TPG4190 Seismic data acquisition and processing
Lecture 17: Multiples - Radon demultiple

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Trondheim fall 2020

Overview

- The effect of the free surface
- The Radon transform
- Radon demultiple

The Radon transform

- 2D Radon transform of a function

$$\hat{f}(p,\tau) = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dt \delta(t - px - \tau) f(x,t), \qquad (1)$$

- p: slope
- τ : intercept
- integration over t:

$$\hat{f}(p,\tau) = \int_{-\infty}^{+\infty} dx f(x, \tau + px). \qquad (2)$$

The Radon transform

- Linear Radon transform or a slant stack
- The line described by

$$t = px + \tau, \tag{3}$$

- is mapped to p, τ in the Radon transformed domain.

The Radon transform

- Fourier transform of equation (2) over τ

$$\hat{f}(p,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} dx f(x, \tau + px) \exp(-i\omega\tau) \qquad (4)$$

- Integration variable τ changed to $u = \tau + px$ equation

$$\hat{f}(p,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dx f(x, u) \exp(-i\omega u + \omega px) \qquad (5)$$

- $k_x = \omega p$, one gets

$$\hat{f}(k_x/\omega,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} du \int dx f(x, u) \exp(-i\omega u + k_x x). \qquad (6)$$

The Radon transform

- Inverse transform

$$f(x,t) = -\frac{1}{2\pi^2}\int dp \int d\tau \frac{\partial_\tau \hat{f}(p,\tau - px)}{\tau - t}. \tag{7}$$

The Radon transform of travel-time hyperbolas

The traveltime of a single primary reflection in a CMP-gather is

$$t^2 = t_0^2 + x^2/c^2. \tag{8}$$

c is the traveltime, and t_0 is the zero-offset traveltime. The slownes p is defined as

$$p = \frac{dt}{dx}, \tag{9}$$

The Radon transform of travel-time hyperbolas

which gives by using equation (8)

$$p = \frac{x}{tc^2}. \tag{10}$$

or

$$t = \frac{x}{pc^2}. \tag{11}$$

Inserting equation (11) into equation (8) gives

$$x = \frac{pt_0c^2}{\sqrt{1 - p^2c^2}}. \tag{12}$$

Also inserting equation (12) into (11) gives

$$t = \frac{t_0}{\sqrt{1 - p^2c^2}}. \tag{13}$$

The Radon transform of travel-time hyperbolas

Using equations (13) and (12) we get

$$\tau = t - px = t_0\sqrt{(1 - p^2c^2)}. \tag{14}$$

The last equation automatically gives

$$\left(\frac{\tau}{ct_0}\right)^2 + p^2 = \frac{1}{c^2}, \tag{15}$$

which shows that the two parameters τ and p lies on an ellipse. An hyperbolic traveltime curve in the x, t space is the transformed to an ellipse in the $\tau - p$ space.

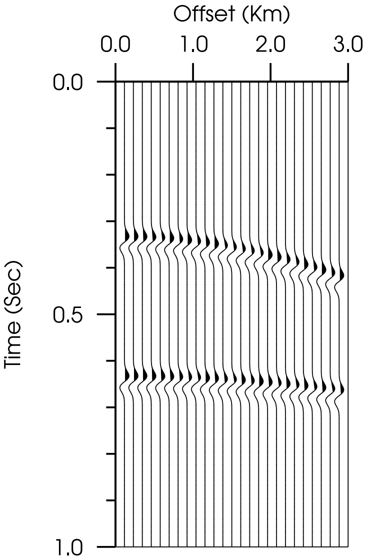

Figure: CMP with two events with velocity equal to 2000 m/s and 2500 m/s.

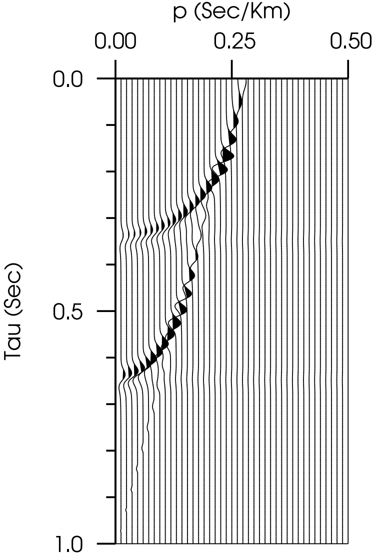

Figure: $\tau - p$ transform of the CMP-gather shown in figure 1

Radon Multiple removal

- Raypaths with multiple reflections
- Multiples are unwanted
- Multiples are usually removed

Principle of Radon demultiple

Traveltime-distance curve for a primary reflection from the bottom of a layer with constant velocity

$$t_p = \sqrt{\tau_0^2 + \frac{4h^2}{c^2}}, \tag{16}$$

and the first multiple reflection from the same reflector

$$t_m = \sqrt{(2\tau_0)^2 + \frac{4(2h)^2}{c^2}}. \tag{17}$$

Principle of Radon demultiple

Consider also a primary reflection arriving from some deeper reflector at the same time

$$t_p = \sqrt{(2\tau_0)^2 + \frac{4(2h)^2}{c_{rms}^2}}. \tag{18}$$

Although arriving at the same time as the multiple reflection, the curvature of this event is different, because the velocity is $c_{rms} > c$.

Principle of Radon demultiple

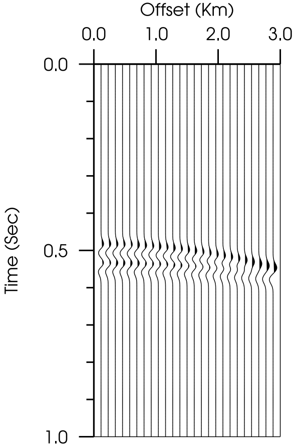

Figure: Cmp with primary reflection and multiple reflection interfering.

Principle of Radon demultiple

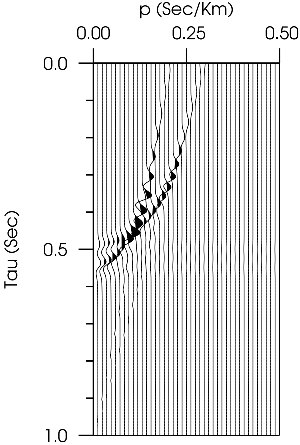

Figure: Cmp with primary reflection and multiple reflection from 3 in the tau-p domain. The apparent velocity for the multiple reflection is lower than the primary reflection, making separation of the two events possible.

Principle of Radon demultiple

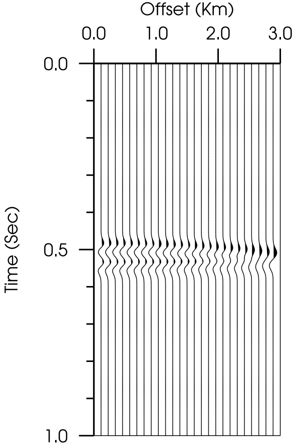

Figure: Cmp with primary reflection and multiple reflection after nmo-correction with a velocity higher than the multiple velocity but lower than the primary velocity.

Principle of Radon demultiple

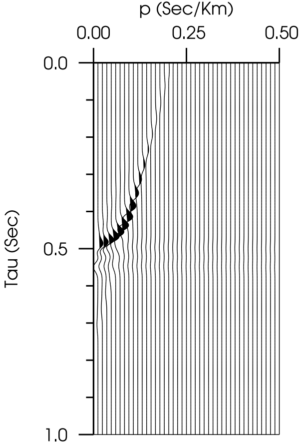

Figure: Cmp with primary reflection and multiple reflection from figure 5 after a *tau* − *p*-p transform. Only the multiple reflection is now visible, the primary appears for small negative p-values (not plotted)

Principle of Radon demultiple

- Straightforward mute and inverse transform is not possible
- Inverse radon gives too many artefacts

Parabolic Radon

$$\hat{f}(p,\tau) = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dt \delta(t - px^2 - \tau) f(x,t). \tag{19}$$

- Transform maps an event in the x-t domain described by a parabola into a point
- Usefull for nmo-corrected data

Parabolic Radon

In general the inverse transform can be considered to be of the general form

$$f(x,t) = \int_{-\infty}^{+\infty} dp \hat{f}(\tau = t - px^2, p), \tag{20}$$

or in discrete form

$$f(x_k,t) = \sum_{l=0}^{N} \hat{f}(\tau = t - p_l x_k^2, p_l), \tag{21}$$

where $p_l = l\Delta p$ and $x_k = k\Delta x$.

Parabolic Radon

We now want to perform a Fourier transform over the time variable

$$f(x_k, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt \sum_{l=0}^{N} \hat{f}(\tau = t - p_l x_k^2, p_l) \exp(-i\omega t), \quad (22)$$

which becomes after substitution of variable $u = t - p_l x_l^2$

$$f(x_k, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} du \sum_{l=0}^{N} \hat{f}(\tau = u, p_l) \exp[-i\omega(u + p_l x_k^2)]. \quad (23)$$

The last equation is then

$$f(x_k, \omega) = \sum_{l=0}^{N} \hat{F}(\omega, p_l) \exp[-i\omega p_l x_k^2]. \quad (24)$$

Parabolic radon

To compute the forward transform we consider the $\hat{F}(\omega, p_l)$ as unknowns, and solve the linear system of equations given in (24). This can easily be done by writing equation (24) as a matrix equation and using the least-squares method.

$$\mathbf{f}(\omega) = \mathbf{L}\hat{\mathbf{F}}(\omega) \tag{25}$$

where $\mathbf{f}$ and $hat\mathbf{F}$ are vectors with elements $f_k = f(x_k, \omega)$ and $\hat{F}_k = \hat{F}(x_k)$. The matrix $\mathbf{L}$ have elements $L_{lk} = \exp[-i\omega p_l x_k^2$.

Parabolic radon

After muting of $\hat{\mathbf{F}}$ to remove multiples we solve the equation

$$\hat{\mathbf{F}}(\omega) = \mathbf{L}^{-1}\mathbf{f}(\omega) \tag{26}$$

with respect to $\mathbf{f}$ to compute the inverse transform.

Parabolic Radon

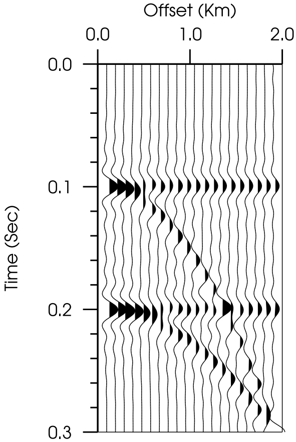

Figure: Input data with primary and multiple reflections. A normal moveout correction has been applied.

Parabolic Radon

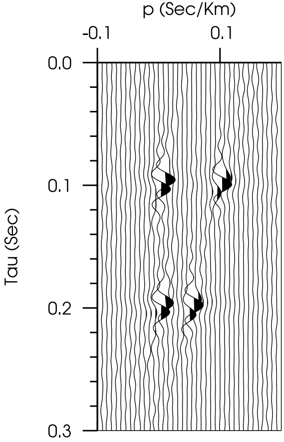

Figure: The parabolic radon transform of the data shown in figure 7.

Parabolic Radon

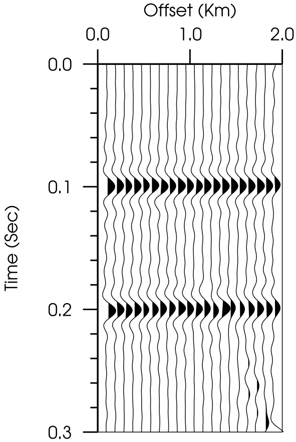

Figure: Primary reflections estimated from the input data in figure 7 using the parabolic radon transform.