Theme D Experiment for fluid dynamics & Incubation of coastal ecosystem

Thumwanit Napat Group 2
16B00133

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Chapter 1

Experiment for fluid dynamics

1.1 Background & Theories

The setting and knowledge are based on the handout of the experiment, Theme D.

1. **Flow rate** - Derived from Bernoulli's equation

$$q_1 = \sqrt{2gh_1^2(h_0 - h_1)} = C_c a\sqrt{2g(h_0 - C_c a)} \quad \left(C_c = \frac{h}{a}\right) \tag{1.1}$$

2. **Froude number & Energy loss** - By conservation of energy, energy loss from hydraulic jump can be formulated as the following.

$$\Delta E = \frac{(h_2 - h_1)^3}{4h_1h_2} \quad (h_2 > h_1) \tag{1.2}$$

By the conservation of momentum, ratio of h_1 and h_2 can be expressed and rewritten as the following formular

$$\frac{h_2}{h_1} = \frac{1}{2}\left[\sqrt{1 + 8F_{r_1}^2} - 1\right] \tag{1.3}$$

3. **Drag force** - Drag force inflicted by the impact of the fluid can be expressed as the following

$$F_D = \frac{1}{2}\rho C_D A U^2 \tag{1.4}$$

where C_D, A, U are the drag coefficient, projected area and flow velocity accordingly.

1.2 Result

1.2.1 Preparatory Experiment

Dimension	Value
h_0	15 cm
h_1	1 cm
h_2	1.2 cm
W (Width of the channel)	38.3 cm
W_{internal} (Width of the channel)	38.3 cm
L (Length of the channel)	2.48 m
a (height of the gate)	1 cm

Table 1.1: list of the dimension of channel and height of water level

Number of measurements	Time (second)
1	7.76
2	7.27
3	7.43

Table 1.2: value of time (s) to fill the bucket (8 L)

1.2.2 Hydraulic jump test

Dimension	Value
h_1	0.9 cm
h_2	4.6 cm

Table 1.3: Hydraulic jump test measured values

1.2.3 Drag force test

Dimension	Value
Width of channel	38.3 cm
Diameter of cylinder	8.9 cm
M_1 (mass of cylinder in the case without porous bed)	9.7 cm
M_2 (mass of cylinder in the case with porous bed)	88 gm
h_3 (in the case without porous bed)	1.5 cm
L (Length of the channel)	2.48 m
a (height of the gate)	1 cm

Table 1.4: list of the dimension of channel and height of water level

1.3 Discussion

1.3.1 Contraction coefficient and flow rate per unit width

From equation (**??**) and information in table **??**, we calculate the contraction coefficient as the following.

$$C_c = \frac{h_1}{a} = \frac{1}{1} = 1$$

then calculate the flow rate per unit width

$$q_1 = C_c a\sqrt{2g(h_0 - C_a a)} = 1\times10^{-2}\times\sqrt{2\times 9.81\times(15\times10^{-2} - 1\times10^{-2})} = 1.66\times10^{-2}\,\frac{\mathrm{m}^2}{\mathrm{s}}$$

From the bucket filling time in table **??**, the average time of filling 8 L of water is $22.46/3 = 7.49$ s. Therefore, the flow rate Q is

$$Q = \frac{8\times10^{-3}}{7.49}\frac{\mathrm{m}^3}{\mathrm{s}}$$

So that the experimental flow rate per unit width

$$q = \frac{Q}{w_{\text{internal}}} = \frac{8\times10^{-3}}{7.49}\times\frac{1}{10.3\times10^{-2}}\frac{m^3}{s} = 1.04\times10^{-2}\frac{m^2}{s}$$

The experimental value is smaller than the theoretical value which might be caused by friction and energy loss.

1.3.2 Energy dissipated owing to the hydraulic jump and Froude numbers (Fr)

We set up the obstacle and measure the height of the water at each point as recorded in the table **??**. From the equation **??**, we can calculate the energy dissipated by hydraulic jump as the following.

$$\Delta E = \frac{(h_2 - h_1)^3}{4h_1h_2} = \frac{(4.6-0.9)^3}{4\times0.9\times4.6}\mathrm{cm} = 3.06\mathrm{cm}$$

And from equation **??**, we can calculate the froude number (F_r) as the following

$$F_{r_1} = \sqrt{\frac{\left(\frac{2h_2}{h_1}+1\right)^2 - 1}{8}} = 3.95$$

Conversely, F_{r_2} can be calculated as the following

$$F_{r_2} = \sqrt{\frac{\left(\frac{2h_1}{h_2}+1\right)^2 - 1}{8}} = 0.34$$

1.3.3 Drag Force

From table **??** and flow rate (Q) from previous section, we obtain the mean velocity as the following

$$\overline{U} = \frac{Q}{h_3 \times w} = \frac{8 \times 10^{-3}}{7.49} \frac{1}{1.2 \times 10^{-2} \times 38.3 \times 10^{-2}} = 2.32 \times 10^{-1} \frac{\mathrm{m}}{\mathrm{s}}$$

From equation **??**, we obtain drag force

$$\begin{aligned}
\text{Without porous bed} \ : \ F_D &= \frac{1}{2}\rho C_D A\overline{U}^2 = \frac{1}{2}\rho C_D (h_3 D)\overline{U}^2 = 2.26 \times 10^{-2}\mathrm{N} \\
\text{With porous bed} \ : \ F_D &= \frac{1}{2}\rho C_D A\overline{U}^2 = \frac{1}{2}\rho C_D (h_3 D)\overline{U}^2 = 5.28 \times 10^{-2}\mathrm{N}
\end{aligned}$$

From the experimental data, we obtain friction coefficient by balancing the weight and calculate by the following equation,

$$\begin{aligned}
F_D &= F_R \\
&= \mu(mg - B) \\
&= \mu(mg - \rho_{\mathrm{water}} v_{\mathrm{sub}} g) \\
\mu &= \frac{F_D}{mg - \rho_{\mathrm{water}} v_{\mathrm{sub}} g}
\end{aligned}$$

where v_{sub} is the volume of the cylinder under the water surface.

	Without porous bed	With porous bed
Drag force	2.26×10^{-2}	5.28×10^{-2}
Buoyancy force	4.58×10^{-1}	1.07
Friction coefficient	1.81×10^{-2}	-2.55×10^{-1}

Table 1.5: Drag force, Buoyancy force and calculated friction coefficient

We can observe that the friction coefficient for the case with porous bed became negative. It might be because we assumed that both cases conserve same flow rate which it should decrease by porous bed and force acting on the cylinder also.

Chapter 2

Incubation of coastal ecosystem

2.1 Result & Discussion

Time	Beaker #	pH	Beaker #	w0	w1	w2	pHa	Intensity
14:10	14	7.985	19	44.2417	93.4206	108.3132	3.515	100%
14:40	17	8.072	15	44.3308	97.70	112.62	3.723	100%
14:50	16	7.902	14	44.6421	94.4369	109.2862	3.553	50%
15:20	12	7.977	15	44.3301	96.6465	111.4785	3.681	50%
15:30	13	7.818	18	45.1818	97.3689	112.1726	3.692	25%
16:00	14	7.868	12	45.6516	97.6967	112.5688	3.667	25%
16:10	18	7.75	11	44.0117	94.7432	109.6703	3.591	0%
16:40	19	7.722	16	43.4848	95.6989	110.5581	3.685	0%

Table 2.1: Experimental values

1. From the table **??**, we calculate according to the equations in the handout and obtain the following result

$$G = -\frac{1}{2}\rho_w \frac{V}{A}\frac{\Delta A_T}{\Delta t}$$
$$P_n = -\rho_w \frac{V}{A}\frac{\Delta C_T}{\Delta t} - G$$

Time	Beaker #	G (mmol/m^{2}h)	P (mmol/m^{2}h)
14:10	14	1.431398231	12.4
14:40	17		
14:50	16	1.324916993	9.86
15:20	12		
15:30	13	0.2226796472	5.43
16:00	14		
16:10	18	0.214260976	-2.46
16:40	19		

Table 2.2: P and G experimental values

2. Using non-linear least square fitting, the result was obtained as the following graph and equation.

E	$P_{observe}$	P_{model}	$(P_{observe} - P_{model})^2$
440	12.4	12.44	2.32×10^{-3}
250	9.86	9.7	1.65×10^{-2}
135.8	5.43	5.54	1.12×10^{-2}
0.4	-2.4	-2.49	6.63×10^{-4}
		sum	3.078×10^{-2}

Table 2.3: experimental and predicted P error table

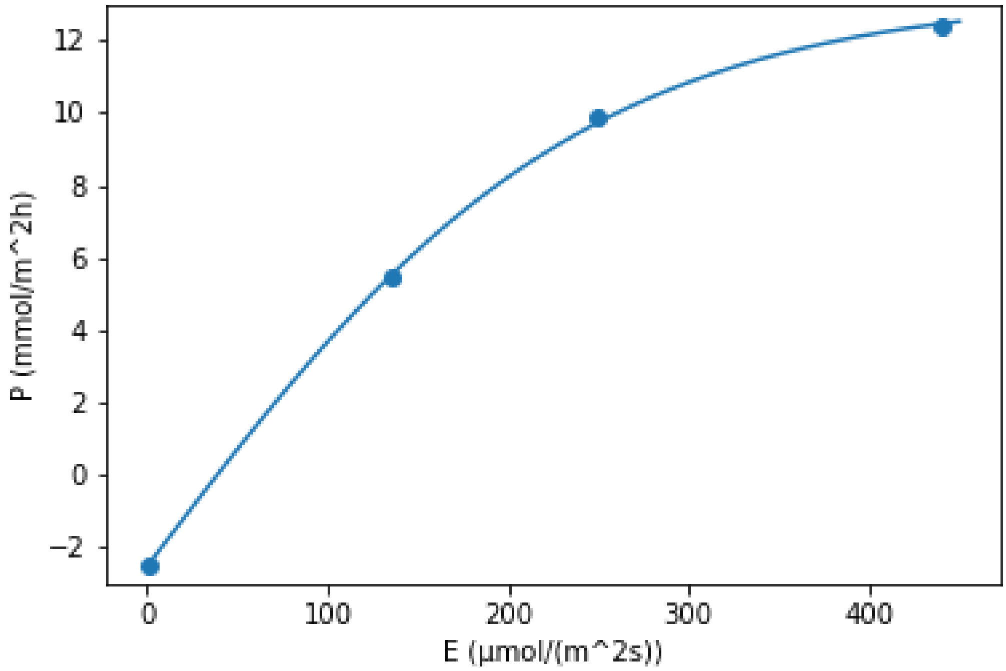

Figure 2.1: E-P Graph

$$P = 15.8 \tanh\left(\frac{E}{2.40 \times 10^2}\right) - 2.51$$

3. Analogously, We obtain the graph and equation for G.

E	$G_{observe}$	G_{model}	$(G_{observe} - G_{model})^2$
440	1.431	1.496	4.151×10^{-3}
250	1.325	1.004	1.028×10^{-1}
135.8	0.2223	0.6102	1.502×10^{-1}
0.4	0.2143	0.08287	1.726×10^{-2}
		sum	2.744×10^{-1}

Table 2.4: experimental and predicted G error table

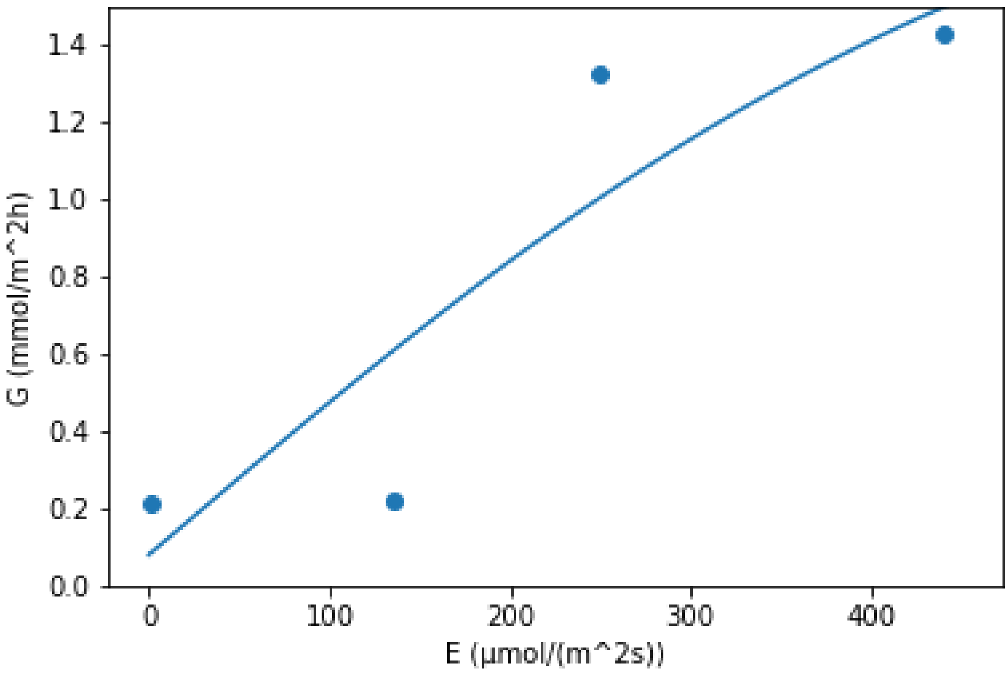

Figure 2.2: E-G Graph

$$G = 2.016 \tanh\left(\frac{E}{5.056 \times 10^2}\right) + 8.128 \times 10^{-2}$$

4. The calcification rate per day of a coral community can be estimated by integrating the calcification rate of a whole day

$$rate = coverage \times \int_0^{24} G(t)dt \text{ ; } t \text{ is in hour} \tag{2.1}$$

where E_b was derived as the following equation (From top to bottom)

$$\begin{aligned}
G &= 2.016 \tanh\left(\frac{E_b}{5.056 \times 10^2}\right) + 8.128 \times 10^{-2} \\
E_b &= E_s(1-\alpha)\exp(-\lambda h) \\
E_s &= 2.1 \times Q \\
Q &= \max\left(\frac{S\cos^2 Z}{(\cos Z + 2.7)e \times 10^{-5} + 1.085\cos Z + 0.10}, 0\right) \\
\cos Z &= \sin\phi\sin\Delta + \cos\phi\cos\Delta\cos HA \\
\Delta &= 23.44^\circ \times \cos[(172 - D) \times 2\pi/365] \\
HA &= (12 - t) \times \pi/12
\end{aligned}$$

With the following constants

$$\begin{aligned} coverage &= 0.3 \\ h &= 0.5 \text{ m} \\ \phi &= 20°\text{N} \\ D &= 71 (\text{March } 21) \\ e &= 1783 \\ \alpha &= 0.07 \\ \lambda &= 0.12 \text{ m}^{-1} \end{aligned}$$

The implementation was done by Python which the code was attached in the Appendix. Then we plot the graph of G.

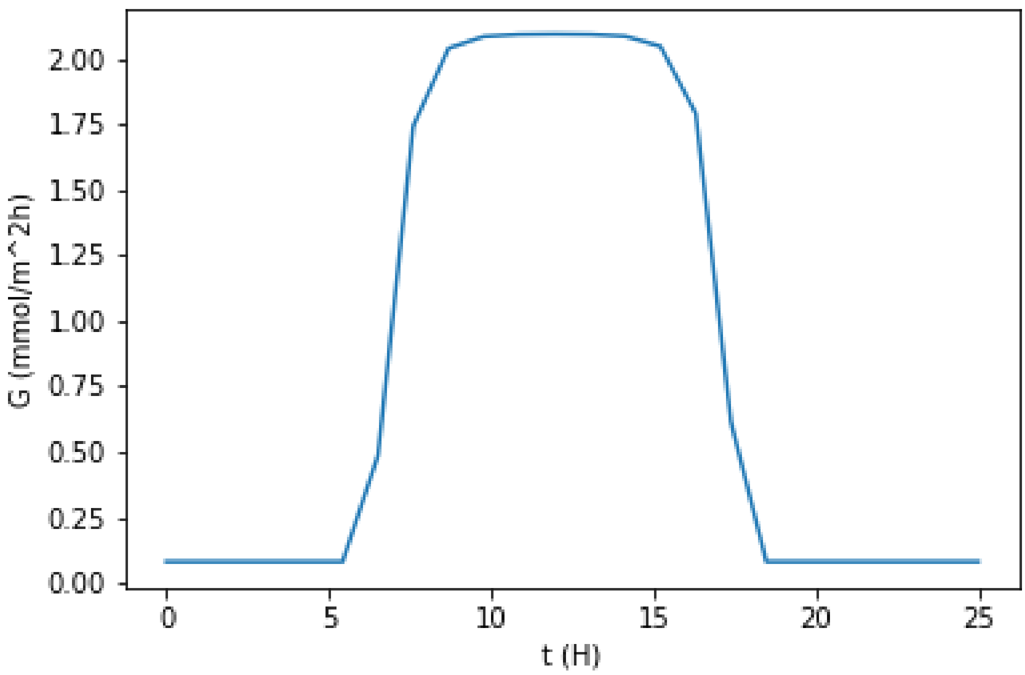

Figure 2.3: G vs time in March 21st

By utilizing simpson method to do integration, we yield the following result

$$rate = coverage \times \int_0^{24} G(t)dt = 6.571 \text{ mmol/m}^2\text{day} \tag{2.2}$$

5. In this question, assume that calcium carbonate ($CaCO_3$) molar weighted 100 g mol^{-1} and 1.4 g cm^{-3} of bulk density of the reef. By these factor, we can calculate the accumulation rate as the following

$$
\begin{aligned}
\text{accumulation rate} &= \frac{6.571 \times 10^{-7}\ \text{mol/cm}^2\text{day} \times 100\ \text{g mol}^{-1}}{1.4\ \text{g cm}^{-3}} \times 365\ \text{day/year} \\
&= 0.1713\ \text{cm/year}
\end{aligned}
$$

6. Sea level rise 0.5 cm/year while the accumulation of coral reef per year is 0.1713 cm. Therefore, for keeping the coastal protection efficiency, the necessary coverage of the coral is

$$coverage_{\text{new}} = coverage_{\text{prev}} \times 0.5/0.1713 = 0.8757$$

or 87.57% coverage required. There are many factors that damaging the coral reef site such as physical damage, from divers, trashes or irresponsible boat. If the diver touch the coral, it may stir up the sediment and cause damage to the coral which education toward tourism should be strengthen. The indirect way is chemical from household or waste water. Another important factor is CO_2. As the result of increase of CO_2, the rate of calcification decrease and the accumulation of coral reef become slower. To solve these problems, we need to reduce our usage of energy, decrease the CO_2 emission, and use less water to reduce the waste water which will return to the ocean eventually.

Appendices

```python
S = 1353
phi = 20 * (2 * np.pi / 360)
delta = 23.44 * np.cos((172 - 71) * 2 * np.pi / 365) * (2 * np.pi / 360)
alpha = 0.07
lmbda = 0.12
h = 0.5
e = 1783
cover = 0.3

def max0(x):
    return (x > 0) * x

def Q(cosz):
    return max0(S * cosz ** 2 / ((cosz + 2.7)*e*10**-5+1.085*cosz+0.1))

def cosz(HA):
    return np.sin(phi) * np.sin(delta) + np.cos(phi) * np.cos(delta) *
        np.cos(HA)

def HA(t):
    return (12 - t) * np.pi / 12.

def Es(Q):
    return 2.1 * Q

def Eb(Es):
    return Es * (1 - alpha) * np.exp(-lmbda * h)

# Eb input by t (time)
def Eb_a(t):
    HA_t = HA(t)
    cosz_t = cosz(HA_t)
    Q_t = Q(cosz_t)
    Es_t = Es(Q_t)
    return Eb(Es_t)
```

Listing 1: Q4 code snippet for calculating $E_b(t)$