

Partial Differential Equations for Science and Engineering

Final Report

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# Problem 2 Burger’s Equation

Using forward-in-time and backward-in-space for the 1st derivative, and centered difference for the 2nd derivative, construct a numerical model for the Burger’s equation. Decide on your own initial conditions and values for v.



Discuss the following (add figures if necessary):

1. Derive and write the discretization as an algebraic equation
2. Investigate by modelling the differences when a (linear) is a constant and when a is u (non-linear) for a cyclic condition.
3. What happens when you set a Dirichlet boundary?
4. List potential applications for Burger’s equation.

## Discretization Algebraic Equation [Q.1]

By these equation







We can derive the algebraic equation



If we let  and , we can rewrite our equation as



## Observation of Modelling and Condition [Q.2+3]

### Initial Value

1. dx = 0.1
2. dy = dx
3. a = 1.
4. v = 2.
5. grid\_x = 100
6. grid\_y = 100
7. nt = 100
8. d1 = 0.1
9. d2 = 0.05
10. dt = d2 \* (dx\*\*2)/v
12. **def** init():
13. T = np.zeros((grid\_x,grid\_y))
14. T[70:90, 70:90] = 80.
15. **return** T

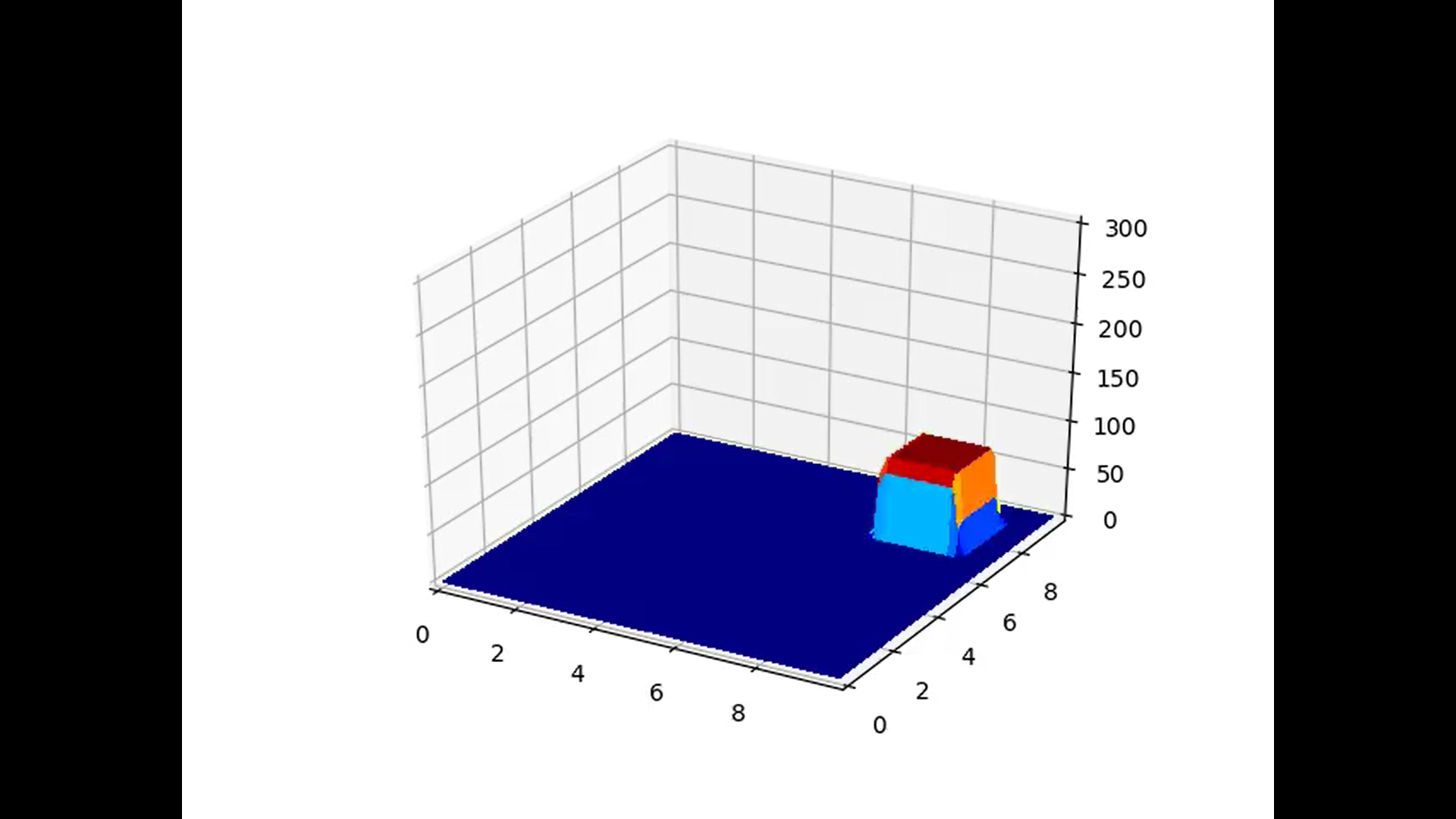


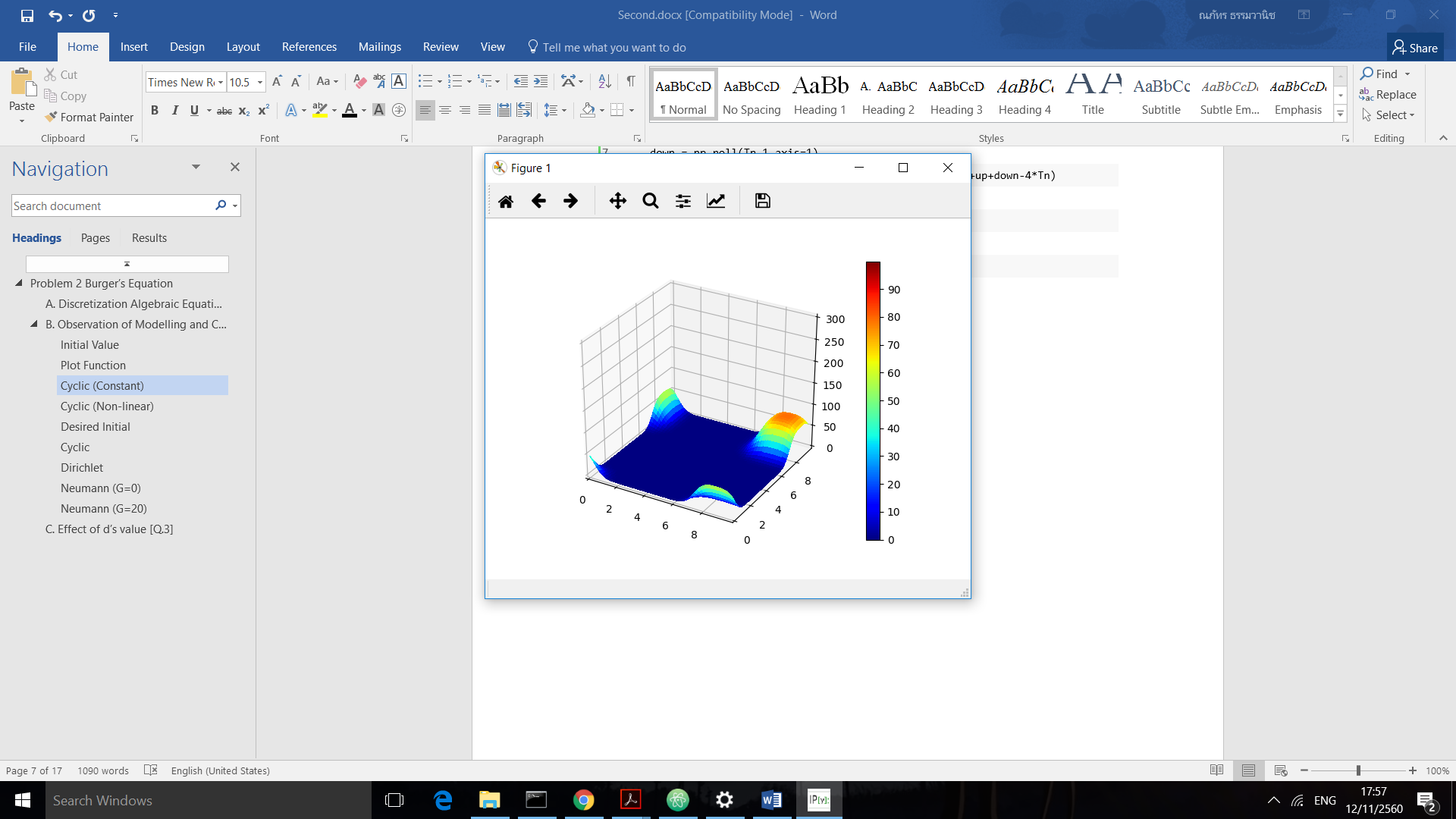
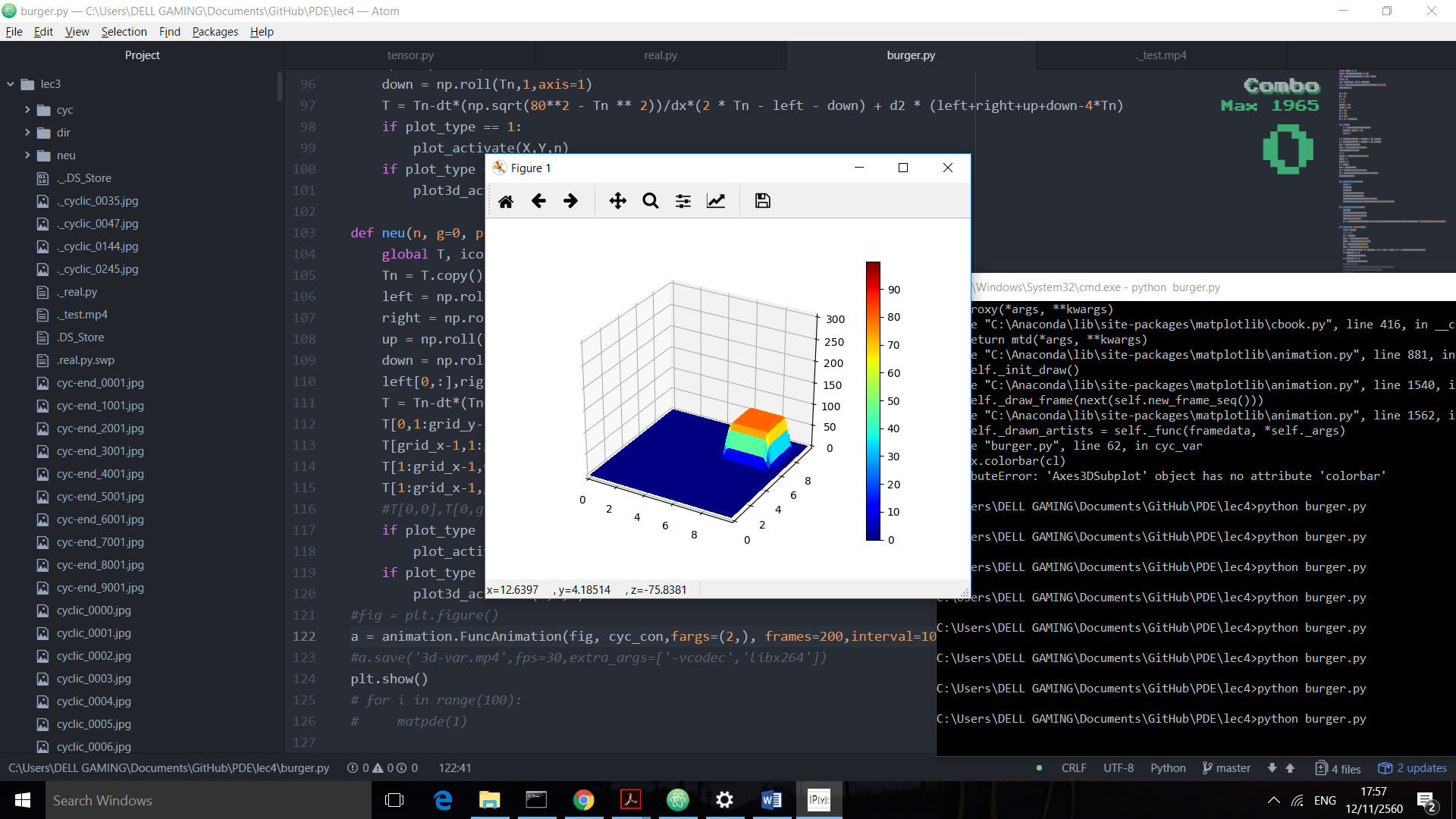
Figure 1 Initial for Observation

### Plot Function

1. fig = plt.figure()
2. ax = fig.gca(projection='3d')
3. cl = plt.contourf(X,Y,T,levels,cmap=cmap)
4. plt.colorbar(cl)
6. **def** plot\_activate(X,Y,n):
7. **global** T
8. plt.cla()
9. plt.clf()
10. plt.xlim(0.,np.max(x))
11. plt.ylim(0.,np.max(x))
12. plt.contourf(X,Y,T,levels,cmap=cmap)
13. plt.text(np.max(x)\*0.8,np.max(y)+dy,"t=%01.5f"%(dt\*n))
15. **def** plot3d\_activate(X,Y,n):
16. ax.cla()
17. ax.set\_xlim(0.,np.max(x))
18. ax.set\_ylim(0.,np.max(y))
19. ax.set\_zlim(0.,300)
20. cl = ax.plot\_surface(X,Y,T,linewidth=0,vmin=np.min(levels),vmax=np.max(levels), cmap=cmap,antialiased=False)

### Cyclic (Constant)

1. **def** cyc\_con(n, plot\_type=1):
2. **global** T,d1,d2
3. Tn = T.copy()
4. left = np.roll(Tn,1,axis=0)
5. right = np.roll(Tn,-1,axis=0)
6. up = np.roll(Tn,-1,axis=1)
7. down = np.roll(Tn,1,axis=1)
8. T = Tn-d1\*(2 \* Tn - left - down) + d2 \* (left+right+up+down-4\*Tn)
9. **if** plot\_type == 1:
10. plot\_activate(X,Y,n)
11. **if** plot\_type == 2:
12. plot3d\_activate(X,Y,n)

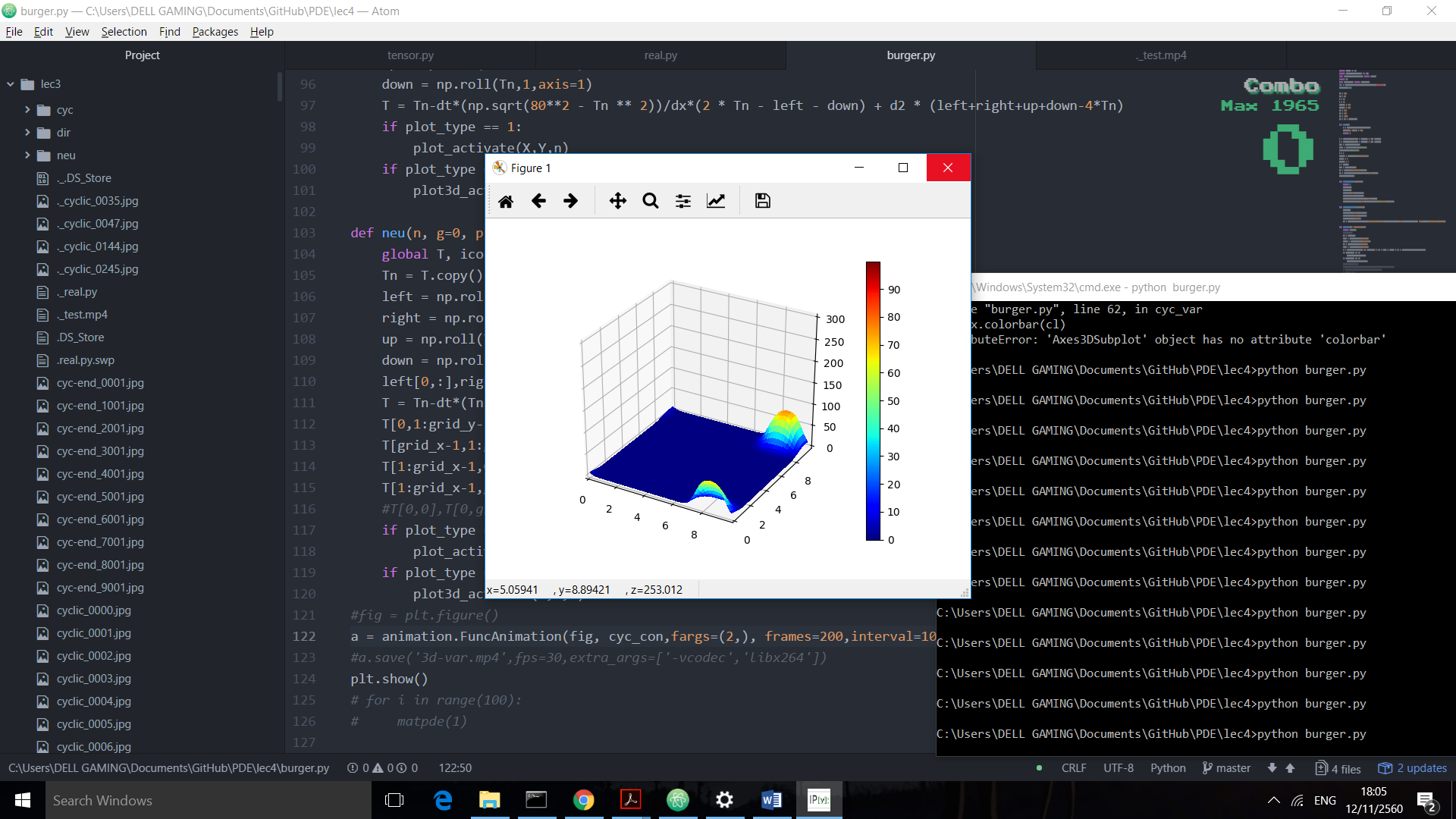


Different to the Diffusion equation, we included advection which cause the blob of heat to move along the  vector ( because the advection term allowed effect from x,y axis equally).

We tried to edit some part of the code following this equation



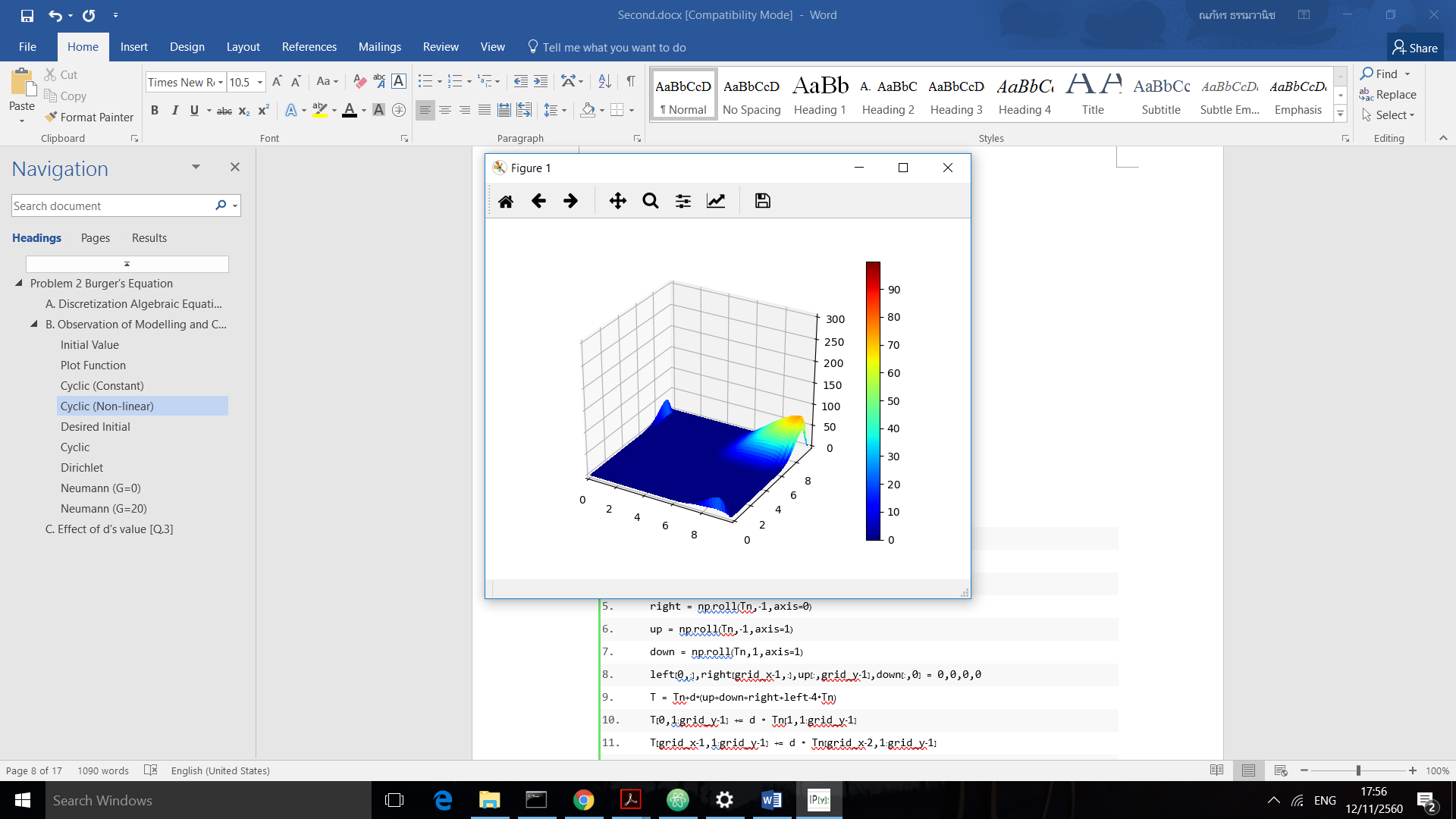
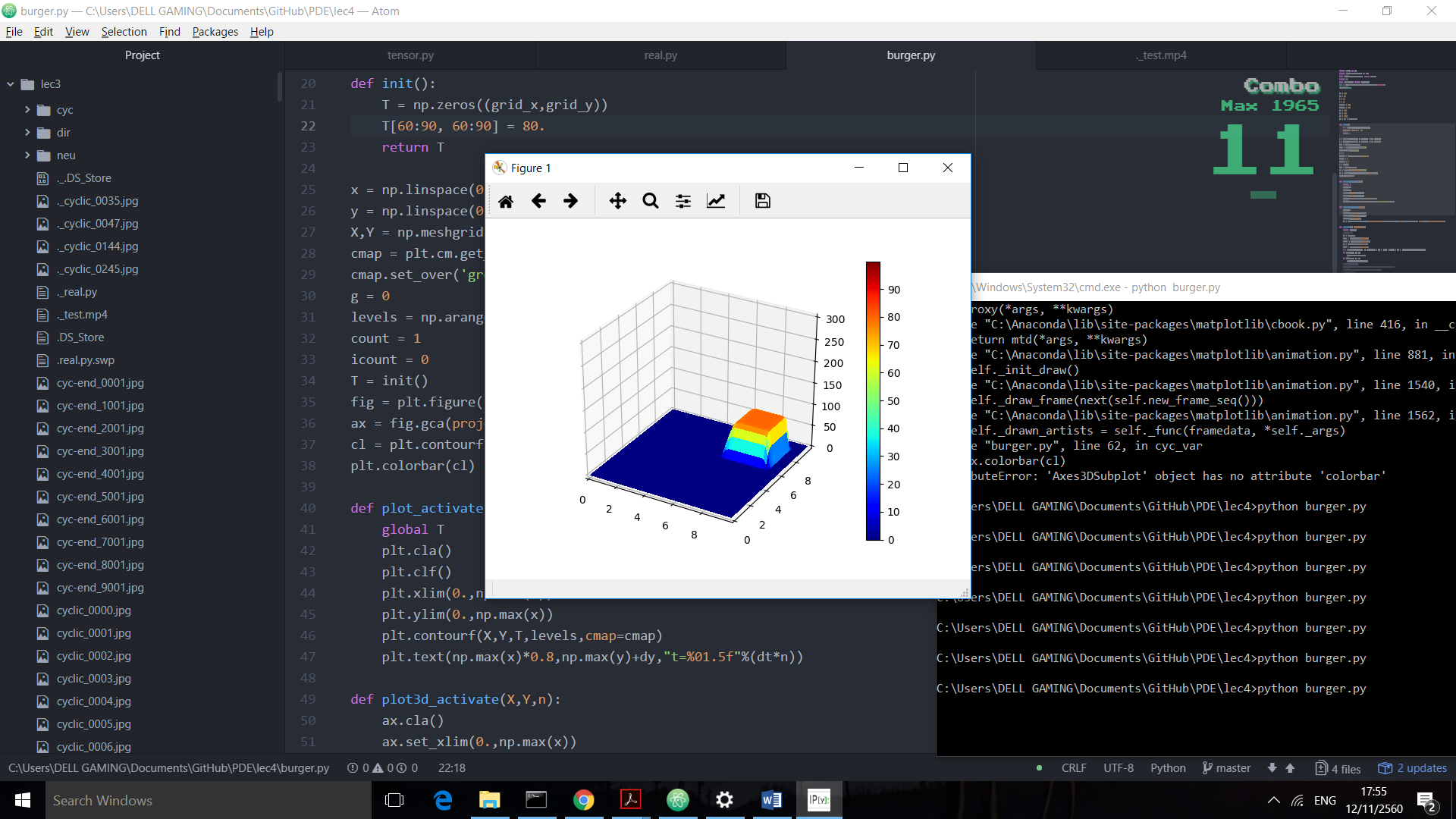
1. **def** cyc\_con(n, plot\_type=1):
2. **global** T,d1,d2
3. Tn = T.copy()
4. left = np.roll(Tn,1,axis=0)
5. right = np.roll(Tn,-1,axis=0)
6. up = np.roll(Tn,-1,axis=1)
7. down = np.roll(Tn,1,axis=1)
8. **T = Tn-d1\*(2 \* Tn - 1.5\*left - 0.5\*down) + d2 \* (left+right+up+down-4\*Tn)**
9. **if** plot\_type == 1:
10. plot\_activate(X,Y,n)
11. **if** plot\_type == 2:
12. plot3d\_activate(X,Y,n)



If look carefully, it can be seen that the blob movement shift to the left compared to the previous result.

### Cyclic (Non-linear – 1)

1. **def** cyc\_var(n, plot\_type=1):
2. **global** T,d1,d2
3. Tn = T.copy()
4. left = np.roll(Tn,1,axis=0)
5. right = np.roll(Tn,-1,axis=0)
6. up = np.roll(Tn,-1,axis=1)
7. down = np.roll(Tn,1,axis=1)
8. T = Tn-dt\*Tn / dx\*(2 \* Tn - left - down) + d2 \* (left+right+up+down-4\*Tn)
9. **if** plot\_type == 1:
10. plot\_activate(X,Y,n)
11. **if** plot\_type == 2:
12. plot3d\_activate(X,Y,n)

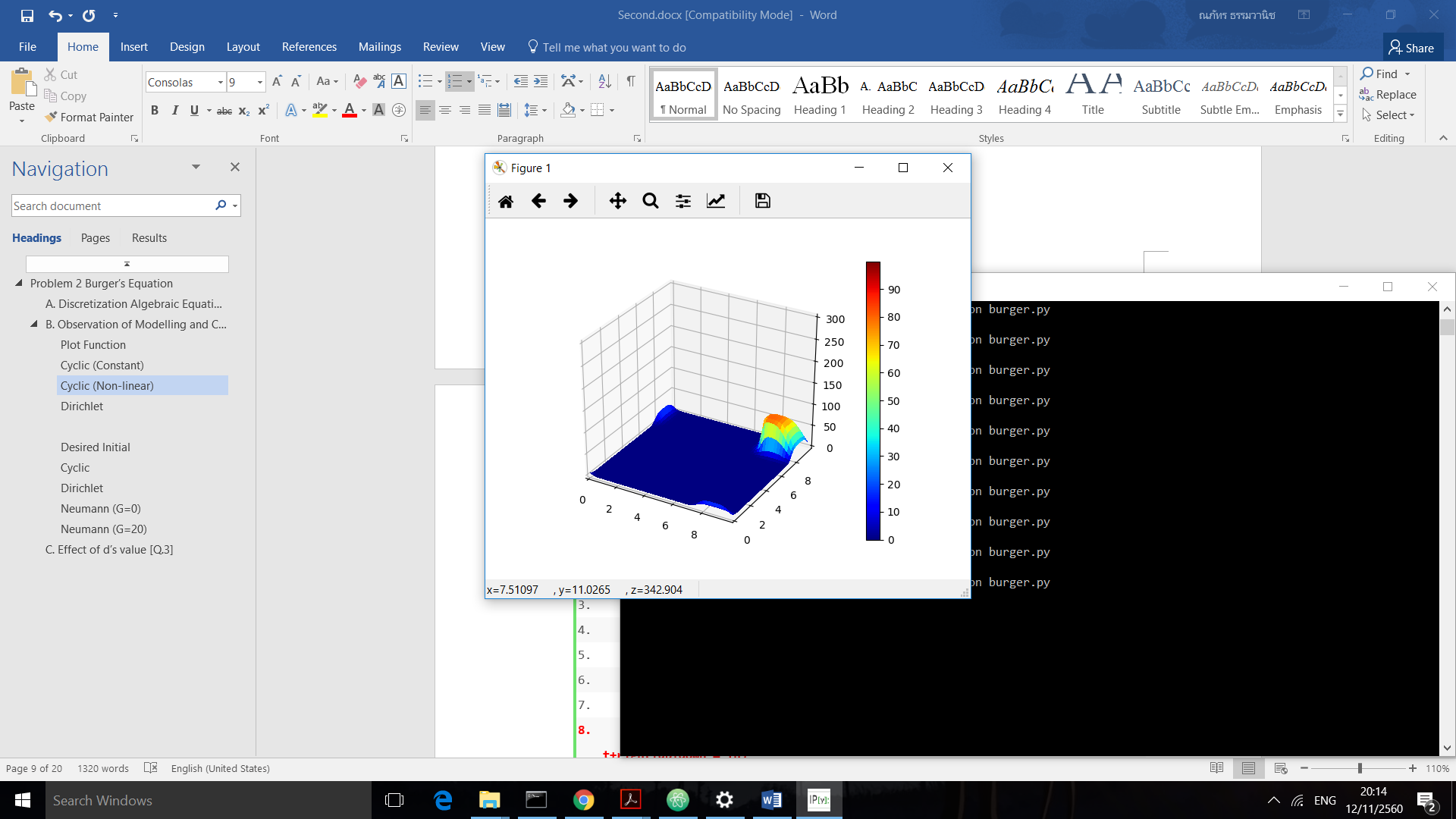


According to the figure, the tip of the heat blob moves faster. This can be assumed by the non-linear factor. By the point where has higher heat, the effect from advection will be stronger be because we multiply itself again from the formula. (is elements wise multiplication)



### Cyclic (Non-linear – 2)

1. **def** cyc\_var(n, plot\_type=1):
2. **global** T,d1,d2
3. Tn = T.copy()
4. left = np.roll(Tn,1,axis=0)
5. right = np.roll(Tn,-1,axis=0)
6. up = np.roll(Tn,-1,axis=1)
7. down = np.roll(Tn,1,axis=1)
8. **T = Tn-dt\*(np.sqrt(80\*\*2 - Tn \*\* 2))/dx\*(2 \* Tn - left - down) + d2 \* (left+right+up+down-4\*Tn)**
9. **if** plot\_type == 1:
10. plot\_activate(X,Y,n)
11. **if** plot\_type == 2:
12. plot3d\_activate(X,Y,n)

In this case, we made the point with more heat moves slower by multiplied with a value of  decreasing conversely to  which we choose (80 is maximum of the heat range)

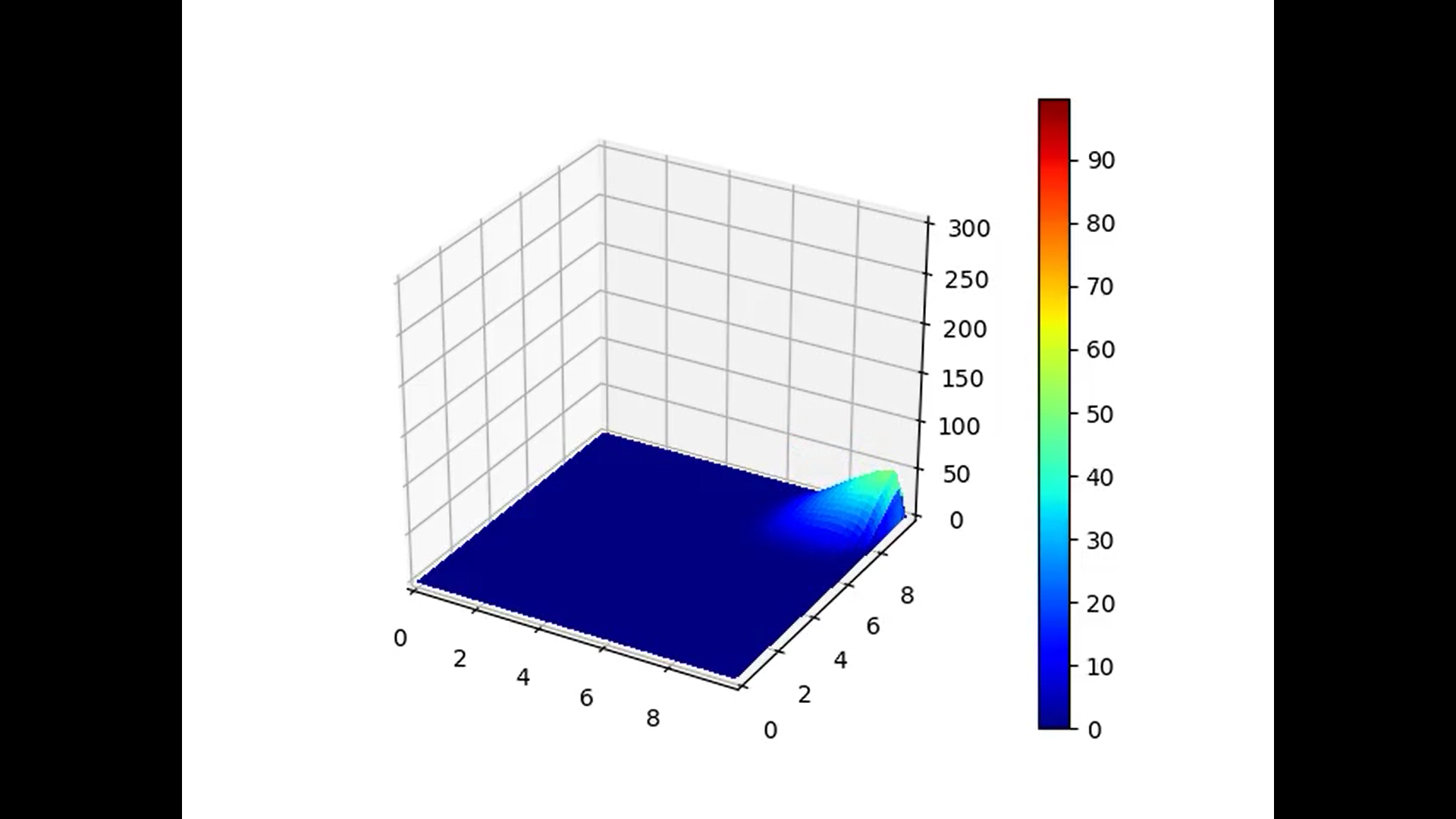
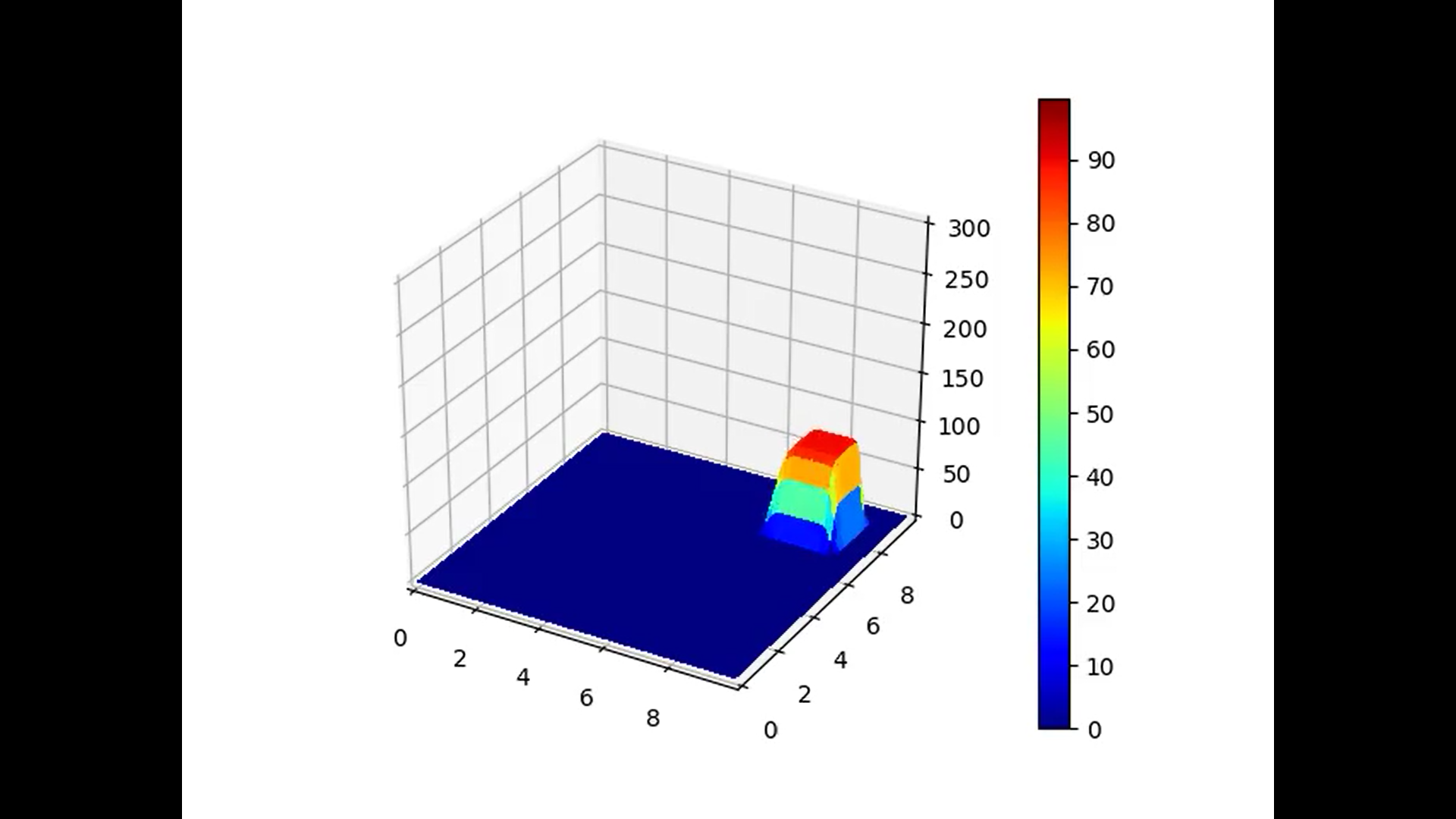


### Dirichlet (Non-linear)

1. **def** der\_var(n, plot\_type=1):
2. **global** T,d1,d2
3. T[0,:],T[:,0], T[grid\_x-1,:], T[:, grid\_y-1] = 0,0,0,0
4. Tn = T.copy()
5. left = np.roll(Tn,1,axis=0)
6. right = np.roll(Tn,-1,axis=0)
7. up = np.roll(Tn,-1,axis=1)
8. down = np.roll(Tn,1,axis=1)
9. T = Tn-dt \*Tn /dx\*(2 \* Tn - left - down) + d2 \* (left+right+up+down-4\*Tn)
10. **if** plot\_type == 1:
11. plot\_activate(X,Y,n)
12. **if** plot\_type == 2:
13. plot3d\_activate(X,Y,n)

and we set new initial condition

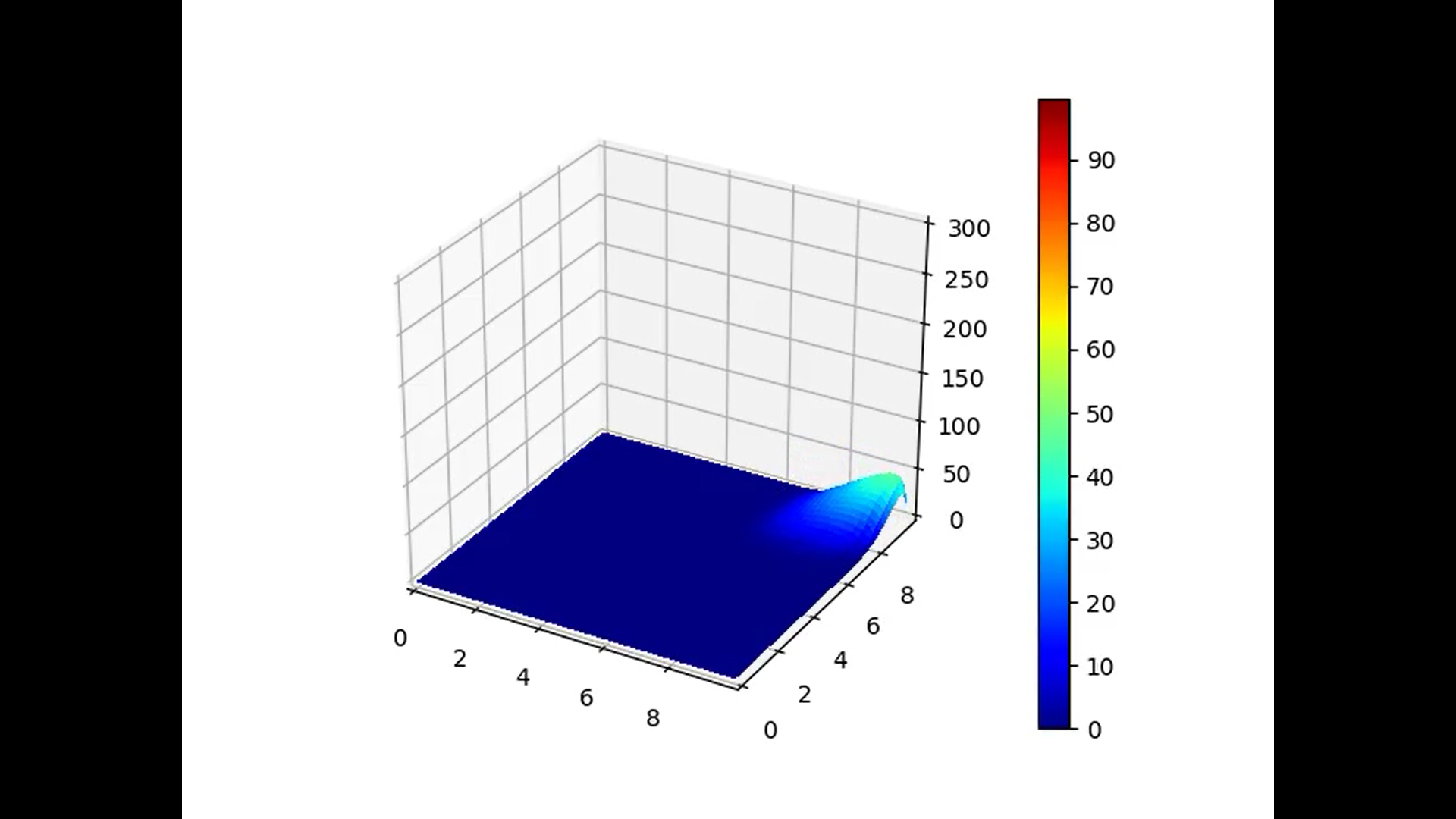
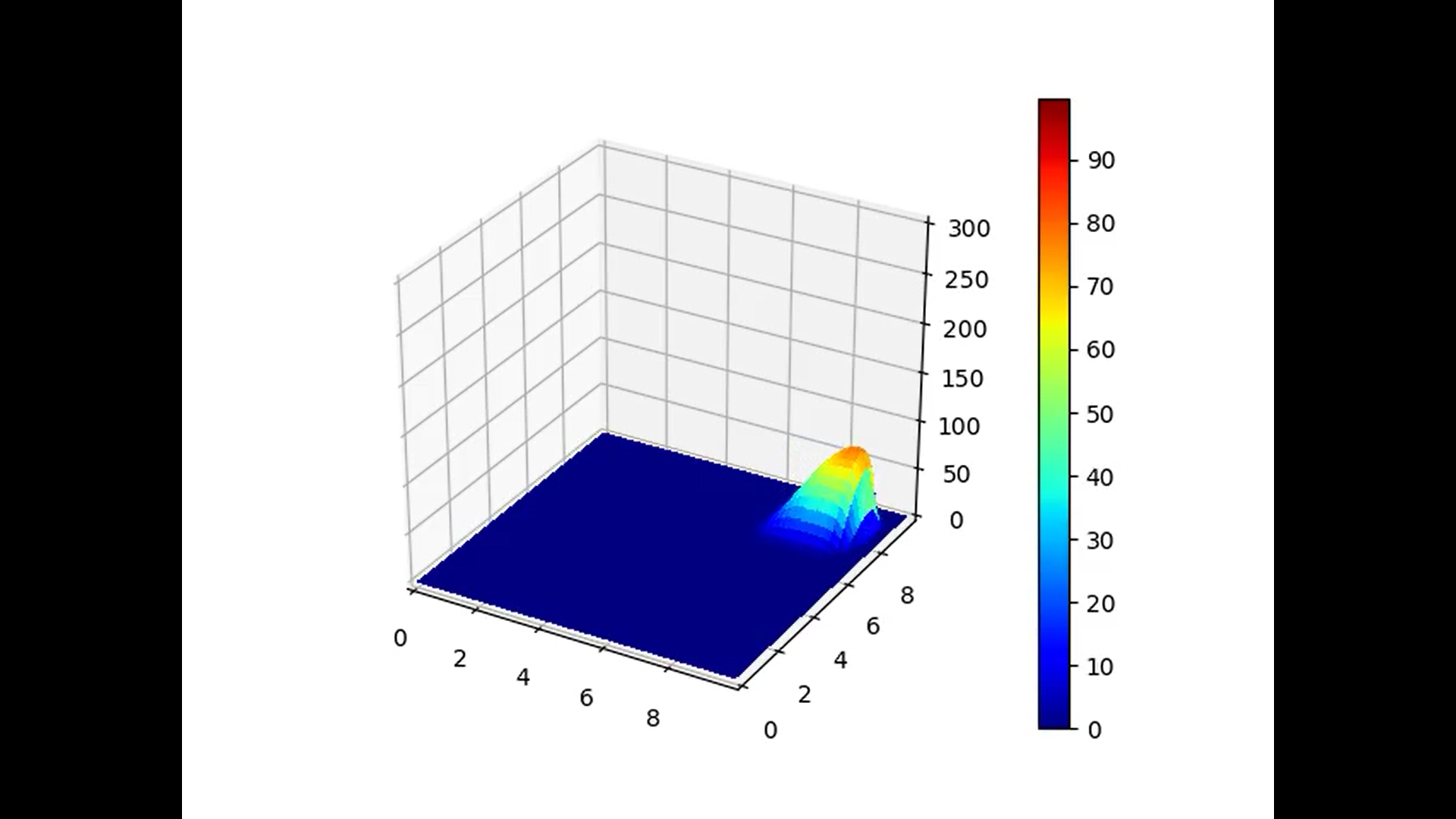
1. a = 2.



No heat leaks around the boundaries and the heat gradually lost from the system even faster that just diffusion equation. (the heat throw itself into the “black hole” wall)

### Neumann (Non-linear, G=5)

1. **def** neu(n, g=0, plot\_type=1):
2. **global** T,d1,d2
3. Tn = T.copy()
4. left = np.roll(Tn,1,axis=0)
5. right = np.roll(Tn,-1,axis=0)
6. up = np.roll(Tn,-1,axis=1)
7. down = np.roll(Tn,1,axis=1)
8. ghost1 = -2 \* dx \* g + Tn[1,1:grid\_y-1]
9. ghost2 = -2 \* dx \* g + Tn[grid\_x-2,1:grid\_y-1]
10. ghost3 = -2 \* dx \* g + Tn[grid\_x-2,1:grid\_y-1]
11. ghost4 = -2 \* dx \* g + Tn[1:grid\_x-1,grid\_y-2]
12. ghost1 = ghost1 > 0 \* ghost1  #prevent negative value
13. ghost2 = ghost2 > 0 \* ghost2
14. ghost3 = ghost3 > 0 \* ghost3
15. ghost4 = ghost4 > 0 \* ghost4
16. left[0,:],right[grid\_x-1,:],up[:,grid\_y-1],down[:,0] = 0,0,0,0
17. T = Tn-dt\* Tn/dx\*(2 \* Tn - left - down) + d2 \* (left+right+up+down-4\*Tn)
18. T[0,1:grid\_y-1] += d2 \* ghost1 + dt \* T[0,1:grid\_y-1]  / dx\*ghost1
19. T[grid\_x-1,1:grid\_y-1] += d2 \* ghost2
20. T[1:grid\_x-1,0] += d2 \* ghost3 + dt \* T[1:grid\_x-1,0]  / dx \* ghost3
21. T[1:grid\_x-1,grid\_y-1] += d2 \* ghost4
22. **if** plot\_type == 1:
23. plot\_activate(X,Y,n)
24. **if** plot\_type == 2:
25. plot3d\_activate(X,Y,n)



Extend from the Dirichlet condition, the wall’s heat was affected by the blob of heat that was approaching the it. In this case G=5 so we can see some gradient or some slopes near the wall.

## Application of Burger’s Equation [Q.4]

1. Used in Fluid Mechanic

We describe the motion of fluid with viscosity by Navier–Stokes equations which is the equation cannot be solved for analytical solution right now. The general formula is too long so we will refer only the case of incompressible fluid Navier-Stokes in convection



If we consider in the simpler case such as homogenous differential equation (no internal and external source), it will become non-linear burger equation.



1. Toy model

Toy model is given a mean as a tool used to understand some behavior of the system but in simpler way rather than a generally complex model. One interesting model is traffic flow.

Given that  is the density of the cars (has meaning as bumper hit the front car!)



1. Nuclear fusion reactor

In the nuclear fusion reactor, there is a part called “Lithium blanket” where playing role as a cooler for the reactor. Lithium blanket is contained of liquid lithium and controlled by magnetic force. Consequently, there is need to study how to control the flow of this blanket.

The model is as similar to Navier-Stokes Burger’s equation with external source.



Where  is applied magnetic field. For example, .

Ref:

*Application of Generalize Burger's Equation.* (n.d.). Retrieved from Shodhganga: http://shodhganga.inflibnet.ac.in/bitstream/10603/37622/11/11\_chapter%204.pdf

Landajuela, M. (2011). Burgers equation. *bcam*, 2-3.

*Viscous Burgers equation physical meaning*. (2014, 7 23). Retrieved from Physics Stackexchange: https://physics.stackexchange.com/questions/127771/viscous-burgers-equation-physical-meaning

## Python Code

1. **import** numpy as np
2. **import** matplotlib.pyplot as plt
3. **from** mpl\_toolkits.mplot3d **import** Axes3D
4. **import** os
5. **from** matplotlib **import** animation
6. dir = os.path.dirname(os.path.realpath(\_\_file\_\_))
7. os.chdir(dir)
9. dx = 0.1
10. dy = dx
11. a = 2.
12. v = 2.
13. grid\_x = 100
14. grid\_y = 100
15. nt = 100
16. d1 = 0.1
17. d2 = 0.05
18. dt = d2 \* (dx\*\*2)/v
20. **def** init():
21. T = np.zeros((grid\_x,grid\_y))
22. T[70:90, 70:90] = 80.
23. **return** T
25. x = np.linspace(0,dx \* (grid\_x - 1), grid\_x)
26. y = np.linspace(0,dy \* (grid\_y - 1), grid\_y)
27. X,Y = np.meshgrid(x,y)
28. cmap = plt.cm.get\_cmap("jet")
29. cmap.set\_over('grey')
30. g = 5
31. levels = np.arange(0.,100.,0.2)
32. count = 1
33. icount = 0
34. T = init()
35. fig = plt.figure()
36. ax = fig.gca(projection='3d')
37. cl = plt.contourf(X,Y,T,levels,cmap=cmap)
38. plt.colorbar(cl)
40. **def** plot\_activate(X,Y,n):
41. **global** T
42. plt.cla()
43. plt.clf()
44. plt.xlim(0.,np.max(x))
45. plt.ylim(0.,np.max(x))
46. plt.contourf(X,Y,T,levels,cmap=cmap)
47. plt.text(np.max(x)\*0.8,np.max(y)+dy,"t=%01.5f"%(dt\*n))
49. **def** plot3d\_activate(X,Y,n):
50. ax.cla()
51. ax.set\_xlim(0.,np.max(x))
52. ax.set\_ylim(0.,np.max(y))
53. ax.set\_zlim(0.,300)
54. cl = ax.plot\_surface(X,Y,T,linewidth=0,vmin=np.min(levels),vmax=np.max(levels), cmap=cmap,antialiased=False)
56. **def** cyc\_var(n, plot\_type=1):
57. **global** T,d1,d2
58. Tn = T.copy()
59. left = np.roll(Tn,1,axis=0)
60. right = np.roll(Tn,-1,axis=0)
61. up = np.roll(Tn,-1,axis=1)
62. down = np.roll(Tn,1,axis=1)
63. T = Tn-dt\*(np.sqrt(80\*\*2-Tn \*\* 2))/dx\*(2 \* Tn - left - down) + d2 \* (left+right+up+down-4\*Tn)
64. **if** plot\_type == 1:
65. plot\_activate(X,Y,n)
66. **if** plot\_type == 2:
67. plot3d\_activate(X,Y,n)
69. **def** cyc\_con(n, plot\_type=1):
70. **global** T,d1,d2
71. Tn = T.copy()
72. left = np.roll(Tn,1,axis=0)
73. right = np.roll(Tn,-1,axis=0)
74. up = np.roll(Tn,-1,axis=1)
75. down = np.roll(Tn,1,axis=1)
76. T = Tn-d1\*(2 \* Tn - 1.5\*left - 0.5\*down) + d2 \* (left+right+up+down-4\*Tn)
77. **if** plot\_type == 1:
78. plot\_activate(X,Y,n)
79. **if** plot\_type == 2:
80. plot3d\_activate(X,Y,n)
82. **def** der\_var(n, plot\_type=1):
83. **global** T,d1,d2
84. T[0,:],T[:,0], T[grid\_x-1,:], T[:, grid\_y-1] = 0,0,0,0
85. Tn = T.copy()
86. left = np.roll(Tn,1,axis=0)
87. right = np.roll(Tn,-1,axis=0)
88. up = np.roll(Tn,-1,axis=1)
89. down = np.roll(Tn,1,axis=1)
90. T = Tn-dt\*(np.sqrt(Tn \*\* 2))/dx\*(2 \* Tn - left - down) + d2 \* (left+right+up+down-4\*Tn)
91. **if** plot\_type == 1:
92. plot\_activate(X,Y,n)
93. **if** plot\_type == 2:
94. plot3d\_activate(X,Y,n)
96. **def** neu(n, g=0, plot\_type=1):
97. **global** T,d1,d2
98. Tn = T.copy()
99. left = np.roll(Tn,1,axis=0)
100. right = np.roll(Tn,-1,axis=0)
101. up = np.roll(Tn,-1,axis=1)
102. down = np.roll(Tn,1,axis=1)
103. ghost1 = -2 \* dx \* g + Tn[1,1:grid\_y-1]
104. ghost2 = -2 \* dx \* g+Tn[grid\_x-2,1:grid\_y-1]
105. ghost3 = -2 \* dx \* g+Tn[grid\_x-2,1:grid\_y-1]
106. ghost4 = -2 \* dx \* g+Tn[1:grid\_x-1,grid\_y-2]
107. ghost1 = ghost1 > 0 \* ghost1
108. ghost2 = ghost2 > 0 \* ghost2
109. ghost3 = ghost3 > 0 \* ghost3
110. ghost4 = ghost4 > 0 \* ghost4
111. left[0,:],right[grid\_x-1,:],up[:,grid\_y-1],down[:,0] = 0,0,0,0
112. T = Tn-dt\*(np.sqrt(Tn \*\* 2))/dx\*(2 \* Tn - left - down) + d2 \* (left+right+up+down-4\*Tn)
113. T[0,1:grid\_y-1] += d2 \* ghost1 + dt \* T[0,1:grid\_y-1]/dx\*ghost1
114. T[grid\_x-1,1:grid\_y-1] += d2 \* ghost2
115. T[1:grid\_x-1,0] += d2 \* ghost3 + dt \* T[1:grid\_x-1,0]/dx\*ghost3
116. T[1:grid\_x-1,grid\_y-1] += d2 \* ghost4
117. **if** plot\_type == 1:
118. plot\_activate(X,Y,n)
119. **if** plot\_type == 2:
120. plot3d\_activate(X,Y,n)
121. a = animation.FuncAnimation(fig, der\_var,fargs=(2), frames=200,interval=10)
122. a.save('3d-der-var.mp4',fps=30,extra\_args=['-vcodec','libx264'])