Applications of Mathematics in urban rainfall-runoff modelling.

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Introduction and Background

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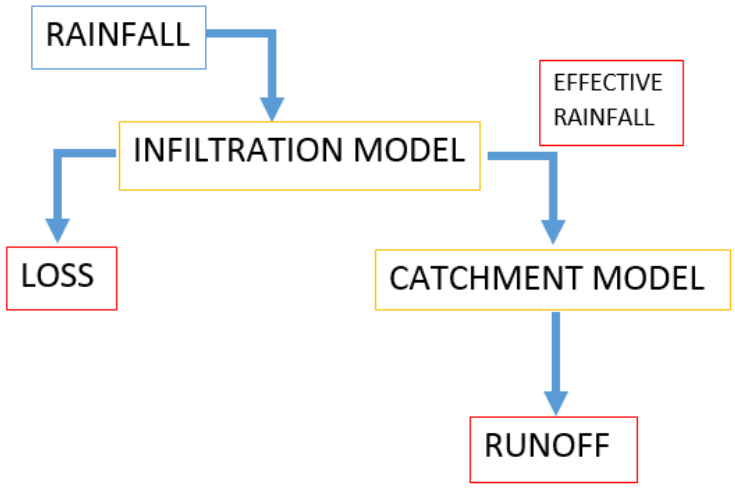
A method to capture these effects is rainfall-runoff modelling.

Range of Conceptual rainfall runoff models



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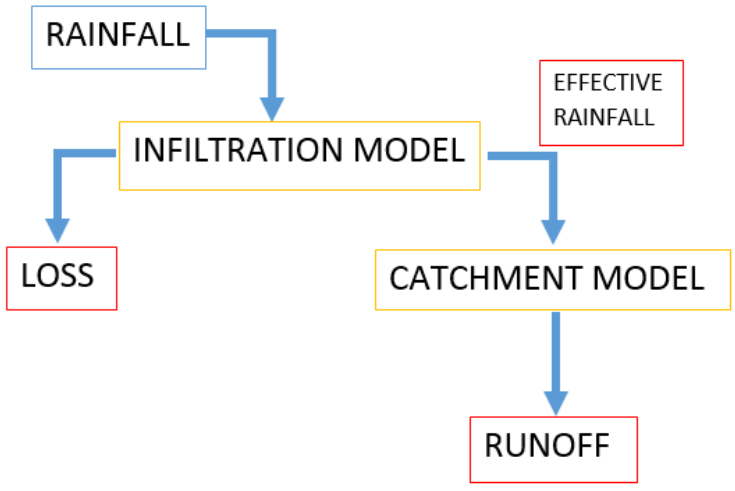

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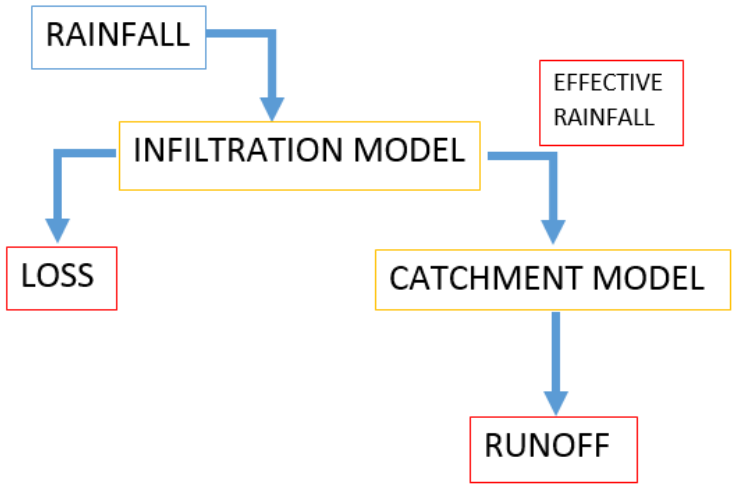

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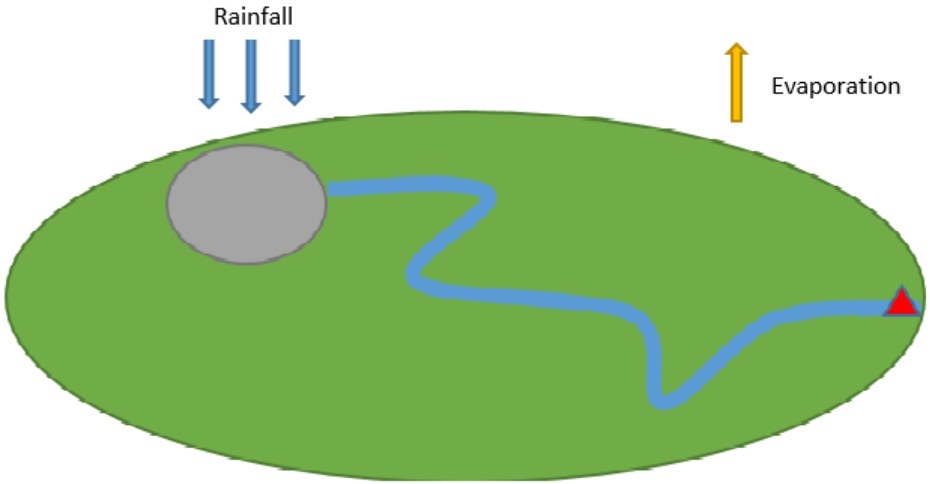
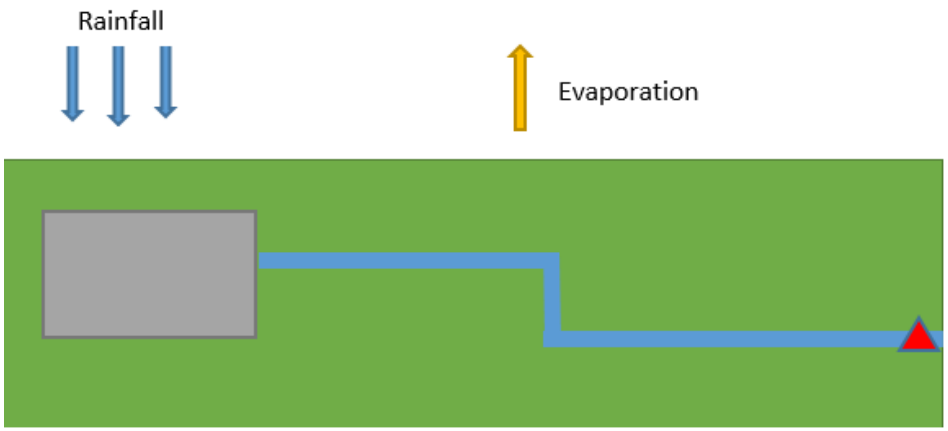
The applications of runoff models ranges from expanding stream climate and flow records, estimating river flow in ungauged catchments and prediction of the effects of land use change.

Difficulties of modelling





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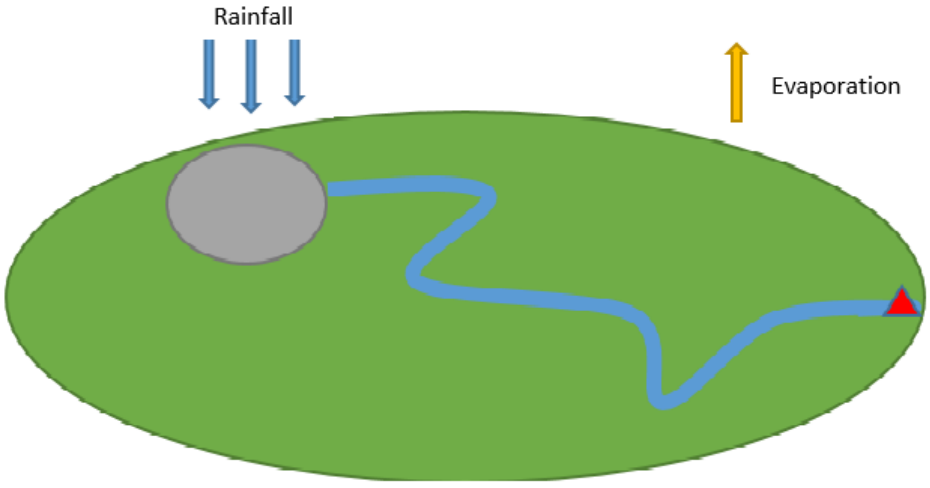

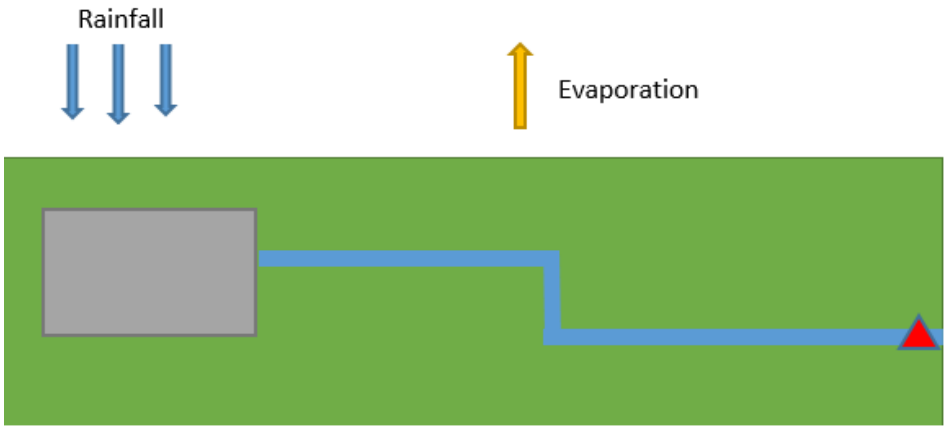

Sealing of previously pervious areas leads to more runoff generated.

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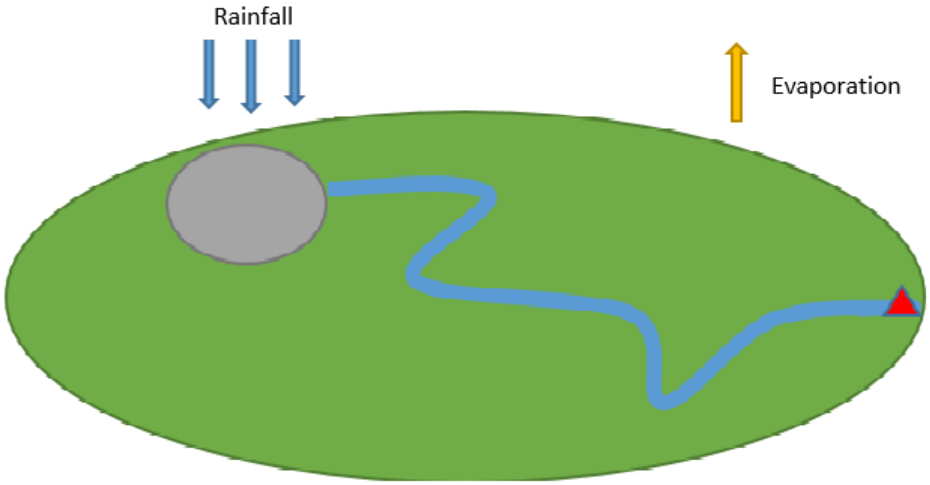

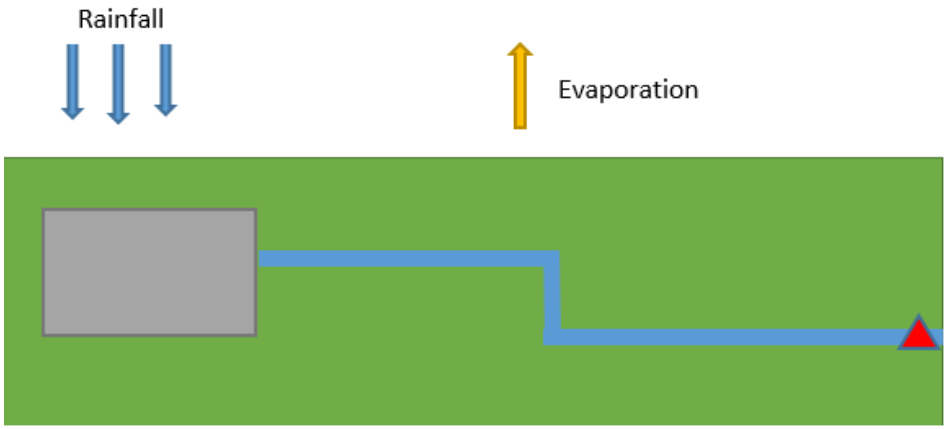

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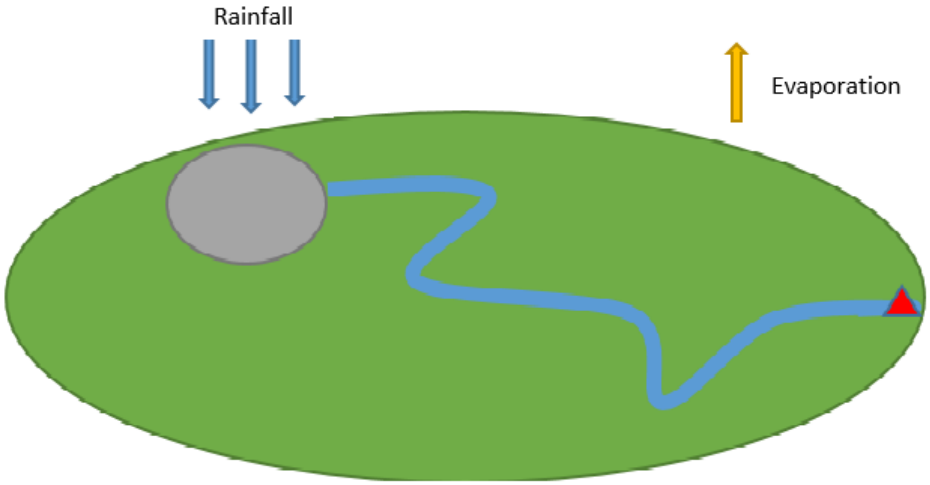

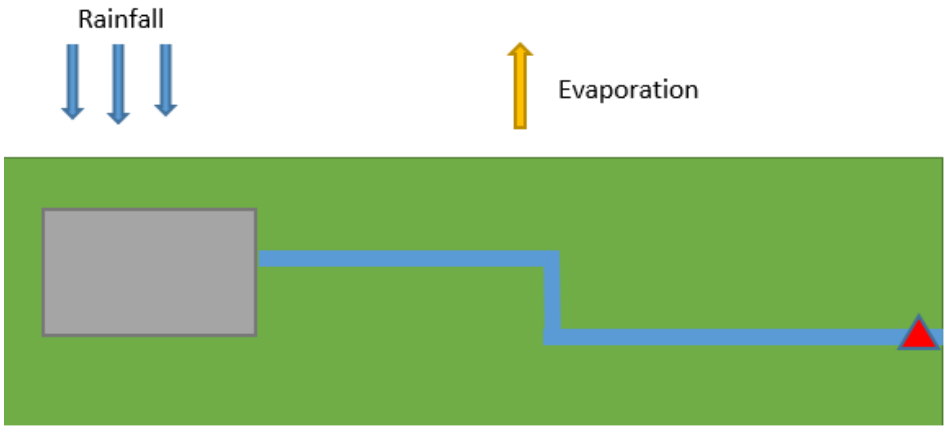

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The location of the urbanisation with relation to the river, can impact transportation times.

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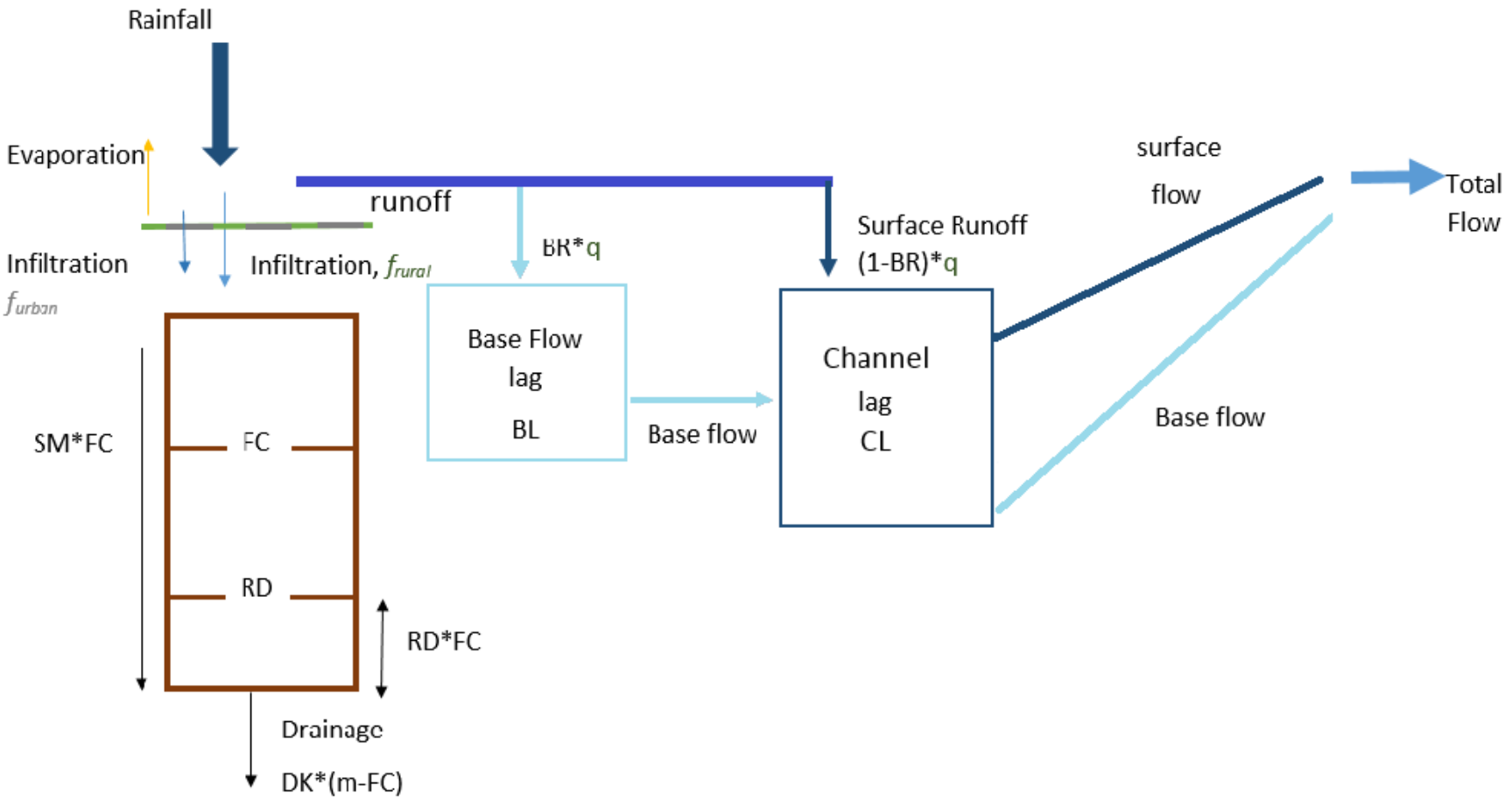
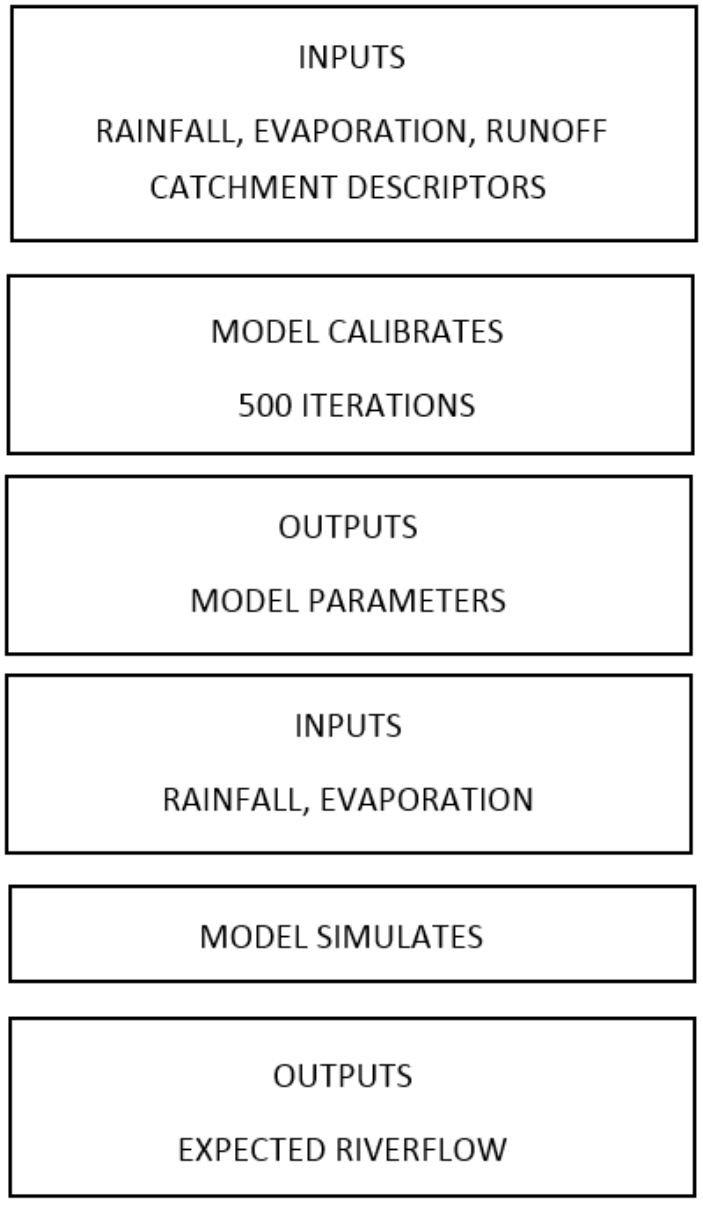
A nested model approach was used, when no urbanisation is present URMOD defaults to a rural model.

Visual representation of URMOD



URMOD processes



Urban soil and runoff equation

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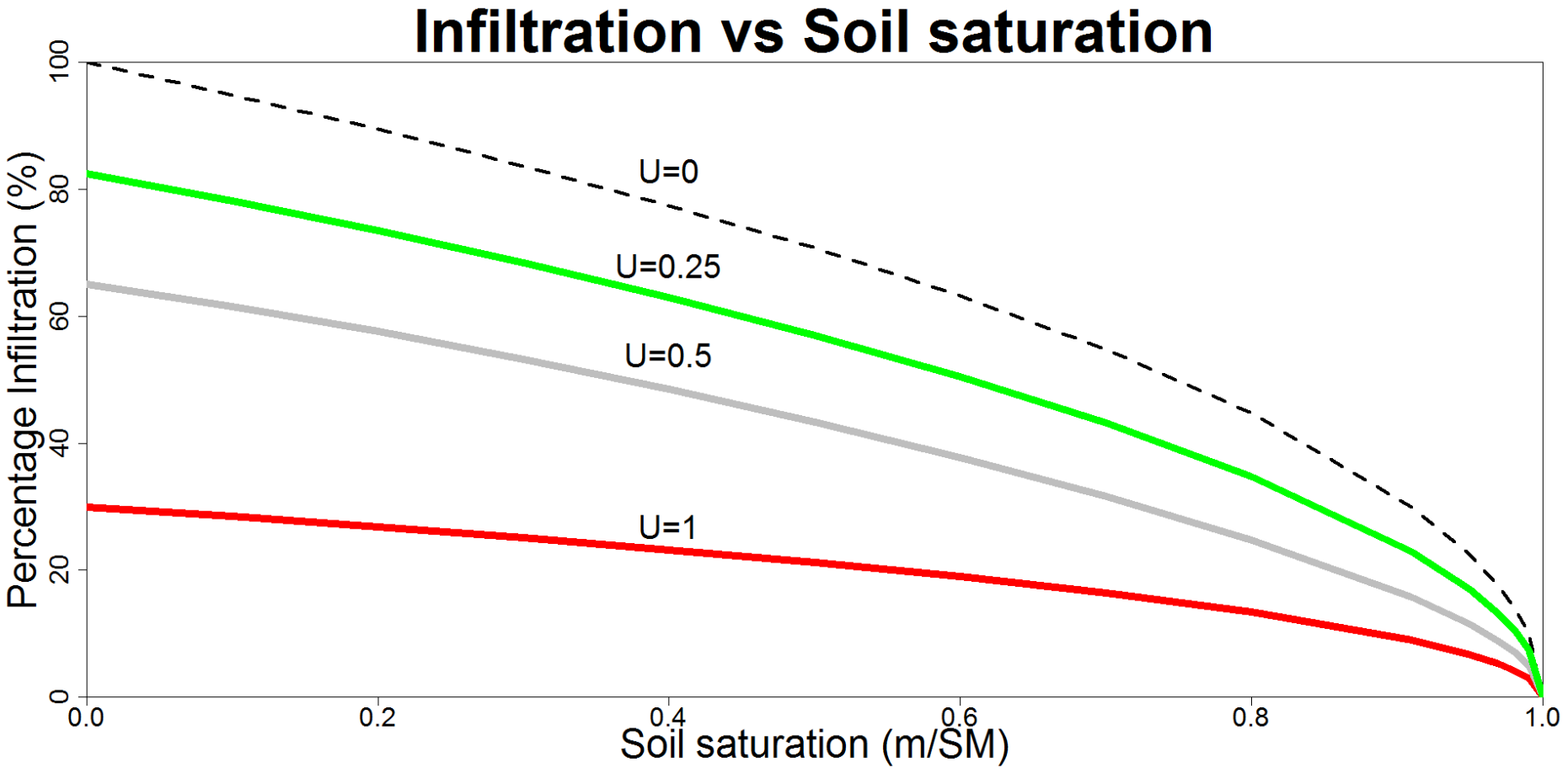
The urban moisture equation

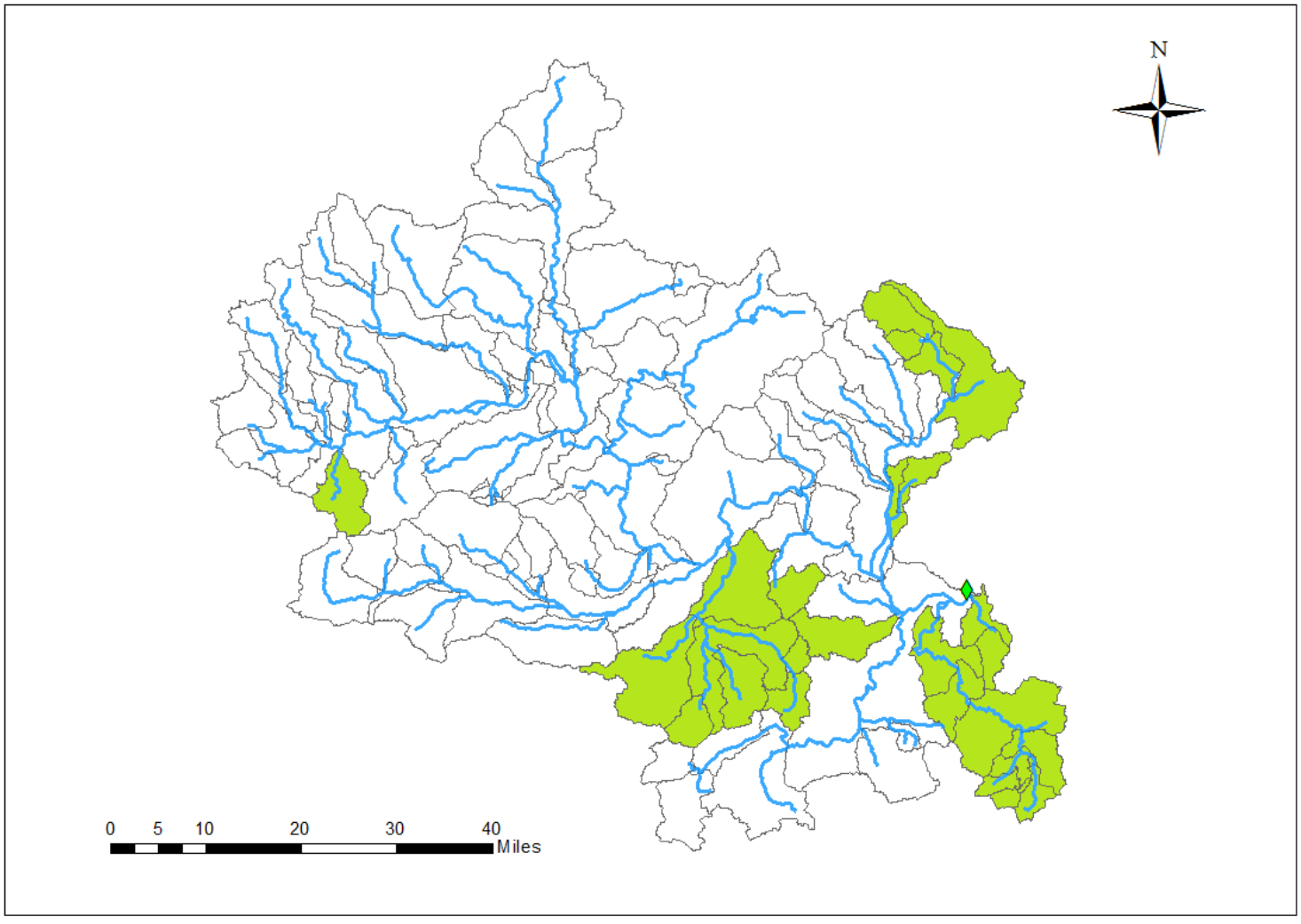
$$\frac{dm}{dt} = \underbrace{(1-u)i\left(1-\frac{m}{SM}\right)^{\frac{1}{2}} + ui(1-\gamma)\left(1-\frac{m}{SM}\right)^{\frac{1}{2}}}_{\text{Infiltration}} - \underbrace{D(m-F)}_{\text{Drainage}} - \underbrace{E_p}_{\text{Evaporatio}}$$

Gamma parameter



This is with a fixed scaling term γ. In the above example $\gamma = 0.7$

Case Study- Thames River



112 catchment's subsetted into 29 catchments in the Thames river.

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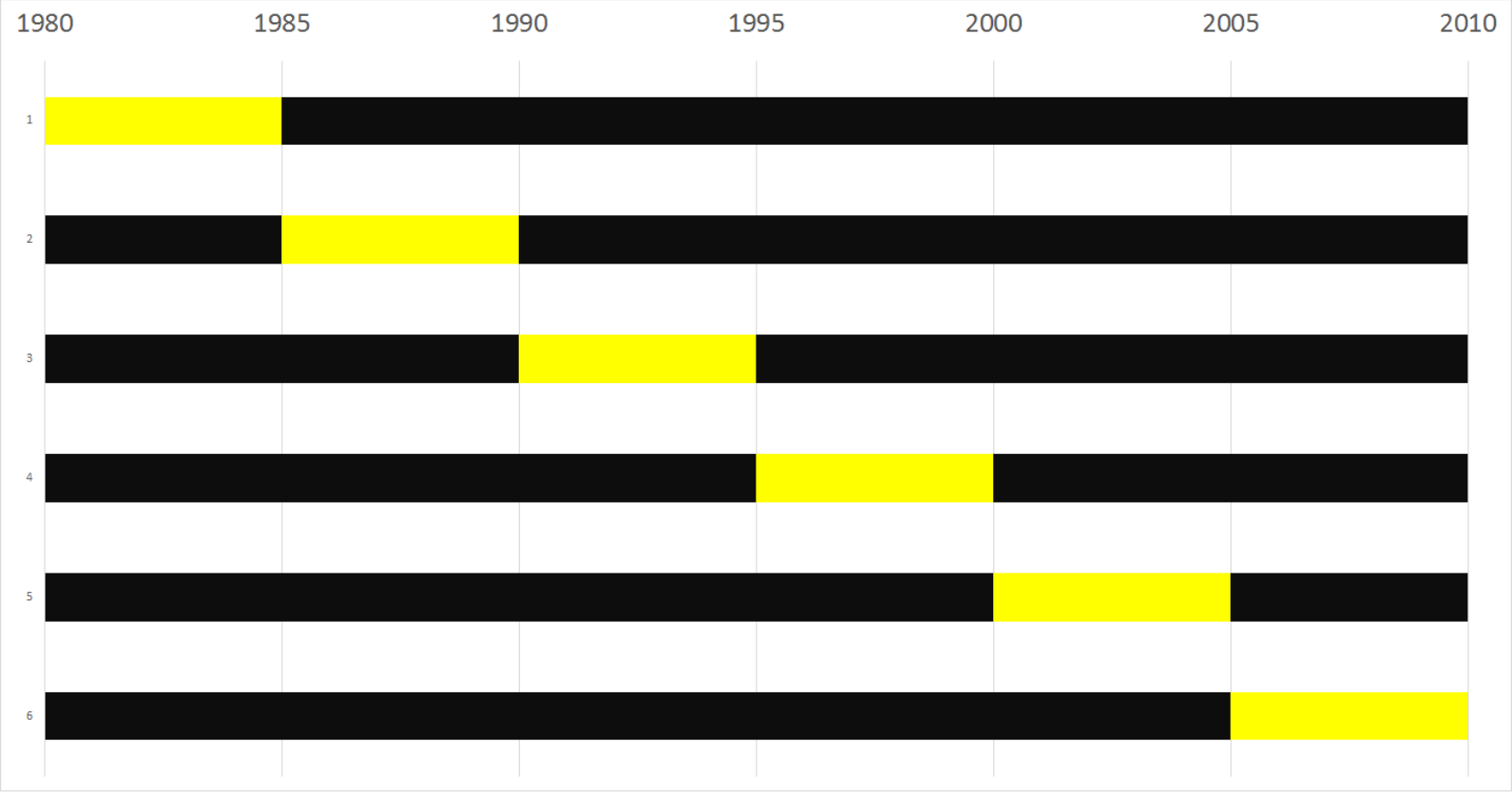
River runoff data used was in a 30 year period from 1980-2010.

Calibration and Validation period



The calibration period for the model was the black years, where as the Validation periods is the yellow years .

Performance criteria

The Nash-Sutcliffe efficiency Statistic E

$$E = 1 - \frac{\sum_{n=1}^{n}(Q_{obs} - Q_{sim})^2}{\sum_{n=1}^{n}(Q_{obs} - \bar{Q}_m)^2}$$

E is bounded between 1 and $-\infty$, so will be transformed.

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A *tanh* transformation was applied to the E statistic. Too constrict the lower bound of the E, bounds now are (-1,0.762).

Jack-knife experiment

The Jack-knife mean is obtained for each catchment.

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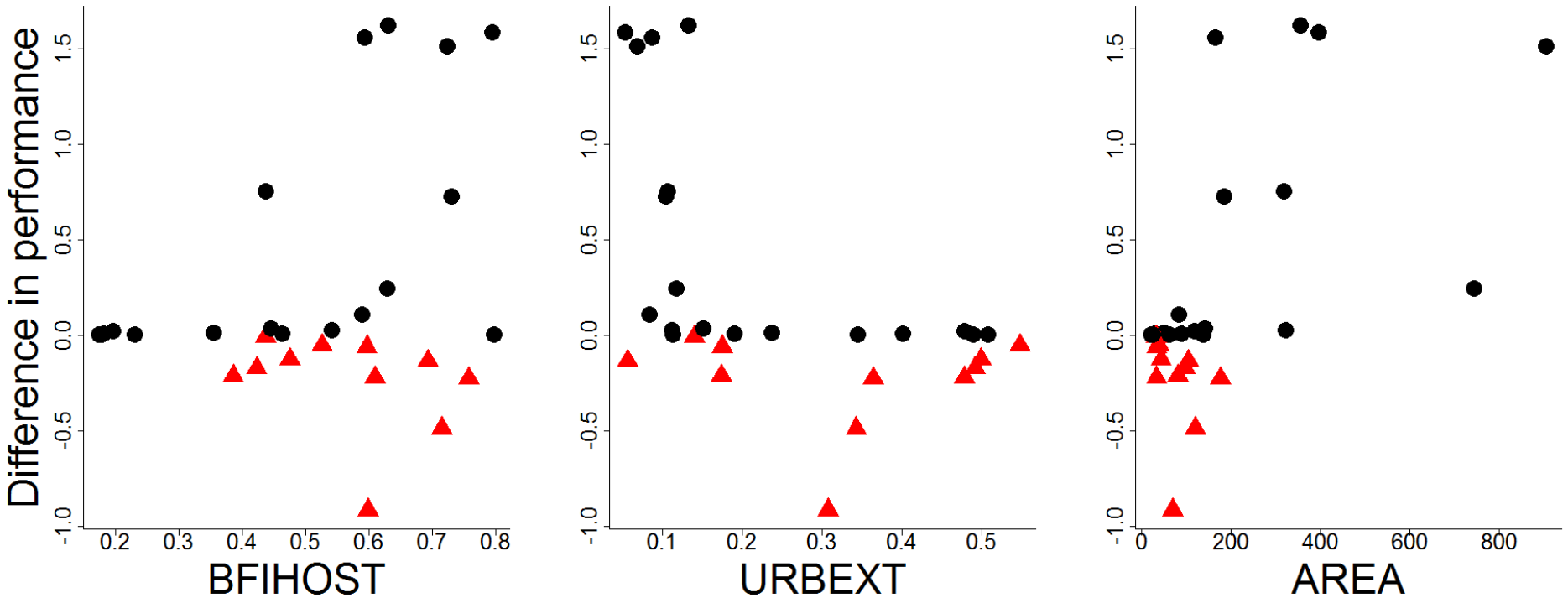
$$V_N = \frac{N-1}{N} \sum_{j=1}^{N} (Z_{N-1,j} - \bar{Z}_N)^2. \tag{2}$$

The 95% confidence intervals are calculated.

$$(\bar{Z_N} - 2 * \sqrt{V_N}, \bar{Z_N} + 2 * \sqrt{V_N}) \tag{3}$$

Comparison of urban and rural model



Comparison between Urban model and default rural. Circles indicate Urban model better performance and triangles indicate default rural model better performance.

Conclusion

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Is URMOD a improvment ?

Further Reading


Bayliss, A., Black, K., Fava-Verde, A., and Kjeldsen, T. (2006). URBEXT2000-a new FEH catchment descriptor. calculation, dissemination and application.


worldbank (2016). Population growth (annual %).