Welcome to instats

The Session Will Begin Shortly

(At the top of the hour, Eastern USA time)

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START

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Nonlinear Time Series Analysis, Part II: Modeling and Phenomenology

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Seminar Overview

- Day 1
  - **Session 1 – Overview of Phenomenology and Tools**
  - Session 2 – Dynamical Systems Analysis
- Day 2
  - Session 3 – Sparse Identification of Nonlinear Dynamics
  - Session 4 – Dynamic Mode Decomposition
- Day 3
  - Session 5 – Hidden Markov Models
  - Session 6 – Machine Learning Approaches
- Day 4
  - Session 7 – Putting it All Together: Lorenz
  - Session 8 – Putting it All Together: Infectious Diseases

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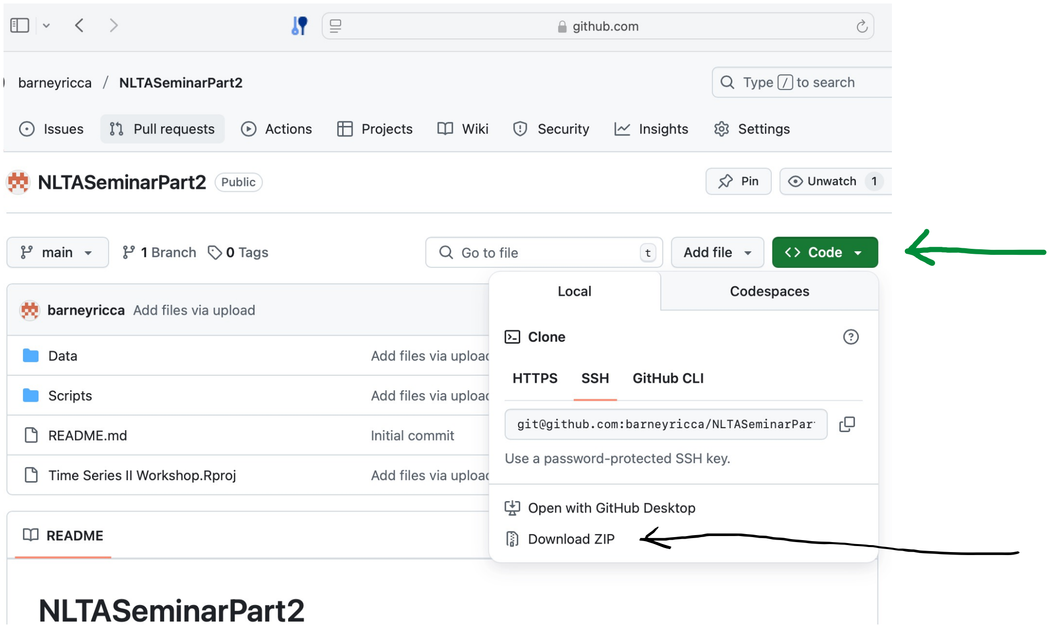
https://github.com/barneyricca/NLTASeminarPart2



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Session Outline

- Introductions
- Modeling
- Phenomenology
- Data wrangling
- Mathematical helpers
  - Estimating derivatives
  - Multilevel Modeling
    - Latent classes
  - Regularization
- Overview of approaches

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Introductions

- You
  - Location and field of interest into the chat, please
- Barney Ricca
  - ”Upstate” New York (USA)
  - Physics, computer science, statistics: Data scientist
  - STEM Education
  - Psychology (trauma and resilience)
  - Idiosyncratic R user
- The schedule
  - Should be reasonably close, but time left at the end because it won’t be…

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Modeling From Theory

- Theory
- Operationalizing
  - Trajectories of theories
  - Being specific
- Coherence of assumptions

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HBR, p. 264

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HBR, p. 264

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HBR, p. 264

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Setting It Up: Data Structures

- Same data structure: Vectors
  - Long data structures (pivoting)
  - Normalization and centering
  - Time column

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Setting It Up: Imputation

- Missingness & imputation
  - Ugh
  - Various types (MCAR, MNAR, etc.)
  - TL;DR: Jäger et al. (2021) find that k-nearest neighbors and random forest imputation methods best

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Is There a Nonlinear Signal?

- Shadow state space
- Tests for alternatives
  - Noise
  - Linearity
  - Stationarity
- Singular Spectrum Analysis (SSA)
- Repeat tests
- Convergent Cross-Mapping (CCM)

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Mathematical Helpers

- Systems of first order equations
  - The *derivative of a derivative* trick to reduce the order to first order
  - Linear or nonlinear
- Estimating derivatives (in detail)
- Linear regression
  - Multilevel modeling
  - Regularization
- Iterative approaches
  - Expectation – Maximization

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Derivatives

- Slopes -> rates of change -> derivative
  - Need animation (Mechanical Universe Episode 3 has derivatives, but not correct for here.)
  - Accel = d(velocity)/dt. Hence, define the slope of a slope, and make things longer. That's tough here, but do it.
- Need to spend some time with derivatives

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Estimating Derivatives: The Problems

- TANSTAAFL
  - "there ain't no such thing as a free lunch"
- Overfitting
- Increase in noise or over-smoothing
- Bias
  - Match levels with derivatives

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Estimating Derivatives

- Finite differences
  - Higher order
  - LLA and GLLA
- Functional data analysis
- Regularized Derivatives
- Empirical Bayes
- Multilevel-based models
  - GOLD

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Finite Differences: Standard

- Standard
  - Good for: frequently sampled data; EDA
  - Pro: Really simple
  - Con: Shifted velocity points
- LLA and GLLA

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Finite Differences: Higher Order

- Higher order
  - More data points
  - Savitzky-Golay, loess
- HBR 9.3 recommends fourth-order centered finite differences:

$$\dot{x} = \frac{8(x_{t+1} - x_{t-1}) - (x_{t+2} - x_{t-2})}{12\delta}$$

  - Delta is an integration step. Should be much less than 1/(sampling frequency)
  - Lose some data

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Finite Differences: LLA and GLLA

- LLA
  - Good for: frequently sampled data; EDA
  - Pro: More flexible smoothing standard finite differences
  - Con: Loss of data (at the ends)
- GLLA
  - More general than LLA
  - Pro: Suitable for end use
  - Con: Must use filtered, not original, level data

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Functional Data Analysis

- Nonparametric approach
  - A la splines
  - Good for: Noise reduction
  - Pro: Very flexible
  - Pro: Many levels of noise reduction possible
  - Con: Must use filtered, not original, data level
  - Pro/Con: Extra researcher degree of freedom

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Some Statistics: Regularization

- Ordinary Least Squares errors
  - Minimize these
- Problems
  - Overfitting
  - Collinearity
- Solution:
  - Add a *regularized* term: Penalize model parameters in some way
  - Don’t care about the details, but will show one example on next slide

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Some Statistics: Regularization

$$L = \sum_{i=1}^{n} \left(y_i + \beta_o - \sum_{j=1}^{p} \beta_j x_{ij}\right)^2 + \lambda \sum_{j=1}^{p} \beta_j^2$$

- Notice: Increases the loss due to every parameter!
  - Tune λ
    - λ=0 provides no help
    - λ small gets a good model
    - λ too high leads to large impact on parameters
  - *Finds solutions with smaller parameters*
- The example uses the L_2 *norm* and is known as *ridge regression*
  - L_1 *norm* is *LASSO* regression (uses absolute value, not square)
    - Can find solutions with parameters pushed to zero (i.e., dropped terms)

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Regularized Derivatives

- OK, this is calculus
  - *Integration* is the inverse of *differentiation*.
  - Regularization penalty: How far the integral of the estimated derivative is from the original function
- Regularized derivatives:
  - Good for: Works best for long data sequences
  - Pro: Better noise characteristics
  - Con: Not currently implemented in *R*
    - But *Python* and *Matlab*
    - Can run *Python* from *R* using *package:reticulate*

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More Statistics: Multilevel Modeling

- Nested data are not i.i.d.
  - Measurements (across time) within a person
  - Students within classrooms within schools
- Goldilocks and the Three Approaches
  - No pooling (ignore the groups)
  - Complete pooling (ignore the individual)
  - Partial pooling (just right)
- Why does pooling matter?
  - Parameter estimates are good regardless
  - Inferential statistics are terrible*, except in partial pooling*

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Generalized Orthogonal Local Derivative

- Is GOLD a derivative?
  - Yeah, but perhaps an odd one
- GOLD
  - Good for: Multilevel modeling
  - Pro: reduces collinearity problems
  - Con: Harder to interpret

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Emprical Bayes

- Use one subject as basis for another
  - Good for: Mixed models (e.g., across subjects)
  - Pro: Keeps noise down
  - Pro: Seems better than GOLD or GLLA or FDA
  - Con: Multivariate normal derivative distribution

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Iterative Approach

- Derivative to model to better idea of derivative to use and rework it.
  - As you learn more about the model, you might choose a better derivative
- Just a thought, though…

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Overview of Modeling Approaches

Method	What it does	When to use it
DSA	Estimate parameters of a model involving derivatives	Some theoretical knowledge built on derivatives
SINDy	Choose a set of functional terms from a large library	Long-term (fixed-points) and trajectories
DMD	Identify the most notable modes of a system	Response to perturbations (modes)
ML	Search widely for possible dynamic relationships	Anytime, but interpretability (fails at phenomenology)
HMM	Search for a ”hidden” model underlying data	Classifying via underlying (hidden) dynamics

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Questions

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STOP

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Next session @ UTC 1900

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