TPG4190 Seismic data acquisition and processing Lecture 15: Imaging 4 - Kirchoff and Angle migration

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Overview

- State-of-the-art processing sequence
- The ray approximation
- The image formula usimg rays
- Kirchhoff migration
- Angle migration
- Kirchhoff computational cost

State-of-the-art processing sequence

1. Load data
2. pre-processing
3. Multiple removal
4. Velocity estimation
5. **Kirchoff Migration**
6. Computation of angle gathers
7. Residual corrections
8. Residual multiple removal
9. Final filters
10. Output of stacks, gathers

The ray approximation

An apperoximate solution of the wave equation can be found by assuming that thye solution is of a special form.

$$\nabla^2 p(\boldsymbol{x}, t) - \frac{1}{c^2(\boldsymbol{x})} \partial_t^2 p(\boldsymbol{x}, t) = 0 \tag{1}$$

In the frequency domain we have

$$\nabla^2 P(\boldsymbol{x}, \omega) + \frac{\omega^2}{c^2(\boldsymbol{x})} \partial_t^2 P(\boldsymbol{x}, \omega) = 0 \tag{2}$$

Now assume that

$$P(\boldsymbol{x}, \omega) = A(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})] \tag{3}$$

Here A is the position-dependent Amplitude of P wile τ is the position-dependent travel time.

The ray approximation

We want to insert the ray approximation ((3)) into the wave equation ((2)) to obtain equations for A and τ. First compute ∇P:

$$\nabla P(\boldsymbol{x},\omega) = \nabla A(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})] + A(\boldsymbol{x})(i\omega\nabla\tau(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})]. \quad (4)$$

$$\begin{aligned}\nabla[\nabla P(\boldsymbol{x},\omega)] &= \nabla A^2(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})] + \nabla A(\boldsymbol{x})(i\omega\nabla\tau(\boldsymbol{x})) \exp[i\omega\tau(\boldsymbol{x})] \\ &+\nabla A(\boldsymbol{x})(i\omega\nabla\tau(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x}))] + A(\boldsymbol{x})(i\omega\nabla^2\tau(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})] \\ &+A(\boldsymbol{x})[-\omega^2(\nabla\tau(\boldsymbol{x}))^2 \exp[i\omega\tau(\boldsymbol{x})]. \quad (5)\end{aligned}$$

The ray approximation

$$
\begin{aligned}
\nabla^2 P(\boldsymbol{x},\omega) + \omega^2/c^2(\boldsymbol{x}) P(\boldsymbol{x},\omega) = & & (6)\\
& \nabla A^2(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})] \\
+ & \; 2\nabla A(\boldsymbol{x})(i\omega\nabla\tau(\boldsymbol{x})) \exp[i\omega\tau(\boldsymbol{x})] \\
+ & \; A(\boldsymbol{x})(i\omega\nabla^2\tau(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})] \\
+ & \; A(\boldsymbol{x})[-\omega^2(\nabla\tau(\boldsymbol{x}))^2 \exp[i\omega\tau(\boldsymbol{x})] \\
+ & \; A(\boldsymbol{x}) \exp[i(\omega\tau(\boldsymbol{x})]\omega^2/c^2(\boldsymbol{x}) \\
= & \; 0 & (7)
\end{aligned}
$$

Divide by ω^2 to obtain:

The ray approximation

$$
\begin{aligned}
& \frac{1}{\omega^2} \nabla A^2(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})] \\
+ \quad & \frac{1}{\omega^2} 2\nabla A(\boldsymbol{x})(i\omega\nabla\tau(\boldsymbol{x})) \exp[i\omega\tau(\boldsymbol{x})] \\
+ \quad & \frac{1}{\omega^2} A(\boldsymbol{x})(i\omega\nabla^2\tau(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})] \\
- \quad & A(\boldsymbol{x})(\nabla\tau(\boldsymbol{x}))^2 \exp[i\omega\tau(\boldsymbol{x})] \\
+ \quad & A(\boldsymbol{x}) \exp[i(\omega\tau(\boldsymbol{x})] 1/c^2(\boldsymbol{x}) \\
= \quad & 0 \qquad (8)
\end{aligned}
$$

The ray approximation

$$
\begin{aligned}
& \frac{1}{\omega^2}\nabla A^2(\boldsymbol{x}) \\
+ \quad & \frac{1}{\omega^2} 2\nabla A(\boldsymbol{x})(i\omega\nabla\tau(\boldsymbol{x})) \\
+ \quad & \frac{1}{\omega^2} A(\boldsymbol{x})(i\omega\nabla^2\tau(\boldsymbol{x}) \\
- \quad & A(\boldsymbol{x})(\nabla\tau(\boldsymbol{x}))^2 \\
+ \quad & A(\boldsymbol{x})/c^2(\boldsymbol{x}) \\
= \quad & 0
\end{aligned}
\tag{9}
$$

The ray approximation

$$\begin{aligned}\frac{1}{\omega^2}\nabla A^2(\boldsymbol{x}) + \frac{1}{\omega^2}2\nabla A(\boldsymbol{x})(i\omega\nabla\tau(\boldsymbol{x}))\\ +\frac{1}{\omega^2}A(\boldsymbol{x})(i\omega\nabla^2\tau(\boldsymbol{x})\\ +A(\boldsymbol{x})(\nabla\tau(\boldsymbol{x}))^2 - A(\boldsymbol{x})\frac{1}{c^2(\boldsymbol{x})} = 0\end{aligned} \tag{10}$$

In the limit $\omega \to \infty$ this reduces to

$$(\nabla\tau(\boldsymbol{x}))^2 - \frac{1}{c^2(\boldsymbol{x})} = 0 \tag{11}$$

The Eikonal equation.

The ray approximation

$$\nabla\tau(\boldsymbol{x})\cdot\nabla\tau(\boldsymbol{x}) = \frac{1}{c^2(\boldsymbol{x})} \tag{12}$$

Solution for $c(\boldsymbol{x}) = c_0 = constant$

$$\tau(\boldsymbol{x}) = r/c_0 \tag{13}$$

where r is the distance from the position of the source.

The ray approximation

$$P(\boldsymbol{x}, \omega) = A(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})] \tag{14}$$

In the time-domain this is

$$P(\boldsymbol{x}, t) = A(\boldsymbol{x})\delta[t - \tau(\boldsymbol{x})] \tag{15}$$

Kirchoff migration

We use Claerbouts approximate imaging formula

$$R(\boldsymbol{x}) = \sum_{\boldsymbol{x}_s} \int dt\, p(\boldsymbol{x}, t) p_0(\boldsymbol{x}, t) \tag{16}$$

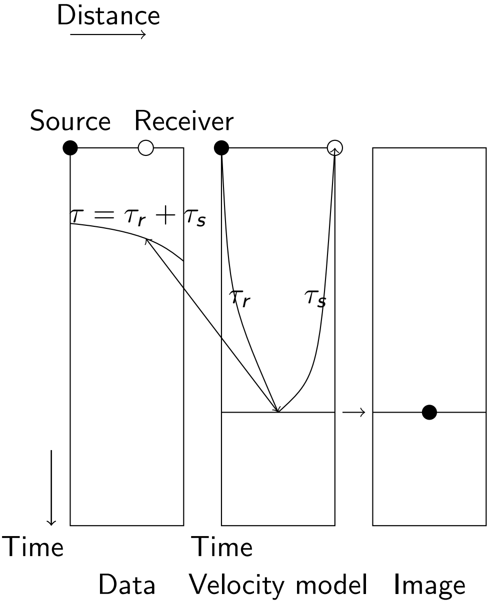
Using the expression for p in the previous slide we get

$$R(\boldsymbol{x}) = \sum_{\boldsymbol{x}_s} \int dt\, A(\boldsymbol{x}) p[\boldsymbol{x}, t + \tau_s(\boldsymbol{x})] A(\boldsymbol{x}) \delta(t - \tau_r(\boldsymbol{x})] \tag{17}$$

$$R(\boldsymbol{x}') = \sum_{\boldsymbol{x}_s} A(\boldsymbol{x}) A(\boldsymbol{x}) p[\boldsymbol{x}, \tau_r(\boldsymbol{x}) + \tau_s(\boldsymbol{x})] \tag{18}$$

$$R(\boldsymbol{x}') \approx \sum_{\boldsymbol{x}_s} p[\boldsymbol{x}, \tau_r(\boldsymbol{x}) + \tau_s(\boldsymbol{x})] \tag{19}$$

Kirchhoff depth migration



Common Image Point (CIP) Gathers

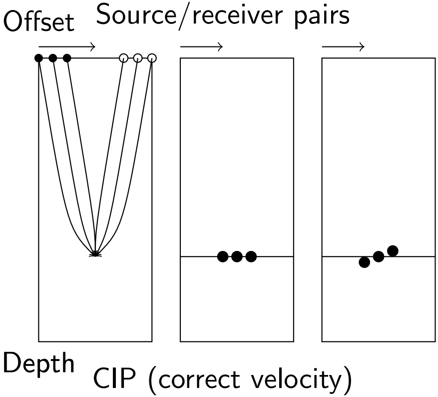
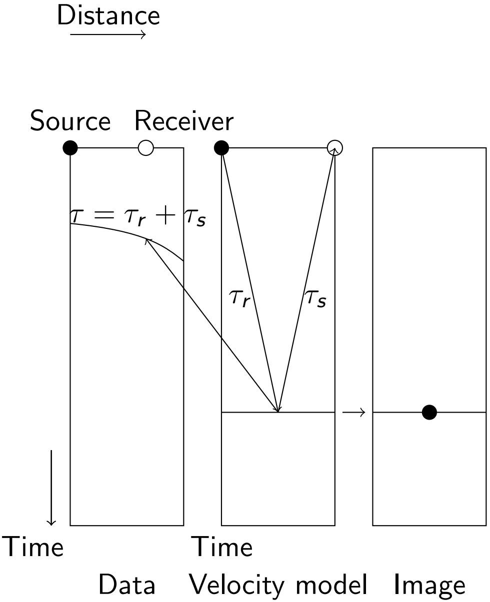


Figure: Common image-point gather

Kirchhoff time migration



Angle migration