$$Ax = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} \begin{bmatrix} V_R \\ V_L \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\alpha} \end{bmatrix}$$

Solve 2 scenarios so I can fill in matrix A.

- If $V_R = -V_L$, the robot will rotate around its center

$\dot{x} = 0$, $\dot{y} = 0$, $\dot{\alpha} = \frac{2\pi}{T}$ (T = time for robot to make a full rotation around center)

$T = \frac{\text{Circumference}}{\text{Wheel Speed}} = \frac{\pi L}{R V_R}$ [seconds], $\dot{\alpha} = \frac{2\pi}{\left(\frac{\pi L}{R V_R}\right)} = \frac{2R}{L} V_R$

- If $V_L = 0$, the robot pivots around its left wheel.

$\dot{\alpha} = \frac{2\pi}{T}$, $T = \frac{2L\pi}{R V_R} \longrightarrow \dot{\alpha} = \frac{R}{L} V_R$

$x = \frac{L}{2}\sin(\alpha)$, $y = -\frac{L}{2}\cos(\alpha)$

$\dot{x} = \frac{L}{2}\dot{\alpha}\cos(\alpha) = L \cdot \frac{R}{L} V_R \cos(\alpha) = \frac{R}{2} V_R \cos(\alpha)$

$\dot{y} = \underbrace{\frac{L}{2}\dot{\alpha}\sin(\alpha)}_{\text{Use chain rule on both}} = \frac{R}{2} V_R \sin(\alpha)$

Use 2 scenarios to solve for A.

$V_R = 1, V_L = -1 \longrightarrow \dot{x} = 0, \dot{y} = 0, \dot{\alpha} = \frac{2R}{L}$

$V_R = 1, V_L = 0 \longrightarrow \dot{x} = \frac{R}{2}\cos(\alpha), \dot{y} = \frac{R}{2}\sin(\alpha), \dot{\alpha} = \frac{R}{L}$

$a_1 - a_2 = 0 \rightarrow a_1 = a_2$

$a_1 + 0a_2 = \boxed{\frac{R}{2}\cos(\alpha) = a_1 = a_2}$

$b_1 - b_2 = 0 \rightarrow b_1 = b_2$

$\boxed{b_1 = \frac{R}{2}\sin(\alpha) = b_2}$

$c_1 - c_2 = \frac{2R}{L}$

$\boxed{c_1 = \frac{R}{L}, \quad c_2 = -\frac{R}{L}}$, $\frac{R}{L} - c_2 = \frac{2R}{L}$

$$\begin{bmatrix} \frac{R}{2}\cos(\alpha) & \frac{R}{2}\cos(\alpha) \\ \frac{R}{2}\sin(\alpha) & \frac{R}{2}\sin(\alpha) \\ \frac{R}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} V_R \\ V_L \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\alpha} \end{bmatrix} \longrightarrow \boxed{\begin{aligned} \dot{x} &= \frac{R}{2}\cos(\alpha)(V_R + V_L) \\ \dot{y} &= \frac{R}{2}\sin(\alpha)(V_R + V_L) \\ \dot{\alpha} &= \frac{R}{L}(V_R - V_L) \end{aligned}}$$