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Bil 511 - Ayrık Matematikte İleri Konular

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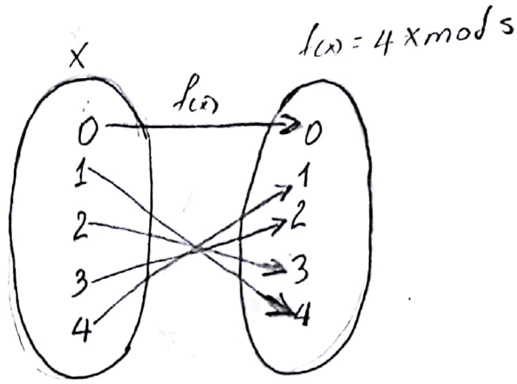
1) $X = \{0,1,2,3,4\}$, $f(x) = 4x \bmod 5$

$$f(x) = \begin{cases} 4(0) \bmod 5 = 0 \\ 4(1) \bmod 5 = 4 \\ 4(2) \bmod 5 = 3 \\ 4(3) \bmod 5 = 2 \\ 4(4) \bmod 5 = 1 \end{cases}$$



$f(x) = \{(0,0), (1,4), (2,3), (3,2), (4,1)\}$

f is one to one (injektif) and onto (surjektif) so f is bijektif

for every x_i we have $x_1 = 0 \neq x_2 = 1$; $f(0) \neq f(1)$

f is injective $\rightarrow$ (1), we know that $f(x)$ is surjective

because $\forall f(x) = y \; \exists x_1 \in X$ such that $f(x) = 4x \bmod 5$

f is surjektive $\rightarrow$ (2)

Since 1 and 2 finally $f(x)$ is bijective.

* $m, n \in \mathbb{Z}^+$, $X = \{0,1,2 \ldots m-1\}$, $f: X \rightarrow X$ $f = n.x \bmod m$

To f function to be bijective has to be:

1) Injective (one to one) $\forall x, x' \in X$, $f(x) = f(x') \Rightarrow x = x'$

Surjective (onto) $\forall y \in Y$, $\exists x \in X$ such that $y = f(x)$

so f is bijective its injective and surjective

2) m and n on $f(x) = nx \bmod m$ to be bijective

m and n should be relatively prime intergers.

such that if they no common factor (divisor) Eg: (4,5), (3,2)

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2) n^2+2n'in tüm n ler için 3'e bölünebildiğini ispatlayin

Answer:

Let's prove it using mathematical induction:

$P(n) = n^3+2n$

step 1:

for $n=1$

$P(1) = 1+2 = 3$ which 3 is divisible by 3

step 2:

for $n=k$

$P(k) = k^3+2k$

Assume that $p(k)$ be divisible by 3

step 3:

for $n=k+1$

$P(k+1) = (k+1)^3+2(k+1)$

$= k^3+2k+3k^2+3k+3$

so $p(k+1) = k^3+3k^2+5k+3 = p(k)\ 3(k^2+k+1)$

Since we assumed $p(k)$ to be divisible by 3 therefore

$p(k+1)$ is also divisible by 3

3) R^* for transitive clousure

Procedure: transitive clousure (M_R : zero-one $n \times n$ matrix)

Algorithm Warshall ($A[1..n, 1..n]$)

// Input the adjacency matrix A

// output the transitive clousure

$R^{(0)} \leftarrow A$

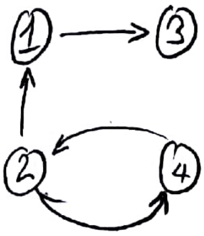
geçişli

for $k \leftarrow 1$ to n do
  for $i \leftarrow 1$ to n do
    for $j \leftarrow 1$ to n do
      $R^{(k)}[i,j] \leftarrow R^{(k-1)}[i,j]$ or $R^{(k-1)}[i,k]$ and $R^{(k-1)}[k,j]$
return R^n

yansiyan

for $x \leftarrow 1$ to n do
  for $y \leftarrow x$ to n do
    if $R^n[x,y] != 1$
      $R^n[x,y] \leftarrow 1$
return R^n

Ex:



$R^{(0)}$ $\begin{matrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$ $R^{(1)}$ $\begin{matrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$ $R^{(2)}$ $\begin{matrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{matrix}$

$R^{(3)}$ $\begin{matrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{matrix}$ $R^{(4)}$ $\begin{matrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{matrix}$ $R^{(5)}$ $\begin{matrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{matrix}$

4) Rastgel Üretilen 4 bit

$\underline{1}\ 0\ 0\ 1$

We know that the firs bit is 1 so:

100
010
000

P1 P2 P3

$\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right)+\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right)+\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right)=3/8=0,375$

the probability is 0,375

5) for each true/false question, the expected number of points is $0.9 * 2 = 1.8$ for the multiple choice questions, the expected is $0.8 \cdot 4 = 3.2$ points each.

therefore, the expected score is

$50 * 1.8 + 25 * 3.2 = 170$

so her expected score is 170

6) Bir partide n kişi şapkalarını vestiyere rastgele şekilde koymuştur.

E(x) is the expeced value
P(s) is the probability of event s
s is the sample space.
X(s) is the random variable

$$E(X)=\sum_{a\in S} P(s)\,X(s)$$

The probality of one person picking his/her hat is $\frac{1}{n}$, two person pickins their hat is $\frac{1}{n}\cdot\frac{1}{n-1}$. let n be the number of people, let $P(n,r)$ be number of r-permutation in elements. to generalize, $P(s)=\frac{1}{P(n,s)}$, where s, the sample is the number of people who picked their own hat.

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7) Ostel Ürctken $X = \{A, B, C, D\}$

a) A, B and C are 1 so D is not 1, could be 0

$$\left(x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)^3 \cdot \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)^1$$

$$= (e^x - 1)^3 \cdot (e^x)$$

$$= (e^{3x} - 3x^{2x} + 3e^x - 1) \cdot e^x$$

$$= e^{4x} - 3e^{3x} + 3.e^{2x} - e^x$$

$$= \sum_{n=0}^{\infty} (4^n - 3.3^n + 3.2^n - 1^n) \cdot \frac{x^n}{n!}$$

b) $\frac{AB}{2}$ $\frac{B}{1}$ $\frac{CD}{0}$

$$= \left(\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}\right) \cdot \left(x + \frac{x^2}{2!} + \frac{x^3}{3!}\right) \cdot \left(1 + x + \frac{x^2}{2!}\right)^2$$

$$= (e^x - 1 - x) \cdot (e^x - 1) \cdot (e^{2x})$$

$$= (e^{2x} - e^x - xe^x - e^x - 1 - x)\, e^{2x}$$

$$= (e^{4x} - 2e^{3x} - xe^{2x} + e^{2x}.(1+x)$$

$$= e^{4x} - e^{3x} \cdot (2+x) + e^{2x}(1+x)$$

$$= \sum_{n=0}^{\infty} 4^n - (2+x) \cdot 3^n + (1+x) \cdot 2^n$$

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8). Theory of productive functions: Apply to find $a+b+c=17$

a) $g(x) = (1+x+x^2+x^3+\dots)^n$ x^r'in katsayısı a^n olsun.

$a^r = C(r+n-1, r)$ ile hesaplanır

b) $(1-x^m)^n = 1 - C(n,1)\,x^m + C(n,2)\,x^{2m} \dots (-1)^n x^{nm}$, dir

c) $(1+x+x^2+\dots x^{m-1}) = (1-x^m)^n \cdot (1+x+x^2+\dots)^n$'dir

$(x^2+x^3+x^4+x^5).(x^3+x^4+x^5+x^6).(x^4+x^5+x^6+x^7)$'deki x^{17}'nin sayısıdır

$= x^2(1+x+x^2+x^3)\ x^3(1+x+x^2+x^3)\ x^4(1+x+x^2+x^3)$

$= x^9(1+x+x^2+x^3)$'daki x^{17}'nin katsayısı

$(1+x+x^2+x^3)$ daki x^8'in katsayısıdır

c'yi kullanarak $= (1-x^4)^3\ (1+x+x^2+\dots)^3$

b'yi kullanarak $= (1-x^4)^3 = 1 - C(3,1)\,x^4 + C(3,2)\,x^8 + \dots$

a'yı kullanarak $= (1+x+x^2+\dots)^3 = 1 + C(4,1)\,x + C(5,2)\,x^2 +$

$C(6,3)\,x^3 + \dots = C(11,8).x^8 + \dots$

$= C(11,8) + C(3,2) - C(3,1).C(7,4)$

$= 168 + 3 - 105$

$= \underline{63}$

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7. $f(n) = 5f(n/2) - 6f(n/4) + n \; ; \; f(1) = 2, \; f(2) = 1$

We can define:

$f(2^k) = g(k)$ and $n = 2^k$

then for g we have that

$g(0) = 2$

$g(1) = 1$

$g(k+2) = 5g(k+1) - 6 \cdot g(k) + 2^k$

$x^2 = 5x - 6$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2 \; , \; x_2 = 3$

$g(k) = C_1 2^k + C_2 \cdot 3^k$

$g(k) = C_1 \cdot k \cdot 2^k + C_2 \cdot 3^k$

$g(0) = 2 \quad C1 \cdot 0 \cdot 2^0 + C_2 \cdot 3^0 = 2 \Rightarrow \boxed{C_2 = 2}$

$g(1) = 1 \quad C1 \cdot 1 \cdot 2^1 + C_2 \cdot 3^1 = 1 \Rightarrow$

$2C_1 = -5$

$\boxed{C1 = -5/2}$

$g(k) = -5/2 \; k \cdot 2^k + 2 \cdot 3^k$

$n = 2^k$

$\log_2 n = \log_2 2^k$

$k = \log_2 n$

$$\boxed{f(n) = -5/2 \log_2 n \cdot n + 2 \cdot 3^{\log_2 n}}$$

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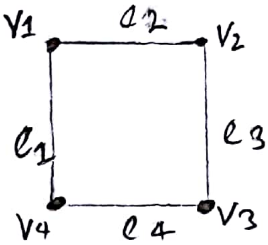
10). N düğümlü bir G grafının komşuluk matrisi ile bağlantı matrix

a) $A \rightarrow m \times m$ matrice
$C \rightarrow n \times m$ matrice

So we have to show example.



Komşuluk Matrix

	V1	V2	V3	V4
V1	0	1	1	0
V2	1	0	0	1
V3	1	0	0	1
V4	0	1	1	0

Bağlantı Matrisi

	e1	e2	e3	e4
V1	1	1	0	0
V2	1	1	0	0
V3	0	1	1	0
V4	0	0	1	1

So for both matris . diagonal element.

B) $A=\{a,b\}$ $L=\{b^m a^n b\}$