed{10}$$

Factorial

$$n! = n * (n-1)!$$
$$0! = 1$$

$$5! = 5 * 4 \times 3 \times 2 * 1 * 1$$
$$= 120$$

Calculate the value of nCr



→ $[{}^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}]$

→ $[{}^{5}C_{3} = {}^{4}C_{2} + {}^{4}C_{3}]$

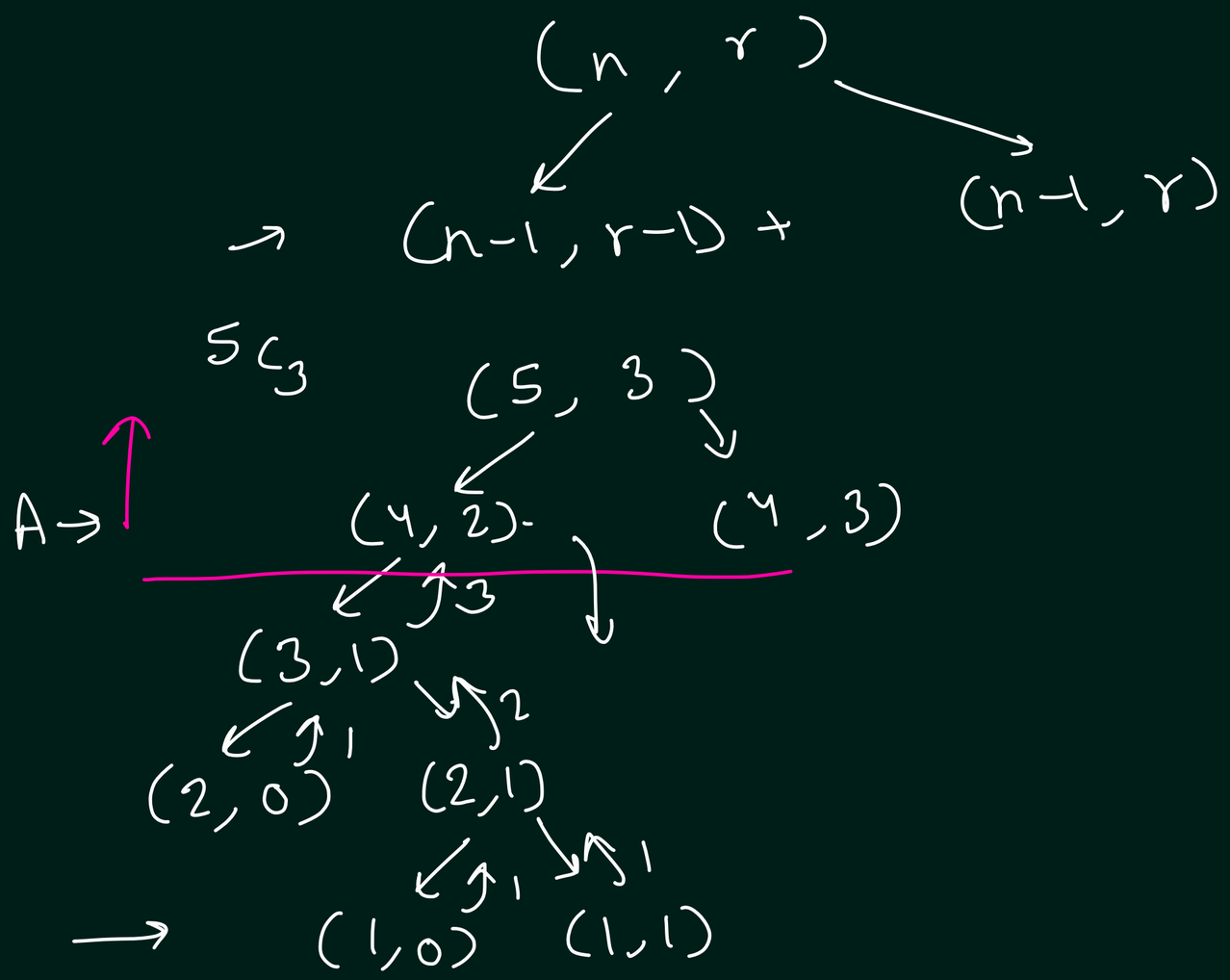
→ $(n == r)$ ${}^{n}C_{r} = 1$

→ $r = 0$ ${}^{n}C_{r} = 1$

↳ $[{}^{n}C_{r} = {}^{n}C_{n-r}]$

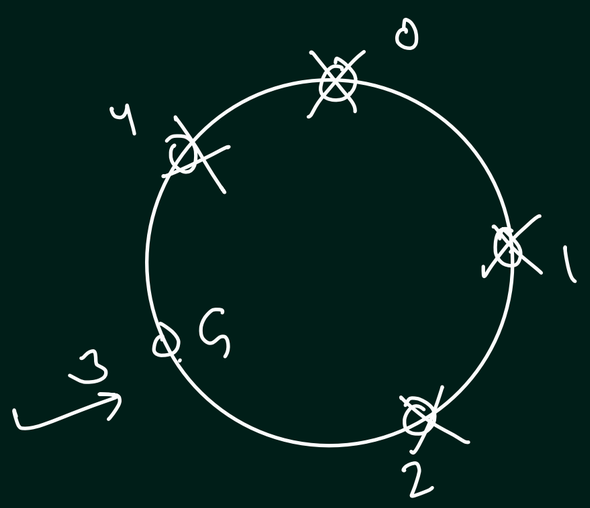
$0 \le r \le n$

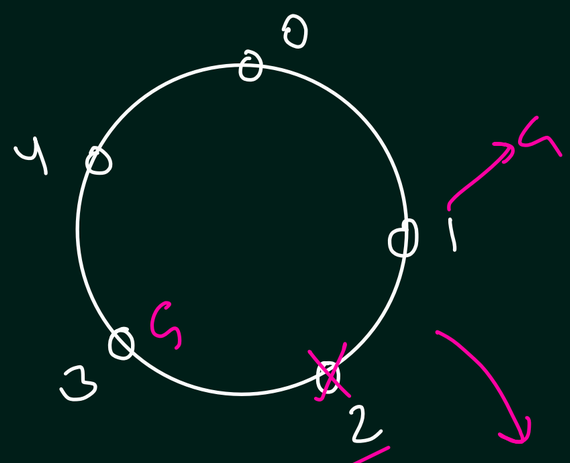
Calculate the value of nCr

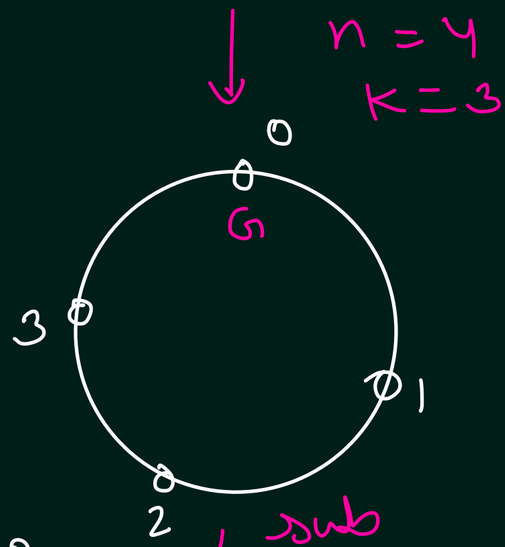



```
3 usages                    3   1
static int nCr(int n, int r) {
    if(n==r || r==0) return 1;
    return nCr( n: n-1,  r: r-1) + nCr( n: n-1, r);
}                                   1          2
```

Josephus Problem

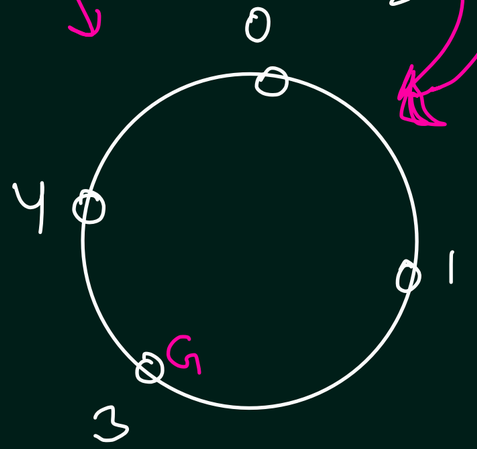




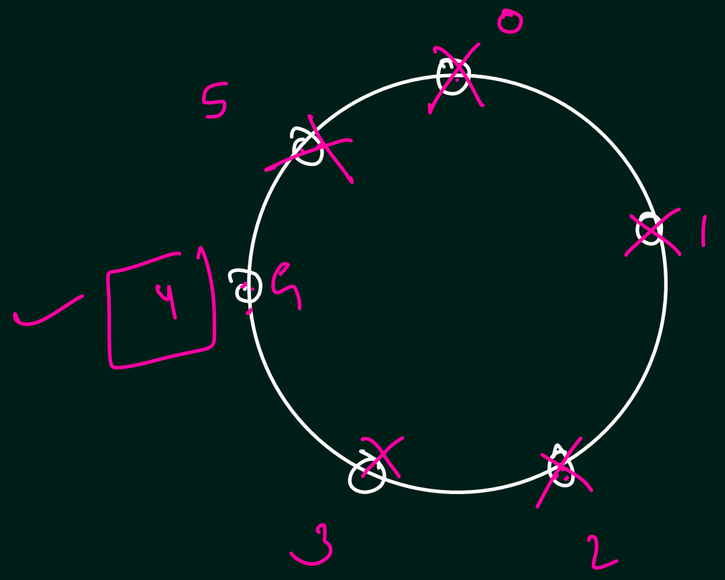


sub

(sub + K) % n



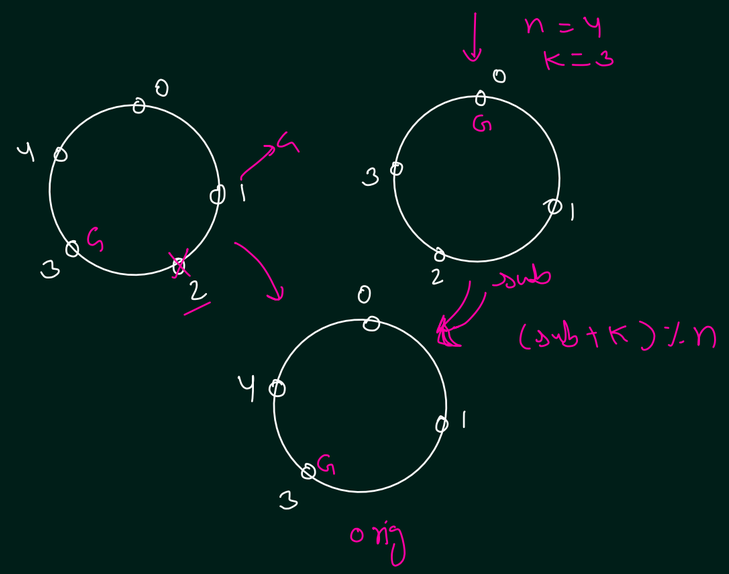
orig

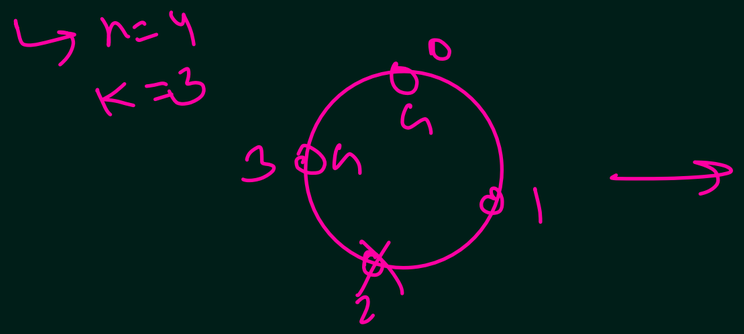


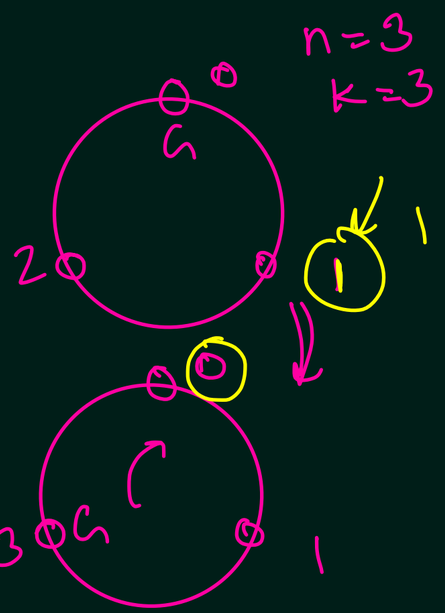
sub	orig
0	3
1	4
2	0
3	1

%

Josephus Problem







sub orig

0 → 3

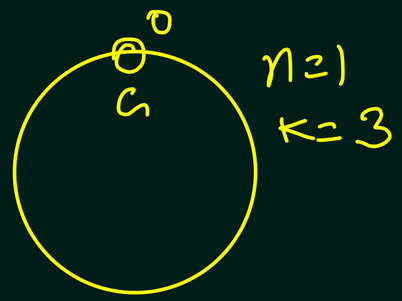
1 → 0

2 → 1

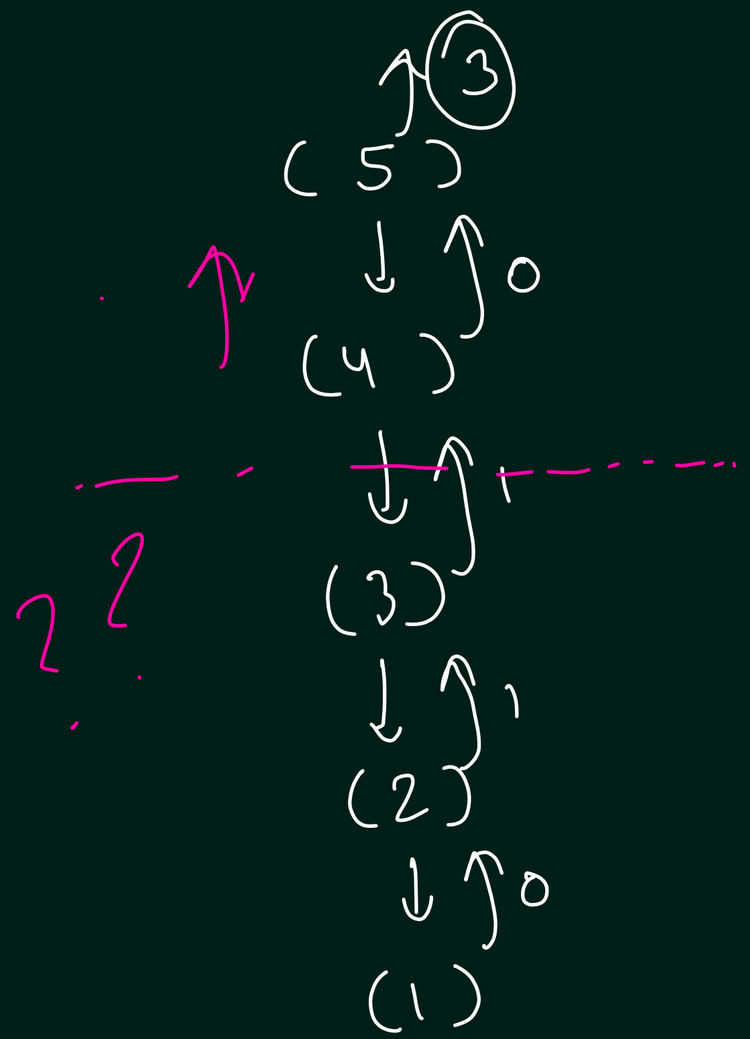
(sub + k) % n

$f(n) = (f(n-1) + k) \% n$

$f(1) = 0$



Josephus Problem



n = 5
k = 3

```
2 usages                                    n=5
static int josephus(int n, int k) {
    if(n==1) return 0;
    return (josephus( n: n-1, k) + k) % n;
}
```

(4) → (0 + 3) % 5

= 3