TPG4190 Seismic data acquisition and processing
Lecture 1: Sources

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Overview

1. Marine seismic data acquisition (sec. 2.2)
2. Bubble time period and primary to bubble ratio (sec. 2.3)
3. Source signature estimation (sec. 2.5)
4. The source ghost spectrum (sec. 2.6)
5. Fighting the ghost (sec. 2.6)

Marine seismic data acquisition

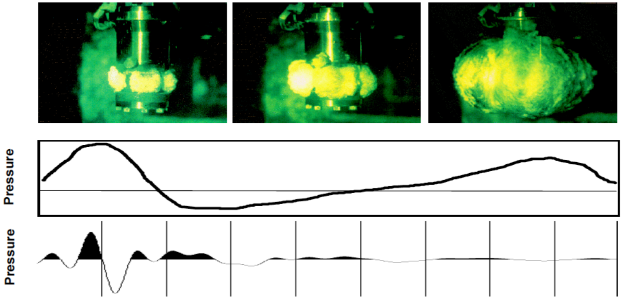


Figure: Firing of airgun

Marine seismic data acquisition

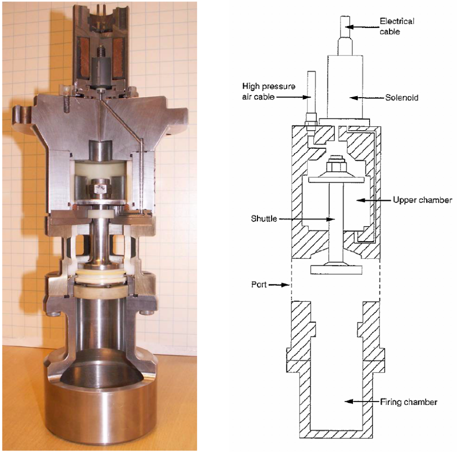
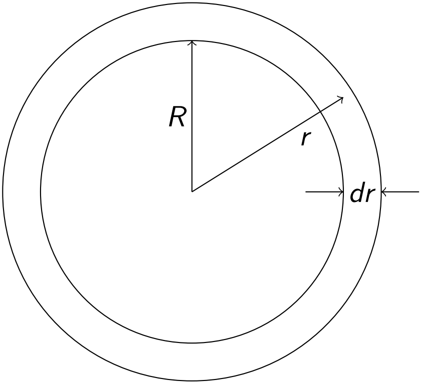


Figure: Airgun design

Motion of expanding water bubble



- R: Bubble wall radius
- $U = \frac{dR}{dt}$: Bubble wall velocity
- r: Water particle radius
- $u = \frac{dr}{dt}$: Water particle velocity

Motion of expanding water bubble

The kinetic energy of a spherical shell with water with thickness dr and mass m is

$$\frac{1}{2}mu^2 = \frac{1}{2}\rho 4\pi r^2 dr u^2 \tag{1}$$

where u is the velocity of a water particel at distance r. The velocity of the surface of the shell (bubble wall) moves at speed U and has distance R.

The amount of mass per time unit moving accross the surface of the shell must be constant (no mass is lost) so we must have

$$\rho 4\pi R^2 \frac{dR}{dt} = \rho 4\pi r^2 \frac{dr}{dt}, \tag{2}$$

or

$$R^2 U = r^2 u, \tag{3}$$

and

$$u = \frac{UR^2}{r^2}. \tag{4}$$

Motion of expanding water bubble

The total kinetic energi of the water motion is (assuming the buble expands freely to infinity)

$$E_k = \int_R^\infty \frac{1}{2} m u^2. \tag{5}$$

using $m = \rho dV = \rho 4\pi r^2 dr$ and equation (4), I get

$$E_k = \int_R^\infty 2\pi\rho U^2 R^4 \int_R^\infty \frac{dr}{r^2} = 2\pi\rho U^2 R^3. \tag{6}$$

Motion of expanding water bubble

The potential energy of the bubble with radius R is

$$E_p = \frac{4}{3}\pi(p - p_\infty)R^3 \tag{7}$$

Here the pressure is p and p_∞ is the hydrostatic pressure. R changes with time, so we are interested in the change in potential energy from an initial position R_0 to a later position R

$$W = \frac{4}{3}\pi(p_0 - p_\infty)R_0^3 - \frac{4}{3}\pi(p - p_\infty)R^3 \tag{8}$$

p_0 is the initial pressure (assumed constant in space, but not time). The total energy is now

$$E = E_k + W = 2\pi\rho\dot{R}^2R^3 + \frac{4}{3}\pi(p_0 - p_\infty)R_0^3 - \frac{4}{3}\pi(p - p_\infty)R^3. \tag{9}$$

where we use $U = \dot{R}$.

Motion of expanding water bubble

Since the energy is conserved, differentiating this equation with respect to time gives

$$0 = 2\pi\rho\left(2\dot{R}\ddot{R}R^3 + \dot{R}^2 3R^2\dot{R}\right) - \frac{4}{3}\pi(p - p_\infty)3R^2\dot{R} \tag{10}$$

which gives Rayleigh's equation for the bubble motion

$$\ddot{R}R + \frac{3\dot{R}^2}{2} = \frac{(p - p_\infty)}{\rho} \tag{11}$$

Motion of expanding water bubble

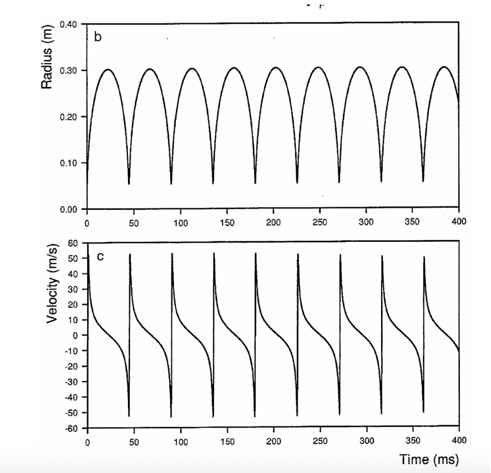


Figure: Solution of Rayleigh's water bubble equation for an initial volume of 40 inch^3, firing pressure 140 bar and water depth 7.5 m

Airgun signature

The Rayleigh equation must be modified to model a realistic airgun. A better equation was develope by Bethe and Kirkwood.

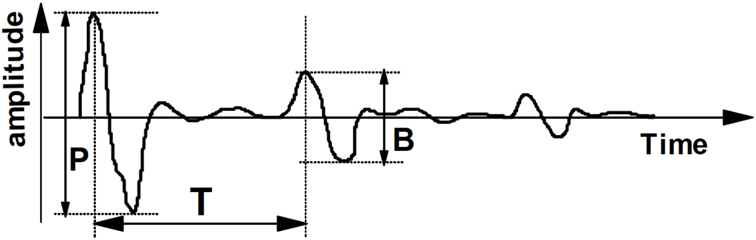


Figure: Measured airgun pressure, with definition of bubble period (T), primary peak amplitude (P) and bubble amplitude.

Airgun signature

- Primar to Bubble ratio = P/B
- P/B should be as large as possible
- Most important method fo achieve high P/B is clustering of airguns close together.
- Another approach is combining airguns with different volumes

Source signature estimation

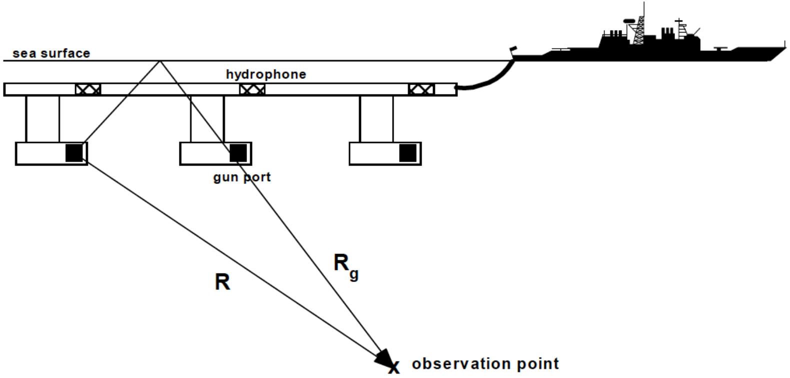


Figure: Notional source measurement

Notional source method

$$p(R,t) = \sum_j \left[\frac{S_j(t - R_j/c)}{R_j} - \frac{S_j(t - R_j^g/c)}{R_j^g} \right]. \quad (12)$$

- R_j: Distance from source to observation point for source no j.
- R_j^g: Distance via sea surface to observation point for source no j.
- S_j: Notional source measurement for source no j.

The source ghost spectrum

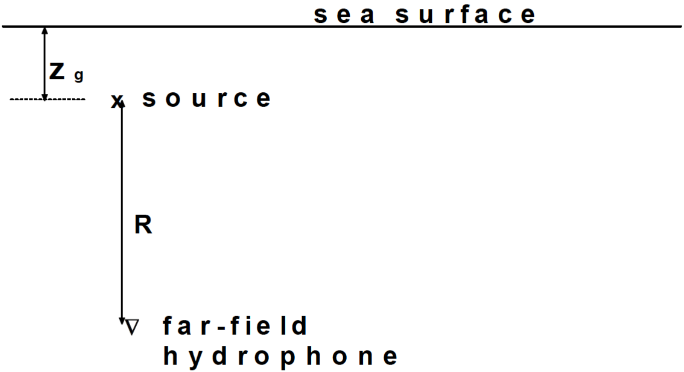


Figure: Ghost measurement setup

The source ghost spectrum

The signal measured at the far-field hydrophone is

$$s(t) = \frac{1}{R} p(t - R/c) - \frac{1}{R_g} p(t - R_g/c) \tag{13}$$

here $R_g = R + 2z_g$. The Fourier transform is defined by

$$S(\omega) = \int_{-\infty}^{+\infty} dt\, s(t) \exp(-i\omega t), \tag{14}$$

and $\omega = 2\pi f$. Fourier transforming equation (13) one gets

$$S(\omega) = \frac{1}{R} P(\omega) \exp(-i\omega R/c) - \frac{1}{R_g} P(\omega) \exp(-i\omega R_g/c). \tag{15}$$

This is approximately ($R \approx R_g$ in the denominator)

$$S(\omega) = \frac{1}{R} P(\omega) \left[1 - exp(-i2\omega z_g/c)\right] \exp(-i\omega R/c) \tag{16}$$

The source ghost spectrum

$$S(\omega) = \frac{1}{R} P(\omega) \exp(-i\omega R/c) \left[1 - exp(-i2\omega z_g/c)\right] = \frac{1}{R} P(\omega) H(\omega). (17)$$

$P(\omega)$ is the Fourier transform of the airgun source signal and the exponential factor is a time delay, but the extra factor

$$H(\omega) = \left[1 - exp(-i2\omega z_g/c)\right], \quad (18)$$

is only due to the influence of the sea surface and describes the so-called ghost-effect.
H can be written in a slightly different way

$$S(\omega) = H(\omega) = \exp(-i\omega z_g/c) \left[\exp(i\omega z_g/c) - exp(-i\omega z_g/c)\right]. (19)$$

The source ghost spectrum

Using the fact that $2i\sin(x) = \exp(ix) - \exp(-ix)$ we can write H in the form

$$S(\omega) = H(\omega) = 2i\exp(-i\omega z_g/c)\sin(\omega z_g/c) \tag{20}$$

The amplitude spectrum of the ghost filter H can be computed by

$$|H(\omega)|^2 = H^*(\omega)H(\omega), \tag{21}$$

where the * denotes complex conjugation ($i \to -i$). We get

$$|H(\omega)|^2 = (-2i)\exp(i\omega z_g/c)\sin(\omega z_g/c)2i\exp(-i\omega z_g/c)\sin(\omega z_g/c) \tag{22}$$

$$|H(\omega)|^2 = 4\sin^2(\omega z_g/c) \tag{23}$$

$$H(\omega) = 2|\sin(2\pi f z_g/c)| \tag{24}$$

The source ghost spectrum

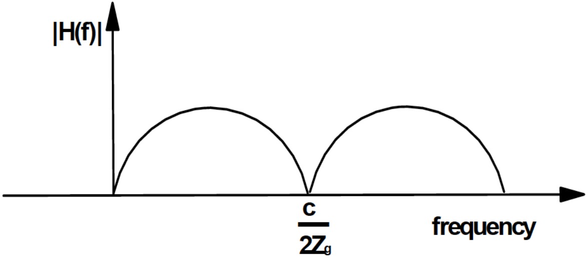


Figure: Ghost filter

The source ghost spectrum

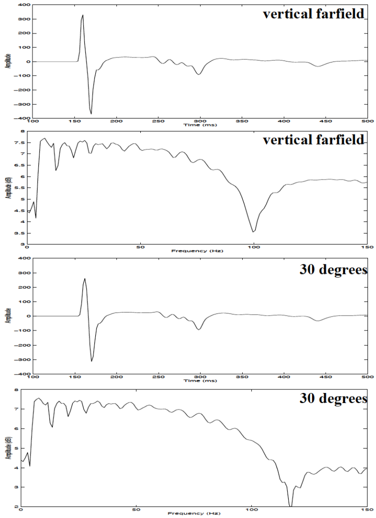


Figure: Ghost filter

The source ghost spectrum

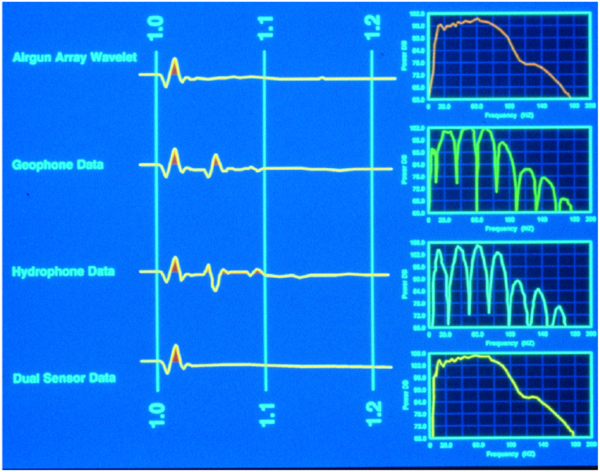


Figure: Ghost filter