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Suppression of multiple reflections using the Radon transform

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ABSTRACT

Multiple suppression using a variant of the Radon transform is discussed. This transform differs from the classical Radon transform in that the integration surfaces are hyperbolic rather than planar. This specific hyperbolic surface is equivalent to parabolae in terms of computational expense but more accurately distinguishes multiples from primary reflections. The forward transform separates seismic arrivals by their differences in traveltime moveout. Multiples can be suppressed by an inverse transform of only part of the data. Examples show that multiples are effectively attenuated in prestack and stacked seismograms.

INTRODUCTION

Removing reverberations from reflection seismograms has been a long standing problem of exploration geophysics. Multiple reflections often destructively interfere with the primary reflections of interest. The most robust and effective way to suppress multiples is stacking normal moveout corrected (NMOC) seismic gathers. Unfortunately, stacking does not eliminate all multiples. Also, stacking attenuates multiples only in stacked seismograms.

Over the years, many techniques for suppressing multiples have been tried. In recent years, generalized Radon transform approaches have attracted attention. The generalized Radon transform integrates (stacks) the data along curved surfaces, whereas the classical Radon transform stacks along planar surfaces (Beylkin, 1987). In practice, the technique is applied in the same way that the Fourier transform and slant stacking are used as dip filters for coherent noise with linear moveout. Hampson (1986) shows that a Radon transform using parabolic stacking surfaces suppresses multiples in NMOC gathers.

In this paper, we describe a procedure for suppressing multiples using a hyperbolic transform. The technique works by stacking NMOC seismic gathers over a range of hyperbolic surfaces. The forward transform is computed by applying a convolutional filter to the stacked data. In this domain, different events are characterized by their hyperbolic traveltime moveout. If the primary and multiple reflections have different moveouts, they will appear in different regions of the transform domain. This type of filtering is effective whenever there is sufficient separation of multiples and primaries in the transform domain. We choose hyperbolic stacking surfaces rather than parabolae because, on NMOC gathers, the residual moveout of multiples is closer to being hyperbolic than parabolic. The more closely the stacking surface matches the moveout of the events, the better the separation of multiples and primaries in the transform domain. The final step of the procedure is a partial inverse transform of the parameters containing multiples and then the subtraction of these data from the original seismogram.

Radon transform-based multiple suppression schemes are fundamentally limited by the transform's ability to resolve different events on the basis of moveout differences. This is particularly true for interbed multiples. Because seismic data consists of a finite number of discrete observation points, a least-squares solution of the transform, rather than the classical formulation, improves resolution (Thorson and Claerbout, 1985). From a practical point of view, the problem with this approach is that the calculation of the convolutional operator is time consuming, therefore expensive to compute.

In general, the cost of computing the least-squares solution is prohibitively expensive. This cost can be brought within a reasonable range by careful formulation of the problem. There are two important aspects to be considered in selecting the stacking surfaces. The first is that the stacking surface should be time invariant. Time invariance of the stacking surfaces excludes the conventional (Dix) stacking hyperbolae. The benefit of time invariance is that the computations can be performed in the frequency and space domain. Computationally, a key step in the calculation of the

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transform is applying the convolutional operator. The operator is found by inverting a matrix. In the space and time domain, this operator is very large and therefore expensive to invert. By operating in the space and frequency domain, the large matrix inversion reduces to a manageable number of much smaller operations. The second important consideration is that the matrix operators should have Toeplitz form, so that fast solvers (Levinson algorithm) can be used. Toeplitz form is achieved in a natural way by selecting uniform parameter spacing in the Radon domain. It is easy to specify hyperbolic surfaces satisfying both of these conditions. There is no difference in computational efficiency between this type of hyperbolic surface and parabolic ones. This means that resolution can be improved without additional expense.

Even with careful implementation, the computer cost of this type of processing is roughly equal to that of prestack migration. Depending upon the specific type of multiple problem, costs can be significantly reduced. Multiples often contaminate velocity analysis (VA). By removing multiples at only the seismic gathers where VA is performed, a better estimate of the stacking velocity can be obtained. A better estimate of the velocity results in better suppression of multiples in the stacked section. Another special application is removing water bottom multiples. In this problem, the parameter selection of the transform can be specialized to efficiently remove one type of multiple. In general multiple problems, each gather needs to be processed with a full range of parameters. This is the most robust way of attenuating multiples but also the most costly. This type of processing can dramatically influence amplitude versus offset (AVO) analysis.

ALGORITHM DESIGN

First, we define a transform

$$y(p, \tau) = \int_{x_{\min}}^{x_{\max}} z(x, \tau + p\theta(x))\, dx, \qquad (1)$$

where z is the original seismogram and the transform is y, x is a spatial variable, p is a ray parameter, and τ is intercept time. The function θ controls the moveout curves used in stacking z. Equation (1) reduces to the classical (slant stack) Radon transform when the integration limits are infinite and $\theta(x) = x$ (Deans, 1983). This equation generalizes the classical Radon transform from straight line integral paths to arbitrarily curved lines. In geophysics, the slant stack is used for decomposing reflected wavefields into plane-wave components while the velocity stack (hyperbolic surface) is used in velocity analysis.

Computationally, the problem becomes more manageable by transforming the time variable to the frequency domain. The 1-D Fourier transformed form of equation (1) is given as

$$\tilde{y}(p, \omega) = \int_{x_{\min}}^{x_{\max}} \tilde{z}(x, \omega) e^{i\omega p\theta(x)}\, dx. \qquad (2)$$

This equation shows that integration along curved lines in the time domain is represented as an integration of phase shifts in the transform domain. Because the seismogram is digitally recorded, a discrete summation replaces the integration, and equation (2) can be expressed as

$$\tilde{y}(p_j, \omega) = \sum_{k=1}^{N} \tilde{z}(x_k, \omega) e^{i\omega p_j \theta(x_k)} \Delta x_k \{k = 1, \ldots, N\}. \qquad (3)$$

This equation can be expressed in the more concise form

$$\mathbf{y}_\omega(p_j) = \underset{\sim}{\mathbf{R}}\mathbf{z}_\omega(\theta(x_k)) \qquad (4)$$

where the elements of the $M \times N$ dimensional matrix $\underset{\sim}{\mathbf{R}}$ are

$$R_{jk} = e^{i\omega p_j \theta(x_k)} \Delta x_k \begin{Bmatrix} j = 1, \ldots, M \\ k = 1, \ldots, N \end{Bmatrix}. \qquad (5)$$

The least-squares inversion formula for equation (4) is

$$\mathbf{z}_\omega(\theta(x_k)) = [\underset{\sim}{\mathbf{R}}^*\underset{\sim}{\mathbf{R}}]^{-1}\underset{\sim}{\mathbf{R}}^*\mathbf{y}_\omega(p_j). \qquad (6)$$

The numerical computation of equation (6) is time consuming when the offset receiver positions are irregular. Only when the receiver positions are regular, is $\underset{\sim}{\mathbf{R}}^*\underset{\sim}{\mathbf{R}}$ shift invariant (Toeplitz), allowing efficient matrix inversion schemes to be used.

Another way of formulating a transform is

$$\mathbf{z}_\omega(\theta(x_k)) = \underset{\sim}{\mathbf{R}}^*\mathbf{y}_\omega(p_j), \qquad (7)$$

where the $N \times M$ dimensional matrix $\underset{\sim}{\mathbf{R}}^*$ is the Hermitian conjugate of $\underset{\sim}{\mathbf{R}}$. Thorson and Claerbout (1985) and Hampson (1986) describe similar transforms. The least-squares inverse solution of equation (7) is

$$\mathbf{y}_\omega(p_j) = [\underset{\sim}{\mathbf{R}}\underset{\sim}{\mathbf{R}}^*]^{-1}\underset{\sim}{\mathbf{R}}\mathbf{z}_\omega(\theta(x_k)). \qquad (8)$$

Using equations (7) and (8) is computationally more efficient than equations (4) and (6) when the acquisition geometry is irregular. $\underset{\sim}{\mathbf{R}}\underset{\sim}{\mathbf{R}}^*$ is Toeplitz whenever the increment Δp_j is chosen to be constant. This property holds regardless of the experimental geometry. The inversion of a Toeplitz matrix is performed with the number of operations proportional to the number of stacking parameters squared (M^2). When Δx_k is variable, the matrix inversion of $\underset{\sim}{\mathbf{R}}^*\underset{\sim}{\mathbf{R}}$ requires computations on the order of the cube of the number of receiver positions (N^3).

Another important aspect of using equations (7) and (8) instead of equations (4) and (6) is the resolution of the forward transform. The convolutional operator helps improve resolution. When using equations (4) and (6), $[\underset{\sim}{\mathbf{R}}^*\underset{\sim}{\mathbf{R}}]^{-1}$ is applied following the inverse transform. This operator improves the accuracy of reconstructed data (estimate of the original data). The problem with this is that the resolution of the forward transform may not be adequate for resolving different events. On the other hand, if one uses equations (7) and (8), $[\underset{\sim}{\mathbf{R}}\underset{\sim}{\mathbf{R}}^*]^{-1}$ is applied as part of the forward transform, the spectra of hyperbolic events are much better resolved. This enhanced resolution is important in the filtering of multiples. Figure 1 demonstrates the focusing power of the convolutional filter and shows the resolution of the forward transform. These results are obtained by calculating the impulse response, which shows the resolution of the trans-

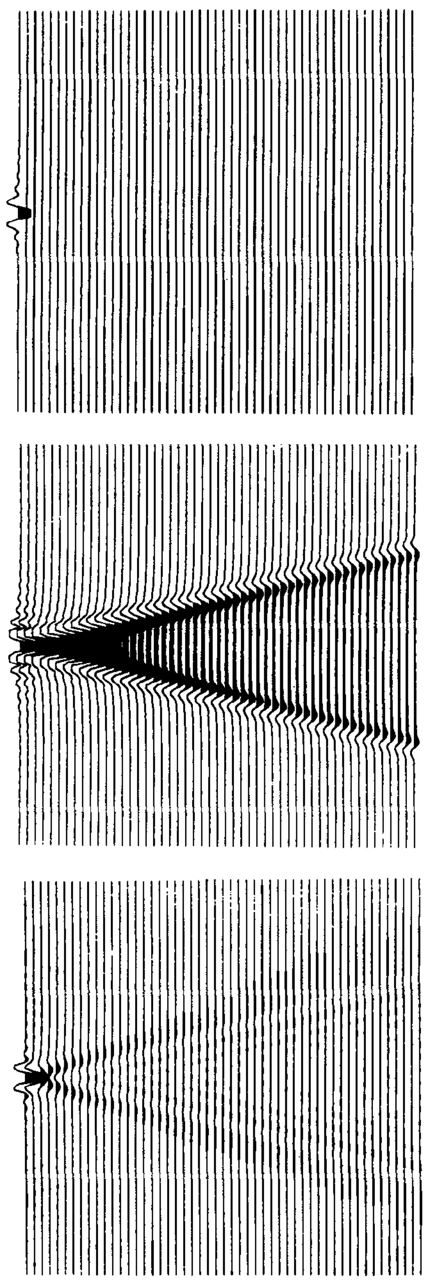

FIG. 1. The top plot is the desired theoretically exact result for an impulse model. The middle and bottom plots show the resolving power of the forward transform without and with the convolutional filter, respectively.

form given finite input data. The larger the input data, the closer the middle and bottom plots become. If the input data were infinite, all three plots would be identical. Figure 1 demonstrates that, given finite input data, the convolutional operator enhances resolution, as well as suppresses linear artifacts (edge effects).

APPLICATIONS

A useful application of the generalized Radon transform [equations (7) and (8)] is filtering coherent noise from seismic data. By parameterizing the moveout of any undesirable events on the seismogram, the unwanted energy can be selectively transformed, reproduced (inverse transformed), and then subtracted from the original data. The specific noise type we address here is multiple reflection. Multiple reflections have moveout curves that are hyperbolic with respect to traveltime and offset distance. Mathematically, the factor of the time delay function (phase shift) is given as

$$\theta(x_k) = \sqrt{(x_k)^2 + (z_{\text{ref}})^2} - z_{\text{ref}}, \tag{9}$$

where x_k is the offset receiver position and z_{ref} is a constant parameter defined as the reference depth. The choice of z_{ref} is not entirely arbitrary because the difference between these hyperbolae and those of reflected waves is controlled by this parameter. The smaller this difference the more compact the events will appear in the transform domain. In all computations presented here, the reference depth equals the maximum offset. With this value of z_{ref}, events reflected from this depth are optimally resolved.

Even on NMOC seismic gathers, the residual moveout of reflected waves are approximately hyperbolic. The closer the stacking surface matches the moveout of the reflections, the better the localization of different events in the transform domain. The basic assumption of this method is that the moveout of multiple reflections is different than that of the primaries. When multiple and primary events separate in the transform domain, the multiples can be distinguished from primaries. Figure 2 shows the errors of parabolic and hyperbolic [equation (9)] stacking surfaces. In this example, the error is calculated for a water bottom multiple reflection that has been moveout-corrected. The medium has a linear increase of velocity as a function of depth. In both curves, the zero offset traveltime and the transform parameter p are determined optimally in the least-squares sense. The reflector depth is 8000 ft and for the hyperbolic surface the reference depth is 10 000 ft (3048 m). The hyperbolic surface has smaller errors than that of the parabola.

The algorithm for removing multiples first requires an estimate of where the multiple energy is found in the transform domain. Then the transformed wavefield is computed by equation (8). The matrix inversion is performed by a complex Levinson recursion scheme. Estimating the multiples is done by inverse transforming [equation (7)] over the range of parameters containing multiple energy. Figure 3 shows the results of this processing. In Figure 3a the raw common-midpoint (CMP) gather is shown, and the estimate of the multiples in this gather is displayed in Figure 3c. The gather shown in Figure 3b is the primary event.

Even with careful considerations, computer costs are approximately equal to those of prestack migration. There-

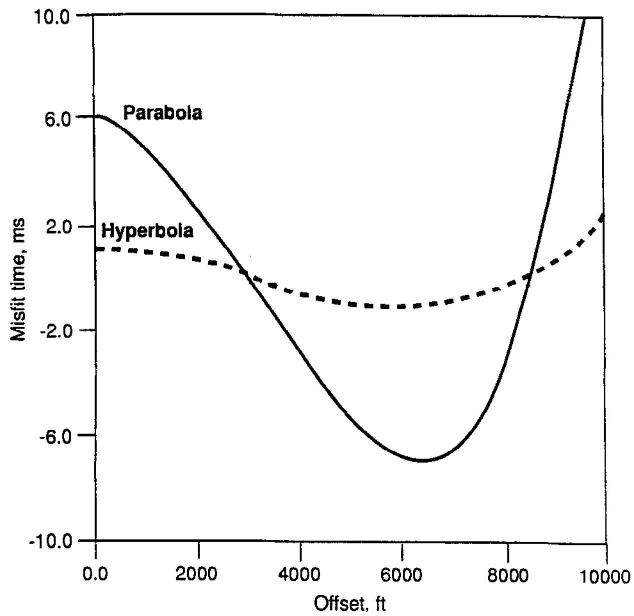

FIG. 2. The traveltime errors of parabolic and hyperbolic stacking surfaces for a water bottom multiple are shown. The multiple (assuming a 8000 ft reflector depth) is moved out with a stacking velocity that increases linearly with depth. The intercept is the velocity of water and the slope is $.5\ s^{-1}$.

fore, applications of the method should be judiciously chosen. For long period multiples, stacking can suppress multiples if a sufficiently accurate stacking velocity is used. Removing multiples from those gathers where velocity analysis (VA) is performed can help in improving the accuracy of the stacking velocity. If only the gathers at VA locations are processed, the prestack processing costs can be significantly reduced.

Figure 4 shows an example of semblance plots before and after multiple removal. These particular multiples were generated by layers of coal in the shallow portion of a seismic section. In this case, the difference in moveout between multiples and primary is large enough that prestack processing of just the VA gathers, repicking the velocity, and then stacking all gathers is adequate for suppressing multiples. There is no need to remove multiples with prestack processing of each gather.

For amplitude versus offset (AVO) analysis, removing multiples beforehand can improve results dramatically. Figure 5 shows the AVO response obtained from a seismic gather from the Gulf of Mexico before and after multiple suppression. The broad distribution in the crossplot before multiple suppression demonstrates the destructive interference of multiples. The intercept and slope of the reflection coefficient as a function of offset are quite different before

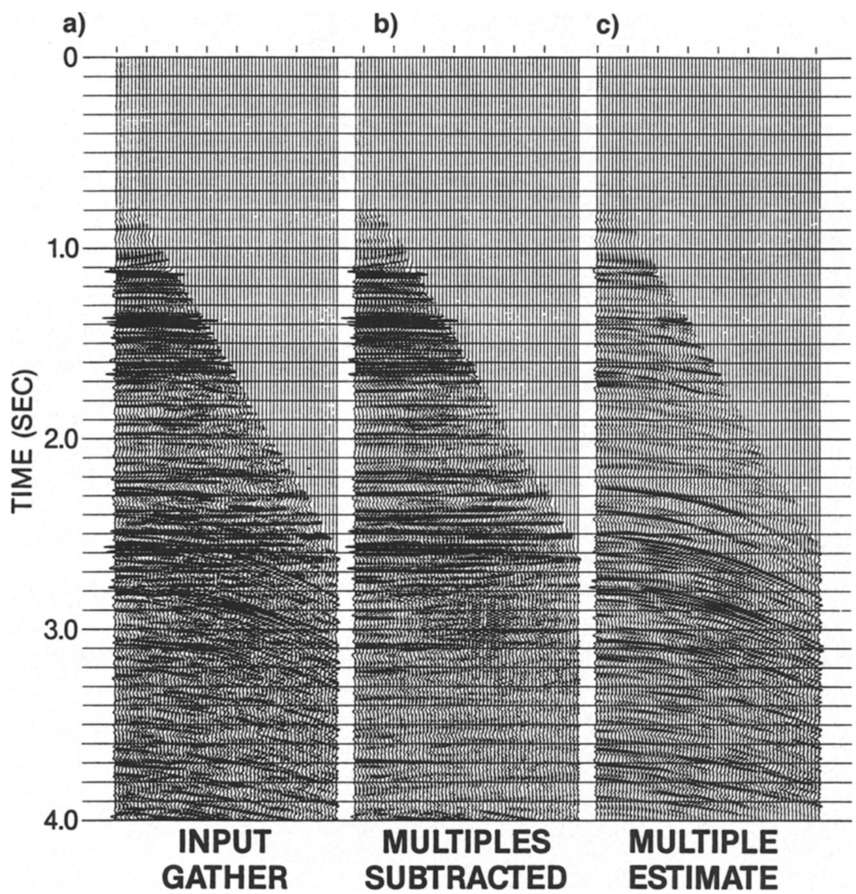

FIG. 3. (a) is on the left and is a normal moveout corrected CMP gather. The right hand plot (c) is the inverse transformed estimate of the multiples contained in (a). The middle plot (b) is the result of subtracting (c) from (a).

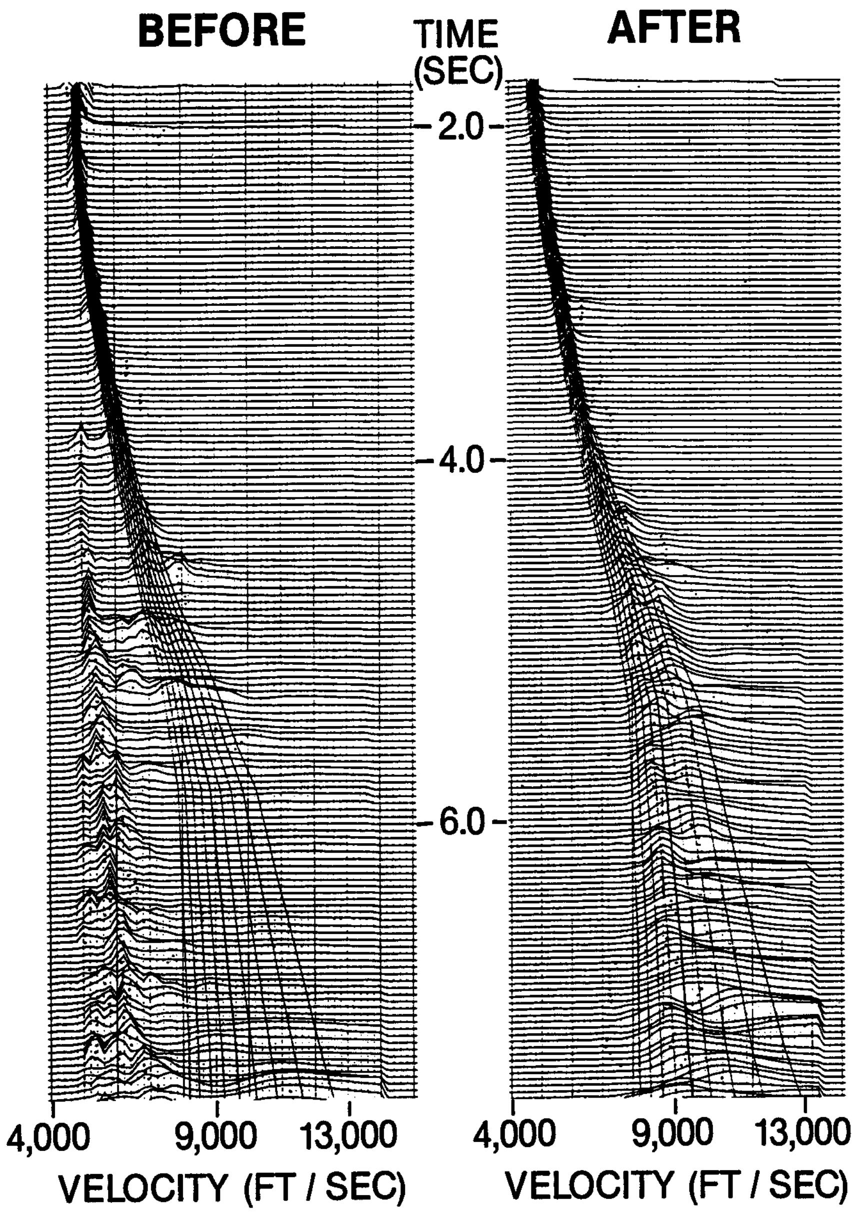

FIG. 4. Velocity analyses are plotted before and after multiple suppression.

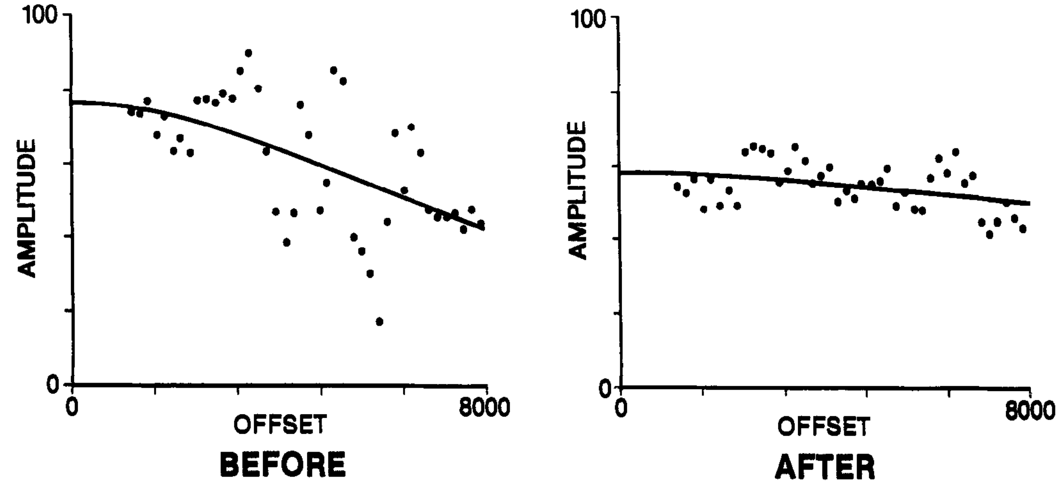

FIG. 5. Graphs of amplitude versus offset (AVO) for an event before and after multiple suppression are shown. The points indicate the amplitude response at the individual receiver locations, and the solid line is the root-mean-squared value.

and after removing the multiples. In this example, the interference of the multiples gave a false indication of the presence of gas. After removing the multiples the correct AVO response is obtained.

In some cases, multiples are not entirely removed by stacking even if the velocity is accurately known. An example is short period multiples (e.g., interbed reverberations), which are not always suppressed by stacking. Another

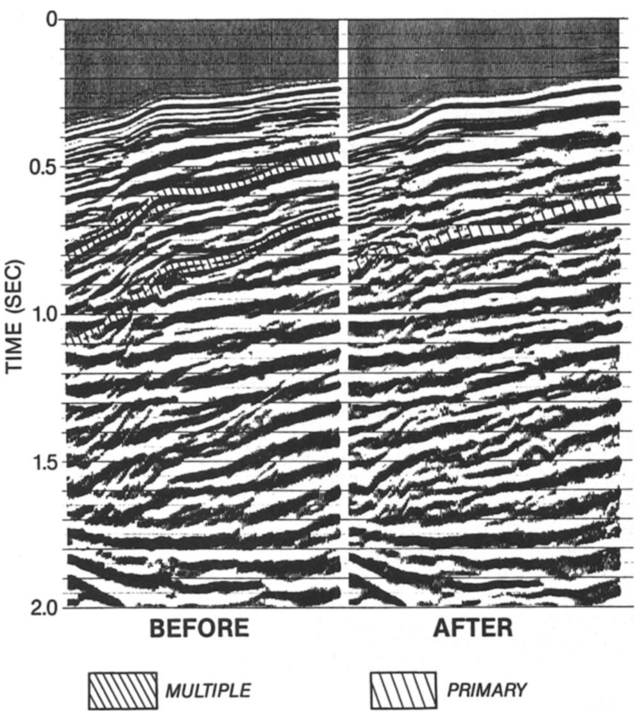

FIG. 6. Stacked data with and without multiple suppression.

example is water bottom multiples from a hard bottom. In these and other situations, each gather needs to be processed individually. Figure 6 shows stacked sections from offshore northern California with and without multiple removal. Each gather is processed in these data. In this profile, the main problem arises from the water bottom multiples that obscure stratigraphic objectives deeper in the section. The conventionally stacked section shows multiples. In these data, multiples appear to have been effectively removed. In the case of water bottom multiples, computer time can be reduced by moving out the CMP gathers at the velocity of water; then the transform focuses the multiples that can be inverse transformed and subtracted from the original gather. The savings comes from using a smaller number of transform parameters. If there is only one type of multiple, it is more efficient to focus the multiples rather than the primary reflections. The calculation of the convolutional filter is proportional to the number of stacking parameters squared.

An example of interbed multiples is shown in Figure 7. These stacked and migrated data are from onshore Gulf of Mexico. The objective is around 4 s and is approximately in the middle of the section. This is a gas prospect and multiples are obscuring the fault blocks. Figure 8 is a clearer picture of the fault after multiples have been suppressed. In this processing, the CMP gathers have been moved out with the primary velocities.

Another example of interbed multiples is shown in Figures 9 and 11. These data are from the Gulf of Mexico (offshore). In both of these data, the effects of multiple interference are subtle. In Figure 9, multiple interference makes it difficult to estimate the displacement along the faults. Figure 10 shows a much better resolved profile from which it is easier to estimate the displacement of the faults. In Figure 11, reflectors should rollover slightly but this is obscured by multiples. Figure 12 shows a much more interpretable section.

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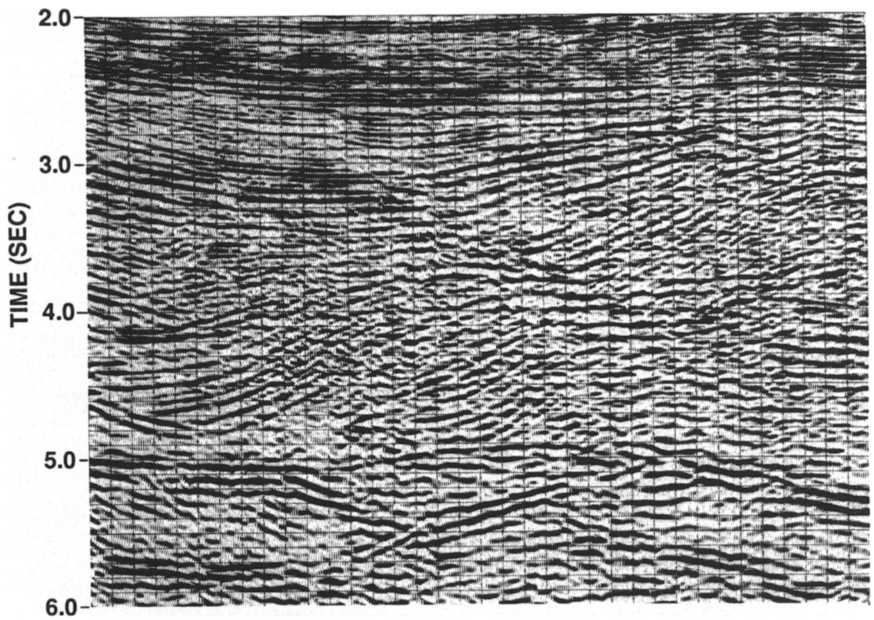

FIG. 7. Stacked and migrated data with multiples.

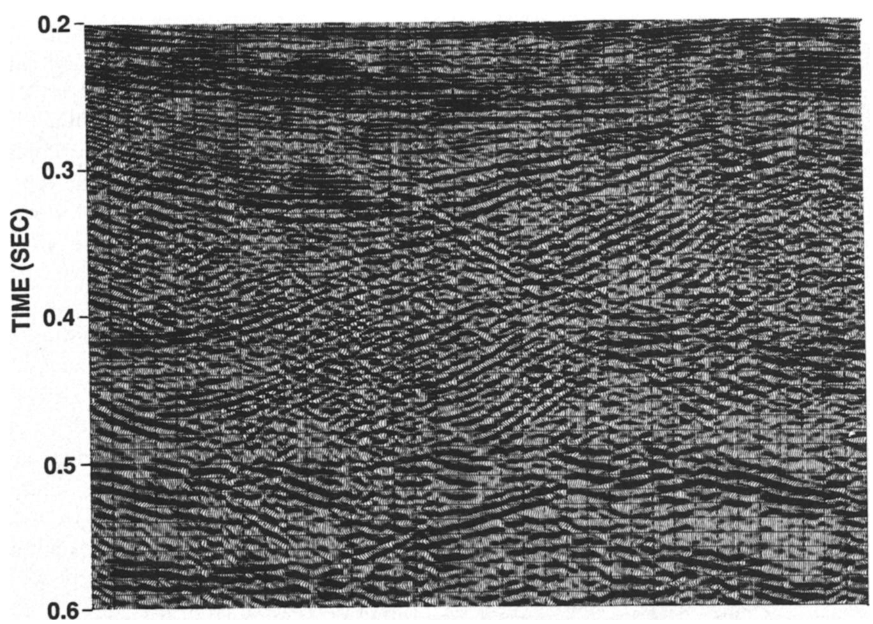

FIG. 8. Stacked and migrated data without multiples.

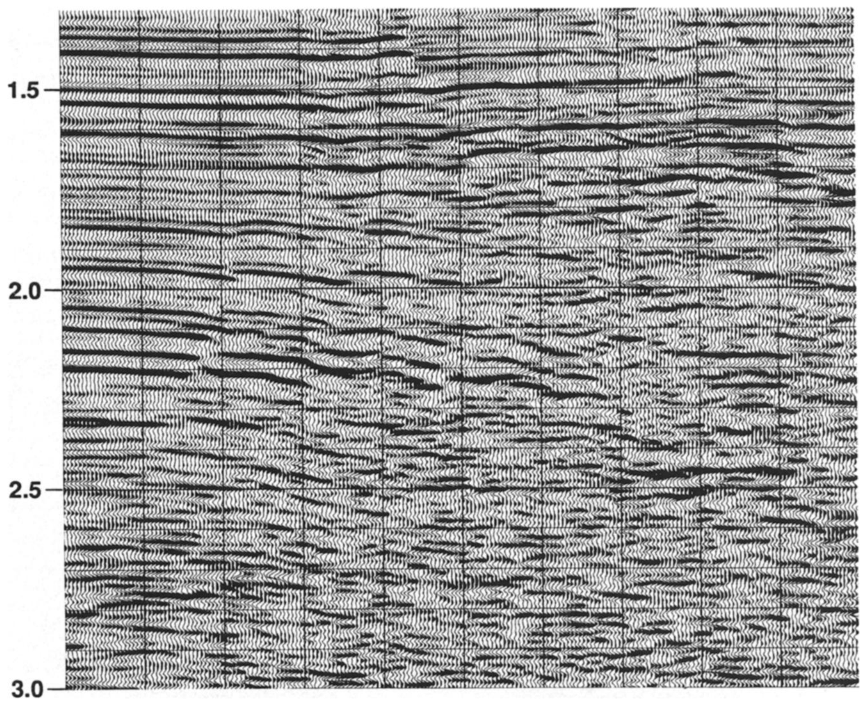

FIG. 9. Stacked data with multiples.

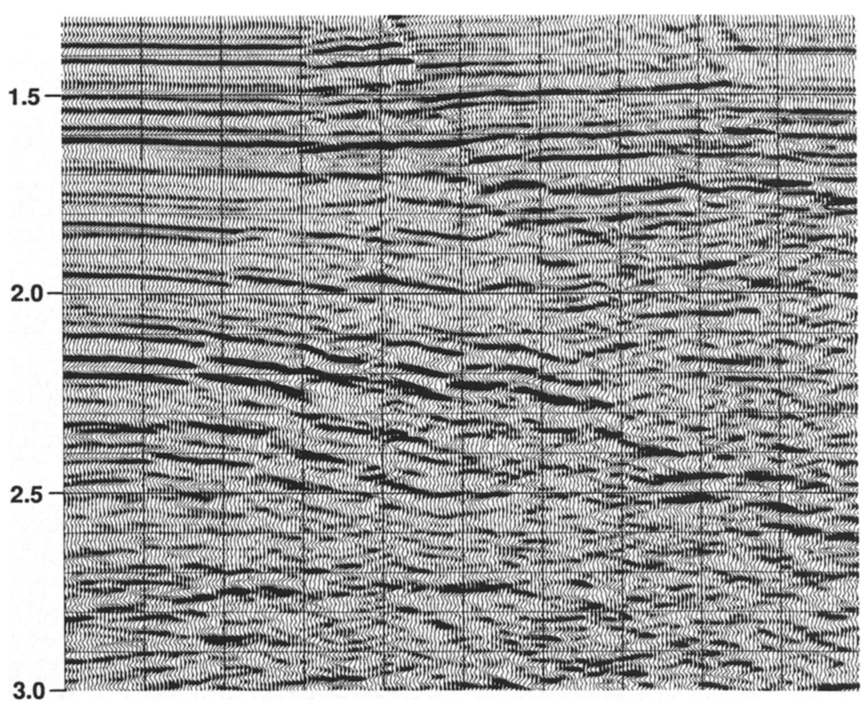

FIG. 10. Stacked data without multiples.

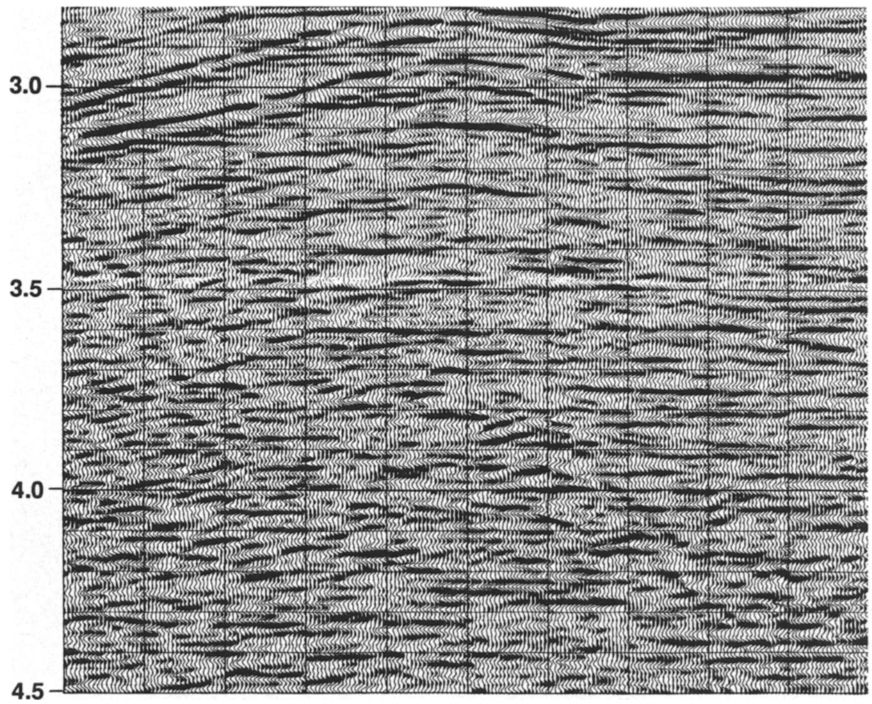

FIG. 11. Stacked data with multiples.

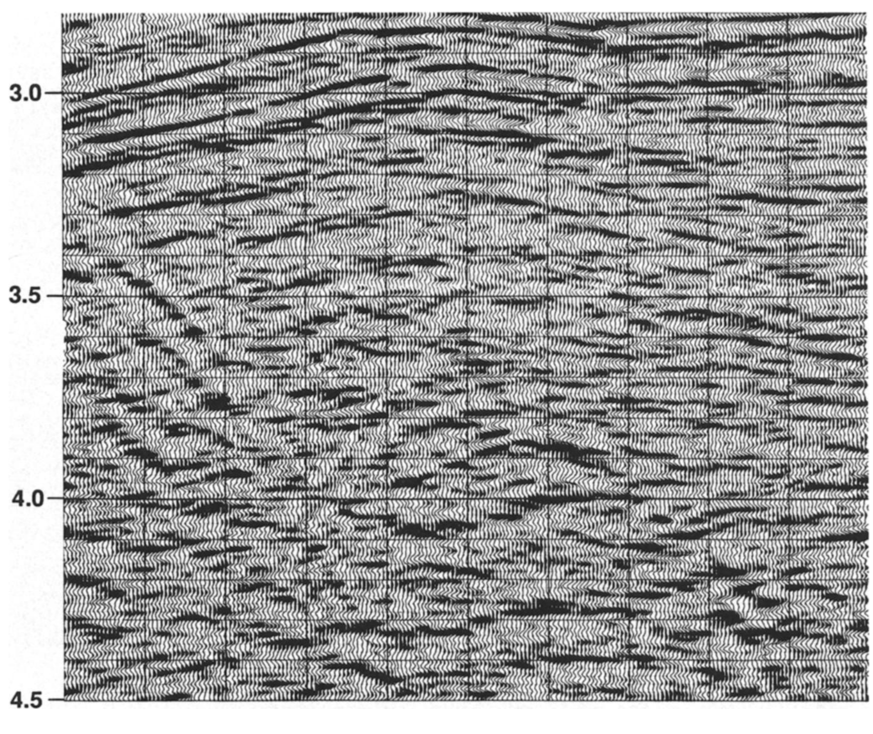

FIG. 12. Stacked data without multiples.

CONCLUSIONS

A coherent noise rejection method using a generalized Radon transform is effective in removing multiple reflections. This generalized transform stacks CMP gathers using hyperbolic integration surfaces. In general, the computer costs are expensive but with careful considerations in formulating the transform, the expense is on the order of prestack migration. One important consideration is the integration surface used in the transform. An efficient hyperbolic surface is given that is more accurate than the more conventional parabolic surface. The computer costs associated with these hyperbolae is the same as parabolic surfaces. Costs can be further reduced by careful application to the specific exploration objectives.

The most cost efficient application is to remove multiples from only those gathers that are being used for velocity analysis. This approach is particularly useful when multiple interference makes it hard to determine the stacking velocity. Given an accurate velocity, stacking will suppress most multiples. In cases where multiples still stack into the final section, the multiple suppression process needs to be applied to each gather. For particularly sensitive prestack analysis like AVO, multiple suppression on each gather is most desirable. Processing each gather is more expensive but is clearly effective in suppressing multiples where conventional stacking does not.

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