TPG4190 Seismic data acquisition and processing Imaging

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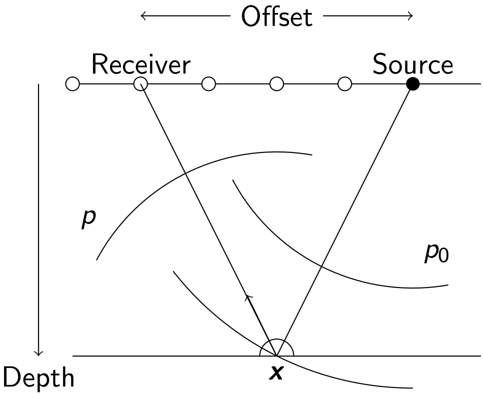
Overview

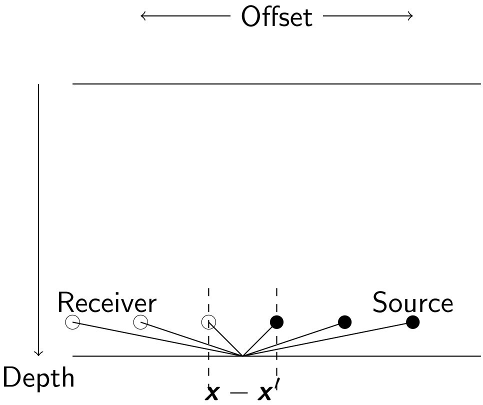
- Imaging conditions
- Reciprocity
- Time reversal
- Imaging formula

Imaging condition I



p: Scattereded wavefield (data)
p_0: Modeled wavefield
$\boldsymbol{x}$: Spatial position

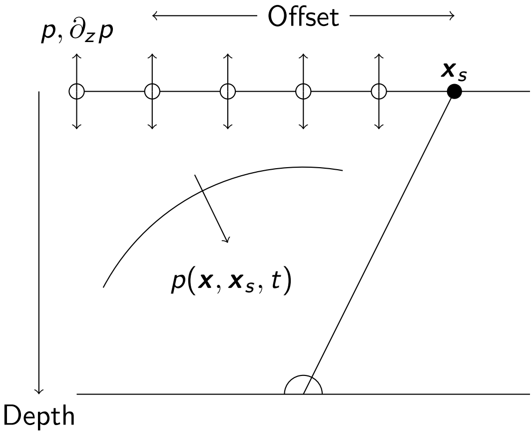
Imaging condition II



$\boldsymbol{x}$: Virtual receiver

$\boldsymbol{x}'$: Virtual source

Imaging condition III

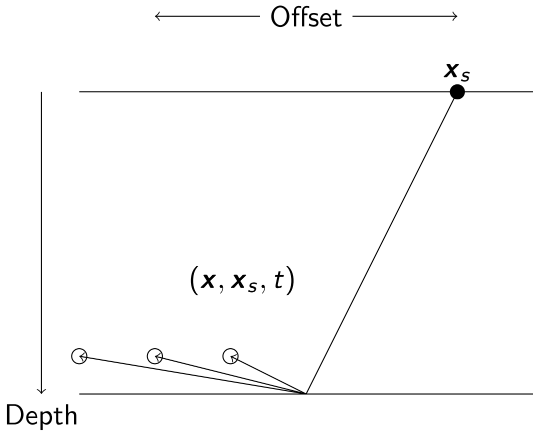


p: Scattered wavefield

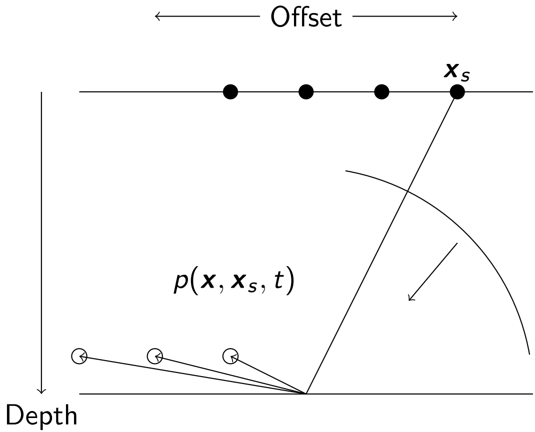
$\boldsymbol{x}_s$: Source position

$\boldsymbol{x}, t$: Position,time

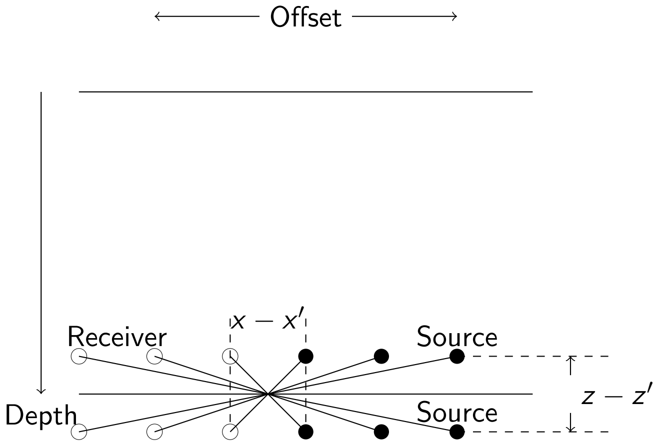
Imaging condition IV



Imaging condition V

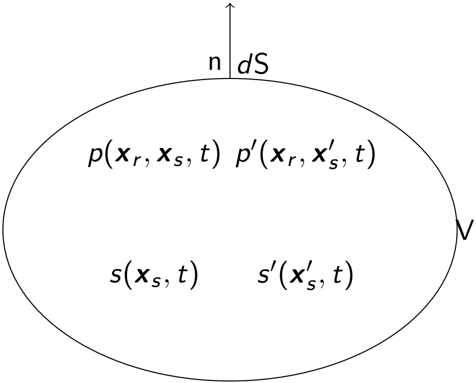


Imaging condition VI

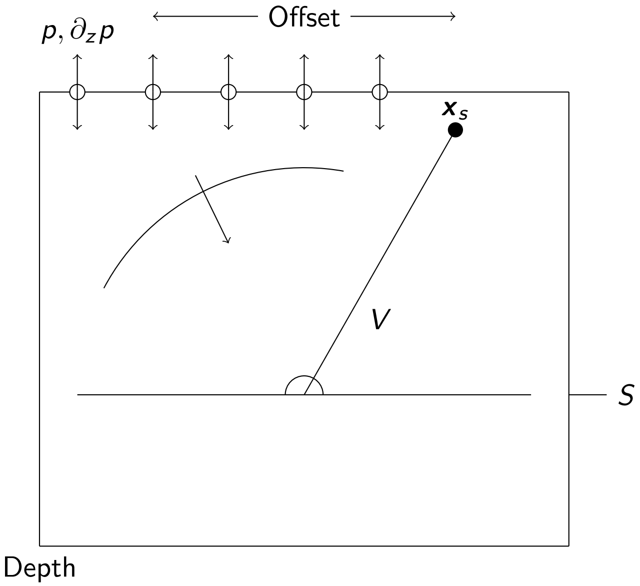


$x - x'$: Horizontal offset
$z - z'$: Vertical offset

Reciprocity



Reciprocity



Reciprocity

Two sources s' and s are located inside a volume V with surface S at positions $\boldsymbol{x}_s$ and $\boldsymbol{x}'_s$. The corresponding wavefields are $p(\boldsymbol{x}, \boldsymbol{x}_s)$ and $p'(\boldsymbol{x}, \boldsymbol{x}'_s)$. The wave equations for these two fields are

$$\nabla^2 p(\boldsymbol{x}, \boldsymbol{x}_s, t) - \frac{1}{c^2(\boldsymbol{x})} \partial_t^2 p(\boldsymbol{x}, \boldsymbol{x}_s, t) = s(\boldsymbol{x}, t), \tag{1}$$

$$\nabla^2 p'(\boldsymbol{x}, \boldsymbol{x}'_s, t) - \frac{1}{c'^2(\boldsymbol{x})} \partial_t^2 p'(\boldsymbol{x}, \boldsymbol{x}'_s, t) = s'(\boldsymbol{x}, t) \tag{2}$$

Reciprocity

Fourier transform over time ($\partial_t^2 \to -\omega^2$)

$$\nabla^2 P(\boldsymbol{x}, \boldsymbol{x}_s, \omega) + \frac{\omega^2}{c^2(\boldsymbol{x})} P(\boldsymbol{x}, \boldsymbol{x}_s, \omega) = S(\boldsymbol{x}, \omega), \tag{3}$$

$$\nabla^2 P'(\boldsymbol{x}, \boldsymbol{x}'_s, \omega) + \frac{\omega^2}{c'^2(\boldsymbol{x})} P'(\boldsymbol{x}, \boldsymbol{x}'_s, \omega) = S'(\boldsymbol{x}, \omega) \tag{4}$$

Reciprocity

Multiply equation (3) with P' and equation (4) with P and integrate over V (suppressing ω as an argument)

$$\int dV(\boldsymbol{x}) \left[P'(\boldsymbol{x}, \boldsymbol{x}'_s)\nabla^2 P(\boldsymbol{x}, \boldsymbol{x}_s) + P'(\boldsymbol{x}, \boldsymbol{x}'_s)\frac{\omega^2}{c^2(\boldsymbol{x})}P(\boldsymbol{x}, \boldsymbol{x}_s) \right] = \int dV(\boldsymbol{x})\, P'(\boldsymbol{x}, \boldsymbol{x}'_s)S(\boldsymbol{x}) \quad (5)$$

$$\int dV(\boldsymbol{x}) \left[P(\boldsymbol{x}, \boldsymbol{x}_s)\nabla^2 P'(\boldsymbol{x}, \boldsymbol{x}'_s) + P(\boldsymbol{x}, \boldsymbol{x}_s)\frac{\omega^2}{c'^2(\boldsymbol{x})}P'(\boldsymbol{x}, \boldsymbol{x}'_s) \right] = \int dV(\boldsymbol{x}) P(\boldsymbol{x}, \boldsymbol{x}_s)S'(\boldsymbol{x}) \quad (6)$$

Reciprocity

Subtract equation (6) from equation (5): (Assume $c' = c$).

$$\int dV(\boldsymbol{x}) \left[P'(\boldsymbol{x}, \boldsymbol{x}'_s)\nabla^2 P(\boldsymbol{x}, \boldsymbol{x}_s) - P(\boldsymbol{x}, \boldsymbol{x}_s)\nabla^2 P'(\boldsymbol{x}, \boldsymbol{x}'_s)\right] = \int dV(\boldsymbol{x}) \left[P'(\boldsymbol{x}, \boldsymbol{x}'_s)S(\boldsymbol{x}) - P(\boldsymbol{x}, \boldsymbol{x}_s)S'(\boldsymbol{x})\right] \quad (7)$$

Reciprocity

Gauss divergence theorem:

$$\int dV(\boldsymbol{x}) \nabla \cdot \boldsymbol{A}(\boldsymbol{x}) = \int dS\, \boldsymbol{A}(\boldsymbol{x}) \cdot \boldsymbol{n}(\boldsymbol{x}) \tag{8}$$

Put $\boldsymbol{A} = P'(\boldsymbol{x}, \boldsymbol{x}_s') \cdot \nabla P(\boldsymbol{x}, \boldsymbol{x}_s)$ to get

$$\begin{aligned} \int dV(\boldsymbol{x}) \nabla \left[P'(\boldsymbol{x}, \boldsymbol{x}_s') \nabla P(\boldsymbol{x}, \boldsymbol{x}_s) \right] = \\ \int dV(\boldsymbol{x}) \left[P'(\boldsymbol{x}, \boldsymbol{x}_s') \nabla^2 P(\boldsymbol{x}, \boldsymbol{x}_s') + \nabla P'(\boldsymbol{x}, \boldsymbol{x}_s') \cdot \nabla P(\boldsymbol{x}, \boldsymbol{x}_s) \right] = \\ \int dS\, P'(\boldsymbol{x}, \boldsymbol{x}_s') \nabla P(\boldsymbol{x}, \boldsymbol{x}_s) \cdot \boldsymbol{n}(\boldsymbol{x}) \end{aligned} \tag{9}$$

Reciprocity

Put $\boldsymbol{A} = P(\boldsymbol{x}, \boldsymbol{x}_s) \cdot \nabla P'(\boldsymbol{x}, \boldsymbol{x}'_s)$ to get

$$\int dV(\boldsymbol{x}) \nabla \left[P(\boldsymbol{x}, \boldsymbol{x}_s) \nabla P'(\boldsymbol{x}, \boldsymbol{x}'_s) \right] =$$

$$\int dV(\boldsymbol{x}) \left[P(\boldsymbol{x}, \boldsymbol{x}_s) \nabla^2 P'(\boldsymbol{x}, \boldsymbol{x}'_s) + \nabla P'(\boldsymbol{x}, \boldsymbol{x}'_s) \cdot \nabla P(\boldsymbol{x}, \boldsymbol{x}_s) \right] =$$

$$\int dS \, P(\boldsymbol{x}, \boldsymbol{x}_s) \nabla P'(\boldsymbol{x}, \boldsymbol{x}'_s) \cdot \boldsymbol{n}(\boldsymbol{x}) \quad (10)$$

Subtract equation (10) fro equation (9) to get

$$\int dV(\boldsymbol{x}) \left[P'(\boldsymbol{x}, \boldsymbol{x}'_s) \nabla^2 P(\boldsymbol{x}, \boldsymbol{x}_s) - P(\boldsymbol{x}, \boldsymbol{x}_s) \nabla^2 P'(\boldsymbol{x}, \boldsymbol{x}'_s) \right] =$$

$$\int dS \, \left[P'(\boldsymbol{x}, \boldsymbol{x}'_s) \nabla P(\boldsymbol{x}, \boldsymbol{x}_s) - P(\boldsymbol{x}, \boldsymbol{x}_s) \nabla P'(\boldsymbol{x}, \boldsymbol{x}'_s) \right] \cdot \boldsymbol{n}(\boldsymbol{x}) \quad (11)$$

Reciprocity

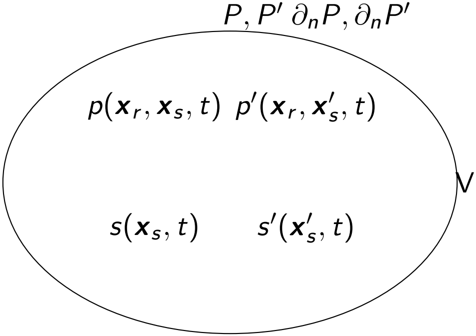
Replace the left hand side of equation (7) with equation (11)to get

$$\int dS(\boldsymbol{x}) \left[P'(\boldsymbol{x},\boldsymbol{x}'_s)\nabla P(\boldsymbol{x},\boldsymbol{x}_s) - P(\boldsymbol{x},\boldsymbol{x}_s)\nabla P'(\boldsymbol{x},\boldsymbol{x}'_s)\right]\cdot \boldsymbol{n}(\boldsymbol{x}) = \int dV(\boldsymbol{x}) \left[P'(\boldsymbol{x},\boldsymbol{x}'_s)S(\boldsymbol{x}) - P(\boldsymbol{x},\boldsymbol{x}_s)S'(\boldsymbol{x})\right] \quad (12)$$

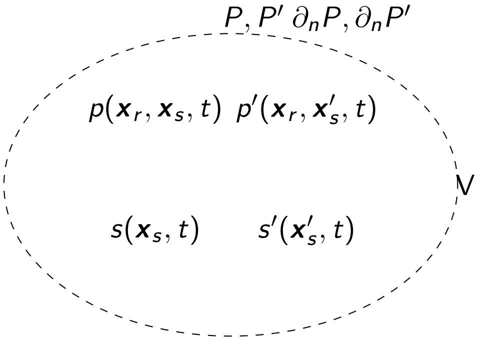
Assume point source $S'(\boldsymbol{x}) = \delta(\boldsymbol{x} - \boldsymbol{x}'_s)$ to get

$$P(\boldsymbol{x}'_s,\boldsymbol{x}_s) = \int dV(\boldsymbol{x})P'(\boldsymbol{x},\boldsymbol{x}'_s)S(\boldsymbol{x}) + \int dS(\boldsymbol{x}) \left[P'(\boldsymbol{x},\boldsymbol{x}'_s)\nabla P(\boldsymbol{x},\boldsymbol{x}_s) - P(\boldsymbol{x},\boldsymbol{x}_s)\nabla P'(\boldsymbol{x},\boldsymbol{x}'_s)\right]\cdot \boldsymbol{n}(\boldsymbol{x}) \quad (13)$$

Reciprocity



Reciprocity



Source-receiver reciprocity

Assume noe that the surface integral is zero and that $S(\boldsymbol{x}) = \delta(\boldsymbol{x} - \boldsymbol{x}_s)$. Equation (12) then gives if we assume $P = P'$ and $S = S'$

$$P(\boldsymbol{x}_s, \boldsymbol{x}'_s) = P(\boldsymbol{x}'_s, \boldsymbol{x}_s) \tag{14}$$

The pressure recorded at $\boldsymbol{x}_s$ due to a source at $\boldsymbol{x}'_s$ is the same as the pressure recorded at $\boldsymbol{x}'_s$ due to a source at $\boldsymbol{x}_s$.

Reciprocity

We now assume that the source of P' is

$$S'(\boldsymbol{x}) = \delta(\boldsymbol{x} - \boldsymbol{x}_s)\delta(t) \tag{15}$$

$P'(\boldsymbol{x}, \boldsymbol{x}'_s) = G(\boldsymbol{x}, \boldsymbol{x}'_s)$ is then called a Green's function.

Reciprocity

Use reciprocity $G(\boldsymbol{x}'_s, \boldsymbol{x}) = G(\boldsymbol{x}, \boldsymbol{x}'_s)$ to get from equation (13)

$$P(\boldsymbol{x}'_s, \boldsymbol{x}_s) = \int dV(\boldsymbol{x}) G(\boldsymbol{x}'_s, \boldsymbol{x}) S(\boldsymbol{x})$$

$$+ \int dS(\boldsymbol{x}) \left[G(\boldsymbol{x}'_s, \boldsymbol{x}) \nabla P(\boldsymbol{x}, \boldsymbol{x}_s) - P(\boldsymbol{x}, \boldsymbol{x}_s) \nabla G(\boldsymbol{x}'_s, \boldsymbol{x}) \right] \cdot \boldsymbol{n}(\boldsymbol{x})$$

Rename $\boldsymbol{x} \to \boldsymbol{x}'$ and $\boldsymbol{x}'_s \to \boldsymbol{x}$.

$$P(\boldsymbol{x}, \boldsymbol{x}_s) = \int dV(\boldsymbol{x}') G(\boldsymbol{x}, \boldsymbol{x}') S(\boldsymbol{x}')$$

$$+ \int dS(\boldsymbol{x}') \left[G(\boldsymbol{x}, \boldsymbol{x}') \nabla P(\boldsymbol{x}', \boldsymbol{x}_s) - P(\boldsymbol{x}', \boldsymbol{x}_s) \nabla G(\boldsymbol{x}, \boldsymbol{x}') \right] \cdot \boldsymbol{n}(\boldsymbol{x}')$$

Reciprocity

Fourier transform back to time

$$
\begin{aligned}
p(\boldsymbol{x}, \boldsymbol{x}_s, t) = &\int dV(\boldsymbol{x}') g(\boldsymbol{x}, \boldsymbol{x}', t) * s(\boldsymbol{x}', t) \\
&+ \int dS(\boldsymbol{x}') \, \Big[g(\boldsymbol{x}, \boldsymbol{x}', t) * \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, t) \\
&\qquad p(\boldsymbol{x}', \boldsymbol{x}_s, t) * \nabla g(\boldsymbol{x}, \boldsymbol{x}', t) \Big] \cdot \boldsymbol{n}(\boldsymbol{x}')
\end{aligned}
$$

Forward modeling

Fourier transform back to time

$$p(\boldsymbol{x}, \boldsymbol{x}_s, t) = \int dV(\boldsymbol{x}')g(\boldsymbol{x}, \boldsymbol{x}', t) * s(\boldsymbol{x}', t) \tag{16}$$

Reciprocity

The interpretation of this equations is

1. The volume integral describes the conribution to the pressure from the source $S(\boldsymbol{x})$ inside the volume V.
2. The surface integral describes the effect of the boundary conditions at the boundary S.

Time reversal

Two sources s' and s' are located inside a volume V with surface S at positions $\boldsymbol{x}_s$ and $\boldsymbol{x}'_s$. The corresponding wavefields are $p(\boldsymbol{x}, \boldsymbol{x}_s, t)$ and $p'(\boldsymbol{x}, \boldsymbol{x}'_s, -t)$. The wave equations for these two fields are

$$\begin{aligned} \nabla^2 p(\boldsymbol{x}, \boldsymbol{x}_s, t) - \frac{1}{c^2(\boldsymbol{x})} \partial_t^2 p(\boldsymbol{x}, \boldsymbol{x}_s, t) &= s(\boldsymbol{x}, t), \\ \nabla^2 p'(\boldsymbol{x}, \boldsymbol{x}'_s, -t) - \frac{1}{c^2(\boldsymbol{x})} \partial_t^2 p'(\boldsymbol{x}, \boldsymbol{x}'_s, -t) &= s'(\boldsymbol{x}, -t), \end{aligned} \tag{17}$$

Time reversal

Assume point sources $S'^*(\boldsymbol{x}) = H'^*\delta(\boldsymbol{x} - \boldsymbol{x}_s')$ and $S(\boldsymbol{x}) = H\delta(\boldsymbol{x} - \boldsymbol{x}_s')$ in equation (12)

$$\int dS(\boldsymbol{x}) \left[P'^*(\boldsymbol{x}, \boldsymbol{x}_s')\nabla P(\boldsymbol{x}, \boldsymbol{x}_s) - P(\boldsymbol{x}, \boldsymbol{x}_s)\nabla P'^*(\boldsymbol{x}, \boldsymbol{x}_s')\right] \cdot \boldsymbol{n}(\boldsymbol{x}) = P'^*(\boldsymbol{x}_s, \boldsymbol{x}_s')H - P(\boldsymbol{x}_s', \boldsymbol{x}_s)H'^* \tag{18}$$

Use reciprocity to get:

$$P'^*(\boldsymbol{x}_s, \boldsymbol{x}_s')H - P(\boldsymbol{x}_s, \boldsymbol{x}_s')H'^* = \int dS(\boldsymbol{x}) \left[P'^*(\boldsymbol{x}, \boldsymbol{x}_s')\nabla P(\boldsymbol{x}_s, \boldsymbol{x}) - P(\boldsymbol{x}_s, \boldsymbol{x})\nabla P'^*(\boldsymbol{x}, \boldsymbol{x}_s')\right] \cdot \boldsymbol{n}(\boldsymbol{x}) \tag{19}$$

Imaging

Renaming $\boldsymbol{x}_s \to \boldsymbol{x}$, $\boldsymbol{x} \to \boldsymbol{x}'$ and $\boldsymbol{x}_s' \to \boldsymbol{x}_s$

$$P^*(\boldsymbol{x}, \boldsymbol{x}_s)H - P(\boldsymbol{x}, \boldsymbol{x}_s)H'^* = \int dS(\boldsymbol{x}') \left[P^*(\boldsymbol{x}', \boldsymbol{x}_s)\nabla P(\boldsymbol{x}, \boldsymbol{x}') - P(\boldsymbol{x}, \boldsymbol{x}')\nabla P^*(\boldsymbol{x}', \boldsymbol{x}_s)\right] \cdot \boldsymbol{n}(\boldsymbol{x}') \tag{20}$$

Fourier transforming back to time gives:

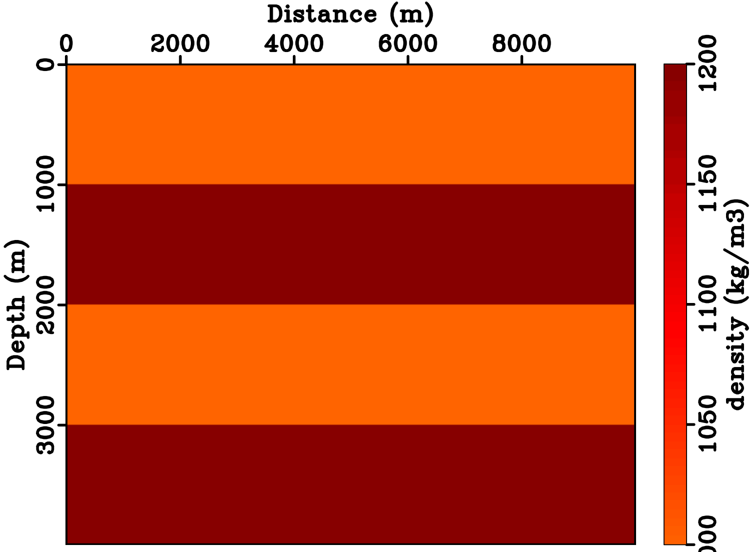
$$\begin{aligned} p(\boldsymbol{x}, \boldsymbol{x}_s, -t) * h(t) - p(\boldsymbol{x}, \boldsymbol{x}_s, t) * h'(-t) = \\ \int dS(\boldsymbol{x}') \left[p(\boldsymbol{x}', \boldsymbol{x}_s, -t) * \nabla p(\boldsymbol{x}, \boldsymbol{x}', t)\right. \\ \left. - p(\boldsymbol{x}, \boldsymbol{x}', t) * \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, -t)\right] \cdot \boldsymbol{n}(\boldsymbol{x}') \end{aligned} \tag{21}$$

Imaging

Interpretation:

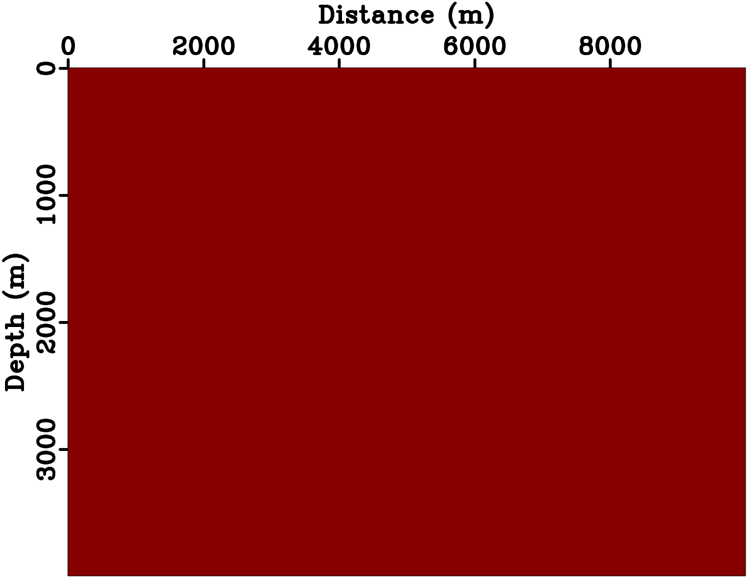
1. The lefthand side of equation (21) is the pressure p for time $t > 0$ and time $t < 0$. The pressure for $t > 0$ is zero for $t < 0$ and the pressure for $t < 0$ is zero for $t > 0$. Both pressures are obtained at a field point $\boldsymbol{x}$.
2. The pressure for negative t on the right hand side describes time-reversed data due to a source at x_s recorded at the surface S. The pressure for positive time on the right hand side describes modeling of data from the receiver positions to an arbitrary field point $\boldsymbol{x}$.
3. The effect of the convolutions and the surface integral is to extrapolate the recorded data at the surface S to an arbitrary field point $\boldsymbol{x}$.

Numerical example



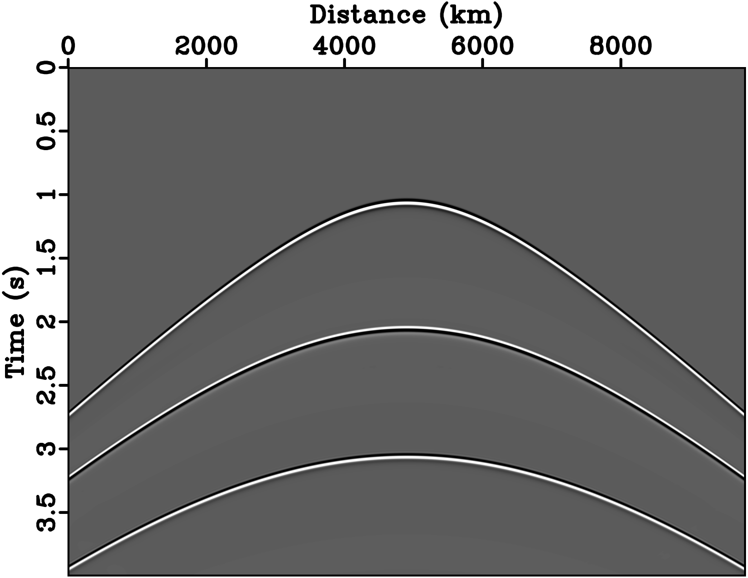
Density

Numerical example



Velocity

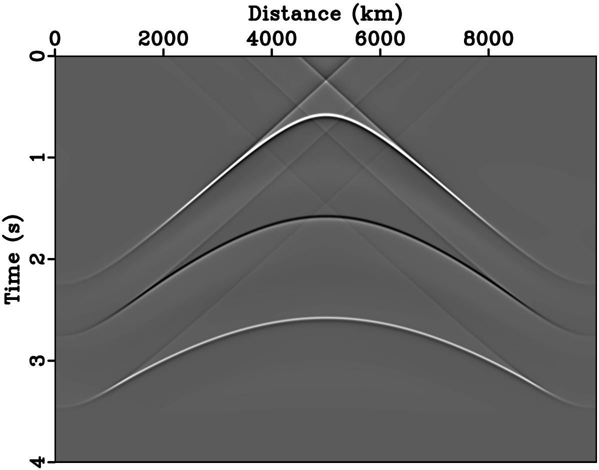
Numerical example



Data

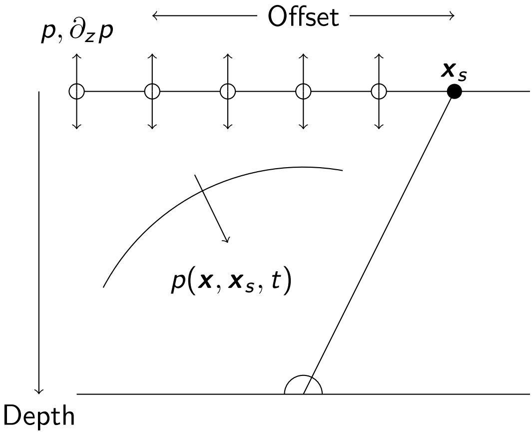
Numerical example

$p(\boldsymbol{x}, \boldsymbol{x}_s, t)$ at depth of 1000 m.



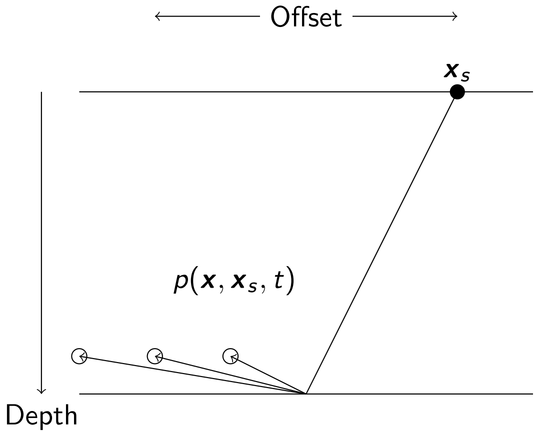
Data at depth 1000m

Imaging condition III

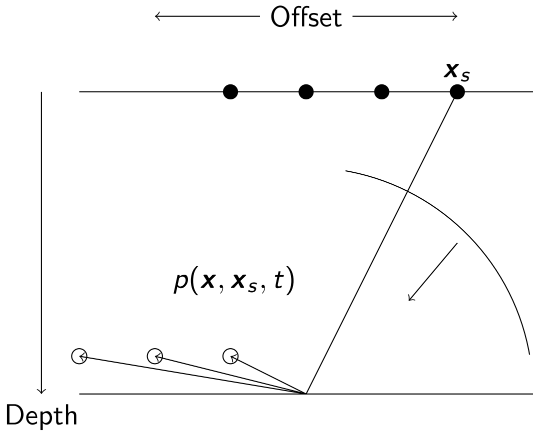


p: Scattered wavefield
$\boldsymbol{x}_s$: Source position
$\boldsymbol{x}, t$: Position,time

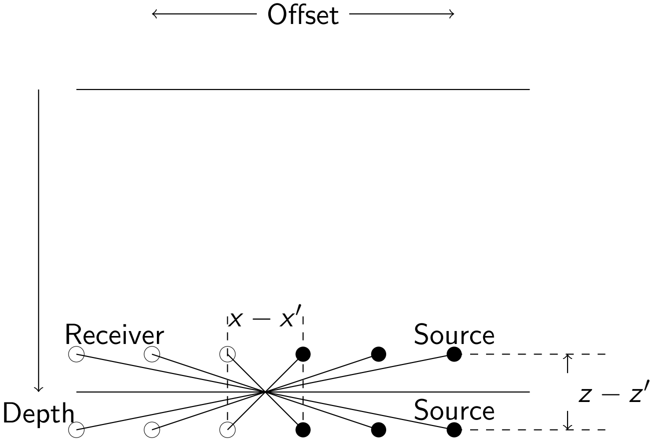
Imaging condition IV



Imaging condition V



Imaging condition VI



$x - x'$: Horizontal offset
$z - z'$: Vertical offset

Imaging

$$
\begin{aligned}
&p(\boldsymbol{x}, \boldsymbol{x}_s, -t) * h(t) - p(\boldsymbol{x}, \boldsymbol{x}_s, t) * h(-t) = \\
&\quad \int dS(\boldsymbol{x}') \, \big[p(\boldsymbol{x}', \boldsymbol{x}_s, -t) * \nabla p(\boldsymbol{x}, \boldsymbol{x}', t) \\
&\qquad - p(\boldsymbol{x}, \boldsymbol{x}', t) * \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, -t) \big] \cdot \boldsymbol{n}(\boldsymbol{x}')
\end{aligned} \tag{22}
$$

Use reciprocity again

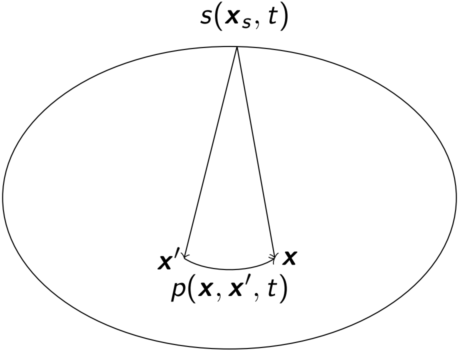
$$
\begin{aligned}
&p(\boldsymbol{x}_s, \boldsymbol{x}, -t) * h(t) - p(\boldsymbol{x}_s, \boldsymbol{x}, t) * h(-t) = \\
&\quad \int dS(\boldsymbol{x}') \, \big[p(\boldsymbol{x}_s, \boldsymbol{x}', -t) * \nabla p(\boldsymbol{x}, \boldsymbol{x}', t) \\
&\qquad - p(\boldsymbol{x}, \boldsymbol{x}', t) * \nabla p(\boldsymbol{x}_s, \boldsymbol{x}', -t) \big] \cdot \boldsymbol{n}(\boldsymbol{x}')
\end{aligned} \tag{23}
$$

Imaging

Rename $\boldsymbol{x}_s$ to $\boldsymbol{x}'$ and vice versa

$$
\begin{aligned}
& p(\boldsymbol{x}', \boldsymbol{x}, -t) * h(t) - p(\boldsymbol{x}', \boldsymbol{x}, t) * h(-t) = \\
& \quad \int dS(\boldsymbol{x}_s) \, \big[p(\boldsymbol{x}', \boldsymbol{x}_s, -t) * \nabla p(\boldsymbol{x}, \boldsymbol{x}_s, t) \\
& \qquad - p(\boldsymbol{x}, \boldsymbol{x}_s, t) * \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, -t) \big] \cdot \boldsymbol{n}(\boldsymbol{x}')
\end{aligned} \tag{24}
$$

Interferometry



Imaging

Define $r(\boldsymbol{x}', \boldsymbol{x}, t) = p(\boldsymbol{x}', \boldsymbol{x}, -t) * h(t) - p(\boldsymbol{x}', \boldsymbol{x}, t) * h(-t)$

$$
\begin{aligned}
&r(\boldsymbol{x}', \boldsymbol{x}, t) = \\
&\int dS(\boldsymbol{x}_s) \int d\tau \, \Big[p(\boldsymbol{x}', \boldsymbol{x}_s, t+\tau) \nabla p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \\
&\quad - p_0(\boldsymbol{x}, \boldsymbol{x}_s, t+\tau) \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \Big] \cdot \boldsymbol{n}(\boldsymbol{x}')
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
&r(\boldsymbol{x}', \boldsymbol{x}, t=0) = \\
&\int dS(\boldsymbol{x}_s) \int d\tau \, \Big[p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \nabla p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \\
&\quad - p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \Big] \cdot \boldsymbol{n}(\boldsymbol{x}')
\end{aligned} \tag{26}
$$

p_0: Forward wavefield

Imaging

The final imaging formula can be interpreted as

1. The left hand side is the wavefield at position $\boldsymbol{x}'$ due to a source at position $\boldsymbol{x}$. These two positions are arbitrary, so the sources (and receivers) are virtual.
2. The two terms on the right hand side is the cross-correlation between the extrapolated data and forward modelled data. Both the pressure and the derivative of the pressure are needed as data.

Imaging

Migration consists in:

1. Compute the forward wavefield $p(\boldsymbol{x}, \boldsymbol{x}_s, t)$ from equation (16)
2. Compute the backward wavefield $p(\boldsymbol{x}', \boldsymbol{x}_s, t)$ from equation (21).
3. Compute the image from equation (26)

Simplification

$$
\begin{aligned}
r(\boldsymbol{x}', \boldsymbol{x}, t = 0) = \\
\int dS(\boldsymbol{x}_s) \int d\tau \, \big[p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \nabla p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \\
- p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \big] \cdot \boldsymbol{n}(\boldsymbol{x}')
\end{aligned}
\tag{27}
$$

Horizontal receiver implies

$$
\begin{aligned}
r(\boldsymbol{x}', \boldsymbol{x}, t = 0) = \\
\int dS(\boldsymbol{x}_s) \int d\tau \, \big[p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \partial_z p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \\
- p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \partial'_z p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \big]
\end{aligned}
\tag{28}
$$

Simplification

$\partial_z p(\boldsymbol{x}', \boldsymbol{x}_s.t) = 0$ (No recorded pressure gradient)

$$r(\boldsymbol{x}', \boldsymbol{x}, t = 0) = \int dS(\boldsymbol{x}_s) \int d\tau \, p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \partial_z p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \tag{29}$$

Classical Imaging condition

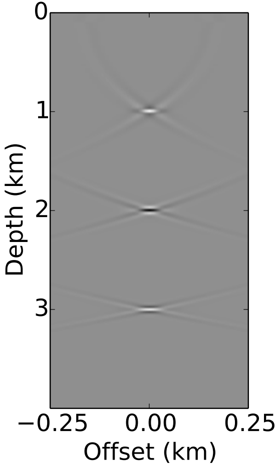
For $\boldsymbol{x}' = \boldsymbol{x}$ and by ignoring ∂_z this is the classical imaging condition (Claerbout, 1971)

$$r_c(\boldsymbol{x}) = \sum_{\boldsymbol{x}_s} \sum_{\tau} p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) p(\boldsymbol{x}, \boldsymbol{x}_s, \tau)$$

Ignoring ∂_z implies an unfocused image with less than optimal resolution and incorrect amplitudes.

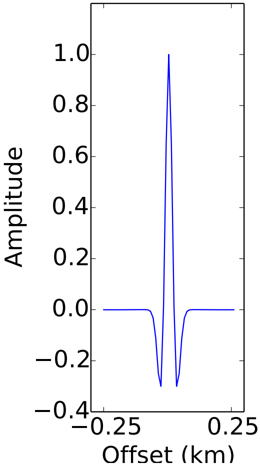
Numerical examples

Common image point gather (CIP) in the center of the model
Classical imaging condition:



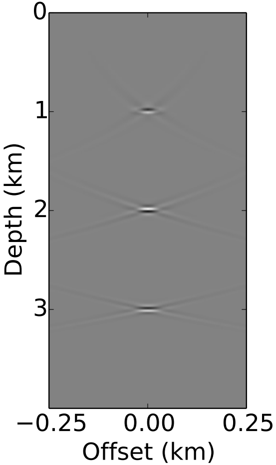
Numerical examples

Horizontal profile through reflector at 1000m depth



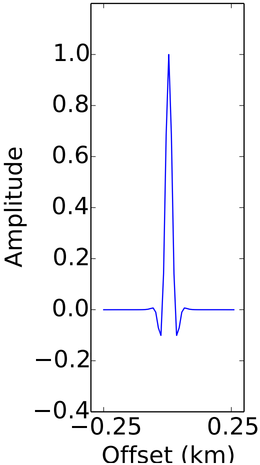
Numerical examples

Common image point gather (CIP) in the center of the model.
New imaging condition:



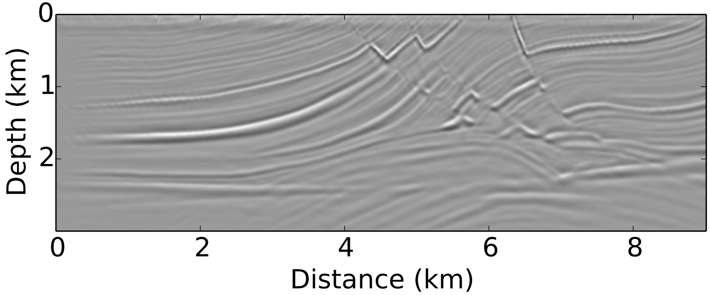
Numerical examples

Horizontal profile through reflector at 1000m depth



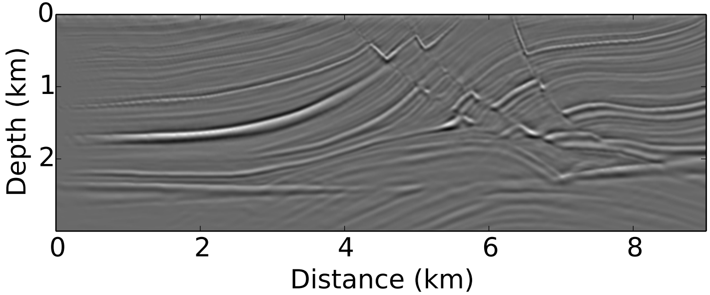
Numerical example

Conventional imaging condition:



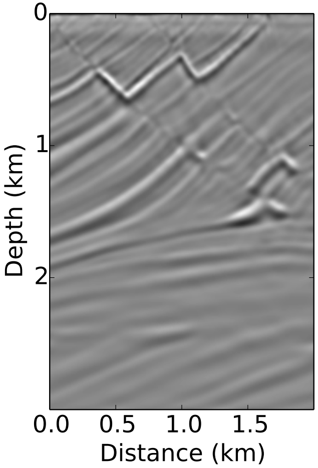
Numerical example

New imaging condition:



Numerical example

Conventional imaging condition:



Numerical example

New imaging condition: