

# Welcome to **instats**

**The Session Will Begin Shortly**  
(At the top of the hour, Eastern USA time)

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# START

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# Nonlinear Time Series Analysis, Part II: Modeling and Phenomenology

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## Seminar Outline

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- Day 1
  - Session 1 – Overview of Phenomenology
  - Session 2 – Dynamical Systems Analysis
- Day 2
  - Session 3 – Sparse Identification of Nonlinear Dynamics
  - **Session 4 – Dynamic Mode Decomposition**
- Day 3
  - Session 5 – Hidden Markov Models
  - Session 6 – Machine Learning Approaches
- Day 4
  - Session 7 – Putting it All Together: Lorenz
  - Session 8 – Putting it All Together: Infectious Diseases

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## Dynamic Mode Decomposition (DMD)

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- DMD
  - Identifies linear dynamics (modes) from high-dimensional data
  - Purely data-driven
  - Computationally and conceptually not-too-hard
  - Foundation for Hankel, Koopman, etc.
- However
  - Limited to linear modes
  - Noise sensitivity
  - Generally requires long data sequences

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## Dynamic Mode Decomposition (DMD)

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- Measurements
  - $x_1, x_2, x_3 \dots x_{m+1}$
  - $t_1, t_2, t_3 \dots t_{m+1}$
- Constraints
  - Equally spaced times
  - No missingness

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## Dynamic Mode Decomposition (DMD)

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- Make 2 matrices from the data
- $X = [x_1 \ x_2 \ \dots \ x_m]$
- $X' = [x_2 \ x_3 \ \dots \ x_{m+1}]$
- DMD equation
- $X' = AX$
- In general, can't find eigenvectors and eigenvalues of  $A$ 
  - But we can approximate them
  - Especially for large  $m$

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## Modes

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- Redux from Part I of series

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## Sample Matrix Creation

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- One dimensional example (simple oscillator, with very slow decay)
- Then do a string to show many different modes and how to work with them

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## Eigenvalues and Eigenvectors

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- Here we can exactly solve the equations (I think!)

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## Reconstruction from Modes

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## DMD v. SINDy

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- DMD: Modes
  - Most modes decay to zero long-term
  - Response to external perturbations given by modes
- SINDy: Equations
  - Trajectories and *fixed points*
  - Doesn't tell us much about perturbations

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## An Intriguing Finding

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- Ricca & Green (in preparation)
- Long-term behavior prediction
  - 30-day EMA
  - 6-month follow up
  - Latent SINDY classes > GMM classes > DMD classes
- Don't make too much of this
  - There are reasons we haven't submitted this for publication yet

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## HAVOK Analysis

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- *Hankel Alternative View Of Koopman*
  - Specific approach to DMD
- Moulder et al. (2023)
  - ECG Behavior during a *go/no-go* task

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## Tangentially Related Tools

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- Burstiness (*package:ndstools*)
  - System memory as a measure of *self-organization*
- Autoregression (*package:forecast*)
  - Includes ARIMA, network psychometrics, etc.
  - Different estimation method than DMD, but same delay setup
- Orbital decomposition (*package:ndstools*)
  - Mesoscale pattern identification
- Detrended fractal analysis (*package:DFA*)
  - Long-range correlation structure

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## Questions

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# STOP

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Next session @ UTC 1900

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