An Introduction to the Neutron Transport Equation

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Supervised by: Alex Cox, Simon Harris and Andreas Kyprianou.

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Notation

- Let N_t be the number of particles alive at time t.
- Then, the system can be described by $x_t^k = (r_t^k, v_t^k)$, for $k = 1, \ldots, N_t$.

- $\sigma_S(x_u)$ = scattering rate;
- $\Theta_S(x_u, \mathrm{d}x_u')$ = probability of particle with configuration x_u, scattering off nucleus with new configuration x_u';
- $\sigma_F(x_u)$ = fission rate;
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Let

- D_0 be the nuclear reactor;
- V be the velocity space.

Consider

$$v_f(x,t) = \mathbb{E}_{\delta_x}\left[\sum_{k=1}^{N_t} f(x_t^k)\right],$$

where $f : D_0 \times V \to \mathbb{R}$ is a bounded measurable function.

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v_f(x,t) = f(x_t) \\
&+ \int_0^t \sigma_S(x_u)\left\{\int_W v_f(x'_u, t-u)\Theta_S(x_u, \mathrm{d}x'_u) - v_f(x_u, t-u)\right\}\mathrm{d}u \\
&+ \int_0^t \sigma_F(x_u)\left\{\int_W v_f(x'_u, t-u)\Theta_F(x_u, \mathrm{d}x'_u) - v_f(x_u, t-u)\right\}\mathrm{d}u
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Not just neutrons!

What if we now had other types of particles/rays flying around in our reactor?

Suppose we now have m different types of particles!

Updated notation:

- $\sigma^i_S(x_u)$ = scattering rate of type i particle;
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Thank you!