THE RADON TRANSFORM

The two-dimensional Radon transform of a function can be defined by

$$\hat{f}(p,\tau) = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dt \delta(t - px - \tau) f(x,t), \tag{1}$$

where p is the slope and τ is the intercept. Performing the integral over t one gets

$$\hat{f}(p,\tau) = \int_{-\infty}^{+\infty} dx f(x, \tau + px). \tag{2}$$

Equation (2) is called a linear Radon transform or a slant stack, and shows together with equation (1) that the effect of the Radon transform is to map the line described by

$$t = px + \tau, \tag{3}$$

to the point p, τ in the Radon transformed domain. We see that the slope p is equal to the inverse of the apparent horizontal velocity, c_x.

The Fourier transform of equation (2) over the τ variable gives

$$\hat{f}(p,\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} dx f(x, \tau + px) \exp(-i\omega\tau) \tag{4}$$

By changing the integration variable from τ to $u = \tau + px$ equation (4) becomes

$$\hat{f}(p,\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dx f(x,u) \exp(-i\omega u + \omega p x) \tag{5}$$

and by identifying $k_x = \omega p$, one gets

$$\hat{f}(k_x/\omega, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} du \int dx f(x,u) \exp(-i\omega u + k_x x). \tag{6}$$

Equation (6) shows that there is a close connection between Fourier transform and the Radon transform.

The inverse of equation (2) is

$$f(x,t) = -\frac{1}{2\pi^2} \int dp \int d\tau \frac{\partial_\tau \hat{f}(p, \tau - px)}{\tau - t}. \tag{7}$$

THE RADON TRANSFORM OF TRAVEL-TIME HYPERBOLAS

The traveltime of a single primary reflection in a CMP-gather is given by

$$t^2 = t_0^2 + x^2/c^2. \tag{8}$$

Here c is the velocity, and t_0 is the zero-offset traveltime. The slowness p is defined as

$$p = \frac{dt}{dx}, \tag{9}$$

which gives by using equation (8)

$$p = \frac{x}{tc^2}. \tag{10}$$

or

$$t = x/pc^2. \tag{11}$$

Inserting equation (11) into equation (8) gives

$$x = \frac{pt_0c^2}{\sqrt{1-p^2c^2}}. \tag{12}$$

Also inserting equation (12) into (9) gives

$$t = \frac{t_0}{\sqrt{1-p^2c^2}}. \tag{13}$$

Using equations (13) and (12) we can then express the intercept parameter τ as

$$\tau = t - px = t_0\sqrt{1-p^2c^2}. \tag{14}$$

The last equation automatically gives

$$\left(\frac{\tau}{ct_0}\right)^2 + p^2 = \frac{1}{c^2}, \tag{15}$$

which shows that the two parameters τ and p lies on an ellipse. An hyperbolic traveltime curve in the x, t space is then transformed to an ellipse in the $\tau - p$ space.

MULTIPLE REMOVAL

Seismic data often contain reflected waves which have raypaths with more than one reflection. These so-called multiples are often considered as noise and are unwanted. A large number of methods have been developed to remove or attenuate multiples. In the following we will describe the use of the Radon transform to remove multiples, and also describe the

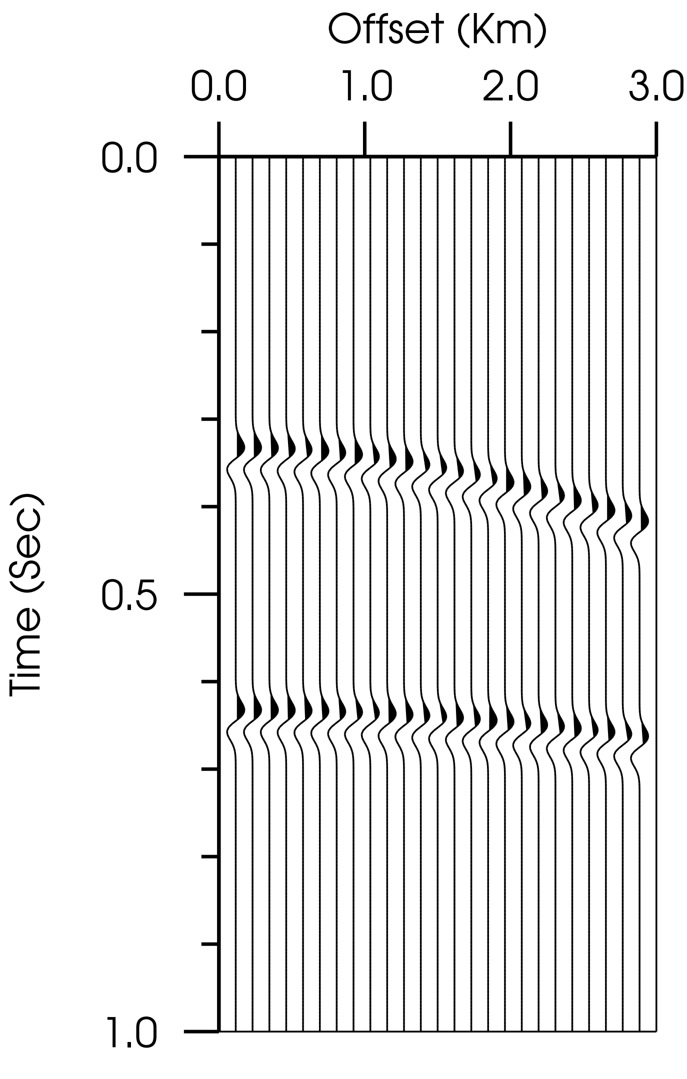

Figure 1: CMP with two events with velocity equal to 2000 m/s and 2500 m/s.

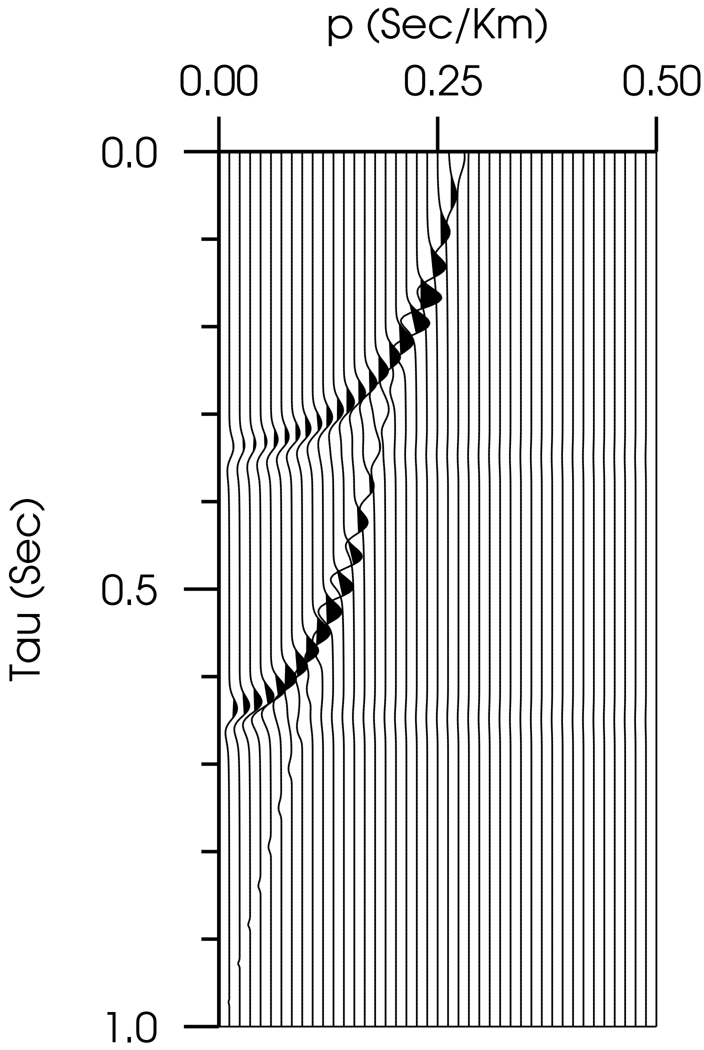

Figure 2: $\tau - p$ transform of the CMP-gather shown in figure 1

so-called 'Surface Related Multiple Elimination' (SRME) method.

PRINCIPLE OF RADON MULTIPLE REMOVAL

Multiple reflections and primary reflections often arrive at the same time and are interfering, but the apparent velocity of the primaries and multiples are often different. Figure 3 shows a cmp-gather with a primary reflection and a multiple reflection with zero-offset traveltime of 0.5 seconds. The two events interfere, and we can not distinguish the multiple reflection from the primary reflection. If we used this cmp to make an image of the subsurface, we would very likely create an incorrect image. Using the linear tau-p transform given by

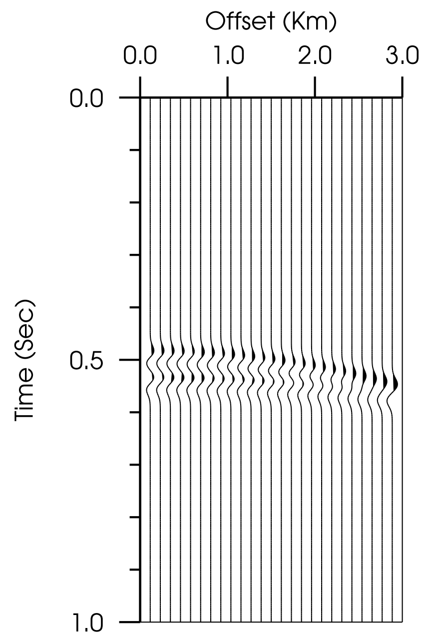

Figure 3: Cmp with primary reflection and multiple reflection interfering.

equation (1), figure 4 shows that the primary and multiple reflection separate in the tau-p domain To increase the separation of the primary and multiple events, a standard trick is to perform a normal moveout correction on the cmp-gather with a velocity which is higher than the multiple velocity and lower than the primary velocity, as shown in figure 5. After a $\tau - p$ transform, we see in figure 6 that only the multiple reflection is visible. The primary now appears for small negative p-values. . In principle the separation between the multiple and primary events can now be utilized by muting in the tau-p domain followed by an inverse tau-p transform to recover only the primary reflection. Unfortunately is this difficult to perform in practice, because the inverse tau-p transform leads to numerical artifacts resulting in a less than desired result.

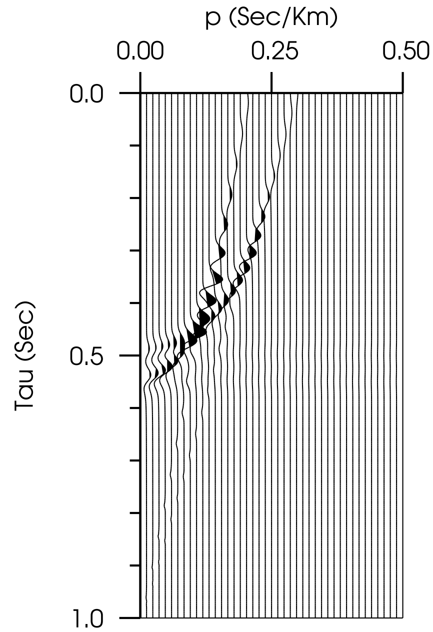

Figure 4: Cmp with primary reflection and multiple reflection from 3 in the tau-p domain. The apparent velocity for the multiple reflection is lower than the primary reflection, making separation of the two events possible.

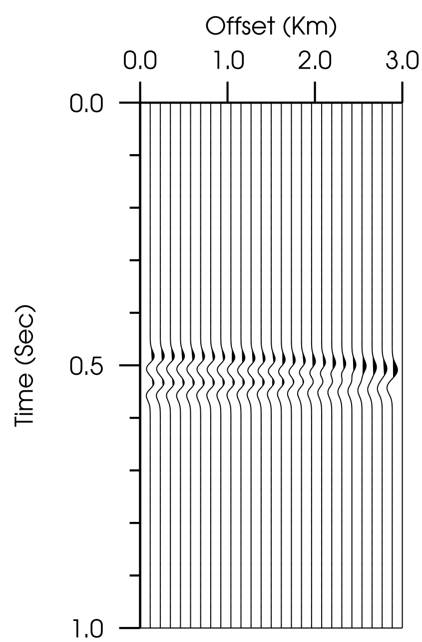

Figure 5: Cmp with primary reflection and multiple reflection after nmo-correction with a velocity higher than the multiple velocity but lower than the primary velocity.

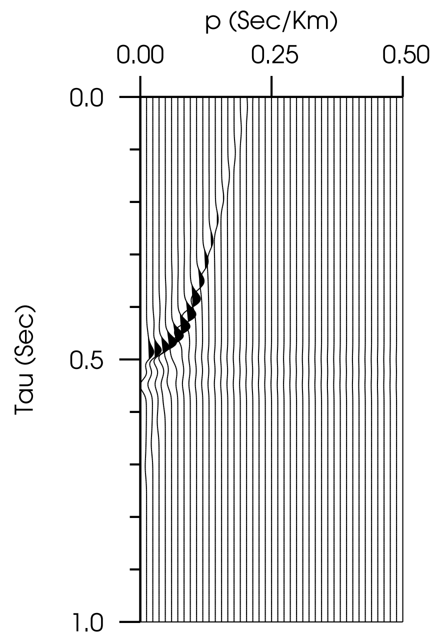

Figure 6: Cmp with primary reflection and multiple reflection from figure 5 after a $tau-p$-p transform. Only the multiple reflection is now visible, the primary appears for small negative p-values (not plotted)

THE PARABOLIC RADON TRANSFORM

Because of the difficulty of computing the inverse Radon transform, the scheme outlined in the section above is usually modified. First, instead of using a linear Radon transform, a common choice is instead to use the so-called parabolic Radon transform, defined by

$$\hat{f}(p,\tau) = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dt\delta(t - px^2 - \tau) f(x,t). \tag{16}$$

This transform maps an event in the x-t domain described by a parabola into a point in the tau-p domain. This has turned out to be a useful approximation to the traveltime curves obtained on cmp-gathers which have been nmo-corrected as described in the previous section.

The problem now is to compute the inverse transform as accurate as possible. In general the inverse transform can be considered to be of the general form

$$f(x,t) = \int_{-\infty}^{+\infty} dp \hat{f}(\tau = t - px^2, p), \tag{17}$$

or in discrete form

$$f(x_k, t) = \sum_{l=0}^{N} \hat{f}(\tau = t - p_l x_k^2, p_l), \tag{18}$$

where $p_l = l\Delta p$ and $x_k = k\Delta x$. We now want to perform a Fourier transform over the time variable

$$f(x_k, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt \sum_{l=0}^{N} \hat{f}(\tau = t - p_l x_k^2, p_l) \exp(-i\omega t), \tag{19}$$

which becomes after substitution of variable $u = t - p_l x_l^2$

$$f(x_k, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} du \sum_{l=0}^{N} \hat{f}(\tau = u, p_l) \exp[-i\omega(u + p_l x_k^2)]. \tag{20}$$

The last equation is then

$$f(x_k, \omega) = \sum_{l=0}^{N} \hat{F}(\omega, p_l) \exp[-i\omega p_l x_k^2]. \tag{21}$$

The above equation can be written as a matrix equation

$$\mathbf{f} = \mathbf{L}\hat{\mathbf{F}}, \tag{22}$$

where the components of the vector vector $\mathbf{f}$ are $f_k = f(x_k, \omega), k = 1, .., N$, the elements of the matrix $\mathbf{L}$ are $L_{lk} = \exp[-i\omega p_l x_k^2]$ and the elements of the vector $\hat{\mathbf{F}}$ are $\hat{F}_l = \hat{F}(\omega, p_l)$. Equation (22) can be solved for $\mathbf{F}$, corresponding to the inverse transform. The forward transform can be found by

$$\hat{\mathbf{F}} = \mathbf{L}^{-1\mathbf{f}}. \tag{23}$$

To remove multiples we can then first solve equation (22) for $\mathbf{F}$ given that the data $\mathbf{f}$ is known. This must be done for all relevant frequencies. Figures 7 and 8 shows the input data and the radon transformed data. Note that after the radon transform according to equation (22), an inverse Fourier-transform over the frequency have been applied and the result shown in figure 8. The multiples can now be identified as the two events to the right in figure 8 and removed. The transform back to the space domain can now be constructed via equation (23) and the results is shown in figure 9 after a Fourier transform from the frequency to the time domain.

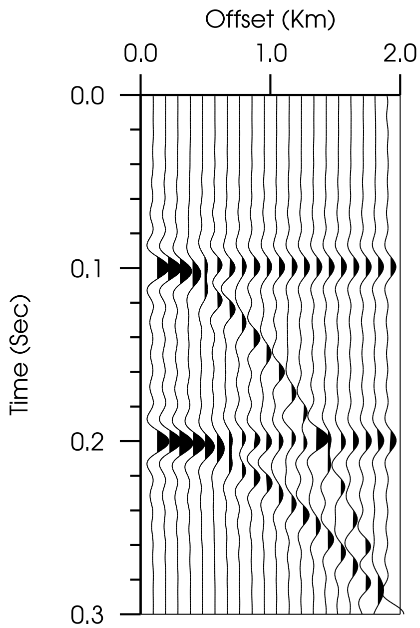

Figure 7: Input data with primary and multiple reflections. A normal moveout correction has been applied.

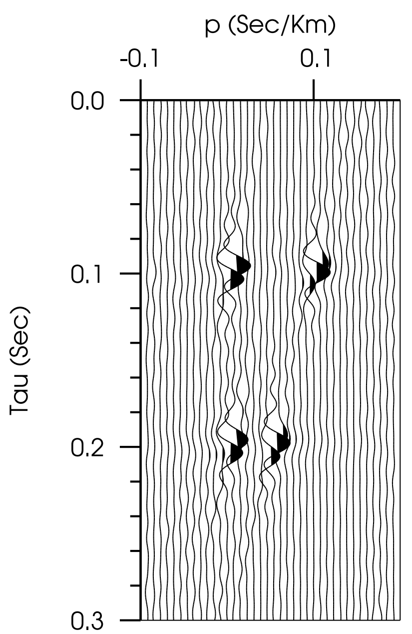

Figure 8: The parabolic radon transform of the data shown in figure 7.

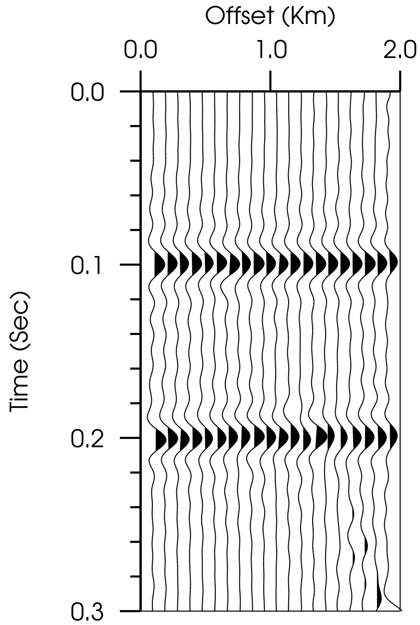

Figure 9: Primary reflections estimated from the input data in 7 using the parabolic radon transform.

SURFACE RELATED MULTIPLE REMOVAL (SRME)

Model for multiple reflections

Multiple reflections (multiples) is usually considered as unwanted signal in seismic processing. Multiples generated by the water layer in marine seismic is particularly troublesome because of the large reflection coefficient of the sea surface.

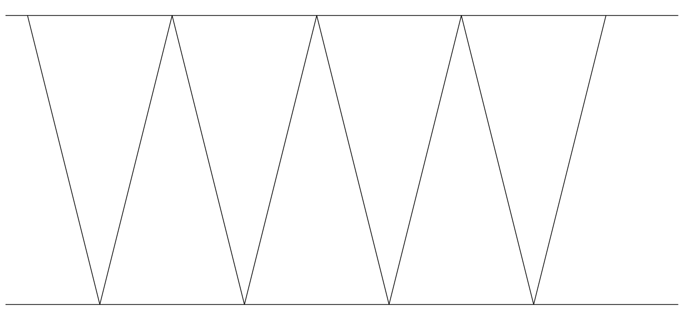

Figure 10: Raypaths for multiple reflections

Figure 10 shows a simplified drawing of ray paths for multiples reflected from the surface and the sea bottom. If we consider only vertically traveling waves, the two-way traveltime in the water layer is τ, the reflection coefficient at the bottom of the layer is r and the reflection coefficient at the top of the layer is -1, we can formulate a simple model for the multiples

$$y(t) = rs(t-\tau) - r^2 s(t-2\tau) + r^3 s(t-3\tau) + \dots \tag{24}$$

Taking the Fourier transform of equation (24), one gets

$$Y(f) = rS(f)\exp(-2\pi f\tau) - r^2 S(f)\exp(-4\pi f\tau) + r^3\exp(-6\pi f\tau) + \dots \tag{25}$$

If we set $R = r\exp(-2\pi f\tau]$, then equation (25) is:

$$Y(f) = S(f)R\left[1 - R + R^2 + \dots\right]. \tag{26}$$

The right hand side of equation (26) is recognized as an infinite geometrical series,

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots \tag{27}$$

implying that equation (26) can be written

$$Y(f) = \frac{S(f)R}{1+R}. \tag{28}$$

Removal of multiples

Equation (28) can now be solved with respect to the primary reflection response R as

$$R = \frac{Y(f)}{S(f)[1 - S^{-1}(f)Y(f)]}, \tag{29}$$

or

$$S(f)R = \frac{Y(f)}{[1 - S^{-1}(f)Y(f)]}. \tag{30}$$

Use of (Rottmann (1960))

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots, \tag{31}$$

gives

$$Y_p(f) = Y(f)\left[1 + S^{-1}Y(f) + S^{-2}Y^2(f) + \dots\right], \tag{32}$$

where $Y_p(f) = S(f)R(f)$, which is the primary reflection response. Equation (32) is truly remarkable, since we can apparently remove all multiple reflections by summing powers of the recorded data itself. Equation (32) does not require any knowledge of the water depth nor the reflection coefficient!

To see how the summation on the right hand side work, truncate the series to

$$Y_p(f) \approx Y(f) + S^{-1}Y^2(f), \tag{33}$$

and insert equation (28) into equation (33)

$$Y_p(f) \approx \frac{S(f)R(f)}{1+R(f)} + S^{-1}(f)\frac{S^2(f)R^2(f)}{[1+R(f)]^2}. \tag{34}$$

Using the following relations

$$\begin{aligned} \frac{1}{1+x} &= 1 - x + \ldots, \\ \frac{1}{(1+x)^2} &= 1 - 2x + \ldots, \end{aligned} \tag{35}$$

we have

$$\frac{S(f)R(f)}{1+R(f)} \approx S(f)R(f)\left[1 - R(f)\right], \tag{36}$$

and

$$S^{-1}(f)\frac{S^2(f)R^2(f)}{[1+R(f)]^2} \approx S^{-1}(f)S^2(f)R^2(f)[1-2R(f)], \tag{37}$$

equation (34) becomes

$$Y_p(f) \approx S(f)R(f)\left[1 - R(f)\right] + S^{-1}(f)S^2(f)R^2(f)(1-2R(f)), \tag{38}$$

which is equal to

$$Y_p(f) \approx S(f)R(f) - S(f)R^2(f) + S(f)R^2(f) - 2S(f)R^3(f), \tag{39}$$

which is equal to

$$Y_p(f) \approx S(f)R(f) - 2S(f)R^3(f). \tag{40}$$

Note that all <u>first order</u> multiples have been removed and only third order and higher multiples remain. If we keep more terms in equation (33) <u>second</u> and higher order multiples will be removed. Although equation (28) was derived assuming vertical only wavepropagation, it can very easily be extended to two-dimensional wave propagation by replacing $Y(f)$ and $R(f)$ with their corresponding 2D equivalents $Y(k_x, \omega)$ and $R(k_x, \omega)$. It also turns out that the limitation to a single water-layer is not essential, and $R(k_x, \omega)$ can be thought of as the reflected data from a stack of layers below the sea-floor.

Equation (40) shows the basic principle for removing multiples according to the so-called 'Surface Related Multiple Removal" (SRME). Figures 11 and 12 shows an example where SRME has been used to remove multiple reflections.

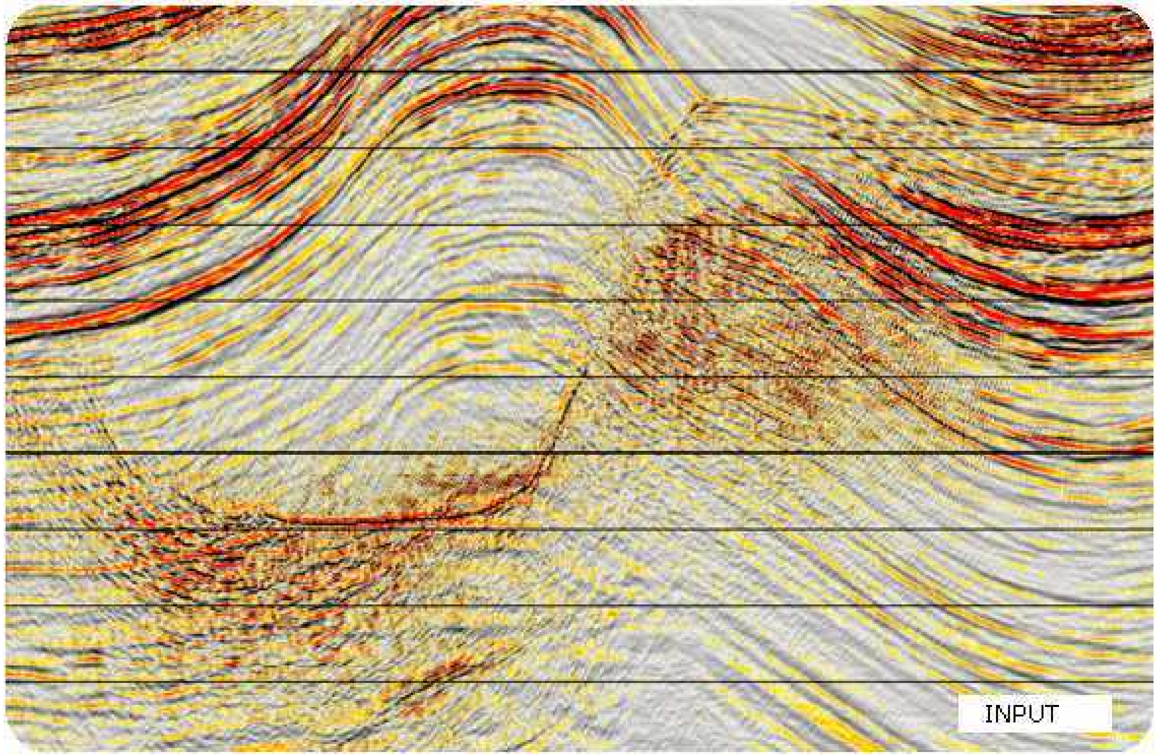

Figure 11: Data with multiples (Courtesy of CGG)

EXERCISE 3

In exercise 1 we set up a simple, but complete, processing sequence for the Snøhvit data set which included velocity analysis, normal-moveout correction and stack. We also clearly saw that the quality of the velocity analysis and the stack were strongly influenced by strong multiple reflections. The objective of this exercise is to study the multiple problem and try to attenuate the multiple reflections as much as possible. We will use the radon transform as a tool to discriminate between multiple reflections and primary reflections based on their different moveout properties.

Removing multiples using the parabolic radon transform

Following the theory presented previously in this chapter, the main procedure for attenuating multiple reflections can be summarized as follows assuming that the multiples have a velocity of approximately 1500 m/s

1. Apply a normal-moveout correction of 1500 m/s to the cmp-gathers.

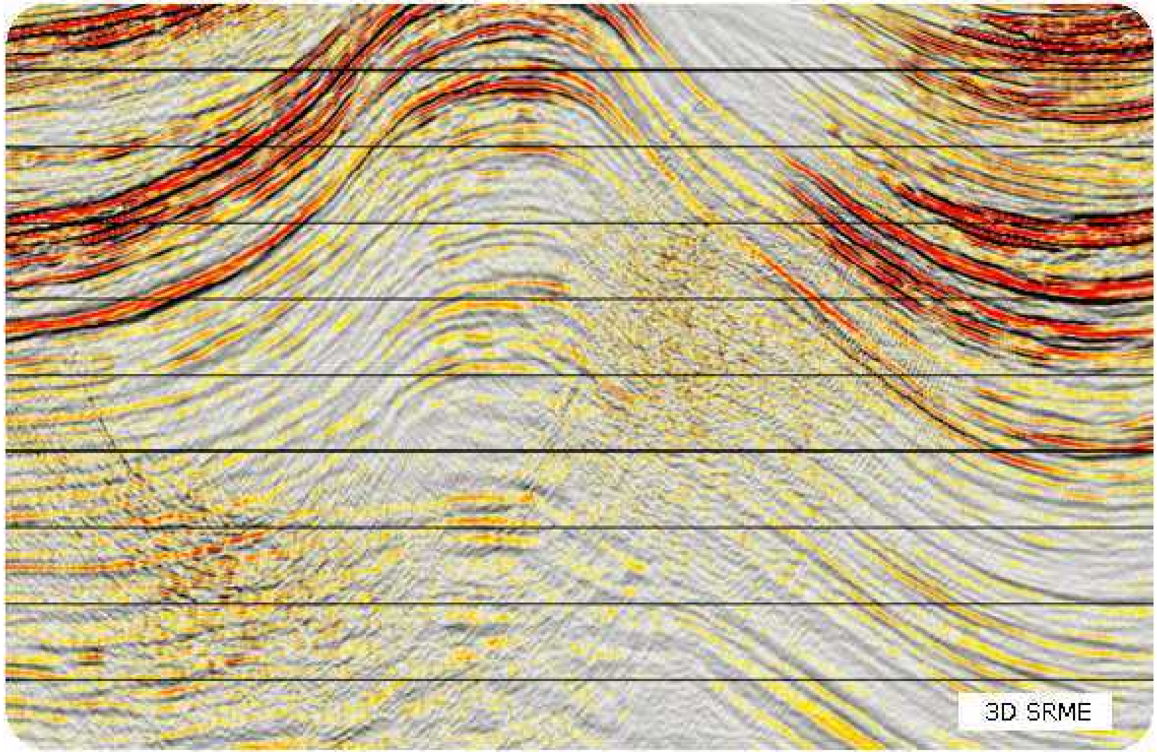

Figure 12: Multiples removed using SRME (Courtesy of CGG)

2. Apply a parabolic Radon transform to the cmp-gathers in order to map the multiple reflections to the $p = 0$ axis.

3. Apply a mute to the radon transformed gathers and remove everything except the multiples. That is, we design a mute function which preserves only data around the $p = 0$ axis in the radon domain.

4. Apply an inverse radon transform to recover only the multiples.

5. Subtract the multiples from the input data. We are the left with only the primary reflections.

6. Apply an inverse normal-moveout correction with velocity of 1500 m/s.

Task 1

Make a new directory called exc3 and copy the Snøhvit data file ans SConstruct file

```
cp /home/vegahag/Data/shots.rsf .
cp /home/vegahag/Data/SConstruct .
```

Study the SConstruct file and identify the processing steps. Note that in this dataset the offset axis contains the full offset. Some of the Madagascar programs require a flag to be set in this case (`offset half=n`). What is the distance between traces in a cdp for this dataset?

Task 2

In this task we are going to try to identify multiple reflections on a single cmp. In the Sconstruct file this is the file called `cmp`. First apply a normal moveout correction with water velocity using the `sfnmostretch` command

```
sfnmostretch half=n v0=1500
```

Plot the nmo-corrected cmp and identify flat events which might be multiples.

Task 3

In this task we are going to apply a parabolic radon transform to the single cmp-file. To check that the parameters for the radon transform is correct we will immediately also perform an inverse radon transform to make sure that no events are lost and that we actually recover the input data. Use the `sfradon` command with the following parameters

```
radon np=251 dp=0.0012 p0=-0.15 x0=1000 parab=y
```

`np` is the number of samples in the radon domain, while `p0` is the smallest (most negative) p-value. The `x0` parameter is an offset where the difference in moveout between the primary reflections and multiple reflections are large enough to be clearly seen. Given that `p0` is measured in seconds pr. km, can you verify that the given parameters are sensible?

The inverse radon radon transform can be performed with the command

```
radon ox=150.0 adj=n x0=1000 nx=60 dx=50.0 parab=y
```

The `ox` parameter is the nearest offset, while `dx` is the spacing between traces in the cmp-gather. What is `nx`?

Now compare the inverse radon transform of the cmp-gather with the input cmp-gather and verify that no major events have been lost.

Task 4

In this task we are going to mute out all events in the radon transform which are not multiples. Use the radon command given above to transform the cmp-gather to the radon domain and the apply the following mute function

```
mutter t0=0.0 v0=0.03
```

After the mute perform an inverse radon transform to get a cmp-gather containing only multiples. Compare the multiple cmp-gather with the cmp-gather you obtained after the inverse radon transform in task 3.

Task 5

In this task we are going to subtract the gather with multiples obtained in task 4 from the cmp-gather we obtained after inverse radon transform in task 3. To subtract to gathers use the following command

```
Flow('cmp-demult',['cmp-mult','cmp-invradon'],'add scale=-1,1 cmp-invrado
```

In this flow the `cmp-mult` file contain only multiples, while the `cmp-invradon` file contain both multiples and primary data. We use `cmp-invradon` instead of the original `cmp` file to make sure that we take into account scaling factors which might be used by the `radon` command. Compare the `cmp-demult` file with the `cmp-invradon` file.

Task 6

In this task we will remove the nmo-correction by using the command

```
Flow('cmp-final','cmp-demult',
     'nmostretch half=n  v0=1500 inv=y| mutter v0=2400 t0=0.20 tp=0.30')
```

Task 7

In order to check that we have actually removed multiples, do a velocity analysis on the output demultippled `cmp-final` file. Plot the velocity scan and the resulting picked velocity function.

Do a corresponding velocity analysis on the `cmp-invradon` file and compare the results. What is your conclusion? Have all multiples been removed

Task 8

This is an optional task if the time permits. The objective of this task is to set up a processing sequence which includes mute, low-pass filtering to remove swell noise, radon to remove multiples, velocity analysis on demultippled cmp-gathers and a final stack. Compare with the stack you got in exercise 1.

REFERENCES