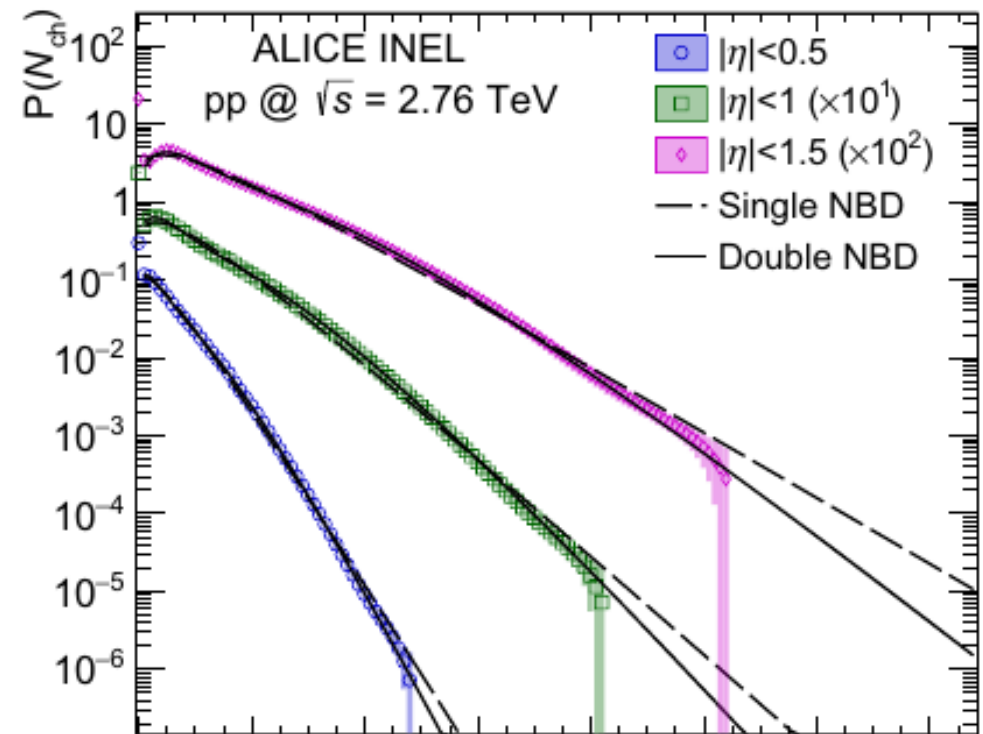


# **Modern Particle Physics Experiments**

Many aspects of particle physics are of stochastic nature:

- final state properties in particle collisions:

- **number of particles**
- type of particles
- particle four-momenta



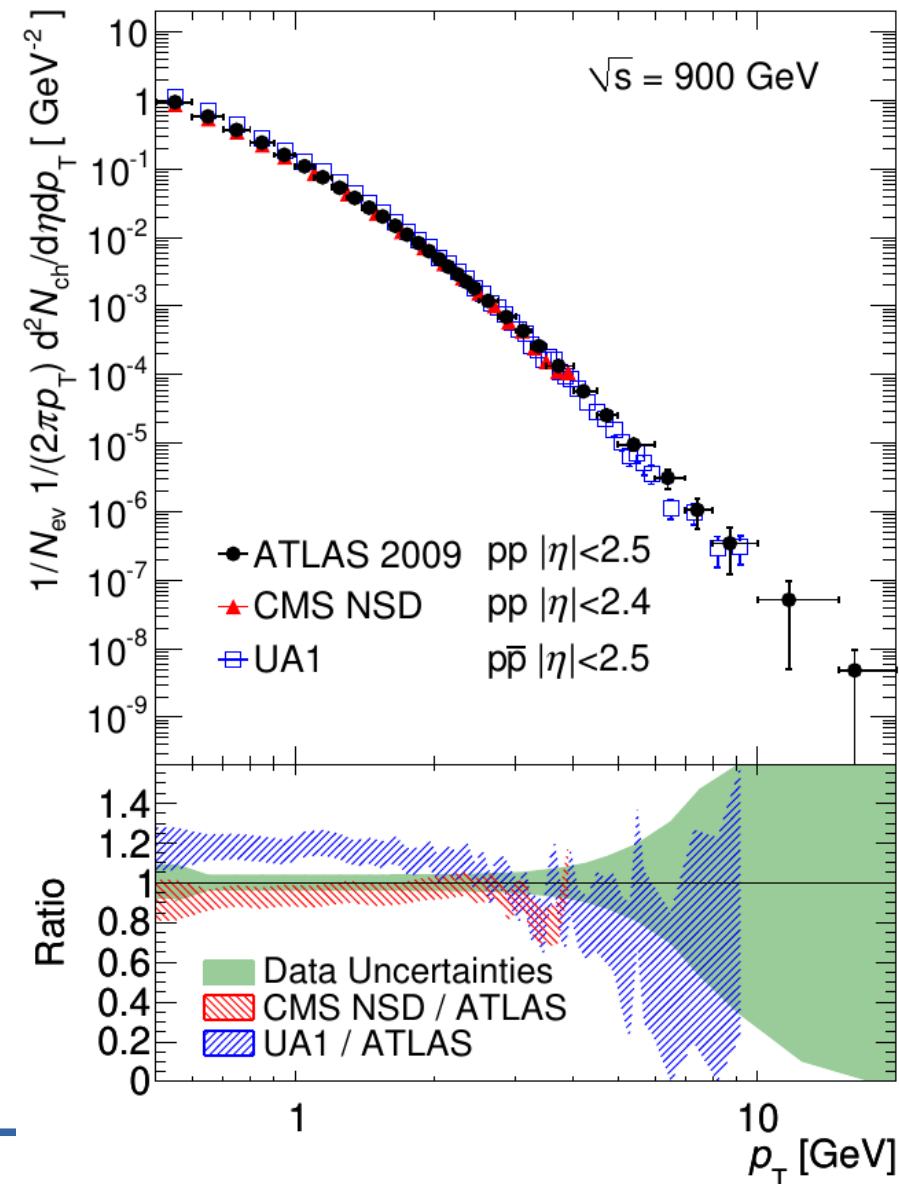
<https://arxiv.org/abs/1509.07541>

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- final state properties in particle collisions:

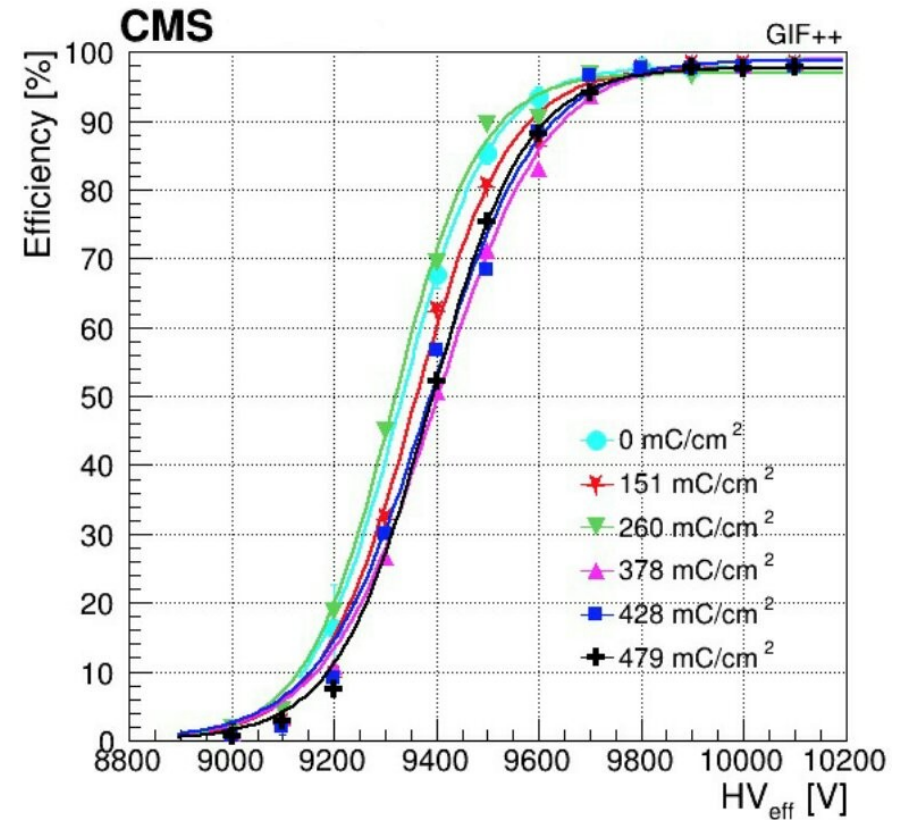
<https://arxiv.org/abs/1003.3124>

- number of particles
- type of particles
- **particle four-momenta**



Many aspects of particle physics are of stochastic nature:

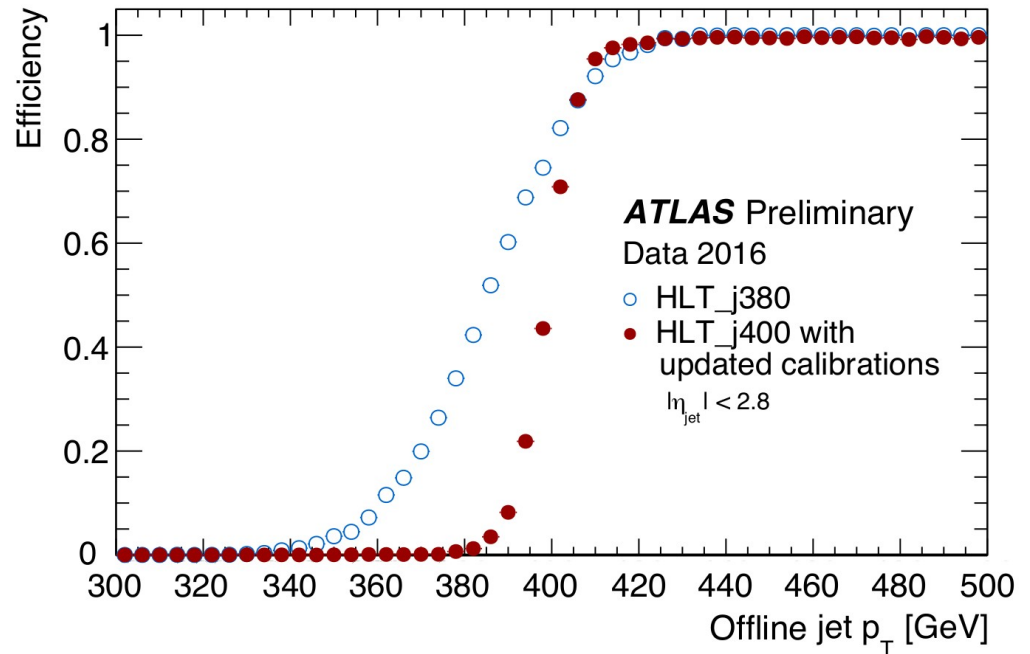
- response of the detector:
  - efficiency of
    - **detector elements**
    - trigger
    - reconstruction algorithms
  - difference of measured values from the true values (resolution)



<https://arxiv.org/abs/2005.11397>

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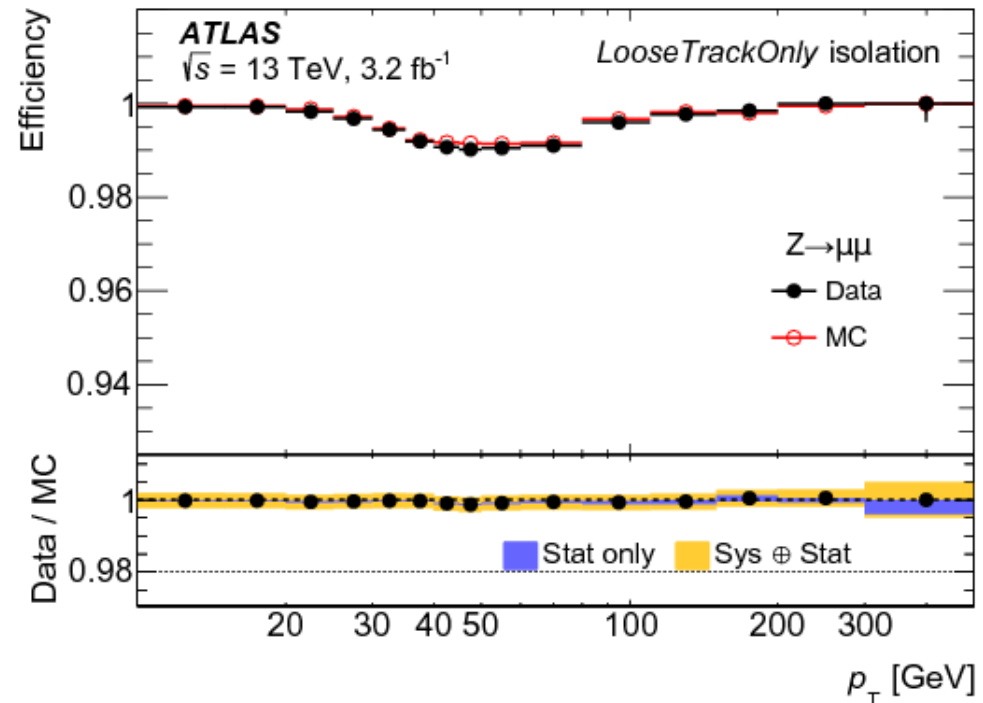
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<https://arxiv.org/abs/1711.02946>

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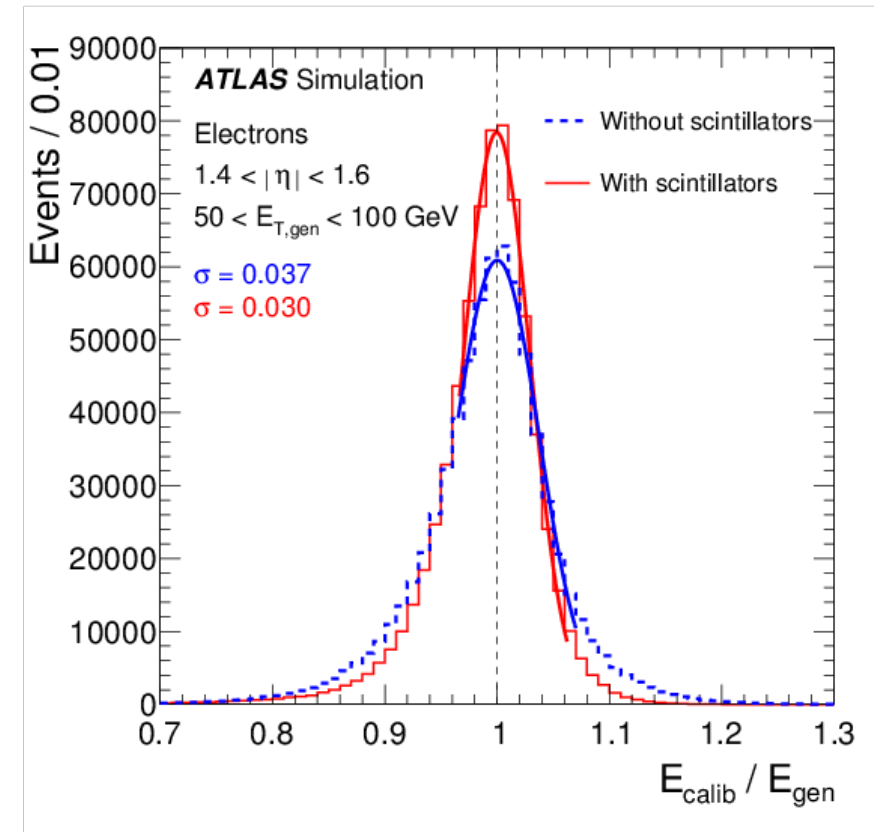
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<https://arxiv.org/abs/1603.05598>

Many aspects of particle physics are of stochastic nature:

- response of the detector:
  - efficiency of
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    - reconstruction algorithms
  - **difference of measured values from the true values (resolution)**



<https://arxiv.org/abs/1812.03848>

Probability distribution, or probability density for results of particle interactions are proportional to differential cross sections:

$$\frac{d\sigma}{dN} = \dots, \quad \frac{d\sigma}{dp_T} = \dots, \quad \frac{d\sigma}{d\eta} = \dots$$

Probability distribution has to be correctly normalized:

$$p(\eta) = \frac{1}{\sigma_{tot}} \frac{d\sigma}{d\eta}$$

total cross section  
– normalization  
constant

differential cross  
section – the  
distribution shape



What is the probability distribution for the electron energy for electrons from muon decay?

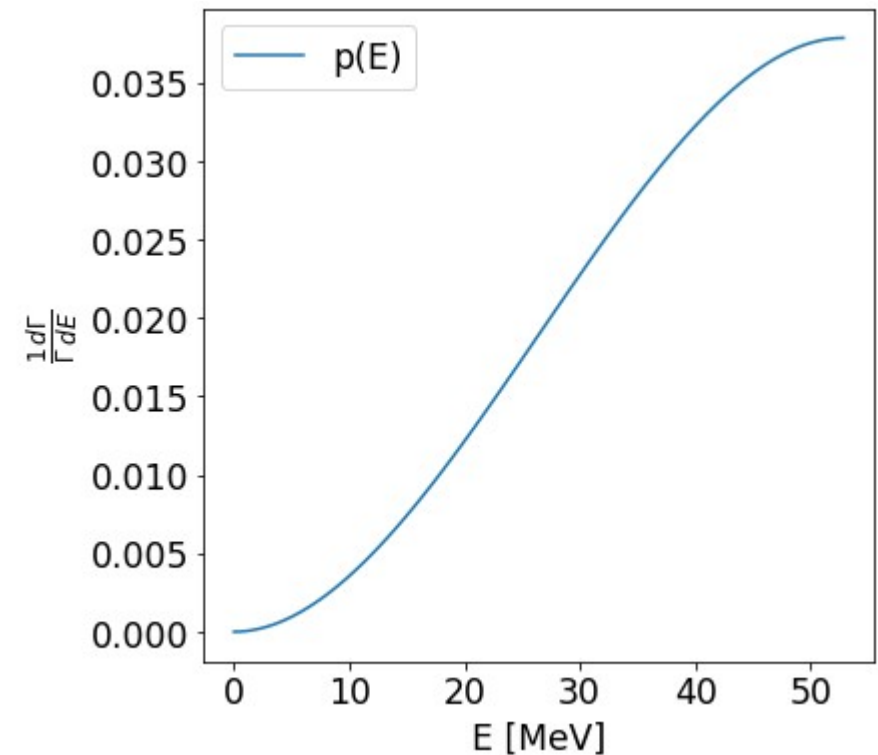
- the differential decay width section is as follows:

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{4\pi^3} m_\mu^2 E_e^2 \left(1 - \frac{4E_e}{3m_\mu}\right)$$

- the total decay width is as follows:

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

- the probability is:  $p(E)dE = \frac{1}{\Gamma} \frac{d\Gamma}{dE_e} dE$



How to generate fake dataset (energy values only for now) of electrons from  $\mu$  decay?

- draw numbers (energy values) from correct probability distribution.

How? Let's look at cumulative distribution function (CDF):

$$F(x) = \int_{-\infty}^x p(x) dx, \quad 0 \leq F(x) \leq 1$$

$$p(x) = \frac{dF(x)}{dx}$$

- what is probability distribution for  $F(x)$ ?

$$P(x_0 < x < x_1) = \int_{x_0}^{x_1} p(x) dx = \int_{F(x_0)}^{F(x_1)} p(F) dF$$

$$\parallel$$

$$\int_{x_0}^{x_1} \frac{dF(x)}{dx} dx =$$

$$\int_{F(x_0)}^{F(x_1)} \frac{dF}{dx} \frac{dx}{dF} dF = \int_{F(x_0)}^{F(x_1)} \frac{dF}{dx} \left( \frac{dF}{dx} \right)^{-1} dF =$$

$$\int_{F(x_0)}^{F(x_1)} 1 dF$$

property of inverse  
function derivative

change of integration  
variable  $x \rightarrow F$

$$p(F) = \frac{dF}{dx} \left( \frac{dF}{dx} \right)^{-1} = 1$$

CDF has a flat distribution over  
[0,1] range.

# How to generate a number from arbitrary distribution?

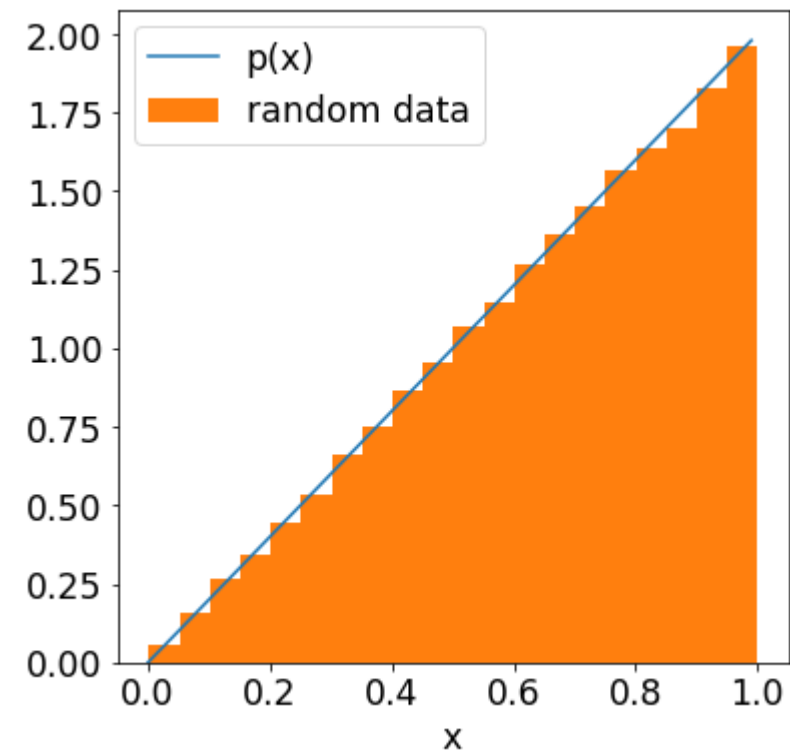
- 1) draw a random number  $z$ , from flat distribution in  $[0,1]$
- 2) pretend this is a CDF value for your distribution at some  $x$ :

$$z = \int_0^x p(x) dx$$

- 3) calculate  $x$  using functional form of  $F^{-1}(z)$ :

$$x = F^{-1}(z)$$

**Problem:**  $F^{-1}(z)$  is often not easy (or possible) to calculate.



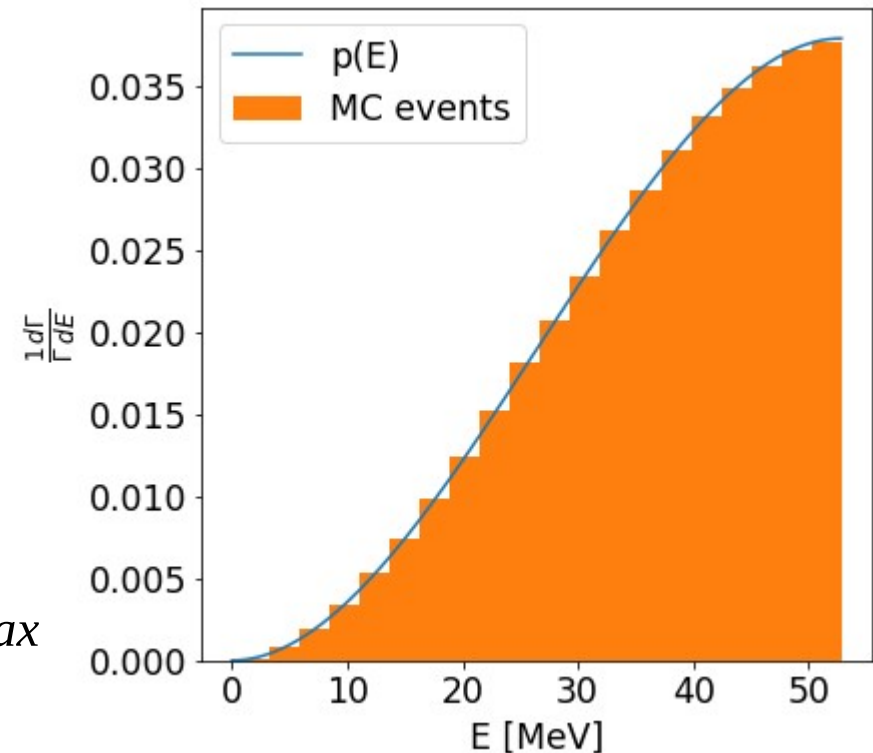
1) draw the value of number of interest (e.g. energy,  $E$ ) from flat distribution in desired range:  $[E_{\min}, E_{\max}]$

2) draw a second number,  $z$ , from flat distribution in  $[0,1]$

3) calculate  $p(E)$

4) accept the “event” if:  $x < p(x)$

or more effectively if:  $x < p(x) / p_{\max}$



**Review articles of probability and statistics on PDG portal:**

- *The Review of Particle Physics*

**Python packages:**

- *Scikit-HEP Project*