The Fresnel Equations

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1. Aim

The experiment aimed to investigate refraction and reflection; specifically using Snell's law to derive the refractive index, critical angle and Brewster's angle for silica glass. The Fresnel equation was then used to analyse data on the reflectance and transmittance of the glass.

2. Experimental Principles

To investigate reflection and refraction through a half-cylinder silica glass, an apparatus like that in Fig. 1(a) was used. A graduated table connected to an indicator arm was used to line up the lasers beam to the normal face of the glass. By aligning the beam with the vertical indicator line on the arm, this ensured the beam was correctly placed at each measurement. The angle of the transmitted beam was then measured using the angle of the indicator arm on the graduated table; with the table having incremental values of 1° through $0 \leq \theta \leq 360°$ This was for the first part of the experiment, where Snell's Law was studied. To investigate the reflectance and transmittance, the apparatus as in Fig. 1(b) was used, with the polariser including increments on it like the graduated table. Once the beam was transmitted, the intensity of the reflected or transmitted beam was measured using a silicon detector attached to a photoreceiver module. The detector was freely movable around the graduated table; with the photoreceiver module giving no units for the intensity. Background light was accounted for in all experiments, by switching the laser off and measuring the ambient light intensity.

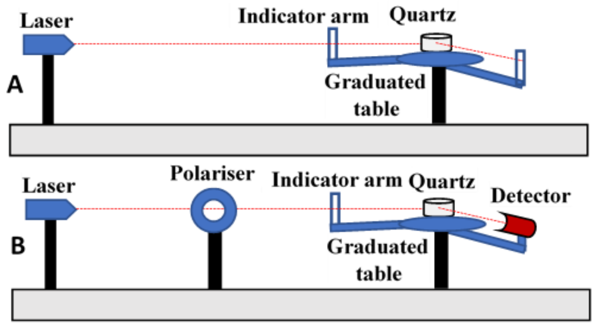

Fig. 1. Schematic of the apparatus'; A) Snell's Law experiment, B) Fresnel Experiment.

For low-high index, the beam was incident on the flat face of the glass. Therefore, using Fig. 1(a), measurements of the transmitted light intensity were taken at incident angles of 10° intervals. This was viable up to 90° as at that point, the laser would be parallel to the flat face of the glass. For high-low index, the same procedure was carried out, though the beam was incident on the curved face of the glass. For this case, the critical angle can be easily observed once the transmitted beam is hardly distinguishable.

With the apparatus in Fig. 1(b), the reflectivity of the glass was investigated similarly. For low-high index, having the beam incident on the flat face of the glass and the polariser set to 0°; measurements of intensity were taken for the reflected beam at 5° intervals. This was then repeated for the polariser at 90°. For high-low index, the procedure was followed again with both polariser states but on the curved surface. Here, the Brewster angle is readily observed as the intensity of the reflected beam falls to a minimum.

3. Results and Discussion

To determine the refractive index of the glass, Snell's Law [1] was used, which is as follows;

$$n_i \sin \theta_i = n_t \sin \theta_t \qquad (1)$$

where n_i and n_t are the refractive indices of the incident material and transmission material respectively. The refractive index of air is considered as 1 [1]. With θ_i and θ_t being the angles of incidence and transmission. This can then be rearranged to find the critical angle for the high-low index, where $\theta_t = 90°$, so $\sin \theta_t = 1$. An initial calculation of the refractive index of the glass was found using this method, with $n_i = 1.45 \pm 0.01$. The critical angle was found to be $\theta_c = 45° \pm 0.5$ as illustrated in Fig. 3.

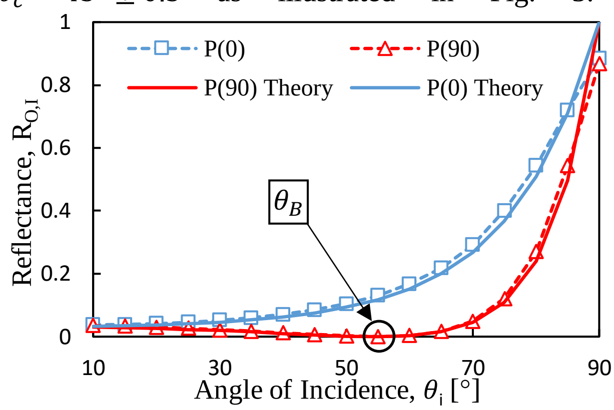

Fig. 2. The reflectance of the silica glass as a function of the laser's incident angle at low-high index, for the polariser at 0° and 90° with the Brewster angle indicated.

The values for Reflectance, R, plotted from measured data are calculated using the equation,

$$R = \frac{P_r^I}{P_i(1-R_n)^2}. \qquad (2)$$

Where P_r^I is the measured power of the reflected beam, P_i is the maximum power measured from the laser directly hitting the detector, and R_n is a value calculated using the normal incidence case of the Fresnel equation. Evaluating the constants in the formula give $R = P_r^I \cdot 1.1 \cdot 10^{-3}$. This is used to calculate the reflectance and plotted against the incident angle for both polariser angles as shown in Fig. 2 and 3.

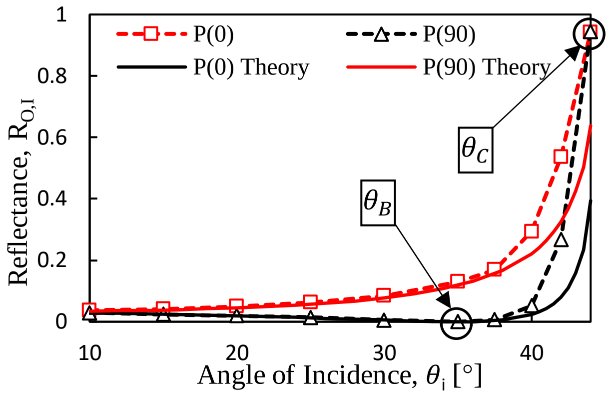

Fig. 3. The reflectance of the silica glass as a function of the laser's incident angle at high-low index, for the polariser at 0° and 90° with the Brewster angle and critical angle indicated.

The Brewster angle, θ_B [2], can be calculated using the equation;

$$\theta_B = \tan^{-1}\frac{n_t}{n_i}. \quad (3)$$

The angle calculated was $\theta_B = 55° \pm 0.5°$ for low-high index and $\theta_B = 35° \pm 0.5°$ for high-low index. This angle can then be used to identify an improved value for the refractive index of the glass; giving $n_i = 1.43 \pm 0.01$ for low-high index and $n_t = 0.70 \pm 0.01$ as the high-low index. The equations to calculate the Reflectance, $R_{I,O}$ [3];

$$R_I = \left(\frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}\right)^2 \quad (4)$$

and R_O,

$$R_O = \left(\frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}\right)^2. \quad (5)$$

These are the Fresnel equations; where, the subscripts I and O denote the angle of the polariser used in each case. With the angle being 0° and 90° respectively, using the improved refractive indices calculated and remaining variables defined before.

These are then plotted as the theoretical curves.

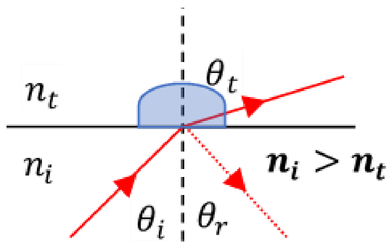

Fig. 4. Diagram of the light path taken when travelling through different mediums at an angle.

The polarisers are set to 0° and 90° to investigate linearly polarised light with the polarisation vector parallel and perpendicular to the incidence plane respectively. The refractive indices of the mediums determine the nature of the transmitted beam as illustrated in Fig. 4. This shows how the angle of incidence is equal to the angle of reflection, but the index geometrically alters the transmission angle. Errors minimisation was largely accounted for in this experiment by consistent alignments. Such as the laser spot being entirely within the sensor of the diode during each measurement. Though, improvements could be made towards discrepancies from theory like that in Fig. 3. This could be due to systematic errors in measurements within the scaling of the tools provided, so it is something to be aware of for future experiments.

4. Conclusions

The polarisation state along with the refractive indices of the mediums and angles of incidence have shown to determine the reflectance and transmission of the light. This has been tested using Snell's Law, Fresnel's equation, and manipulation of the critical and Brewster angles. In particular, the refractive index of the silica glass was found to be $n_i = 1.43 \pm 0.01$.

References

[1] P. A. Ribeiro *et al.* Optics, Photonics and Laser Technology, (2018).

[2] A. Lahiri. Basic optics: principles and concepts, (2016).

[3] W. W Wendlandt. Modern Aspects of Reflectance Spectroscopy, (1968).