TPG4190 Seismic data acquisition and processing
Lecture 5: Deghosting

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Trondheim fall 2021

Overview

- The wave equation
- Up- and downgoing waves
- Removal of the ghost effect

The acoustic wave equation

The acoustic wave equation descries wave motion in a material with no shear forces, i.e. only P-waves can exist.

$$\rho(\boldsymbol{x})\ddot{u}_i(\boldsymbol{x}, t) = \partial_i \sigma(\boldsymbol{x}, t) + f_i(\boldsymbol{x}, t), \tag{1}$$
$$\sigma(\boldsymbol{x}, t) = \kappa(\boldsymbol{x})\partial_i u_i(\boldsymbol{x}, t) + q(\boldsymbol{x}, t). \tag{2}$$

- ρ: Density
- u_i: Component i of particle dispalcement
- σ : $-p$ where p is the pressure.
- κ : Bulk modulus
- f_i: component i of source body force
- q: source of volume injection type

The acoustic wave equation

Divide equation (2) with density and differentiate equation (2) two times w.r.t. time:

$$\ddot{u}_i(\boldsymbol{x},t) = \frac{1}{\rho(\boldsymbol{x})}\partial_i \sigma(\boldsymbol{x},t) + \frac{f_i(\boldsymbol{x},t)}{\rho(\boldsymbol{x})}, \tag{3}$$

$$\ddot{\sigma}(\boldsymbol{x},t) = \kappa(\boldsymbol{x})\partial_i \ddot{u}_i(\boldsymbol{x},t) + \ddot{q}(\boldsymbol{x},t). \tag{4}$$

Insert equation (3) into equation (4) to obtain:

$$\ddot{\sigma}(\boldsymbol{x},t) = \kappa(\boldsymbol{x})\partial_i \left[\frac{1}{\rho(\boldsymbol{x})}\partial_i \sigma(\boldsymbol{x},t)\right] + \partial_i \left[\frac{f_i(\boldsymbol{x},t)}{\rho(\boldsymbol{x})}\right] + \ddot{q}(\boldsymbol{x},t). \tag{5}$$

The acoustic wave equation

Assume $f = 0$ and constant density $\rho = \rho_0$

$$\ddot{\sigma}(\boldsymbol{x}, t) = c^2(\boldsymbol{x})\partial_i\partial_i\sigma(\boldsymbol{x}, t) + \ddot{q}(\boldsymbol{x}, t). \tag{6}$$

where $c(\boldsymbol{x}) = \sqrt{\kappa(\boldsymbol{x})/\rho_0}$ or

$$\nabla^2\sigma(\boldsymbol{x}, t) - \frac{1}{c^2(\boldsymbol{x})}\ddot{\sigma} = s(\boldsymbol{x}, t) \tag{7}$$

where $s(\boldsymbol{x}, t) = -\ddot{q(\boldsymbol{x}, t)}/c^2(\boldsymbol{x})$

The acoustic wave equation

We now assume that the velocity is independent of x and y coordinates and only depends on depth; $c(\boldsymbol{x}) = c(z)$

$$\nabla^2 \sigma(\boldsymbol{x}, t) - \frac{1}{c^2(z)} \ddot{\sigma} = s(\boldsymbol{x}, t) \tag{8}$$

∇^2 operator is

$$\nabla^2 \sigma(\boldsymbol{x}, t) = \frac{\partial^2 \sigma}{\partial_x^2}(\boldsymbol{x}, t) + \frac{\partial^2 \sigma}{\partial_y^2}(\boldsymbol{x}, t) + \frac{\partial^2 \sigma}{\partial_z^2}(\boldsymbol{x}, t) \tag{9}$$

The Fourier transform over x- and y is obtained by replacing $\partial_x^2 \rightarrow (-ik_x)^2 = -k_x^2$ and $\partial_y^2 \rightarrow (-ik_y)^2 = -k_y^2$.

The acoustic wave equation

$$\nabla^2\sigma(\boldsymbol{x},t) \rightarrow (-k_x^2 - k_y^2)\sigma(k_x,k_y,z,t) + \frac{\partial^2\sigma}{\partial_z^2}(k_x,k_y,z,t) \quad (10)$$

We also have for the fourier transform over time

$$\frac{\partial^2\sigma}{\partial_t^2}(\boldsymbol{x},t) \rightarrow -\omega^2\sigma(k_x,k_y,\omega) \quad (11)$$

Putting this together we get for the wave equation

$$-(k_x^2 + k_y^2)\sigma(\boldsymbol{x},t) + \frac{d^2\sigma}{d^2z}(k_x,k_y,z,\omega) + \frac{\omega^2}{c^2(z)}\sigma(k_x,k_y,z,\omega) \quad (12)$$

$$= s(k_x.k_y,z,\omega) \quad (13)$$

The acoustic wave equation

Using

$$k_z^2 = \omega^2/c^2(z) - (k_x^2 + k_y^2) = \omega^2/c^2(z) - k_r^2, \tag{14}$$

where $k_r^2 = k_x^2 + k_y^2$.
we have

$$\frac{d^2\sigma(k_r, z, \omega)}{d^2z} + k_z^2(k_r, z, \omega)\sigma(k_r, z, \omega) = s(k_r, z, \omega) \tag{15}$$

The acoustic wave equation

The solution of the this wave equation is

$$\sigma(k_r, z, \omega) = U(k_r, \omega)\exp(ik_z z) + D(k_r, \omega)\exp(-ik_z z), \qquad (16)$$

Where the constants U and D are interpreted as the amplitude of the upgoing wave and the downgoing wave, respectively.
We also have from the equation of motion

$$\rho_0 \ddot{u}_z(\boldsymbol{x}, t) = \rho_0 \dot{v}_z(\boldsymbol{x}, t) = \frac{\partial \sigma(\boldsymbol{x}, t)}{\partial_z}. \qquad (17)$$

Or in the wave-number frequency domain

$$-i\omega\rho_0 v_z(k_r, \omega, z) = \frac{\partial \sigma(k_r, \omega, z)}{\partial_z}, \qquad (18)$$

$$v_z(k_r, \omega, z) = \left(\frac{-1}{i\omega\rho_o}\right)\frac{\partial \sigma(k_r, \omega, z)}{\partial_z}. \qquad (19)$$

The acoustic wave equation

Using the last equation together with equation (16)

$$v_z(k_r,\omega,z) = \left(\frac{-ik_z}{i\omega\rho_o}\right)\left[U(k_r,\omega)\exp(ik_z z) - D(k_r,\omega)\exp(-ik_z z)\right], \quad (20)$$

For simplicity we set $z = 0$ at the receiver depth. Equation (16) and (20) then gives

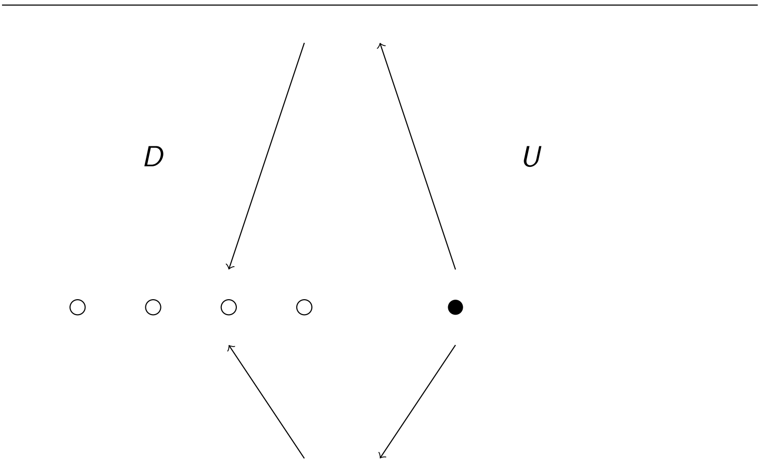
$$\begin{aligned} \left(\frac{i\omega\rho}{-ik_z}\right) v_z(k_r,\omega) &= U(k_r,\omega) - D(k_r,\omega), && (21) \\ \sigma(k_r,\omega) &= U(k_r,\omega) + D(k_r,\omega). && (22) \end{aligned}$$

U-D separation

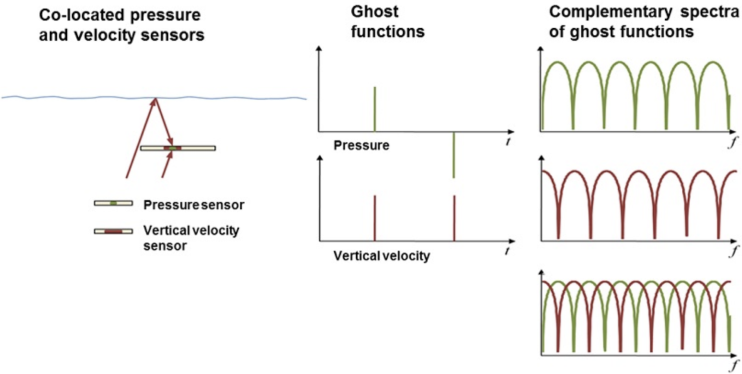
Adding and subtracting equations (22) and (22) gives

$$
\begin{aligned}
U(k_r, \omega) &= \left(\frac{i\omega\rho}{-2ik_z}\right) v_z(k_r, \omega) + \sigma(k_r, \omega), && (23) \\
D(k_r, \omega) &= -\left(\frac{i\omega\rho}{-2ik_z}\right) v_z(k_r, \omega) + \sigma(k_r, \omega). && (24)
\end{aligned}
$$

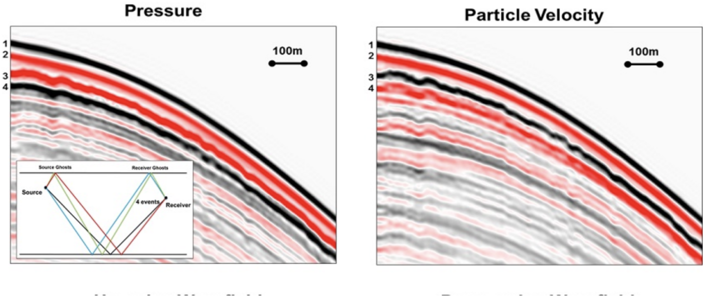
U-D deghosting



U-D deghosting



U-D deghosting



U-D deghosting