$$m \xrightarrow{k_{tr}} p \qquad \text{where } m(t) = [\text{mRNA}] \text{ at time } t$$
$$\downarrow k_{md} \qquad \downarrow k_{pd} \qquad p(t) = [\text{protein}] \text{ at time } t$$

$$\frac{dm}{dt} = -k_{md} \cdot m \longrightarrow m = m_0 \cdot e^{-k_{md} \cdot t} \quad \text{where } m_0 = m(t=0)$$

$$\frac{dp}{dt} = k_{tr} \cdot m - k_{pd} \cdot p$$

$$\underbrace{\frac{dp}{dt}}_{\cdot e^{k_{pd} t}} = \underbrace{k_{tr} \cdot m_0 e^{-k_{md} \cdot t} - k_{pd} \cdot p}_{\cdot e^{k_{pd} \cdot t}}$$

$$e^{k_{pd} t} \frac{dp}{dt} = k_{tr} \cdot m_0 \, e^{(k_{pd} - k_{md}) t} - \underbrace{e^{k_{pd} \cdot t} \cdot k_{pd} \cdot p}$$

$$\int \Big[\underbrace{e^{k_{pd} \cdot t}}_{u} \underbrace{\frac{dp}{dt}}_{dv} + \underbrace{k_{pd} \cdot e^{k_{pd} t}}_{du} \cdot \underbrace{p}_{v} \Big] dt = \int \Big[k_{tr} \cdot m_0 \cdot e^{(k_{pd} - k_{md}) \cdot t} \Big] dt$$

$\downarrow$ integration by parts

$$e^{k_{pd} \cdot t} \cdot p = \frac{k_{tr} \cdot m_0}{k_{pd} - k_{md}} e^{(k_{pd} - k_{md}) t} + C$$

$$p = \frac{k_{tr} \cdot m_0}{k_{pd} - k_{md}} e^{-k_{md} \cdot t} + C \cdot e^{-k_{pd} t}$$

$$p(t=0) = 0 = \frac{k_{tr} \cdot m_0}{k_{pd} - k_{md}} \cancel{e^{-k_{md} \cdot 0}}^{1} + C \cdot \cancel{e^{-k_{pd} \cdot 0}}^{1}$$

$$\longrightarrow C = -\frac{k_{tr} \cdot m_0}{k_{pd} - k_{md}}$$

$$\boxed{p = \frac{k_{tr} \cdot m_0}{k_{pd} - k_{md}} \left[e^{-k_{md} \cdot t} - e^{-k_{pd} \cdot t} \right]}$$