Worksheet 5: Spatial modelling and management objectives

Due in noon Wednesday 6th April 2022

This example sheet is for credit. Example sheets 9 and 10 can help you. Please make sure code is well commented and shows your working for these answers. Code must be uploaded alongside your pdf solutions.

20 marks will be awarded for presentation (including LaTeXed solutions and clarity of answers). Your write up (including figures) should be no more than 8 pages using size 12 font and 2.5cm margins. You do not have to copy out the questions asked, but the solutions must make sense as a stand alone document. 15 marks will be given for appropriate use of code with comments.

In this worksheet we will consider the control of a Rift Valley fever virus (RVFV) outbreak in livestock. Before starting this worksheet please watch this video:
`https://www.youtube.com/watch?v=n6PyXNYmnt0` and
and read this webpage by the World Organisation for Animal Health:
`https://www.oie.int/en/disease/rift-valley-fever/`

Model

We will use a deterministic, "Ross-Macdonald"-type (vector-borne disease), spatial kernel model to describe the spread of infection between livestock (hosts) and mosquitoes (vectors) within and between different geographical locations during an outbreak.

The model includes cumulative counters for deaths of livestock, D_h, as well as failed pregnancies (abortions), A_h, in livestock which were infected during their pregnancy. In the mosquito vector we account for "vertical vectorial transmission" which is direct transmission of infection from infected adults I_v to their offspring in some proportion (q) of cases. The larval stage of mosquitoes, L_v, is modelled by partitioning larvae into non-infected, P_v, and infected, Q_v, sub-populations. Infected larvae do not contribute to directly transmission but will emerge as infected adults which can. We assume that due to limitations on habitat suitability and predation, there is some carrying capacity on the larval stages governed by the parameter C.

Other notation is as standard (S, E, I and R represent susceptible, exposed, infectious and recovered individuals) with subscripts h for animal hosts and v for mosquito vectors.

$$
\text{Livestock} \quad \begin{cases}
\dfrac{\mathrm{d}S_{hi}}{\mathrm{d}t} &= \mu_h S_{hi} - \mathrm{FOI}_{hi} - \mu_h S_{hi} \\
\dfrac{\mathrm{d}E_{hi}}{\mathrm{d}t} &= \mathrm{FOI}_{hi} - (\sigma_h + \mu_h) E_{hi} \\
\dfrac{\mathrm{d}I_{hi}}{\mathrm{d}t} &= \sigma_h E_{hi} - (\gamma_h + \mu_h) I_{hi} \\
\dfrac{\mathrm{d}R_{hi}}{\mathrm{d}t} &= (1-\delta)\gamma_h I_{hi} - \mu_h R_{hi} \\
\dfrac{\mathrm{d}D_{hi}}{\mathrm{d}t} &= \delta \gamma_h I_{hi} \\
\dfrac{\mathrm{d}A_{hi}}{\mathrm{d}t} &= \mu_h (E_{hi} + I_{hi} + R_{hi})
\end{cases} \tag{1}
$$

$$
\text{Mosquitoes} \quad \begin{cases}
\dfrac{\mathrm{d}P_{vi}}{\mathrm{d}t} &= B_{vi}(1 - q\dfrac{I_{vi}}{N_{vi}}) - \theta P_{vi} - \dfrac{P_{vi}(P_{vi} + Q_{vi})}{C_i} \\
\dfrac{\mathrm{d}Q_{vi}}{\mathrm{d}t} &= B_{vi} q \dfrac{I_{vi}}{N_{vi}} - \theta Q_{vi} - \dfrac{Q_{vi}(P_{vi} + Q_{vi})}{C_i} \\
\dfrac{\mathrm{d}S_{vi}}{\mathrm{d}t} &= \theta P_{vi} - \mathrm{FOI}_{vi} - \mu_v S_{vi} \\
\dfrac{\mathrm{d}E_{vi}}{\mathrm{d}t} &= \mathrm{FOI}_{vi} - (\sigma_v + \mu_v) E_{vi} \\
\dfrac{\mathrm{d}I_{vi}}{\mathrm{d}t} &= \sigma_v E_{vi} + \theta Q_{vi} - \mu_v I_{vi}
\end{cases}
$$

where for the force of infection (FOI) terms are:

$$\mathrm{FOI}_{hi} = a p_h \frac{S_{hi}}{N_{hi}(t)} \sum_j K(d_{ij}) I_{vj}$$

and

$$\mathrm{FOI}_{vi} = a p_v S_{vi} \sum_j K(d_{ij}) \frac{I_{hj}}{N_{hj}(t)}$$

and the spatial kernel, K_{ij} is given by:

$$K(d_{ij}) = \exp(-b d_{ij}^2)$$

where d_{ij} is the distance (in km) between farms i and j and b is a constant dictating the strength of spatial connectedness. N.B. The denominator in the force of infection terms is dependent on the current number of livestock in location i at time t.

Parameter values and descriptions are given in Table 1.

We will simulate this outbreak using this spatially explicitly model. Starting with 1 initial infection (in the I class) on Farm 1 at time 0, and all other livestock in the susceptible class. We will assume that the first detection will happen once 1 animal dies (one or more farms have $D_{hi} \geq 1$) and that the outbreak is over when each farm has < 1 infected (E_{hi} or I_{hi}) animal remaining.

We assume the first controls could be put in place 7 days after initial detection. The interventions that could be used are:

1. (NC) No control (let the epidemic burn out without action)
2. (VC) Performing vector control to reduce the carrying capacity of mosquito breeding habitat. We assume this can only be done for farm 1 and that we are uncertain how successful it will be. This intervention costs \$500 per week.

The effectiveness of vector control could be measured by monitoring the "apparent trap density" (ATD) of adult mosquitoes: By placing traps before the vector intervention we can learn a baseline ATD. Then, following control to reduce larval carrying capacity, we can measure the new relative ATD (given as a proportion of the original ATD) after we believe the mosquito dynamics have settled at their new equilibrium. We will assume here that we can fully learn the relative ATD after 15 weeks of vector control deployment.

Analysis

1. Before you simulate an RVFV outbreak first consider the infection-free mosquito population dynamics in a single location (we will drop the subscripts i for convenience here).

   (a) Using the following equations:

$$\begin{aligned} \text{Larvae} \quad \frac{\mathrm{d}L_v}{\mathrm{d}t} &= B_v - \theta L_v - \frac{L_v^2}{C} \\ \text{Adults} \quad \frac{\mathrm{d}N_v}{\mathrm{d}t} &= \theta L_v - \mu_v N_v \end{aligned} \tag{2}$$

   give the larval population size at disease-free equilibrium, L_v^*, in terms of the adult disease free equilibrium, N_v^*, and other model parameters. Next give the expression for B_v which is needed to "balance" the other terms assuming $C = N_v^*$. [*Hint: "Balance" = What is needed to ensure we have a non-zero stable population of mosquitoes?*] [2 marks]

   (b) If vector control to reduce larval habitat is deployed (corresponding to changing C from its initial value to $\tilde{C}$) and results in a a relative ATD of $\tilde{n}_v$ once the system has reached its new equilibrium, give $\tilde{C}$ as a function $N_v^*, \tilde{n}_v, \theta$, and μ_v. Assume B_v remains the same value as you computed in Q1a. Plot $\tilde{C}$ against $\tilde{n}_v \in [0, 1]$. What is the corresponding larval population size at this equilibrium, $\tilde{L}_v^*$? [3 marks]

2. Now we will return to thinking about infections, but use the answers to Question 1 to parameterise B_{vi} for each farm i and our vector control intervention on farm 1.

Parameter	Description	Value	
$(N_{h1}(0), N_{h2}(0), N_{h3}(0))$	Number of livestock on each farm before the outbreak	$(1000, 500, 500)$	
μ_h	Per capita animal death rate	$1/10$	years^{-1}
p_h	Probability of a susceptible animal becoming infected from a single bite by an infectious mosquito	1	
σ_h	Inverse of the latency period in animals	$1/4$	days^{-1}
γ_h	Rate of moving from I to R or D (recovery or death)	$1/6$	days^{-1}
δ	Probability of death	0.3	
$N_{vi}(0)$	Initial adult vector population size in location i	$N_v(0) = mN_{hi}(0)$	
a	Vector bite rate	$1/4$	days^{-1}
p_v	Probability of a susceptible vector becoming infected from a single bite on an infectious animal	0.1	
q	Probability of new larva being “born” infected if its parent was infectious	0.05	
m	Relative vector to host ratio	10	
μ_v	Per capita mosquito death rate	$1/16$	days^{-1}
B_{vi}	Farm-specific “birth rate” of larvae	To be computed	
θ	Inverse of larval period	$1/15$	days^{-1}
C_i	Parameter linked to larval carrying capacity	$N_{vi}(0)$	
σ_v	Inverse of the latency period in mosquitoes	$1/14$	days^{-1}
d_{ij}	The distances between farms i and j, which is computed from the geolocations of farms		km
b	Parameter governing the decay in transmission strength by distance	0.05	
K_{ij}	Spatial kernel describing the infectious pressure *from* farm j *to* farm i		

Table 1: Model parameters. The geolocations of the 3 farms can be found as 3×2 matrix in the `locations.mat` file in Moodle.

(a) What is the within farm R_0 for farm 1? [*Hint: Use the next generation matrix approach and assume* $K_{ij} = 0$ *for expect for* $K_{11} = 1$.] [4 marks]

(b) Include a plot of the spatial kernel K (as a function of distance), and another of the location of the three farms, marking on the strength of the connections between farms. [2 marks]

(c) Run your model with no control (NC) and generate some illustrative plots of the infection dynamics. What impact does vertical vectorial transmission have on the dynamics? [10 marks]

(d) Compute the duration and cost of this outbreak if each direct death of an animal costs \$250 and each failed pregnancy (abortion) costs \$50. [2 marks]

3. Now we will examine the impact of interventions on our outbreak.

(a) Run you model again, this time looking at VC interventions for example relative ATDs of $\tilde{n}_V = 0.1$ and $\tilde{n}_V = 0.4$ (remember VC only happens on farm 1). Consider the static VC intervention – continued from 7 days after detection of the outbreak to the end of the outbreak (VC→VC) – and a passive adaptive management intervention – starting VC from 7 days and lasting for 15 weeks before stopping (VC→NC). Include a plot of the adult and larval vector dynamics in each case. What is the impact on the fundamental objectives of duration and cost? Compare this to your NC scenario. [12 marks]

(b) If we initially believe that the relative ATD after vector control will be about 0.3, with a distribution of our uncertainty given by $\tilde{n}_v^* \sim \mathrm{Beta}(2, 5)$, compute the expected duration and cost under the VC→VC and VC→NC strategies. Rank your NC, VC→VC and VC→NC strategies. [5 marks]

(c) If we can resolve our uncertainty in the relative ATD we would achieve at 15 weeks after deploying VC, what strategy would we now pick? Use active adaptive management to justify your answer, assuming that if we monitor ATD we gain perfect information. [13 marks]

4. (a) Describe other considerations that you might want to take into account if you were trying to make a policy recommendation. You should mention: other interventions against RVFV, timing of interventions, policy-maker objectives, different ways in which data could influence strategy recommendation. [6 marks]

(b) Explain three modifications you would make to your model to make it more realistic/practical for guiding RVFV policy [6 marks]