Full Waveform Inversion in the data and image space

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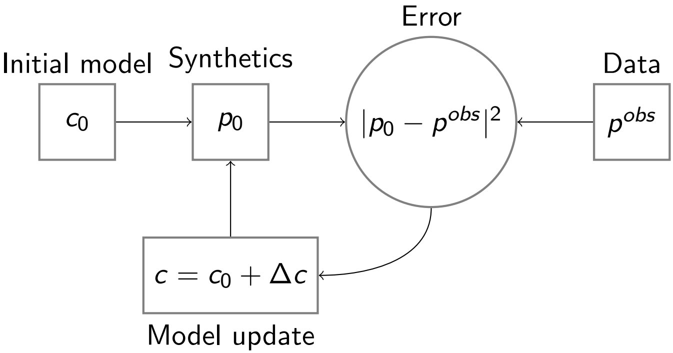
Overview

1. Introduction
2. Initial models for FWI
3. Wave Equation Migration Velocity Analysis
4. Joint Inversion in the image and data spaces
5. Numerical Example
6. Conclusions

Introduction

Full Waveform Inversion loop



Introduction

Full Waveform Inversion (FWI) minimization the least-squares error w.r.t. velocity (Tarantola, 1984)

$$e_l = |p - p^{obs}|^2 \tag{1}$$

Linearization leads to a Newton-Raphson Scheme where the first iteration is

$$\boldsymbol{J}^T[p_0 - p^{obs}] = \boldsymbol{J}^T\boldsymbol{J}\Delta c \tag{2}$$

where $\boldsymbol{J}$ is the Jacobi operator and the Born approximation is

$$\Delta p = p_0 - p^{obs} = \boldsymbol{J}\Delta c \tag{3}$$

$$\Delta c \approx \alpha\nabla_c e_l = \alpha\boldsymbol{J}^T[p_0 - p^{obs}] \tag{4}$$

Introduction

The gradient is given as

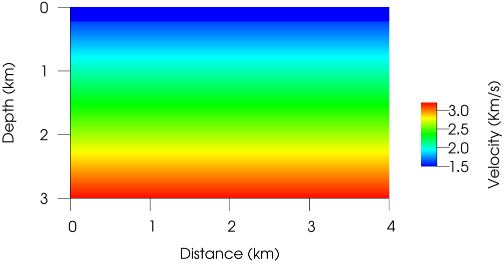
$$\boldsymbol{J}^T[p_0 - p^{obs}](\boldsymbol{x}) = \frac{\partial e_l}{\partial c(\boldsymbol{x})} = \int dt\, p_0(\boldsymbol{x}, t) p(\boldsymbol{x}, t) \tag{5}$$

The time-reversed pressure p is computed by solving

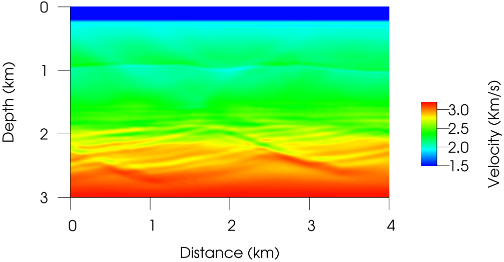
$$\nabla^2 p(\boldsymbol{x}, t) - \frac{1}{c^2(\boldsymbol{x})} \partial_t^2 p(\boldsymbol{x}, t) = \sum_{x_r} [p_0(\boldsymbol{x}_r, t) - p^{obs}(\boldsymbol{x}_r, t)] \tag{6}$$

Introduction

Initial model A

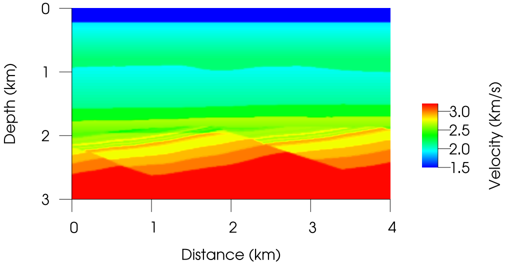


FWI

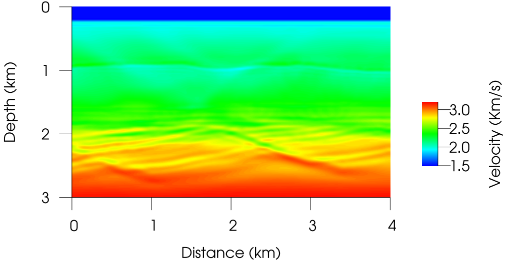


Introduction

Exact model

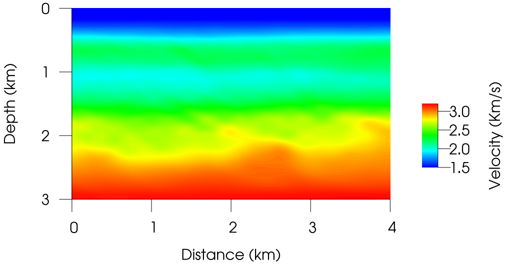


FWI

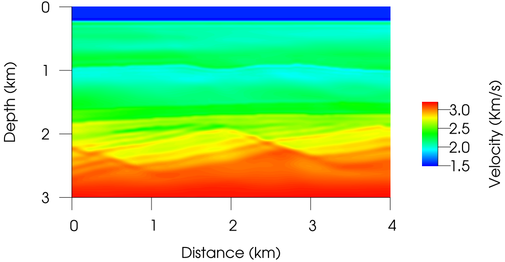


Introduction

Initial model B

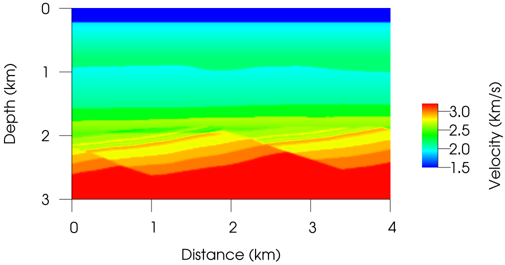


FWI

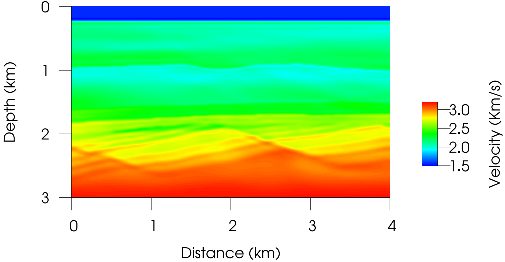


Introduction

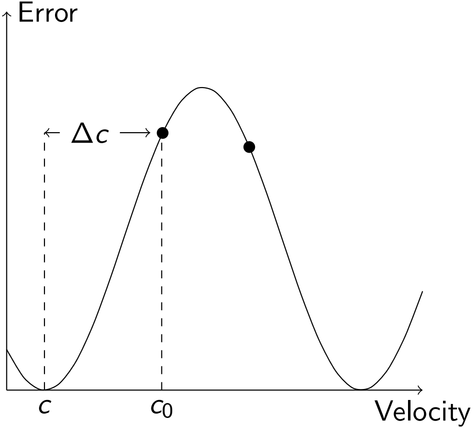
Exact model



FWI



Introduction



Initial models for FWI

Born approximation holds (Beydoun and Tarantola, 1988)

$$\Delta T < \frac{1}{2f_0} \tag{7}$$

- Δt : Traveltime error between model and data
- f_0: Dominant frequency

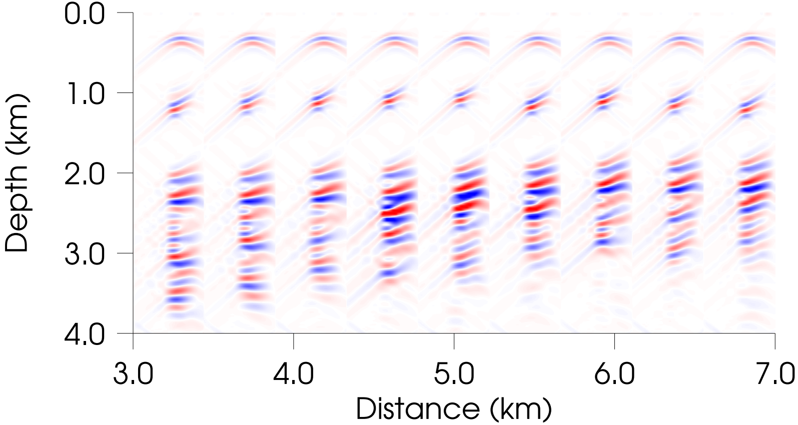
or (Pratt et al. 2008)

$$\frac{\Delta T}{T} < \frac{1}{N_\lambda} \tag{8}$$

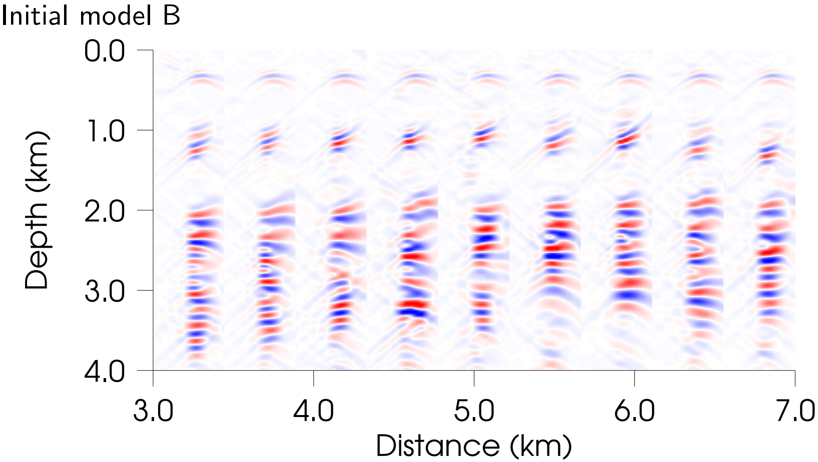
- N_λ : No of wavelengths
- T : Record time

Initial models for FWI

Initial model A



Initial models for FWI



Initial models for FWI

Jian-Bing et al. (2009) for

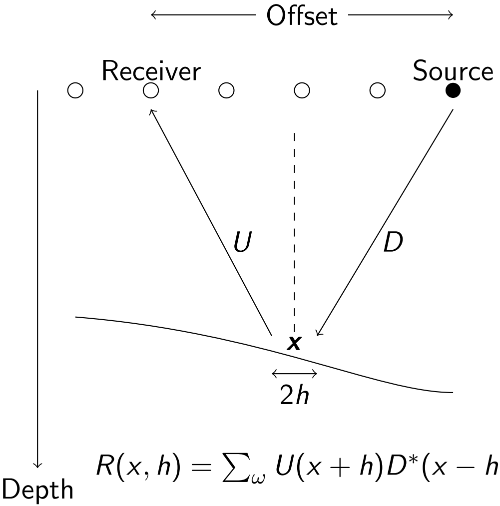
$$(\omega/c_0)^2 I < 1, \tag{9}$$

$$I = \max|\int g(x_r, \omega, x)\frac{\Delta c(x)}{c_0(x)} d^3x|. \tag{10}$$

g is the acoustic Green's function.
Give max frequencies $\approx 1 - 10Hz$ for both model A and B

WEMVA

Wave Equation Migration Velocity Analysis (WEMVA)



$$R(x, h) = \sum_{\omega} U(x+h) D^{*}(x-h)$$

WEMVA

Minimize e_s w.r.t c

$$e_s = \sum_x \sum_h h^2 \left[\frac{\partial R(\boldsymbol{x}, \boldsymbol{h})}{\partial z} \right]^2, \tag{11}$$

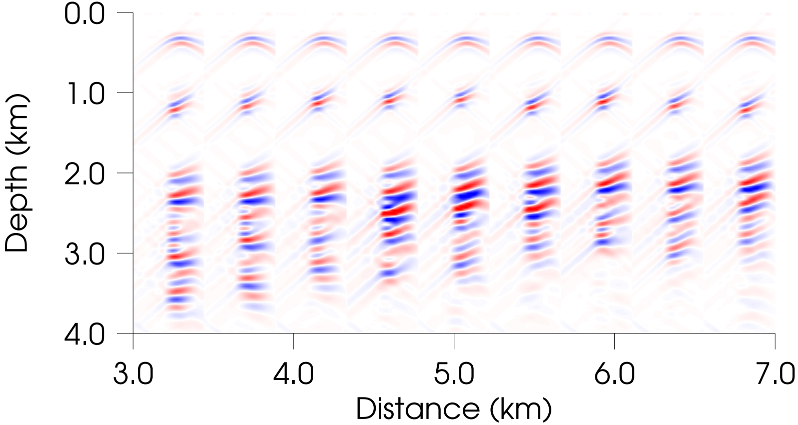
Iterative solution

$$\begin{aligned} c &= c_0 + \Delta c \\ \Delta c &\approx \alpha \nabla_c e_s \end{aligned} \tag{12}$$

- e_s is mainly sensitive to travel-time
- Low resolution
- Relies on the Born Approximation

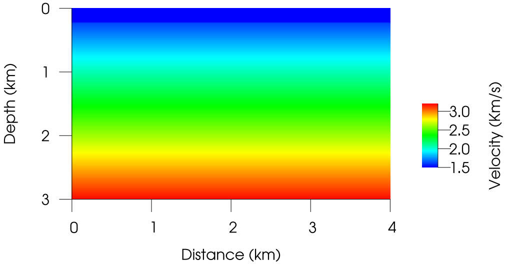
WEMVA

Initial model

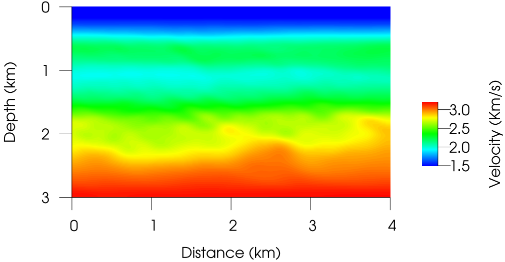


WEMVA

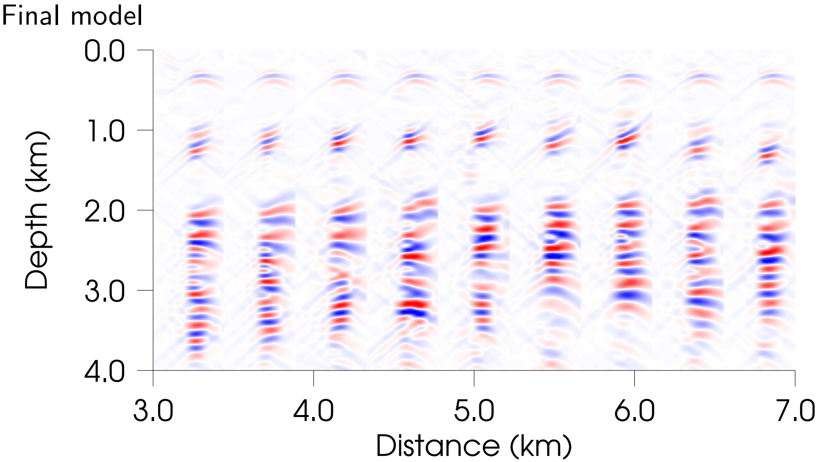
Initial model



WEMVA 25 iterations

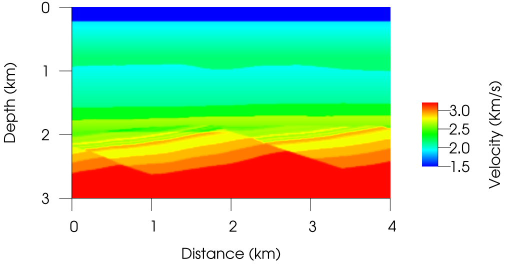


WEMVA

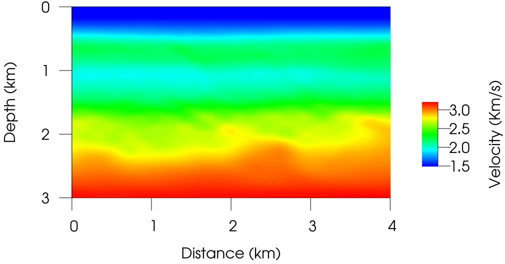


WEMVA

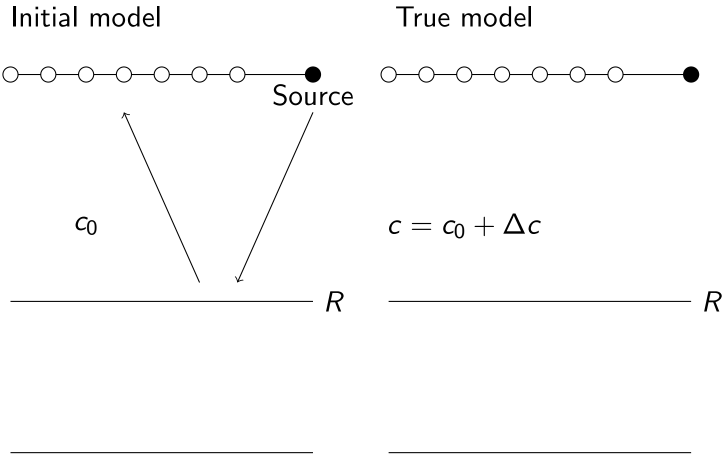
Exact model



WEMVA 25 iterations

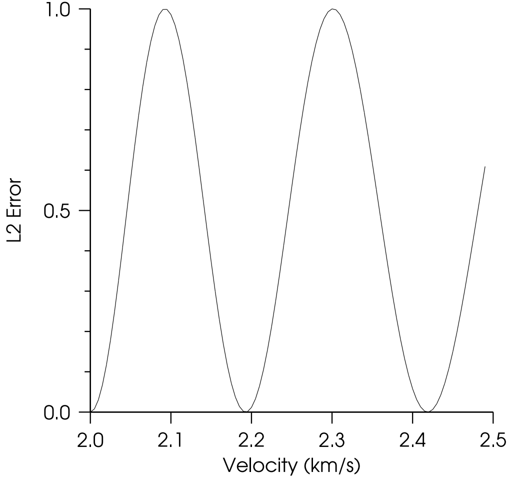


Joint Inversion



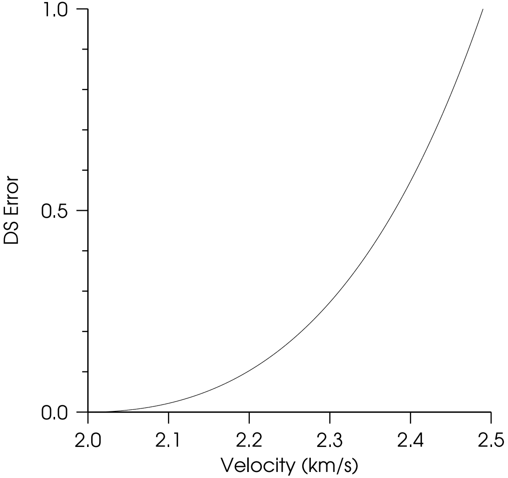
Joint Inversion

L2 Error



Joint Inversion

Differential Semblance Error



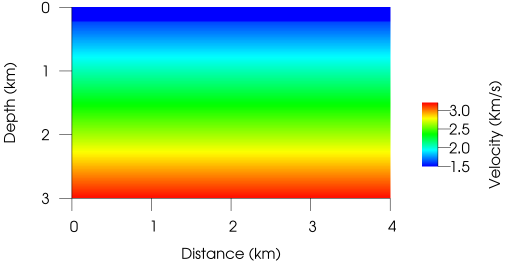
Joint Inversion

$$e = w_I e_I + w_s e_s \tag{13}$$

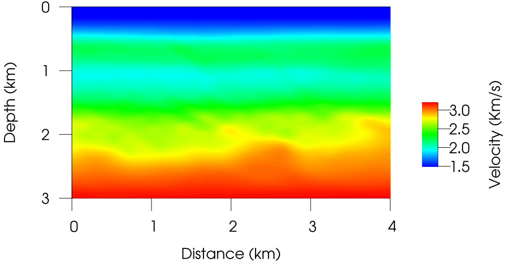
- w_I, w_s: Weights
- e_I: Least-squares Inversion error
- e_s: Differential semblance error

Joint Inversion

Initial model A

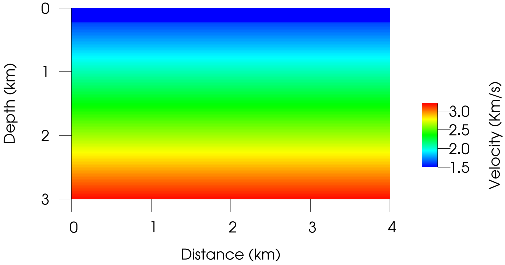


WEMVA after 25 iterations

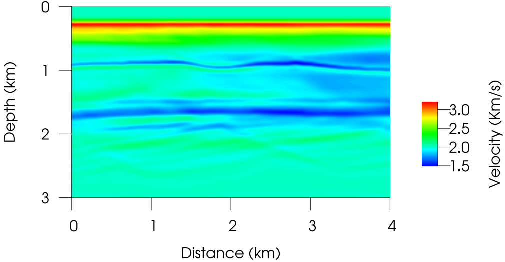


FWI resolution

Initial model from WEMVA

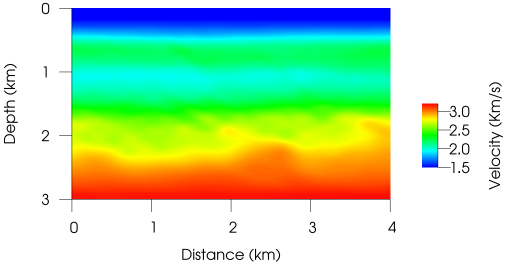


FWI Iteration 1 - Initial model $= \Delta c$

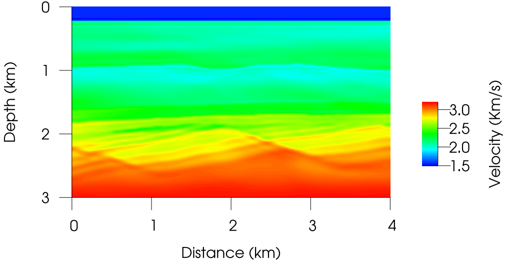


Joint Inversion

WEMVA model

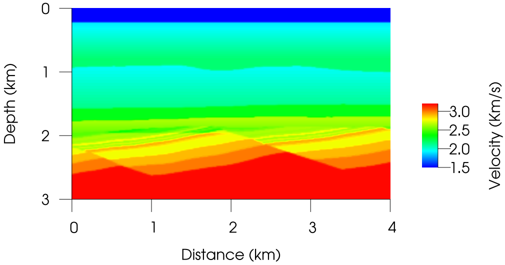


FWI after 25 iterations



Joint Inversion

Exact Model



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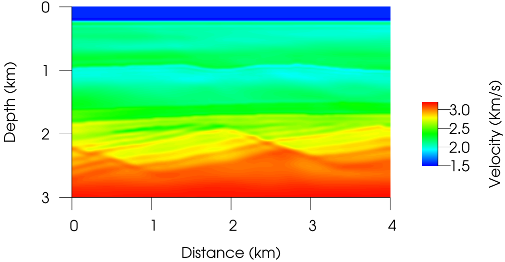

Conclusions

- WEMVA produces low resolution velocity models with reasonable good kinematic properties from simple initial models
- WEMVA velocity models can be used as initial models for FWI to obtain high resolution velocity models

Acknowledgements

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