Wiener spiking deconvolution and minimum A tutorial

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While working on the problem of enemy missile fire prediction at MIT during World War II, Norbert Wiener developed a statistical process which separated radar signals from noise. The process was originally known as smoothing or Wiener prediction filtering. The inverse process of unsmoothing, or prediction error filtering, was called decomposition and was later termed deconvolution. Today, prediction error filtering is the most commonly used technique for processing reflection seismic data. Prediction error filtering can be classified as either spiking or predictive deconvolution. This tutorial explains spiking deconvolution and elucidates how the abstract mathematics used to design spiking filters are actually easy to understand. First, the convolution model upon which deconvolution theory is based is briefly reviewed. This model breaks down the seismic trace into individual components. Spiking deconvolution makes assumptions concerning each of these components. It is clearly shown how the required assumptions are interwoven with the mathematical development of spiking filter design. Finally, a few examples illustrate the importance of minimum phase wavelets for acceptable filter performance.

The convolutional model. Wiener deconvolution is based on the simple, one-dimensional, plane-wave convolutional model of the seismic trace. Texts abound which explain the convolutional process and how it portrays seismic wave propagation as a linear filtering process. In this model, an input acoustic signal is transmitted through the earth and a filtered version of this signal is recorded at a later time. The earth is assumed to consist of a finite number of horizontal layers upon which the signal is directed at normal incidence (vertically). The simplest trace representation consists of an average wavelet w(t) convolved with a reflection coefficient series r(t). This noise-free trace is

$$x(t) = w(t)*r(t).$$

Obviously, many mathematical simplifications are incorporated here. Nevertheless, the convolutional model provides a good visualization and understanding of the reflection process. When a reflector is far enough away from a point source so that wavefront curvature is minimal, the simplifications are valid. Even though the one-dimensional model is most applicable to poststack data, it can also be used to represent prestack data when applied to small angles of incidence.

The linear convolutional model represents the ideal earth as a cascade of filters, one after the other. Because convolution is commutative, these individual filters can be rearranged regardless of their order of occurrence. The beauty of the convolutional model is that, instead of being considered individually, various complex earth filtering effects can be lumped together into a seismic wavelet. Out of the many wavelet components, a few important ones are the source signature, inelastic attenuation, and short-period multiples. The above simplistic convolutional grouping facilitates visualization of the task of spiking deconvolution: the removal of the composite wavelet filtering effect in order to uncover the earth reflection coefficient series.

Need for deconvolution. In radar and sonar applications, the velocity of the medium (be it air or water) is known and the arrival time of the reflected signal is used to give an indication of the distance to the object of interest. For these applications the filtering of the medium is negligible. In seismic reflection work, the medium is interpreted in a different way, in the sense that a single unknown component of the earth filter (i.e., the reflectivity) is the information the geophysicist would like to know. This alternative viewpoint, initiated by MIT's Geophysical Analysis Group in the 1950's, set the stage for the development of deconvolution and digital signal processing (see Robinson's "Predictive decomposition of seismic traces", GEOPHYSICS 1957). Deconvolution was refined in the field of exploration geophysics and presently is utilized in many other fields which require signal processing techniques. Each acoustic impedance [seismic velocity (v) x density (p)] boundary between different rock types generates a reflection. The reflection coefficient series consists of reflections located at each boundary according to the formula

$$r(t)= \frac{\nu_2 \rho_2 - \nu_1 \rho_1}{\nu_2 \rho_2 + \nu_1 \rho_1}$$

the wave is traveling from layer 1 to layer 2. These sharp spikes contain all possible seismic frequencies (as limited by the data sampling rate). Each spike location is convolved with the seismic wavelet as it passes through the earth.

The important concept of a seismic wavelet was developed by Ricker (see "The form and laws of propagation of seismic wavelets," GEOPHYSICS 1953). A wavelet is a transient with a definite start time and length. The instant each shot is fired, the source signature begins propagating through the earth. The signature is earth-filtered, a process which especially attenuates high frequencies. As a result of this filtering, the source signature (now known as the seismic wavelet) becomes more and more elongated in time. The convolutional model depicts wavelet propagation by positioning a wavelet beginning at each spike location within the trace. Since reflection spikes are closer together than a wavelet length, the spike locations become confused. Adjacent wavelets overlap each other, producing a tangle of constructive and destructive interference. Spiking deconvolution attempts to compress these wavelets in time, unravel the confusion, and resolve each sharp reflection. The convolutional model represents these innumerable wavelets as an average wavelet. Spiking deconvolution utilizes the convolutional model and, thus, assumes that wavelet properties do not change with time (are stationary).

Spiking filter design. Least-squares deconvolution filter design can be represented in the following way. The filter attempts to shape the input seismic trace x(t) into the desired output r(t) by minimizing the mean-squared error I between the desired output and the actual filter output y(t). The actual output is simply the input x(t) convolved with the filter f(t). The least-squares error is

$$I = \Sigma^{t}(r(t) - y(t))^2$$

which becomes, when substituting for y(t)

$$I = \Sigma^{t}(r(t) - x(t)*f(t))^2$$

is minimized by setting its partial derivatives, with respect to each filter coefficient, equal to zero. Differentiation and simplification results in the normal equations, which are a matrix representation of a series of simultaneous linear equations. These equations define the filter.

$$\phi_{xx}(\tau) * f(\tau) = \phi_{rx}(\tau)$$

To solve for the filter requires knowledge of the two correlation functions: the crosscorrelation function $\phi_{rx}(\tau)$ between the reflectivity and the trace, and the trace autocorrelation function $\phi_{xx}(\tau)$.

Crosscorrelation (CCF) and autocorrelation (ACF) functions are frequently used in seismic data processing programs. CCF is used to indicate the degree of correlation between two different time series as a function of time shift (or lag) between them. ACF is conveniently used to indicate any periodicities (repetitive convolutional components) present within a seismic trace. As Anstey states in “Correlation techniques - a review” *(Geophysical Prospecting* 1964): “The autocorrelation function of a waveform is a graph of the similarity between the waveform and a time-shifted version of itself, as a function of this time shift.” The ACF also contains the frequency content of the trace, and displays the energy distribution of these frequencies as a function of time shift. A signal with limited bandwidth (or frequency range), such as a single frequency sine wave, has an infinitely long sinusoidal ACE Any white time series (broadband, containing all possible frequencies) has an ACF which is nonzero only at zero time lag.

In general, Wiener filter theory is elegant in the sense that a filter can be determined only when an ACF and a CCF are known. In the spiking deconvolution case, the left side of the equation (ACF) can easily be calculated but the right side (CCF) can not because r(t) is unknown. An educated guess, or assumption, needs to be made concerning the statistical properties of the reflection coefficient series. This assumption is now used to develop the normal equations towards a solvable form.

Since convolution is commutative, $\phi_{xx}(\tau)$ and $\phi_{rt}(\tau)$ can each be conveniently rearranged into their r(t) and w(t) components. The ACF becomes

$$\begin{aligned}\phi_{xx}(\tau) &= [x(\tau)] * [x(-\tau)] \\ &= [w(\tau) * r(\tau)] * [w(-\tau) * r(-\tau)] \\ &= [r(\tau) * r(-\tau)] * [w(\tau) * w(-\tau)] \\ &= \phi_{rr}(\tau) * \phi_{ww}(\tau)\end{aligned}$$

And, the CCF between the reflectivity and the trace becomes

$$\begin{aligned}\phi_{rx}(\tau) &= r(\tau) * x(-\tau) \\ &= r(\tau) * [r(-\tau) * w(-\tau)] \\ &= [r(\tau) * r(-\tau)] * w(-\tau) \\ &= \phi_{rr}(\tau) * w(-\tau).\end{aligned}$$

The normal equations can now be rewritten

$$[\phi_{rr}(\tau) * \phi_{ww}(\tau)] * f(\tau) = \phi_{rr}(\tau) * w(-\tau).$$

Notice that the reflectivity ACF appears on both sides. In order to simplify further, it is now assumed: *The reflectivity series has the statistical properties of random white noise.*

In simpler terms, the reflectivity is assumed to have a reflection at each sample location (some of which must be zero), each of which has a random amplitude which in no way depends upon the amplitudes of any other reflection. When such a white time series is also stationary, the series will have an ACF which equals a spike at zero time shift.

With these ideal statistical properties, the reflectivity ACF now equals a spike at zero lag scaled by its power P, or

$$\phi_{rr}(\tau) = P\delta(0).$$

The symbol S(0) represents an idealized zero lag spike. Because the reflectivity ACF is only a scalar, it can be averaged out of the normal equations. This important simplification effectively transforms the normal equations from the “trace” to the “wavelet” domain.

$$\phi_{ww}(\tau) * f(\tau) = w(-\tau).$$

Since the reflectivity is assumed to have ideal statistical properties, it can be ignored for filter design. This is a paradox of spiking deconvolution: One first makes a questionable statistical assumption concerning the reflectivity of the earth in order to subsequently estimate this reflectivity. Studies of numerous well log-derived reflection coefficient series show that they can be “somewhat close” to having ideal statistical properties. Earth filters remove high and low frequencies from the seismic trace, so that the reflectivity cannot contain all frequencies as assumed. At any rate, I now ignore the influence of the reflectivity series on filter design.

A wavelet can be characterized by its amplitude spectrum (another way of displaying frequency content) and its phase spectrum (or phase delay). One can conceptualize a suite of causal wavelets (no amplitudes occur before the start time) all of which contain the same frequencies. These wavelets differ by phase delay only. There is a limit to how close the energy within any given wavelet can be compressed towards the start time. A unique minimum phase wavelet defines this limit. Even though the minimum phase concept is often defined mathematically, it is intuitively easy to grasp. Any minimum phase wavelet has the least amount of phase delay possible (it is the most front-loaded) as a function of a given frequency content (Figure 1).

Minimum phase filters are commonly encountered in naturally occurring causal systems. The elastic response of any sudden impact with a solid is minimum phase. There is one important relationship which connects the frequency content with the phase delay of a minimum phase wavelet. This is the Hilbert transform which can be used to reconstruct the minimum phase wavelet from its frequency content or from its ACE

Referring back to the "wavelet domain" normal equations, the wavelet ACF occurs on the left side and the wavelet itself (reversed in time) occurs on the right side. The wavelet frequency content can be determined from the left side wavelet ACE Since the wavelet remains unknown, the right side is unknown and the filter cannot be computed. It is now assumed: *The average wavelet is minimum phase.*

The wavelet on the right side of the normal equations is assumed to be minimum phase. Using the Hilbert transform, this minimum phase wavelet can be reconstructed from its left side ACE The spiking filter can now be computed.

A spiking filter is a finite length approximation to the exact inverse of a minimum phase wavelet. It is interesting to note that the spiking filter is also minimum phase. Any inverse of a minimum phase function is by definition itself minimum phase. Also, the convolutional product of any number of minimum phase functions is minimum phase.

The Wiener spiking deconvolution filter has been developed in light of a few key assumptions. By assuming that the earth reflectivity has ideal statistical properties, the trace ACF is reduced to the average wavelet ACE Subsequently, by assuming that the wavelet is minimum phase, this wavelet can be determined from the wavelet ACE Due to these assumptions, the filter is based solely on the trace ACE This simplicity of design has contributed significantly to the popularity of the spiking filter.

Wavelet analysis. From practical experience, most processing geophysicists would say that spiking deconvolution is robust. In other words, the process works well on a wide range of data sets despite questionable assumptions. As previously shown, visualization of filter action can be simplified by averaging the earth reflectivity out of the defining equations. Even though this is a gross simplification, the following analysis uses individual wavelets to promote a better understanding.

The minimum phase assumption concerns the statistical nature of an average seismic wavelet. This wavelet is equal to the convolutional product of the source signature and any subsequent earth filtering effects. All of these earth filters have been shown to be close to minimum phase. Since the convolutional product of various minimum phase filters is itself minimum phase, the minimum phase assumption would appear satisfied. Except that, in general, marine source signatures are mostly not minimum phase due to bubble pulse problems. Dynamite source signatures are very close to minimum phase. Consequently, the minimum phase assumption is probably more valid for land dynamite data than for marine data.

Any real world wavelet lacks many of the frequencies that were input into the source signature. Remaining frequencies

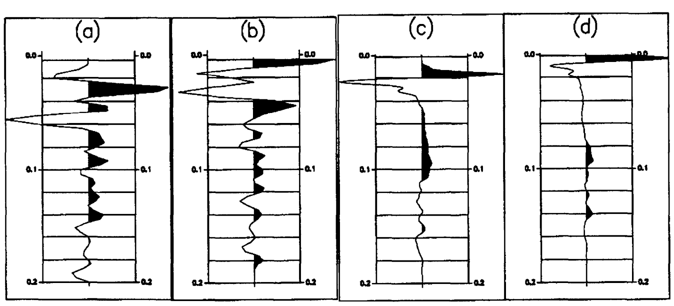

Figure 1. Inputs to a spiking deconvolution. (a) Prestack reflection. (b) Minimum phase prestack reflection. (c) Airgun signature. (d) Minimum phase airgun signature.

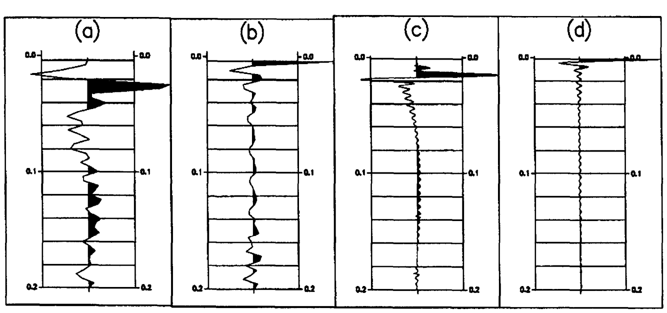

Figure 3. Outputs from spiking deconvolution applied to Figure 1.

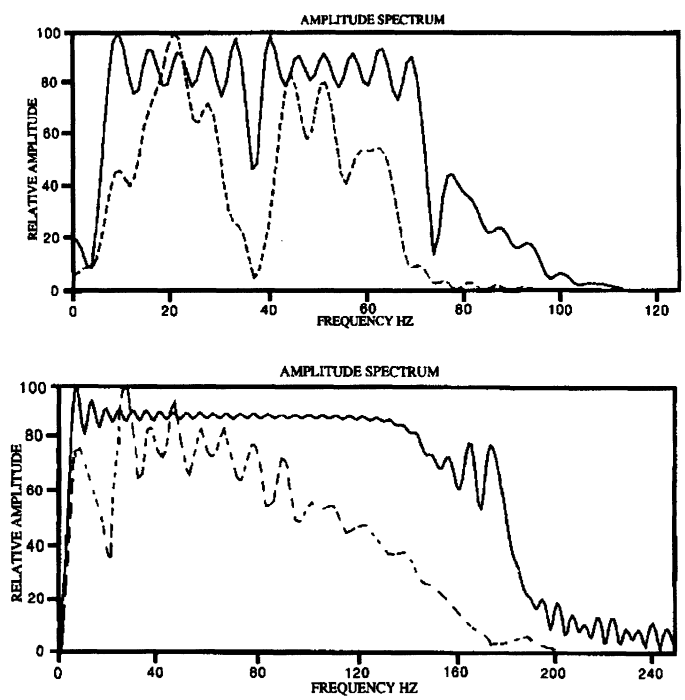

Figure 2. (a) Prestack reflection wavelet pair (dotted curve). Deconvolved prestack reflection wavelet pair (solid curve). (b) Airgun signature wavelet pair (dotted curve). Deconvolved airgun signature wavelet pair (solid curve).

are delayed in time. The spiking filter attacks this wavelet in two separate ways. The filter attempts to restore lost wavelet frequencies, and to remove the delay caused by minimum phase earth filtering. An ideal output would possess a broad-band and flat (white) frequency range with all frequencies restored, and a zero phase spectrum with all phase delays removed. Such an idealized output equals a spike at the wavelet start time.

If the input wavelet is minimum phase, the output equals

$$y(t) = f(t)*w_{min}(t).$$

Since $w_{mm}(t)$ is bandlimited, the output is bandlimited. The output, therefore, bears no resemblance to an ideal impulse located at the wavelet arrival time. The spiking filter is known to be minimum phase. Because the output is the product of two minimum phase functions, it is also minimum phase. In reality, any recorded seismic wavelet is mixed phase to some degree. This mixed phase wavelet can be conceptualized as the convolution of a minimum phase wavelet with a unique all-pass (does not attenuate any frequencies) phase delay filter p(t). The phase delay filter only causes a time delay:

$$W_{min}(t) = w_{min}(t) * p(t).$$

The mixed phase wavelet contains the same frequencies as the minimum phase wavelet. The spiking filter output will be a minimum phase function convolved with a phase delay filter. The output will be mixed phase.

$$\begin{aligned} y(t) &= f(t)*w_{mix}(t) \\ &= [f(t)*w_{min}(t)]*p(t). \end{aligned}$$

I now utilize simple examples to illustrate how the spiking filter compresses minimum and mixed phase wavelets. In the process, distortion resulting from wavelet phase delay is quantified.

Two pairs of input wavelets are shown in Figure 1. The spiking deconvolution outputs from these four wavelets will be analyzed. An unprocessed prestack reflection (PSR) extracted from an airgun marine survey and its minimum phase version constitute the first pair. A high frequency airgun subarray signature (AGS) from another survey and its minimum phase version constitute the second. The PSR is earth filtered. The AGS was recorded at a depth of 100 m below the array and is not earth filtered. Due to earth filtering, the PSR has a narrower frequency range than the AGS (Figure 2). Since a broader bandwidth wavelet can more closely approximate a spike, the minimum phase AGS is more compressed toward zero time (has better resolution) than the minimum phase PSR.

The input wavelets within each pair have the same frequency content and thus differ by phase delay only. These mixed phase delays create dramatic differences (compare Figure la with lb, and lc with 1d). Since the spiking filter is ACF (frequency content) based, the same filter is designed from and applied to each input pair. The wavelets of each output pair will therefore differ by the same phase delay as the corresponding inputs.

Spiking filters restore missing frequencies while attempting to whiten the output amplitude spectra (Figure 2). This partial spectral whitening, along with removal of the minimum phase delays, improves resolution in all four cases (Figure 3).

The resolution increase is subtle for the mixed phase wavelet outputs. Notice the locations of the major wavelet peaks at approximately 30 and 15 ms within the PSR and the AGS, respectively (Figure la and lc). Upon output, the positions of these peaks do not noticeably time shift, although amplitudes for greater times are reduced (Figure 3a and 3c). These major peak locations are influenced by the mixed phase delays. The delays characterize the mixed phase wavelet shapes before and after deconvolution, emphasizing the importance of minimum phase for good filter performance.

The resolution increase is dramatic for both minimum phase wavelets. The major wavelet peaks within the inputs are already compressed toward zero time (Figure lb and 1d). The outputs (Figure 3b and 3d) clearly exhibit drastically reduced amplitudes for times greater than these major wavelet peaks. The spiking filter insures that these minimum phase outputs are good approximations to zero time peaks, as limited by the frequency content of the inputs.

Since minimum phase wavelet results pinpoint the wavelet arrival time well, why not convert the seismic wavelet to minimum phase before spiking deconvolution? Of course, the wavelet has to be known to do this. When the source signature has been recorded, a minimum phase conversion is sometimes performed before subsequent Wiener spiking or predictive deconvolution.

Because the source signature is likely to be the only (or the most) nonminimum phase component of any seismic wavelet, and it is the one component which can most easily measure, it should be recorded whenever possible. Land seismic sources are notoriously difficult to measure, while marine source signatures are easy to measure. Today marine surveys often record the source signature and this knowledge is used to implement deterministic signature deconvolution. Deterministic filters take some of the mystery out of deconvolution by minimizing reliance on a statistical seismic wavelet assumption. This improves confidence in processing results.

Summary. This tutorial has presented the mathematical nuts and bolts of single trace spiking deconvolution by clearly explaining how the required assumptions relate to filter design. With the aid of simplifying assumptions concerning the reflection coefficient series and the average seismic wavelet, the spiking filter is designed from the trace autocorrelation function. The method's years of popularity in the petroleum exploration industry are attributable to easy implementation as well as its robust nature despite questionable assumptions. Deconvolution results often dramatically improve temporal resolution. We have no way of knowing the solution to the problem, yet we like what we see. Today, more elaborate multichannel schemes using surface consistency are popular, although the basic assumptions remain the same.

Nonminimum phase source signatures and earth filtering often invalidate the minimum phase assumption. A few simple examples, which ignore the all-important influence of the reflection series, have shown that Wiener spiking deconvolution successfully handles reasonable wavelet departures from minimum phase. LE

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