Finite-Difference modelling

B. Arntsen

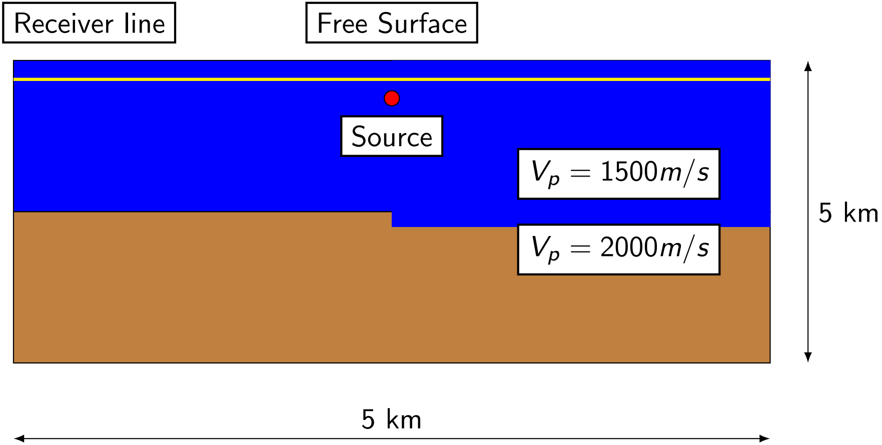
NTNU
Department of Geoscience
borge.arntsen@ntnu.no

UNIS, Svalbard April 2024

Content

1. Why do we need Finite Difference modelling?
2. Newton and Hook's law
3. Finite Difference Grids
4. The Finite Difference Method

Diffracted waves



Diffracted waves

Movie-3

Newton's and Hook's laws

Seismic waves can be completely described by Newton's law

$$\partial_t v(x,t) = \rho^{-1}(x)\partial_x \sigma(x,t) + f(x,t),$$

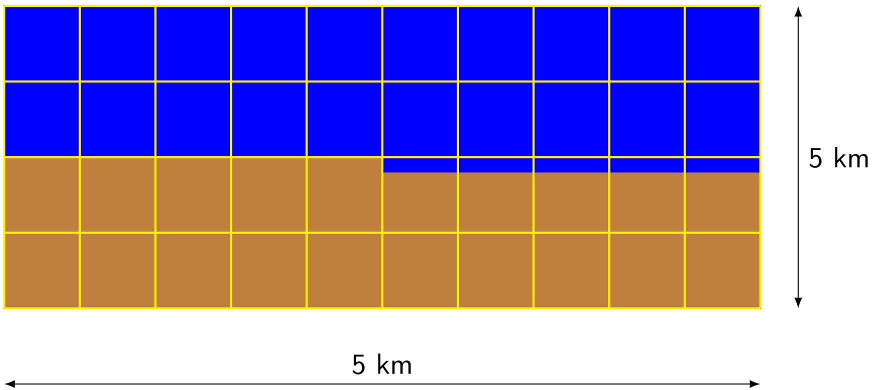
and Hook's law

$$\partial_t \sigma(x,t) = \kappa(x)\partial_x v(x,t).$$

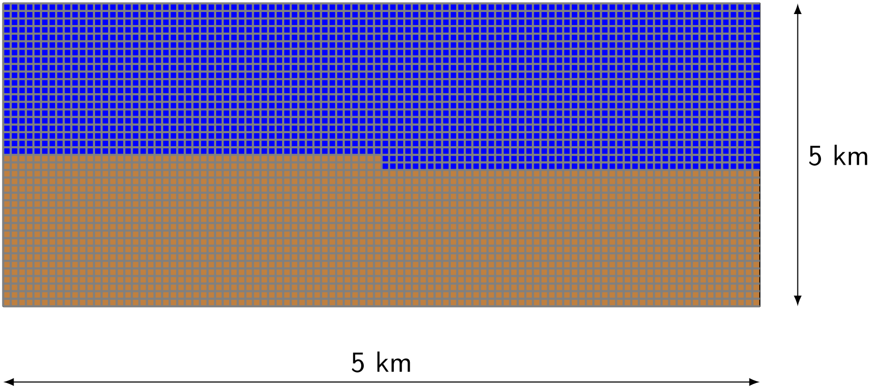
Here :

- $\rho(x)$: Density
- $v(x,t)$: Particle velocity
- $\sigma(x,t)$: Stress
- $f(x,t)$: External force (Source of energy)
- $\kappa(x)$: Bulk modulus (Stiffnes)

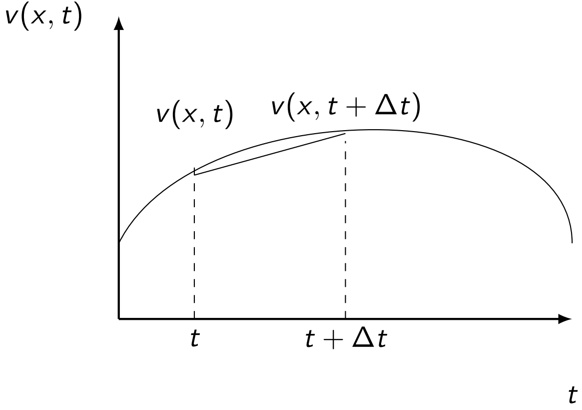
Gridded Models



Gridded Models



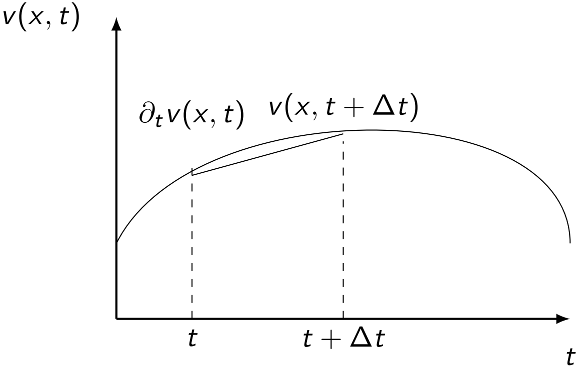
Time derivative of v(x,t)



$$\partial_t v(x,t) \approx \frac{v(x,t+\Delta t) - v(x,t)}{\Delta t}$$

$\partial_t v(x,t)$ is the slope of the line above.

Predict ahead in time



$$v(x,t+\Delta t) \approx v(x,t) + \partial_t v(x,t)\Delta t$$

It is possible to predict future values of $v(x,t)$ if the derivative (slope) is known.

Finite Difference solution

Seismic waves can be completely described by Newton's law

$$\partial_t v(x,t) = \rho^{-1}\partial_x \sigma(x,t) + f(x,t),$$

If we know $\sigma(x,t)$ and $v(x,t)$ we can compute $v(x,t+\Delta t)$ by using the formula from the previous slide:

$$v(x,t+\Delta t) \approx v(x,t) + \Delta t[\rho^{-1}\partial_x \sigma(x,t) + f(x,t)].$$

The derivative of σ on the right hand side can be computed by the same formula

$$\partial_x \sigma(x,t) \approx \frac{\sigma(x+\Delta x,t) - \sigma(x,t)}{\Delta x}$$

Finite Difference solution

Seismic waves can be completely described by Newton's law

$$\partial_t \sigma(x,t) = \rho^{-1} \partial_x v(x,t),$$

If we know $\sigma(x,t)$ and $v(x,t)$ we can compute $\sigma(x,t+\Delta t)$ by using the formula from the previous slide:

$$\sigma(x,t+\Delta t) \approx \sigma(x,t) + \Delta t[\kappa(x)\partial_x v(x,t)].$$

The derivative of v on the right hand side can be computed by the same formula

$$\partial_x v(x,t) \approx \frac{v(x+\Delta x,t) - v(x,t)}{\Delta x}$$

Initial conditions

By repating the formulas on the previous slides we can compute the particle velocity and stress at any future time t.
The computation is started by assuming that the particle velocity and stress is zero at the begining, only the external force f (source) is acting.

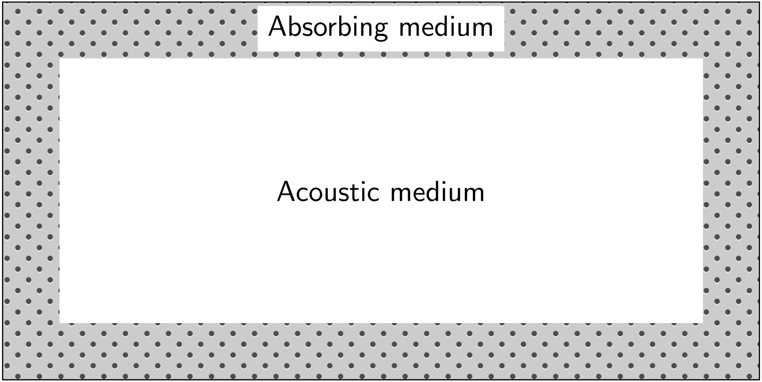
Outer Boundaries

Fig-1

Outer Boundaries

Fig-1

Perfectly Matched Layer (PML)

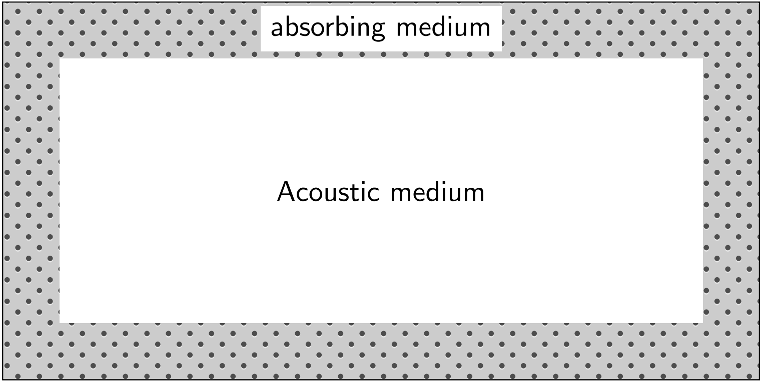


Fibrous Sound absorbing material

β: 0.5	Porosity
ρ_f:1000 kg/m^3	Fluid density
ρ_s:2600 kg/m^3	Fiber density
η: $10^{-3} kg/m \times s$	Viscosity
κ: $0.5 \times 10^{-7}\ m^2$	Permeability
α: 1.2	Tortuosity

Gives Q $\approx$ 1.0 Wave velocity: 1500 m/s

Perfectly Matched Layer (PML)



Perfectly Matched Layer (PML)