

Quantum Information and Computing

Assignment 6 (due in two weeks)

November 29, 2022

1. **Density Matrices** Consider a quantum system composed by N subsystems (spins, atoms, particles etc..) each described by a wave function $\psi_i \in \mathcal{H}^D$ where \mathcal{H}^D is a D -dimensional Hilbert space. How do you write the total wave function of the system $\Psi(\psi_1, \psi_2, \dots, \psi_N)$?
 - (a) Write a code (Fortran or Python) to describe the composite system in the case of N -body non interacting, separable pure state;
 - (b) and in the case of a general N -body pure wave function $\Psi \in \mathcal{H}^{DN}$;
 - (c) Comment and compare their efficiency;
 - (d) Given $N=2$, write the density matrix of a general pure state Ψ , $\rho = |\Psi\rangle\langle\Psi|$;
 - (e) Given a generic density matrix of dimension $D^N \times D^N$ compute the reduced density matrix of either the left or the right system, e.g. $\rho_1 = \text{Tr}_2 \rho$.
 - (f) Test the functions described before (and all others needed) on two-spin one-half (qubits) with different states.