$$\frac{f(n+1)}{f(n)} = \frac{\log(n+1)}{\log n} = \frac{\log(n+1)}{\log(n)}$$

$$\frac{f(n+1)}{f(n)} = \frac{\sqrt{n+1}}{\sqrt{n}} \quad \text{increases by } \frac{\sqrt{n+1}}{\sqrt{n}}$$

$$\frac{f(n+1)}{f(n)} = \frac{(n+1)!}{n!} = n+1$$

$$\frac{f(n+1)}{f(n)} = \frac{(n+1)^2}{n^2} = \frac{n^2+2n+1}{n^2} = 1 + \frac{2}{n} + \frac{1}{n^2}$$

$$\frac{f(n+1)}{f(n)} = \frac{n+1}{n} = 1+\frac{1}{n}$$

$$\frac{f(n+1)}{f(n)} = \frac{2^{n+1}}{2^n} = 2$$

$$\frac{f(n+1)}{f(n)} = \frac{3^{n+1}}{3^n} = 3$$

We divide $f(n+1)$ to $f(n)$ to find growth rate when the input increased by one.

$n! > 3^n$ because $n+1 \geq 3$ for all n which is defined $n \geq 2$

$3^n > 2^n$ because $3 > 2$ for all n which is defined $n \geq 2$

$n^2 > n$ because $\frac{1}{2} + \frac{2}{n} + \frac{1}{n^2} > 1 + \frac{1}{n}$ for all n which is defined $n \geq 2$

$n \neq 0$

$n > \sqrt{n}$ because $\frac{1}{1} + \frac{1}{n} > \frac{\sqrt{n+1}}{\sqrt{n}}$ for all n which is defined $n \geq 2$

$\sqrt{n} > \log_2 n$ because $\frac{\sqrt{n+1}}{\sqrt{n}} > \frac{\log(n+1)}{\log(n)}$ when n is bigger than 2 $\underline{\underline{n \neq 0}}$

for all $n \geq 2$

$$\log_2 n < \sqrt{n} < n < n^2 < 2^n < 3^n < n!$$