Welcome to instats

The Session Will Begin Shortly

(At the top of the hour, Eastern USA time)

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Nonlinear Time Series Analysis, Part I: Detecting Nonlinearity

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Seminar Overview

- Day 1
  - Session 1: Introduction to Nonlinear Time Series (NTLS)
  - Session 2: Behaviors and State Spaces
- Day 2
  - Session 3: State Spaces (continued)
  - Session 4: Recurrences
- Day 3
  - Session 5: Tests
  - **Session 6: Singular Spectrum Analysis and Noise**
- Day 4
  - Session 7: Surrogate Data
  - Session 8: Convergent Cross Mapping

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The Challenge

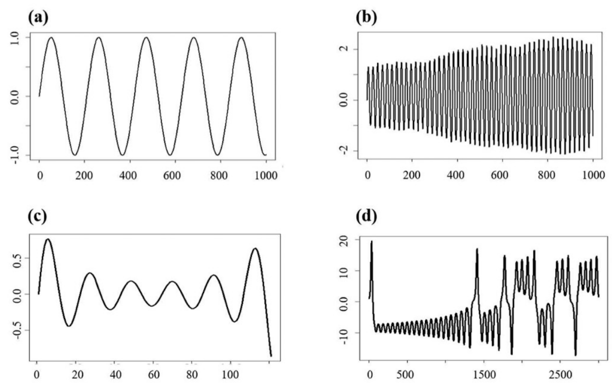

Fig. 6.1 Time series exemplifying the range of possible explanations for volatile dynamics: (a) periodic cycling in the linear model; (b) aperiodic (nonrepeated) cycling created exogenously by randomly shocking the linear model; (c) aperiodic behaviour created by introducing nonstationarity into the linear model; (d) aperiodic cycling endogenous to nonlinear Lorenz equations.

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Pre-process Data

- Background
  - Spectra
  - Eigenvalues and eigenvectors
- Singular spectrum analysis (SSA)
  - Separates some noise from signal
  - Separates some slow variations (e.g., some types of nonstationarity) from signal
- Filters the signal
  - Pushes data away from their original state

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Spectra

- Oscillating behavior
  - Fourier: Sum of sines and cosines
  - Differential equations: Fourier plus decaying amplitude
- Mode
  - Oscillation frequency with decay rate
  - Complex number
    - Frequency = imaginary part
    - Decay rate = real part
- {Mode amplitudes} = Spectra
  - Phases: Each frequency has 2 components

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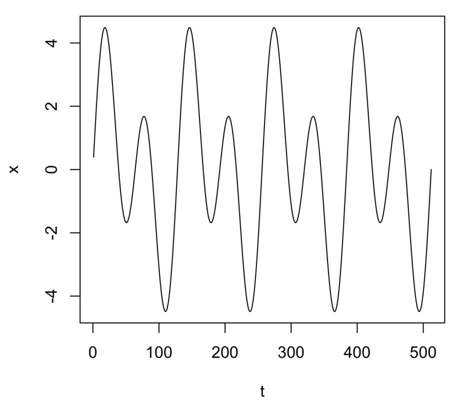
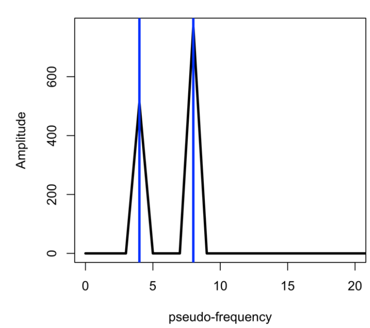
Example: x(t) = 2 sin(t) + 3 sin(2t)

Time Domain



Frequency Domain



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Example: Differential equation

$$m\ddot{x} + \beta\dot{x} + kx = 0$$

- Guess x(t) = A exp(λt)
  - Calculate $\dot{x}$ and $\ddot{x}$
  - Plug in and solve the algebra (eigenvalue) equation
$$m\lambda^2 + \beta\lambda + k = 0$$
- Solutions
$$\lambda = \frac{-\beta \pm}{2m} \pm \frac{\sqrt{\beta^2 - 4mk}}{2m}$$

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Example: Differential equation

- Solutions
$$\lambda = \frac{-\beta \pm}{2m} \pm \frac{\sqrt{\beta^2 - 4mk}}{2m}$$
- These are the modes of the system
- Real roots:
  - Decay/growth only
- Complex roots:
  - First term gives decay rate
  - Second term gives oscillation frequencies
- More details, but don't care about them here

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Nonlinear Issue

- Frequencies get mixed
  - Duh…nonlinearity removes separability
  - Secondary school trigonometry (Ugh)

$$sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$$

- Spectra are only partially useful
  - Skip the details…don't need them

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Matrices and Vectors

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

- Matrix multiplication takes the point (vector) $(x\ y)^T$ and transforms it to another point (*x' y'*)T in the x-y plane.
  - Can do with *n* dimensions; conceptually the same
- So what?

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Eigenvalues and Eigenvectors

- There are special vectors, known as *eigenvectors* do not change their orientation when multiplied by the matrix
  - They are only stretched or shrunk by $\boldsymbol{\lambda}$

$$\lambda \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- Hence $\boldsymbol{\lambda}$ is an *eigenvalue* of the matrix and $(x\ y)^T$ is an *eigenvector* of the matrix.
  - One eigenvector-eigenvalue pair for each dimension (row) of the matrix
    - Sort of…

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Solving

$$\lambda \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- Either the matrix or the vector must have a magnitude of zero
- Determinant allows us to find λ

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So What?

- Depends upon what the matrix represents
- Jacobian (slopes): we get stability (and the direction hyperbolic features) for fixed points
- Modes of a (linear) differential equation: we get the characteristic frequencies of the solutions
  - Noise doesn't have a characteristic frequency
  - Very low frequencies look like nonstationary (trend) behavior
  - Remove these modes from the data to "clean" it

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Singular Spectrum Analysis

- One of several approaches
  - Detects structural variation
  - **Doesn't impose assumptions**
  - Separates signal (e.g., trend, oscillation, etc.) + noise
- Three steps
  - Matrix decomposition (SVD or Toeplitz)
  - Grouping (signal, trend, noise)
  - Time series reconstruction (without trend or noise)

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SSA Benefits

- Filters noise
- Find slow-moving trends
  - Eases nonsationarity somewhat
- Impute data
  - Ugh
  - Can be better than other imputation methods
- Requires some learning of the diagnostics and choices
  - We'll do a lot in R.

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Singular Value Decomposition SSA

- Embed the data
  - Delay = 1
  - Embedding dimension, K
  - Matrix dimensions: (K) x (N-K+1)
- Matrix Decomposition
  - Sum of matrices (mode frequency/decay, amplitude, "vector")
  - λ_i gives relative contribution of mode to the signal

- SSA calculations

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Toeplitz SSA

- Embed the data
  - Delay = 1
  - Embedding dimension, K
  - Matrix dimensions: (K) x (N)
    - Pad the series with zeroes
- Matrix Decomposition
  - Sum of matrices (mode frequency/decay, amplitude, “vector”)
  - λ_i gives relative contribution of mode to the signal
- SSA calculations

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Questions

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STOP

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Day 4 @ 1600 UTC

See you tomorrow (for most time zones)

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