TPG4190 Seismic data acquisition and processing
Imaging

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Overview

- Imaging condition
- Simplification
- Comparison with classical imaging condition

Imaging condition

For the forward modeling we have in the time domain (Equation (13) in lecture 10).

$$p(\boldsymbol{x}, \boldsymbol{x}_s, t) = \int dV(\boldsymbol{x}') g(\boldsymbol{x}, \boldsymbol{x}', t) * s(\boldsymbol{x}', t)$$

$$+ \int dS(\boldsymbol{x}') \left[g(\boldsymbol{x}, \boldsymbol{x}', t) * \nabla p(\boldsymbol{x}', \boldsymbol{x}_s) - p(\boldsymbol{x}', \boldsymbol{x}_s, t) * \nabla g(\boldsymbol{x}, \boldsymbol{x}') \right] \cdot \boldsymbol{n}(\boldsymbol{x}')$$

However, for modelling we can neglect the surface integral

$$p(\boldsymbol{x}, \boldsymbol{x}_s, t) = \int dV(\boldsymbol{x}') g(\boldsymbol{x}, \boldsymbol{x}', t) * s(\boldsymbol{x}', t). \tag{1}$$

Imaging condition

For the time reversed focusing we have (Equation (19) in lecture 10), but with reinsertion of volume integral for the $s'(\boldsymbol{x}, t)$ source

$$p(\boldsymbol{x}, \boldsymbol{x}_s, -t) * h(t) = \int dV \, p(\boldsymbol{x}, \boldsymbol{x}_s, t) * s'(\boldsymbol{x}_s, -t) + \int dS(\boldsymbol{x}') \left[p(\boldsymbol{x}', \boldsymbol{x}_s, -t) * \nabla p(\boldsymbol{x}, \boldsymbol{x}', t) - p(\boldsymbol{x}, \boldsymbol{x}', t) * \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, -t) \right] \cdot \boldsymbol{n}(\boldsymbol{x}') \quad (2)$$

Assume $h(-t) = \delta(-t)$. Then $g(\boldsymbol{x}, \boldsymbol{x}', t) = p(\boldsymbol{x}, \boldsymbol{x}', t)$ and neglecting the volume term (is actually a sink)

$$p(\boldsymbol{x}, \boldsymbol{x}_s, -t) = + \int dS(\boldsymbol{x}') \left[p(\boldsymbol{x}', \boldsymbol{x}_s, -t) * \nabla g(\boldsymbol{x}, \boldsymbol{x}', t) - g(\boldsymbol{x}, \boldsymbol{x}', t) * \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, -t) \right] \cdot \boldsymbol{n}(\boldsymbol{x}') \quad (3)$$

Imaging condition

For the computaion of the image we have from lecture 10 (Equation 24) Define
$r(\boldsymbol{x}', \boldsymbol{x}, t) = p(\boldsymbol{x}', \boldsymbol{x}, -t) * h(t) - p(\boldsymbol{x}', \boldsymbol{x}, t) * h(-t)$

$$
\begin{aligned}
r(\boldsymbol{x}', \boldsymbol{x}, t = 0) = \\
\int dS(\boldsymbol{x}_s) \int d\tau \, \Big[p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \nabla p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \\
- p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \Big] \cdot \boldsymbol{n}(\boldsymbol{x}')
\end{aligned}
\tag{4}
$$

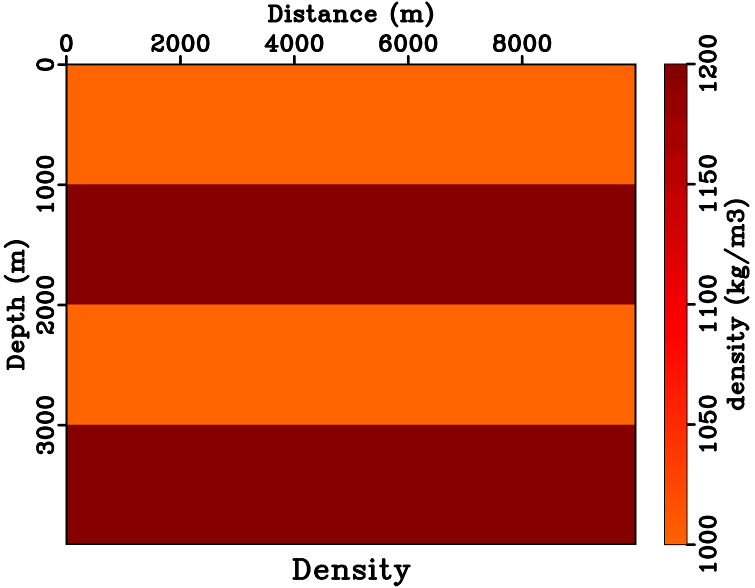
p_0: Forward modeled data
p: Backpropagated data

Imaging

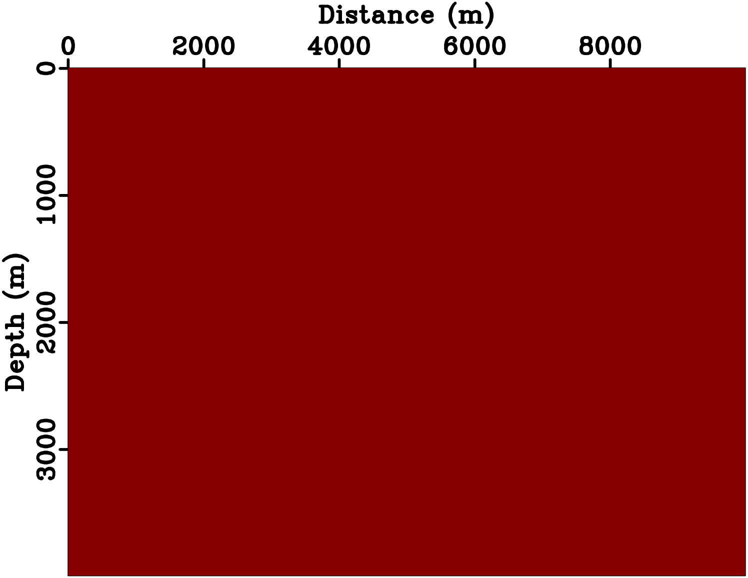
Migration consists in:

1. Compute the forward wavefield $p(\boldsymbol{x}, \boldsymbol{x}_s, t)$ from equation (2)
2. Compute the backward wavefield $p(\boldsymbol{x}', \boldsymbol{x}_s, t)$ from equation (3).
3. Compute the image from equation (4)

Numerical example

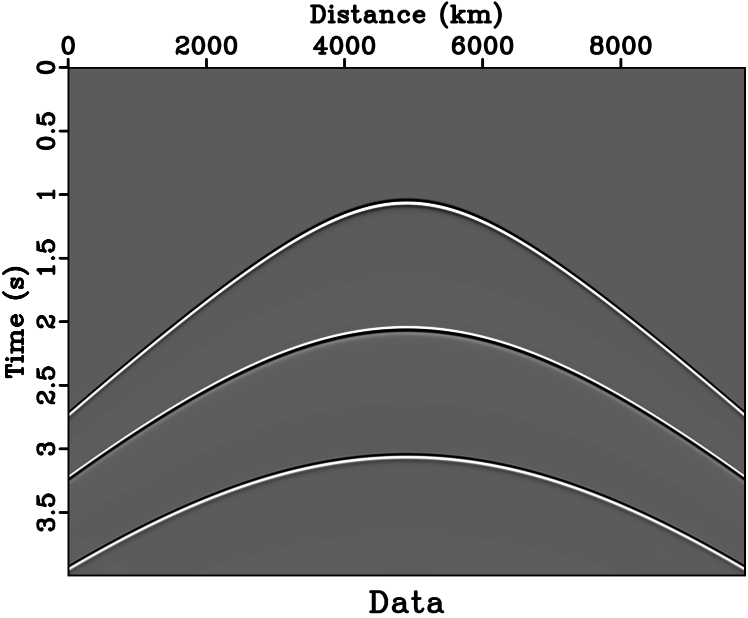


Numerical example



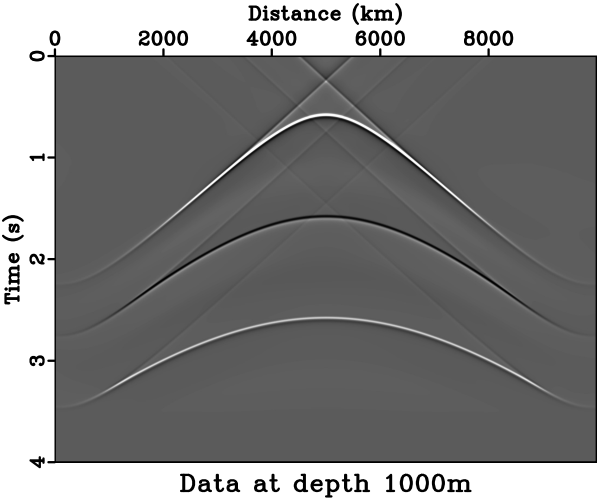
Velocity

Numerical example

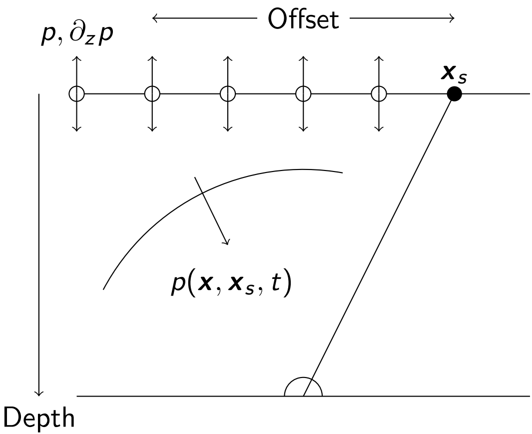


Numerical example

$p(\boldsymbol{x}, \boldsymbol{x}_s, t)$ at depth of 1000 m.



Imaging condition III

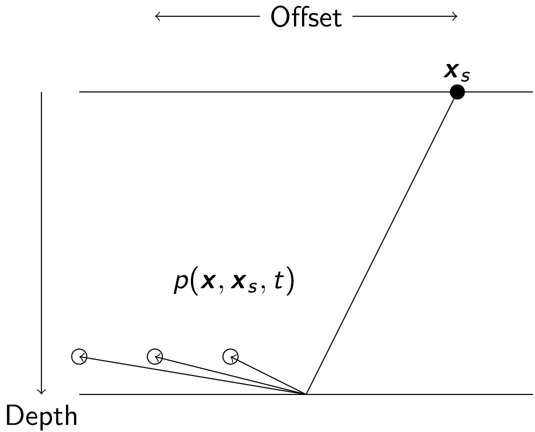


p: Scattered wavefield

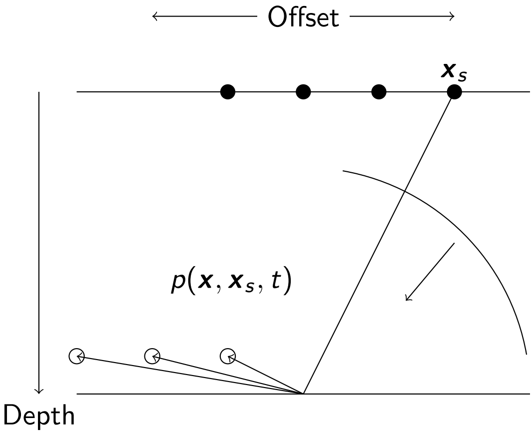
$\boldsymbol{x}_s$: Source position

$\boldsymbol{x}, t$: Position,time

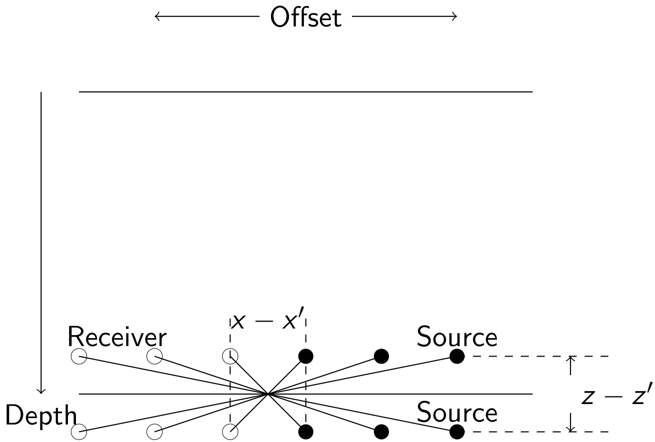
Imaging condition IV



Imaging condition V



Imaging condition VI

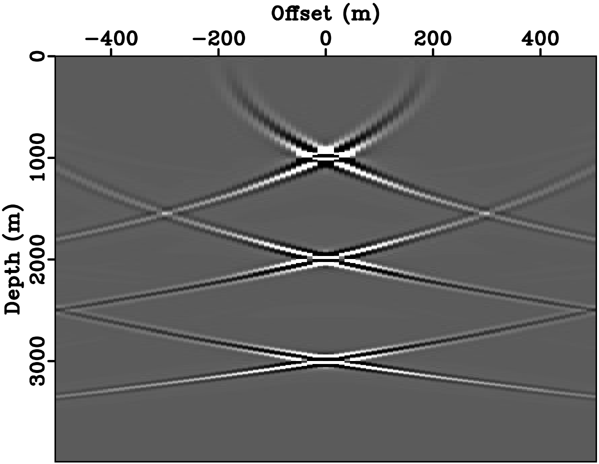


$x - x'$: Horizontal offset

$z - z'$: Vertical offset

Numerical example

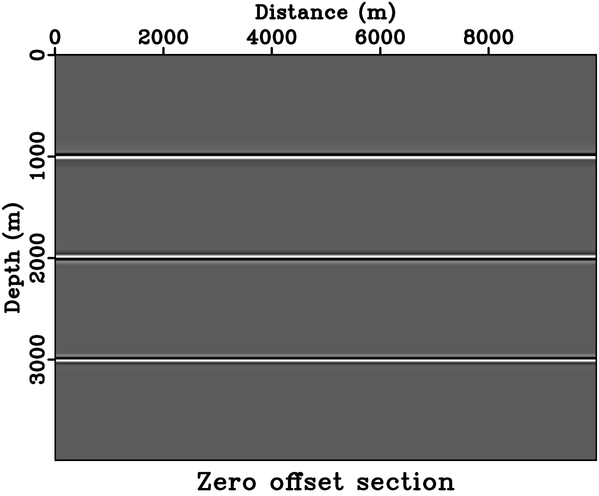
Reflectivity $p(\boldsymbol{x} - \boldsymbol{x}', t = 0)$ at all depths using new imaging condition



Offset CIP gather

Numerical example

Full section $p(\boldsymbol{x} - \boldsymbol{x}' = 0, t)$ at all depths using new imaging condition



Simplification

$$
\begin{aligned}
&r(\boldsymbol{x}', \boldsymbol{x}, t = 0) = \\
&\int dS(\boldsymbol{x}_s) \int d\tau \, \big[p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \nabla p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \\
&\quad - p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \nabla p(\boldsymbol{x}', \boldsymbol{x}_s, \tau)\big] \cdot \boldsymbol{n}(\boldsymbol{x}')
\end{aligned} \tag{5}
$$

Horizontal receiver implies

$$
\begin{aligned}
&r(\boldsymbol{x}', \boldsymbol{x}, t = 0) = \\
&\int dS(\boldsymbol{x}_s) \int d\tau \, \big[p(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \partial_z p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \\
&\quad - p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) \partial_z' p(\boldsymbol{x}', \boldsymbol{x}_s, \tau)\big]
\end{aligned} \tag{6}
$$

Simplification

$\partial_z p(\boldsymbol{x}', \boldsymbol{x}_s.t) = 0$ (No recorded pressure gradient)

$$r(\boldsymbol{x}', \boldsymbol{x}, t = 0) = \int dS(\boldsymbol{x}_s) \int d\tau \, \left[p(\boldsymbol{x}', \boldsymbol{x}_s, \tau)\partial_z p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau)\right. \tag{7}$$

Classical Imaging condition

For $\boldsymbol{x}' = \boldsymbol{x}$ and by ignoring ∂_z this is the classical imaging condition (Claerbout, 1971)

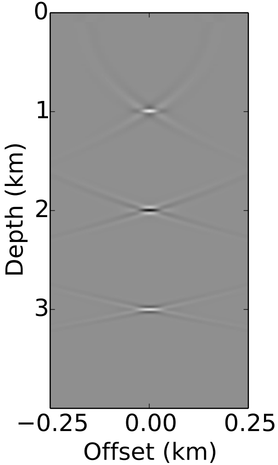
$$r_c(\boldsymbol{x}) = \sum_{\boldsymbol{x}_s} \sum_{\tau} p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau) p(\boldsymbol{x}, \boldsymbol{x}_s, \tau)$$

Ignoring ∂_z implies an unfocused image with less than optimal resolution and incorrect amplitudes.

Numerical examples

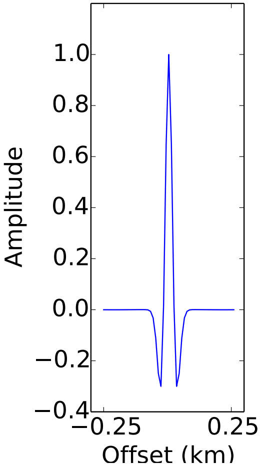
Common image point gather (CIP) in the center of the model

Classical imaging condition:



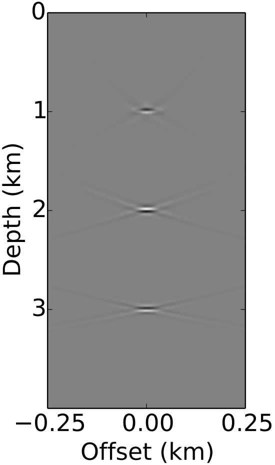
Numerical examples

Horizontal profile through reflector at 1000m depth



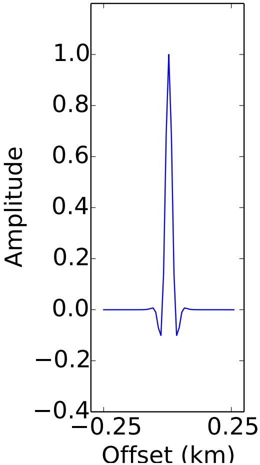
Numerical examples

Common image point gather (CIP) in the center of the model.
New imaging condition:



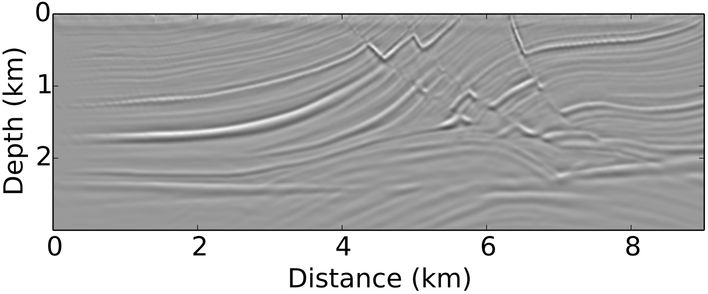
Numerical examples

Horizontal profile through reflector at 1000m depth



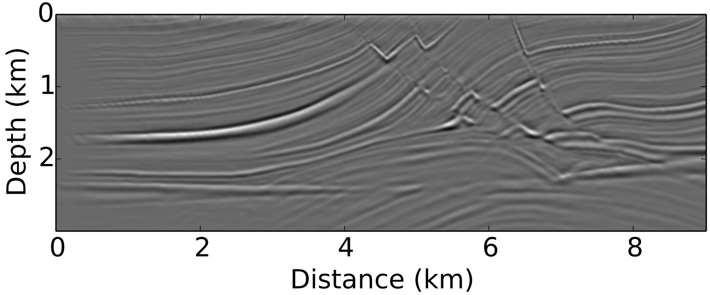
Numerical example

Conventional imaging condition:



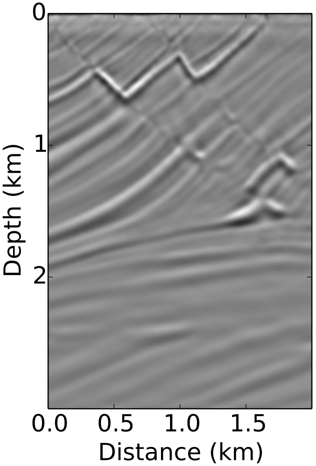
Numerical example

New imaging condition:



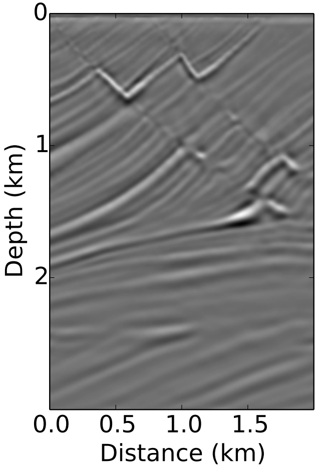
Numerical example

Conventional imaging condition:



Numerical example

New imaging condition:



Numerical example

From reflectivity to plane wave reflection coefficient

$$\partial_z r(\boldsymbol{x}, \boldsymbol{x}', t) \;\; = \;\; 2 \sum_{\boldsymbol{x}_s} \sum_{\tau} \partial^2_{z_s} p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau + t) p_{sc}(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \quad (8)$$

Plane wave reflection coefficient by mapping to $p - \tau$ (deBruin 1991**?**)

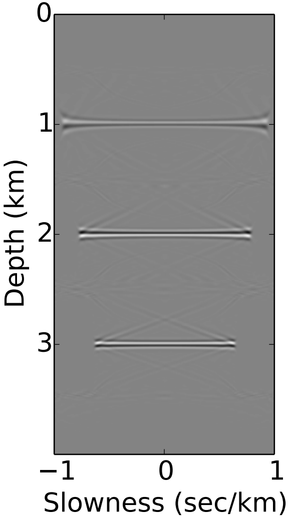
Conventional approach:

$$r(\boldsymbol{x}, \boldsymbol{x}', t) \;\; = \;\; 2 \sum_{\boldsymbol{x}_s} \sum_{\tau} p_0(\boldsymbol{x}, \boldsymbol{x}_s, \tau + t) p_{sc}(\boldsymbol{x}', \boldsymbol{x}_s, \tau) \quad (9)$$

Plane wave reflection coefficient by mapping to $p - \tau$ (deBruin 1991**?**)

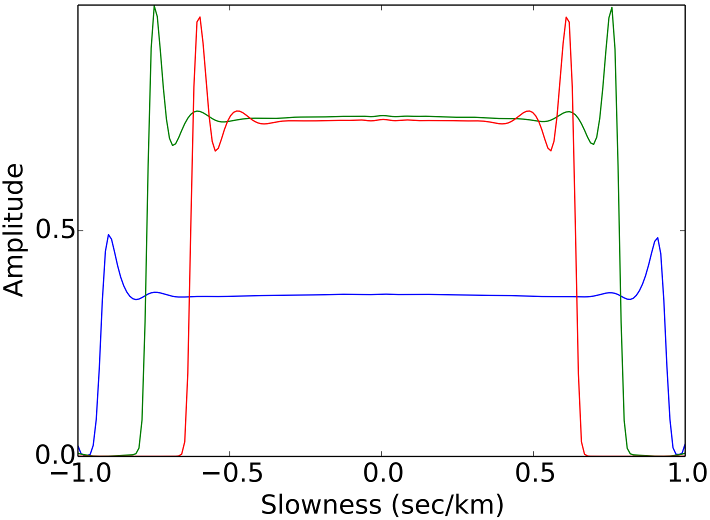
Numerical example

p-gather at the center of the model



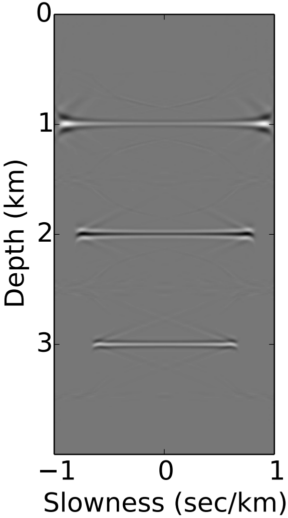
Numerical example

Amplitude picks along p-gather



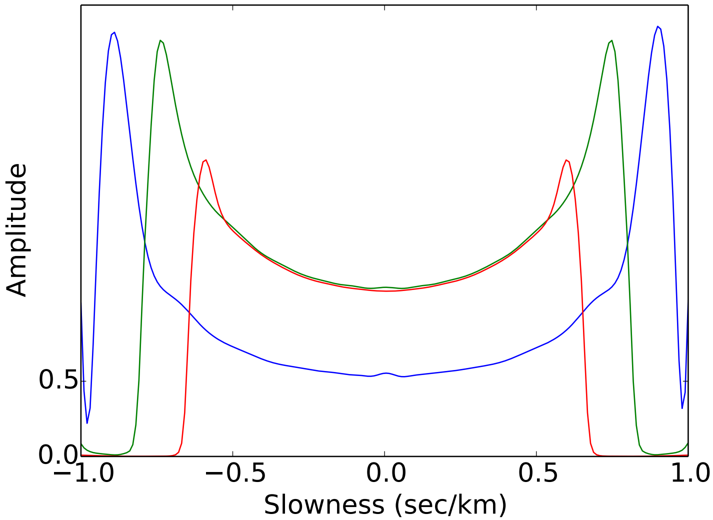
Numerical example

Conventional approach:



Numerical example

Amplitude picks along p-gather



Conclusions

Simple (trivial) modification of the classical imaging condition for Reverse-time migration gives

- Better resolution
- Reflectivity with correct angle behavior