Sensorless Control of an SPMSM drive using a first-order Sliding Mode Observer implemented in dq-reference frame

Barna Temesi

Abstract—This short design documentation presents the used system model, parameters and rotor position estimator. First the machine model is transformed into the so called $dq*$-reference frame. The acquired structure is called Sensorless control of a PMSM using a first-order Sliding Mode Observer (SMO) implemented in dq-reference frame. The rotor position and speed signals are acquired with a PLL. An adaptive observer gain calculation algorithm is also presented.

Index Terms: dynamic modelling of syncronous motors, *dq*-reference frame, PMSM, Sensorless control, Rotor position estimation, Rotor speed estimation, Sliding Mode Observer, b-EMF

1 Introduction

As in every design process, after the problem statement, the system has to be modeled and analyzed. This starts with an overview of the motor parameters. Next, the motor voltage equations, which are already given in the dq-reference frame, are examined. The mechanical equation of the motor is also presented.

This documentation does not consider the FOC used in the system. The focus is strictly on the rotor position and speed estimator.

2 Model of the system

The motor, in the scope of this thesis, is an SPMSM. This means that the permanent magnets are located on the surface of the rotor. Due to this, the motor is non-salient, and also the reluctance path is equal on the d- and q-axis. This results in equal inductance on the d- and q-axis. For easier understanding, the machine inductance will be denoted as L_s [2].

$$L_d = L_q = L_s \tag{1}$$

The most important parameters of the motor and the other necessary system parameters are listed in the table 1.

As can be seen from the table, the motor has 4 pole pairs. Generally speaking, this means that the machine is more geared towards high-speed operation. In high-torque operation applications, like in the case of a steering motor, the number of poles might exceed a 100.

In simulations, the total system resistance will be used, which takes into account the resistance of every possible component in the setup [1].

Table 1: *System parameters, from previous projects such as*

Description	Notation	Value	Unit
Number of pole pairs	N_{pp}	4	-
Winding resistance	R_w	0.19	Ω
Total system resistance	R_s	0.268	Ω
q and d-axis inductance	L_m	2.2	mH
Rotor PM flux linkage	λ_{mpm}	0.12258	wB
Rated speed, SPMSM	$\omega_{m,rated}$	4500	rpm
Rated torque, SPMSM	$\tau_{m,rated}$	20	Nm
Rated power, SPMSM	$P_{m,rated}$	9.4	kW
Rated speed, IM	$\omega_{IM,rated}$	1400	rpm
Rated torque, IM	$\tau_{IM,rated}$	14	Nm
Rated power, IM	$P_{IM,rated}$	2.2	kW
Rated current, VSI	I_{VSI}	35	A
IM machine inertia	J_{IM}	0.0069	$kg \cdot m^2$
SPMSM machine inertia	J_{SPMSM}	0.0048	$kg \cdot m^2$
Total system inertia	J_{sys}	0.0146	$kg \cdot m^2$
Coulomb friction	C	0.2295	Nm
Viscous friction	B	0.0016655	N

The motor voltage equations are shown in equation (2). Due to the assumption that the system is symmetrical and balanced, the zero term (v_0) is zero.

$$\begin{aligned} v_d &= R_s i_d + p\lambda_d - \omega_r \lambda_q \\ v_q &= R_s i_q + p\lambda_q + \omega_r \lambda_d \\ v_0 &= 0 \end{aligned} \tag{2}$$

In the abc-reference frame, the machine flux-linkage is dependent on position and the machine inductance is constant for a non-salient pole machine, but because the model is already transformed into the $dq0$-reference frame, the machine inductance is constant.

The stator $dq0$-reference frame is aligned with the rotor reference frame, which is naturally in the $dq0$-reference frame. The rotor d-axis is chosen to be aligned with the maximum flux density line at no load condition. The q-axis is always leading the d-axis by 90 degrees electric. This way, it is aligned with the minimum flux density line [1].

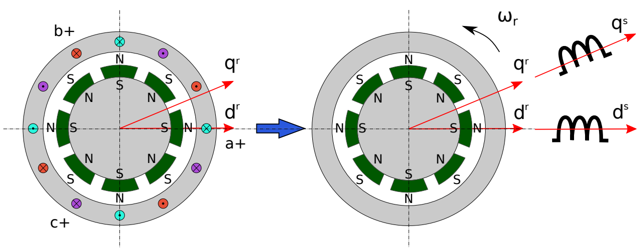

Figure 1: *Reference frame transformation from* ***abc*** *to dq. The structure of the motor is also shown. Inspiration: [3]*

The two d-axis is in line now. This is convenient because, it results in the d-axis and the q-axis flux-linkage as shown in equation (3).

$$\begin{aligned} \lambda_d &= (L_{ls} + L_{md})\, i_d + \lambda_{mpm} = L_d i_d + \lambda_{mpm} \\ \lambda_q &= (L_{ls} + L_{mq}) = L_q i_q \\ \lambda_0 &= 0 \end{aligned} \tag{3}$$

After substitution, the voltage equations may be rewritten as seen in equation (4).

$$\begin{aligned} v_d &= R_s i_d + p(L_d i_d + \lambda_{mpm}) - \omega_r L_q i_q \\ v_q &= R_s i_q + p(L_q i_q) + \omega_r (L_d i_d + \lambda_{mpm}) \end{aligned} \tag{4}$$

Where p is the differential operator $\frac{d}{dt}$. Differentiating the equation, keeping in mind that the derivative of a constant is zero, will result in the following.

$$\begin{aligned} v_d &= R_s i_d + L_d p i_d - \omega_r L_q i_q \\ v_q &= R_s i_q + L_q p i_q + \omega_r (L_d i_d + \lambda_{mpm}) \end{aligned} \tag{5}$$

In one more step, the homogeneous first-order differential equation of the system is acquired.

$$\begin{aligned} \frac{d}{dt} i_d &= -\frac{R_s}{L_d} i_d + \frac{1}{L_d} v_d + \omega_r \frac{L_q}{L_d} i_q \\ \frac{d}{dt} i_q &= -\frac{R_s}{L_q} i_q + \frac{1}{L_q} v_q - \omega_r \frac{L_d}{L_q} i_d - \frac{1}{L_q} \omega_r \lambda_{mpm} \end{aligned} \tag{6}$$

Equations (4) also contain the back-EMF voltage components which are very important in position estimation, hence they are highlighted here:

$$\begin{aligned} e_d &= -\omega_r L_q i_q \\ e_q &= \omega_r (L_d i_d + \lambda_{mpm}) \end{aligned} \tag{7}$$

The governing torque equation can be derived from the equation of the input power of the windings. Simplifying this equation, using the attributions of the SPMSM machine, yields the following expression:

$$T_e = \frac{3}{2} N_{pp} \left(\lambda_d i_q - \lambda_q i_d \right) \tag{8}$$

$$T_e = \frac{3}{2} \frac{N_{poles}}{2} (\lambda_{mpm} i_q + (L_d - L_q) i_d i_q) \tag{9}$$

$$T_e = \frac{3}{2} N_{pp} \left(\lambda_{mpm} i_q \right) \tag{10}$$

Using Newton's second law, the mechanical equation of the system can be derived as shown in equation (11) [1].

$$T_e = J \frac{d\omega_m}{dt} + B_m \omega_m + T_{dist} \tag{11}$$

Where J is the total system inertia and T_{dist}, the disturbance torque, consists of the load torque and Coulomb friction. The total system inertia includes the inertia of both the IM and PMSM machine and also the coupling and fastening components between them.

The first term is related to the torque needed to accelerate the system without friction, the last two terms are related to the torque which is needed to overcome the viscous friction and the disturbance torque, respectively.

3 Rotor position estimation

The goal of this estimator is to estimate the b-EMF voltages in dq-reference frame and then calculate the rotor position from it. The rotor speed can also be acquired by a simple integration of the rotor position state. The implemented estimator is a sliding mode observer. The theory behind non-linear observer can be found in: [4]. The design is based on [1] but it will be also detailed here.

The modified structure of the system, now including the observers, is presented in figure 2.

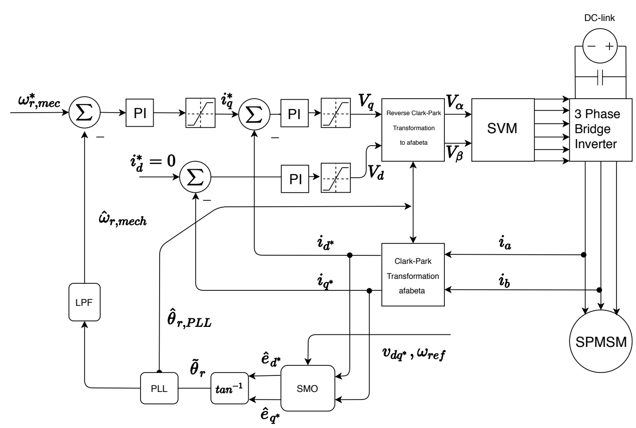

Figure 2: *Structure of control system with a first-order sliding mode observer in dq*-reference frame included*

3.1 First-order sliding mode observer in *dq*-reference frame

Another option is to design the sliding mode observer in the estimated dq-reference frame denoted as dq^* [5]. The advantage of this is that in dq-reference frame the current, voltage, and back-EMF signals are DC signals. However, the

position cannot be directly acquired from the estimated back-EMF signals, only the position error ($\tilde{\theta}$). This new state, position error, will become the focus of this method. Using a PLL, the rotor position and speed can be calculated in the next steps.
Due to the signals being DC, it can be assumed that a simpler filtering structure will be sufficient and 0 phase lag will be added. This was verified in: [1], and you can also check it using the included simulation model.

As a first step, the voltage equations expressed in dq-reference frame are transformed into dq^* reference frame. Using the position error angle, as it is shown in figure 3, the transformation is done by using simple 2 dimensional projections [5].

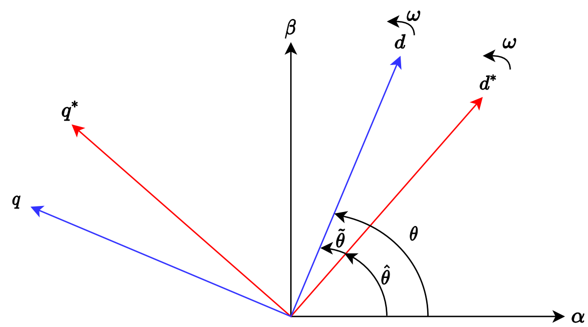

Figure 3: *The definition of the* dq^* *reference frame*

The transformations are done by utilizing the definition of the *sin* and *cos* functions. This is summarized in equation (12).

$$T(\tilde{\theta}) = \begin{bmatrix} cos(\tilde{\theta}) & -sin(\tilde{\theta}) \\ sin(\tilde{\theta}) & cos(\tilde{\theta}) \end{bmatrix} \tag{12}$$

And with the supporting matrix (J_m) seen in equation (13).

$$J_m = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{13}$$

First, the voltage is transformed from dq to dq^*.

$$v_{dq^*} = \begin{bmatrix} v_{d^*} \\ v_{q^*} \end{bmatrix} = T(\tilde{\theta}) \begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} v_d cos(\tilde{\theta}) - v_q sin(\tilde{\theta}) \\ v_d sin(\tilde{\theta}) + v_q cos(\tilde{\theta}) \end{bmatrix} \tag{14}$$

This means that even if we only have d component in the voltage vector, in the dq^*-reference frame a q^* component will also appear. The resulting rotor flux-linkage in dq^* is the following :

$$\lambda_{mpm,dq^*} = \begin{bmatrix} \lambda_{mpm,d^*} \\ \lambda_{mpm,q^*} \end{bmatrix} = \lambda_{mpm} \begin{bmatrix} cos(\tilde{\theta}) \\ sin(\tilde{\theta}) \end{bmatrix} \tag{15}$$

Next, the rest of the system states are transformed in equations (16) and (17).

$$i_{dq^*} = \begin{bmatrix} i_{d^*} \\ i_{q^*} \end{bmatrix} = T(\tilde{\theta}) \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} i_d cos(\tilde{\theta}) - i_q sin(\tilde{\theta}) \\ i_d sin(\tilde{\theta}) + i_q cos(\tilde{\theta}) \end{bmatrix} \tag{16}$$

$$\lambda_{dq^*} = \begin{bmatrix} \lambda_{d^*} \\ \lambda_{q^*} \end{bmatrix} = L_{pm} T(\tilde{\theta}) \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \lambda_{mpm} T(\tilde{\theta}) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{17}$$

$$= L_{pm} \begin{bmatrix} i_{d^*} \\ i_{q^*} \end{bmatrix} + \begin{bmatrix} \lambda_{mpm,d^*} \\ \lambda_{mpm,q^*} \end{bmatrix} \tag{18}$$

As the last step, the derivative of the flux-linkages (λ_{dq^*}) are calculated. This can be seen in equation (19).

$$\dot{\lambda}_{dq^*} = \begin{bmatrix} \dot{\lambda}_{d^*} \\ \dot{\lambda}_{q^*} \end{bmatrix} = L_{pm} \begin{bmatrix} \dot{i}_{d^*} \\ \dot{i}_{q^*} \end{bmatrix} + \begin{bmatrix} \dot{\lambda}_{mpm,d^*} \\ \dot{\lambda}_{mpm,q^*} \end{bmatrix} \tag{19}$$

Now, let's calculate the last unknown term in the voltage equation, $\dot{\lambda}_{mpm,dq^*}$, using equation (20). A term based on saliency is ignored because the given system does not have saliency.

$$\dot{\lambda}_{mpm,dq^*} = \begin{bmatrix} \dot{\lambda}_{mpm,d^*} \\ \dot{\lambda}_{mpm,q^*} \end{bmatrix} = \lambda_{mpm} \dot{\tilde{\theta}} \begin{bmatrix} -sin(\tilde{\theta}) \\ cos(\tilde{\theta}) \end{bmatrix} \tag{20}$$

$$= \lambda_{mpm} \dot{\tilde{\theta}} J_m \begin{bmatrix} cos(\tilde{\theta}) \\ sin(\tilde{\theta}) \end{bmatrix} \tag{21}$$

The first-order differential equation of the system is repeated here in equation (22).

$$\begin{aligned} \frac{di_d}{dt} &= f i_d + g v_d + \omega_r i_q \\ \frac{di_q}{dt} &= f i_q + g v_q - \omega_r i_d - g \omega_r \lambda_{mpm} \end{aligned} \tag{22}$$

Finally, the total system equations, now transformed into dq^*-reference frame, are shown in equation (23). Here, the last terms are the back-EMF terms.

$$\begin{aligned} \frac{di_{d^*}}{dt} &= f i_{d^*} + g v_{d^*} + \omega_r i_{q^*} + g \omega_r \lambda_{mpm} sin(\tilde{\theta}) \\ \frac{di_{q^*}}{dt} &= f i_{q^*} + g v_{q^*} - \omega_r i_{d^*} - g \omega_r \lambda_{mpm} cos(\tilde{\theta}) \end{aligned} \tag{23}$$

The observer law is then derived similarly as it was done for the SMO in $\alpha\beta$-reference frame.

$$\begin{aligned} \frac{d\hat{i}_{d^*}}{dt} &= f i_{d^*} + g v_{d^*} + \hat{\omega}_r i_{q^*} - g k sgn(\sigma_{d^*}) \\ \frac{d\hat{i}_{q^*}}{dt} &= f i_{q^*} + g v_{q^*} - \hat{\omega}_r i_{d^*} - g k sgn(\sigma_{q^*}) \end{aligned} \tag{24}$$

Where $f = -\frac{R_s}{L_s}$, $g = \frac{1}{L_s}$, and $\sigma_{d^*}, \sigma_{q^*}$ are sliding variables for the first-order system and are defined in equation (26).

$$\sigma_{d^*} = \tilde{i}_{d^*} = i_{d^*} - \hat{i}_{d^*} \tag{25}$$

$$\sigma_{q^*} = \tilde{i}_{q^*} = i_{q^*} - \hat{i}_{q^*} \tag{26}$$

It is assumed that the controller can perfectly follow the reference, therefore $\hat{\omega} = \omega_{ref}$. The resistance and inductance are also assumed to be time-invariant [1]. In the next section, the stability of the observer is proved.

3.1.1 Stability of the observer

A detailed stability proof is presented now. The stability of the sliding mode observer is proven by using the Lyapunov stability theory. This will also determine how to select the observer gains.
The chosen Lyapunov candidate function is Positive-definite (P.D.), radially unbounded and shown in equation (27) [4].

$$\boldsymbol{V}(\boldsymbol{\sigma}) = \frac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{\sigma} = \frac{1}{2}(\tilde{i}_{d^*}^2 + \tilde{i}_{q^*}^2) \tag{27}$$

Where $\boldsymbol{\sigma} = [\sigma_{d^*}, \sigma_{q^*}]$.

In order to prove stability of the equilibrium the time derivative, $\dot{\boldsymbol{V}}$, shown in equation (28), needs to be at least Negative-semi definite (N.S.D.). If however, the time derivative,$\dot{\boldsymbol{V}}$, is Negative-definite (N.D) the system is also globally asymptotically stable [4].

$$\dot{\boldsymbol{V}}(\boldsymbol{\sigma}) = \sigma_{d^*}\dot{\sigma}_{d^*} + \sigma_{q^*}\dot{\sigma}_{q^*} \tag{28}$$

The derivative is calculated by subtracting equations (23) and (24) which yields, for the estimated d-axis:

$$\begin{aligned}\dot{\sigma}_{d^*} = \dot{i}_{d^*} - \dot{\hat{i}}_{d^*} &= f i_{d^*} + g v_{d^*} + \hat{\omega}_r i_{q^*} + g\omega_r\lambda_{mpm} sin(\tilde{\theta}) \\ &\quad - f\hat{i}_{d^*} + g v_{d^*} + \hat{\omega}_r \hat{i}_{q^*} - gksgn(\sigma_{d^*}) \\ &= f\sigma_{d^*} + g(\omega_r\lambda_{mpm} sin(\tilde{\theta}) - ksgn(\sigma_{d^*})) + \hat{\omega}_r\sigma_{q^*}\end{aligned} \tag{29}$$

And for the estimated q-axis:

$$\begin{aligned}\dot{\sigma}_{q^*} = \dot{i}_{q^*} - \dot{\hat{i}}_{q^*} &= f i_{q^*} + g v_{q^*} - \hat{\omega}_r i_{d^*} - g\omega_r\lambda_{mpm} cos(\tilde{\theta}) \\ &\quad - f\hat{i}_{q^*} + g v_{q^*} - \hat{\omega}_r \hat{i_{d^*}} - gksgn(\sigma_{q^*}) \\ &= f\sigma_{q^*} - g(\omega_r\lambda_{mpm} cos(\tilde{\theta}) + ksgn(\sigma_{q^*})) - \hat{\omega}_r\sigma_{d^*}\end{aligned} \tag{30}$$

These two equation above are then substituted into equation (28), which yields:

$$\begin{aligned}\dot{\boldsymbol{V}}(\boldsymbol{\sigma}) &= \sigma_{d^*}(f\sigma_{d^*} + g(\omega_r\lambda_{mpm} sin(\tilde{\theta}) - ksgn(\sigma_{d^*})) + \hat{\omega}_r\sigma_{q^*}) \\ &\quad + \sigma_{q^*}(f\sigma_{q^*} - g(\omega_r\lambda_{mpm} cos(\tilde{\theta}) + ksgn(\sigma_{q^*})) - \hat{\omega}_r\sigma_{d^*}) \\ &= f(\sigma_{d^*}^2 + \sigma_{q^*}^2) + g(\sigma_{d^*}\omega_r\lambda_{mpm} sin(\tilde{\theta}) - k\sigma_{d^*} sgn(\sigma_{d^*})) \\ &\quad + g(-\sigma_{q^*}\omega_r\lambda_{mpm} cos(\tilde{\theta}) - k\sigma_{q^*} sgn(\sigma_{q^*}))\end{aligned} \tag{31}$$

Where $\sigma sgn(\sigma) = |\sigma|$, and the back-EMF is defined as:

$$e_{d^*} = \omega_r\lambda_{mpm} sin(\tilde{\theta}) \tag{32}$$

$$e_{q^*} = -\omega_r\lambda_{mpm} cos(\tilde{\theta}) \tag{33}$$

Simplification yields the following:

$$\dot{\boldsymbol{V}}(\boldsymbol{\sigma}) = f(\sigma_{d^*}^2 + \sigma_{q^*}^2) + g(|e_{d^*}||\sigma_{d^*}| - k|\sigma_{d^*}|) + \tag{34}$$

$$+ g(|e_{q^*}||\sigma_{q^*}| - k|\sigma_{q^*}|) \tag{35}$$

As mentioned above for the system to be globally asymptotically stable, equation (28) has to be N.D and then stability would be proven. The gain k is designed under the worst case scenario. The first term of equation (35) is always N.D due to $f = -\frac{R_s}{L_s}$. Next two terms are N.D if k is chosen to be:

$$k > max(|e_{d^*}|, |e_{q^*}|) \tag{36}$$

For the purpose of this stability proof, the gain k is chosen to be a constant which satisfies equation (36). With this chosen k, the equilibrium point of the system becomes globally asymptotically stable. Furthermore, when the sliding surface is reached by the trajectory, then $\dot{\sigma} = \sigma = 0$ in finite time. The back-EMF voltages (e_{dq^*}), and consequently the angle error converges to zero since e_{dq^*} depends on the angle error [5].

The gain may be calculated based on the estimated b-EMF voltages in the previous sampling time *(k-1)*. It can be seen that this not change the stability of the observer, just reduces the stability margin of it.

$$k_k = 100 + \sqrt{e_{\alpha,k-1}^2 + e_{\beta,k-1}^2} \tag{37}$$

Where the first term represents a safety, transient term. The measure of it is in voltages.

3.1.2 Chattering attenuation

In practice, the sign function will be changed to a sigmoid function, equation (38), to attenuate the chattering present in the system. The parameter of the sigmoid function (ϕ) will be tuned experimentally. The effects of it were discussed in [1].

$$F = \frac{\sigma}{\mid \sigma \mid + \frac{\phi}{100}} \tag{38}$$

Using the two estimated back-EMF signals ($\hat{e}_{dq^*}$), there are two possible options to calculate the estimated rotor position. The position estimation error can be directly calculated using the tangent function, such as $\hat{\theta} = tan^{-1}(\frac{-\hat{e}_{d^*}}{\hat{e}_{q^*}})$. The positive thing about using the tangent function is that it is robust against magnitude error in the signals, as long as both of them contain the same error. Using the now known $\tilde{\theta}$ and a simple PLL structure, the estimated position ($\hat{\theta}_r$) and speed ($\hat{\omega}_{r,mech}$) can be acquired. The speed signal is filtered by an LPF before it is fed back to the speed controller.

4 Conclusion

For further information, simulation results, please see [1].

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