TPG4190 Seismic data acquisition and processing
Lecture 12: Imaging 3 - Reverse Time Migration (RTM)

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Overview

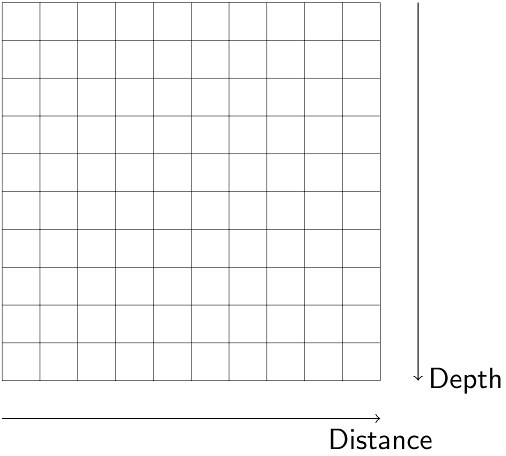
- State-of-the-art processing sequence
- Finite-difference numerical engine
- Boundary condition as sources
- Storage of forward and backward wavefields
- RTM input files
- RTM low frequency noise
- Angle gathers
- RTM computational cost

State-of-the-art processing sequence

1. Load data
2. pre-processing
3. Multiple removal
4. Velocity estimation
5. **Reverse-time-migration**
6. Computation of angle gathers
7. Residual corrections
8. Residual multiple removal
9. Final filters
10. Output of stacks, gathers

Numerical engine

P-wave velocity, Density, Data, Image represented on a regular grid with grid cells of size Δx. Time axis sampled with interval Δt.



Numerical engine

Finite-difference solution of the acoustical wave equation

$$\begin{aligned}\sigma(x,z,t+\Delta t) &= 2\sigma(x,z,t)-\sigma(x,z,t-\Delta t) \\ &+ \Delta t^2\kappa(x,z)[\partial_x \ddot{u}_x(x+\Delta x/2,z,t)+ \\ &+ \partial_z \ddot{u}(x,z+\Delta z/2,t)]+\Delta t^2 \ddot{I}(z,t).\end{aligned} \tag{1}$$

Input to RTM

1. P-wave velocity model on a 3D grid covering surface area and depth range to be imaged. Typical values (depending on area) could be 10x20x5km.
2. Preprocessed shot records typical 1000000 shots.

Numerical engine

Key parameters to be determined

1. Δt of the data
2. Δt for the time stepping
3. Δx for the data
4. Δx for the grid

The sampling interval for the data is determined by the effective frequency content. A sampling interval of 4ms gives a nyquist frequency of

$$f_n = 1/2\Delta t = 1/(2 * 0.004) = 125Hz \tag{2}$$

which is probably adequat for most exploration cases.

Numerical engine

The time stepping δt is determined by two factors

- Stability
- Numerical accuracy (Dispersion)

Stability is given approximately by the largest velocity

$$c_{max}\Delta t \leq 2\Delta x/\sqrt{3} \tag{3}$$

Dispersion is dependent on the approximation for the second order time derivative, but a rough rule is

$$\frac{\lambda}{c\Delta t} \approx 10 - 20 \tag{4}$$

If λ=20m and c =2000m/s this implies Δt <0.001 seconds

Boundary conditions as source

The imaging formula is:

$$r(\boldsymbol{x}', \boldsymbol{x}, t = 0) = \int dS(\boldsymbol{x}_s) \int d\tau \, p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \partial_z p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \tag{5}$$

p_0 is a forward modeled wavefield which is the solution of the acoustic wave equation

$$\nabla^2 p(\boldsymbol{x}, t, \boldsymbol{x}_s) - \frac{1}{c^2(\boldsymbol{x})} \partial_t^2 p(\boldsymbol{x}, \boldsymbol{x}_s, t) = \delta(\boldsymbol{x} - \boldsymbol{x}_s) h(t) \tag{6}$$

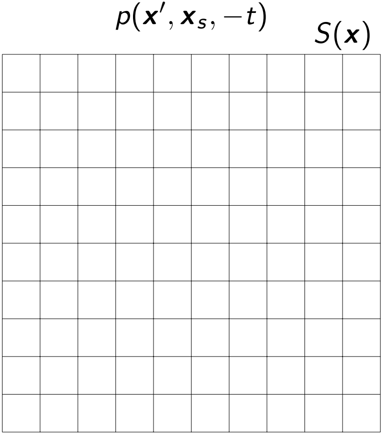
Here $c(\boldsymbol{x})$ is the velocity model.

Boundary conditions as source

$p(\boldsymbol{x}, \boldsymbol{x}_s, t)$ is a wavefield with time-reversed boundary conditions

$$p(\boldsymbol{x}, \boldsymbol{x}_s, t) = -\int dS(\boldsymbol{x}')\, p(\boldsymbol{x}', \boldsymbol{x}_s, -t) * \partial_z' g(\boldsymbol{x}, \boldsymbol{x}', t) \qquad (7)$$

Boundary conditions as source



It is difficult to introduce the data as a pure boundary condition in the finite-difference method (but not impossible). Instead it is easier to replace the boundary condition with sources.

Boundary conditions as source

$$p(\boldsymbol{x}, \boldsymbol{x}_s, t) =$$
$$-\int dV(\boldsymbol{x}'') \left[\int dS(\boldsymbol{x}')\, p(\boldsymbol{x}', \boldsymbol{x}_s, -t) \partial_z \delta(\boldsymbol{x}'' - \boldsymbol{x}') \right] * g(\boldsymbol{x}, \boldsymbol{x}'', t) \quad (8)$$

Here we have used the fact that

$$f'(0) = \int dx\, \partial_x \delta(x) \quad (9)$$

The solution of the acoustic wave equation can be written

$$p(\boldsymbol{x}, \boldsymbol{x}_s, t) = \int dV(\boldsymbol{x}'') g(\boldsymbol{x}, \boldsymbol{x}'', t) * s(\boldsymbol{x}'', t) \quad (10)$$

Comparing equations (10) and (8) we see that the surface integral over the data and the derivative of the delta function is simply a source where each receiver point is a source with strength:

$$p(\boldsymbol{x}', \boldsymbol{x}_s, -t) \partial_z \delta(\boldsymbol{x}'' - \boldsymbol{x}'). \quad (11)$$

Storage requirements

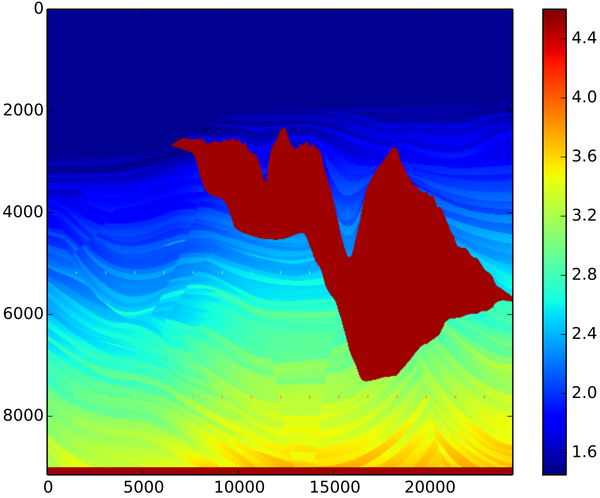
The algorithm for RTM can be summarized as:

1. Forward model a single shot record with a point source Store wavefield for all timesteps
2. Backward model a single shot record with the data acting as sources Store wavefield for all timesteps
3. Cross-correlate (zero-time lag) the forward and backward wavefields
4. repeat for all shots and sum over shots

Storage is needed because the backward field is computed in reverse time order and need to be reordered before cross-correlation

Storage requirements

Migration of the sigsbee 2D model (Very small example compared to 3D industrial migrations).



Storage requirements

Synthetic example with parameters

- No of gridpoints Nx=1496, Nz=601
- Length of model: $\approx$ 37km
- Depth of model: $\approx$ 9 km
- Grid sampling : $\Delta x \approx$ 15m.
- Time sampling : Δt : 0.0005 sec
- Receiver group spacing: 23m
- Number of receiver gropups: 348
- Streamer length: 8km
- Shot distance: 45m
- No of shots: 500
- Migration aperture: 15km
- Shot record length: 12sec

Storage requirements

Synthetic example with parameters

- Size of one snapshot: $1000 \times 600 \times 4 = 2.4 \times 10^6$
- No of snapshots: 24000
- Total storage req for the forward modelled wavefield: $2.4 \times 10^6 \times 24000 : 5.76 \times 10^9 =$58Gb
- Total storage req $\times$58Gb $\approx$ 120Gb
- Cpu time for one shot: 30 minutes implies 10 days for 500 shots.
- Migrate 100 shots in parallel: 2.5 Hours.
- Storage req increase to 120Gb $\times$ 100 $=$ 12Tb.

Storage is a problem with this approach.

Storage requirements

Modified storage approach

- Only store some snapshots, i.e every 0.005 seconds
- Reduced storage to 1.2Tb
- Use compression to further reduce storage
- Interpolation is needed for high quality

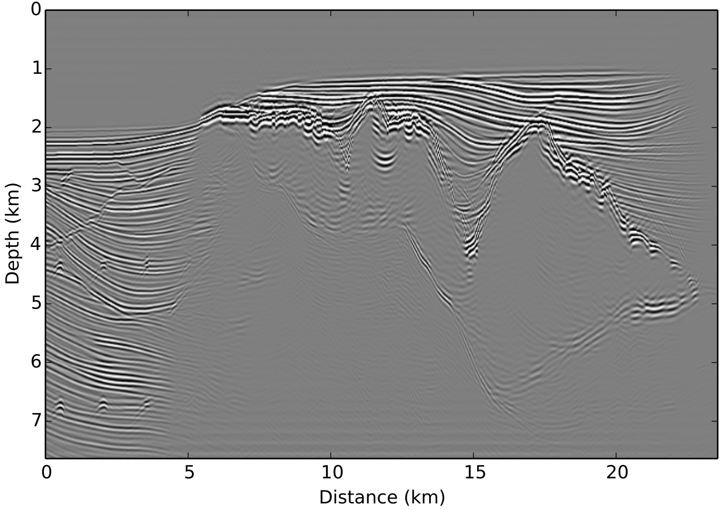
Storage requirements

Reconstruction method

- Store wavefield recorded at the surface when doing forward modeling
- Storage is reduced to small amounts of data
- Perform backward modeling of data and forward wavefield
- Forward wavefield no comes out in reverse time
- Cross-correlation can then be performed immediately

Requires 50% more computer time, but little storage.

Storage requirements



RTM Input files and parameters

- Shot records
- Velocity model
- Δx and Δt
- Migration aperture

Aliasing

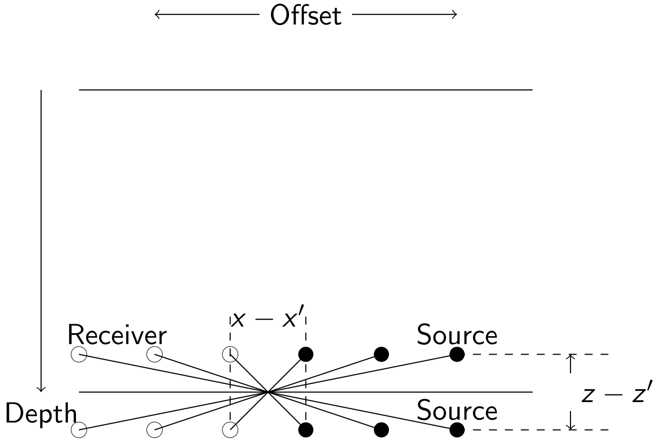
$$\frac{\omega}{c_{min}} \leq \frac{\pi}{\Delta x} \tag{12}$$

$$f_{max} = \frac{c_{min}}{2\Delta x} \tag{13}$$

$$f_{max} = \frac{1500}{2 \times 15} = 30Hz \tag{14}$$

RTM Angle gathers

In addition to the output (zero-offset) image, so-called angle gathers contain additional information on the angle behaviour of the reflectivity. This information is often used for identifying hydrocarbons or classifying rocktypes.



RTM Angle gathers

Use the offset and time delay formula

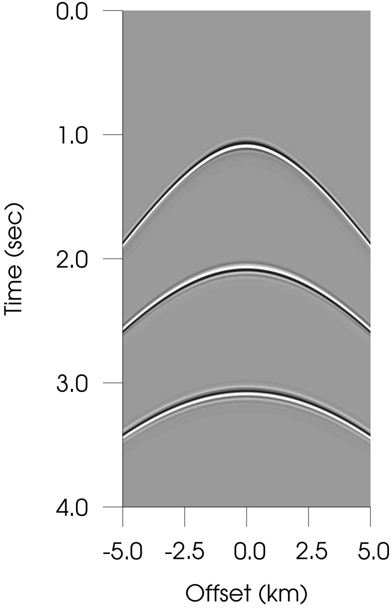
$$r(\boldsymbol{x}', \boldsymbol{x}, t) = r(\boldsymbol{x}' = 0, \boldsymbol{x},$$

$$\int dS(\boldsymbol{x}_s) \int d\tau \, p(\boldsymbol{x}', \boldsymbol{x}_s, \tau + t) \partial_z p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \tag{15}$$

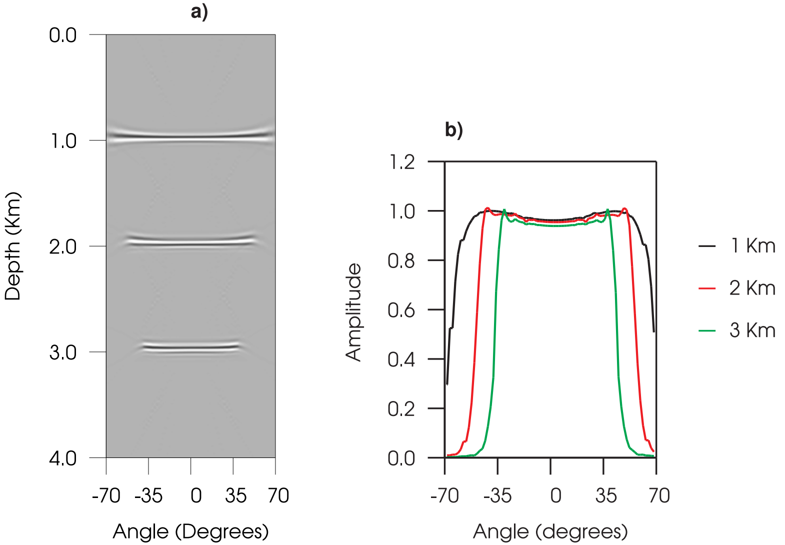
We want to compute the plane-wave reflectivity. The simplest way is to use the so-called $\tau - p$ transform, which is a decomposition into plane waves.

$$r(p, \tau) = \int dx \, r(x, \tau + px) \tag{16}$$

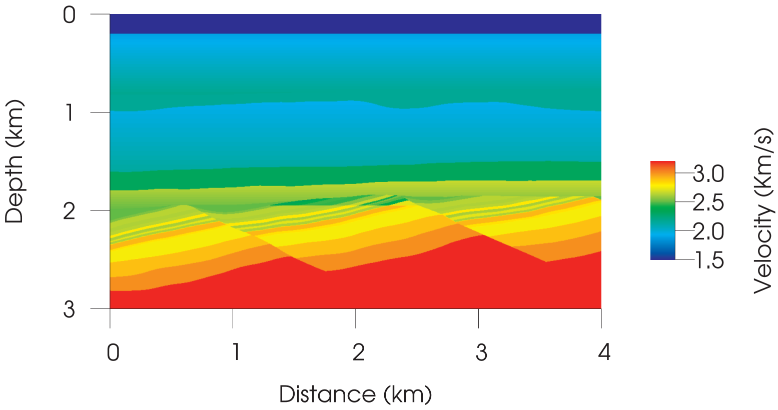
RTM Angle gathers



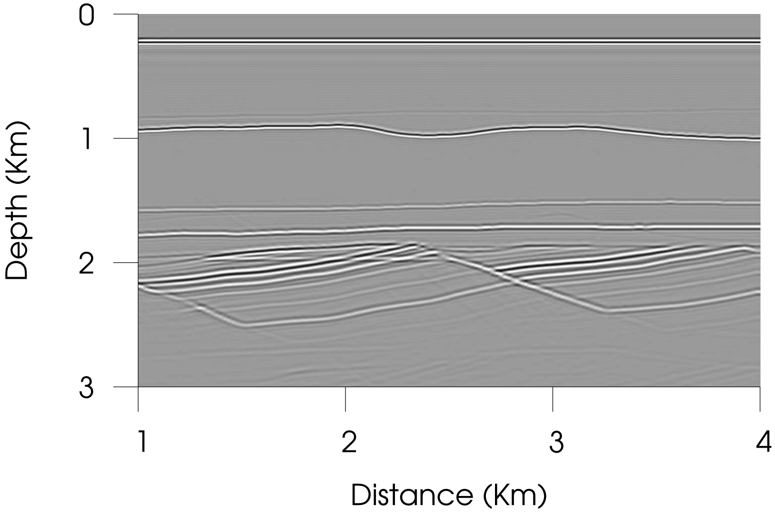
RTM Angle gathers



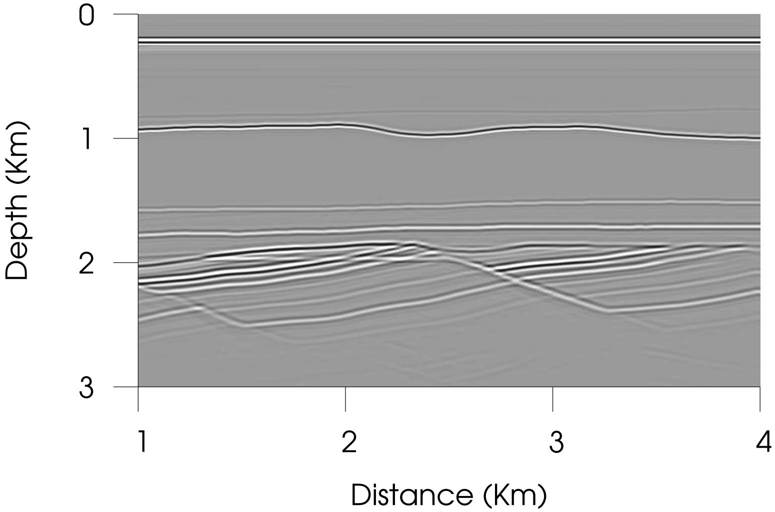
RTM Angle gathers



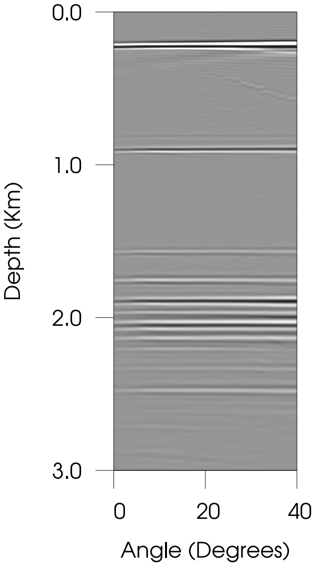
RTM Angle gathers



RTM Angle gathers



RTM Angle gathers



RTM Angle gathers