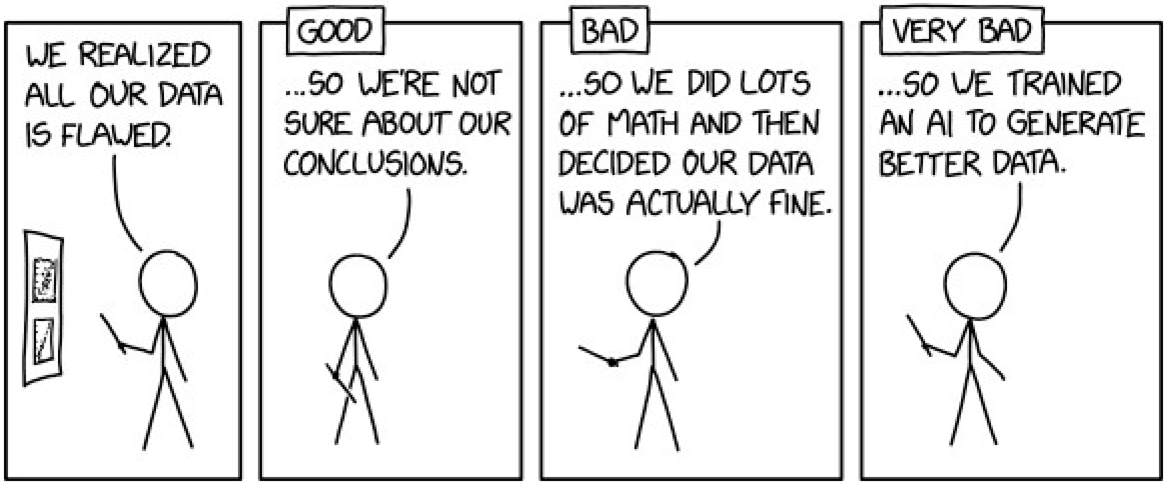
Modern Particle Physics Experiments

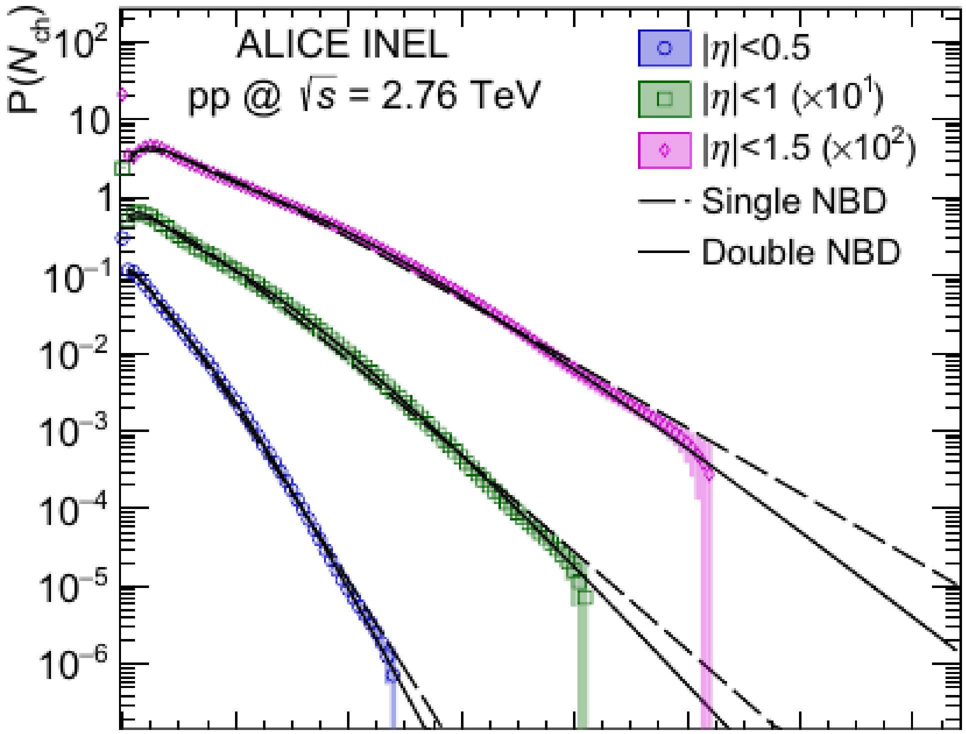


https://xkcd.com/2494/

The probability

Many aspects of particle physics are of stochastic nature:

- final state properties in particle collisions:
  - **number of particles**
  - type of particles
  - particle four-momenta



https://arxiv.org/abs/1509.07541

The probability

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https://arxiv.org/abs/1003.3124

  - number of particles
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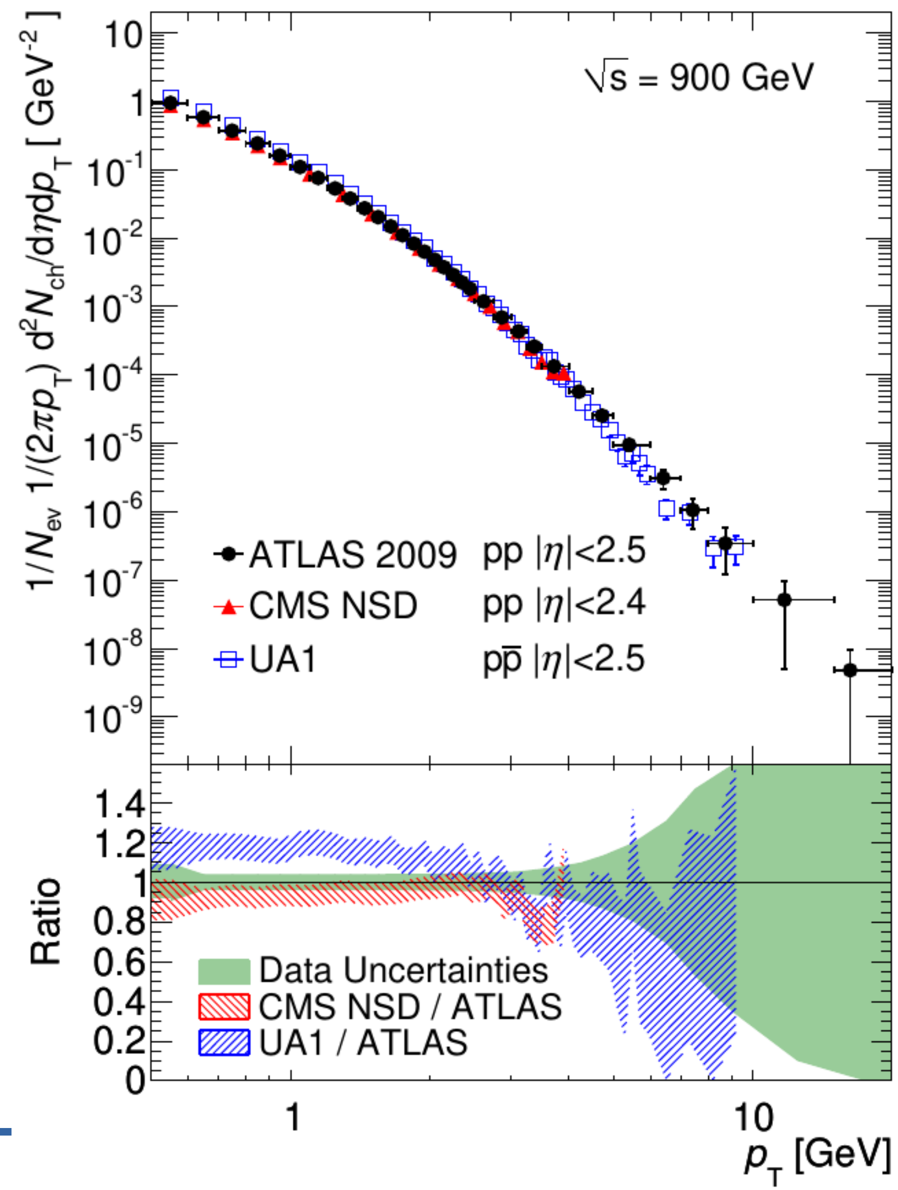
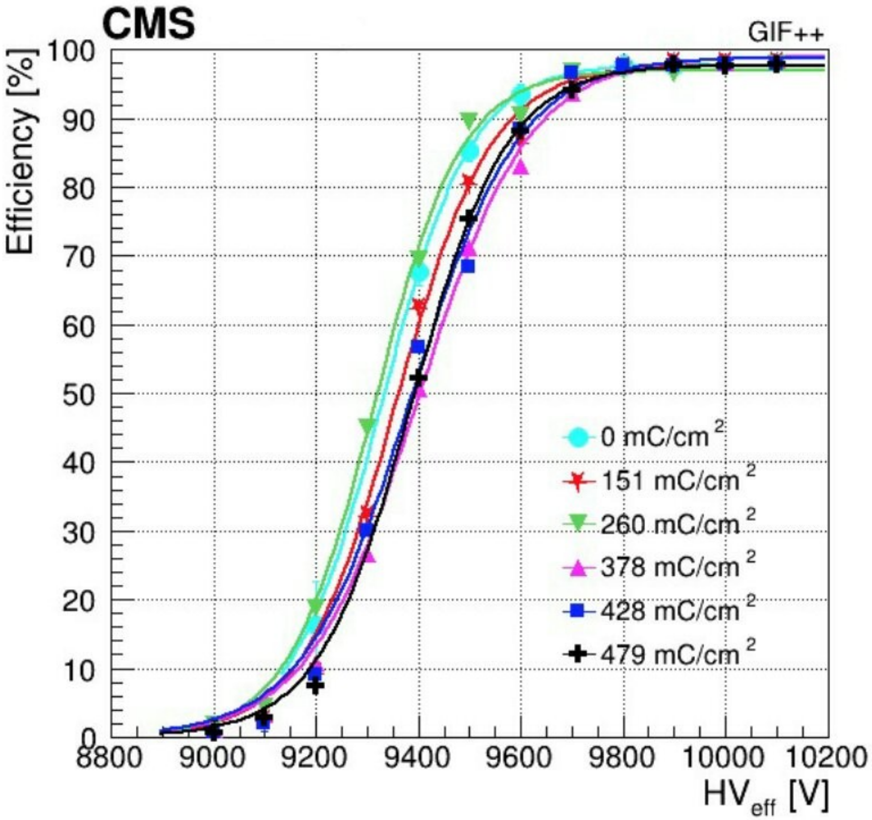


The probability

Many aspects of particle physics are of stochastic nature:

- response of the detector:
  - efficiency of
    - **detector elements**
    - trigger
    - reconstruction algorithms
  - difference of measured values from the true values (resolution)



https://arxiv.org/abs/2005.11397

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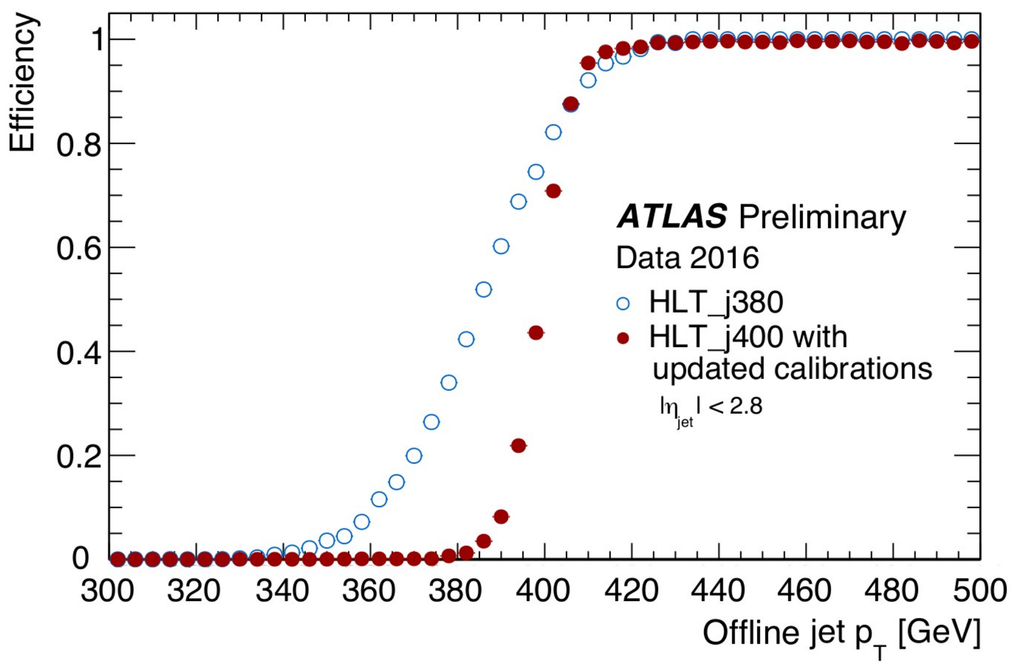

https://arxiv.org/abs/1711.02946

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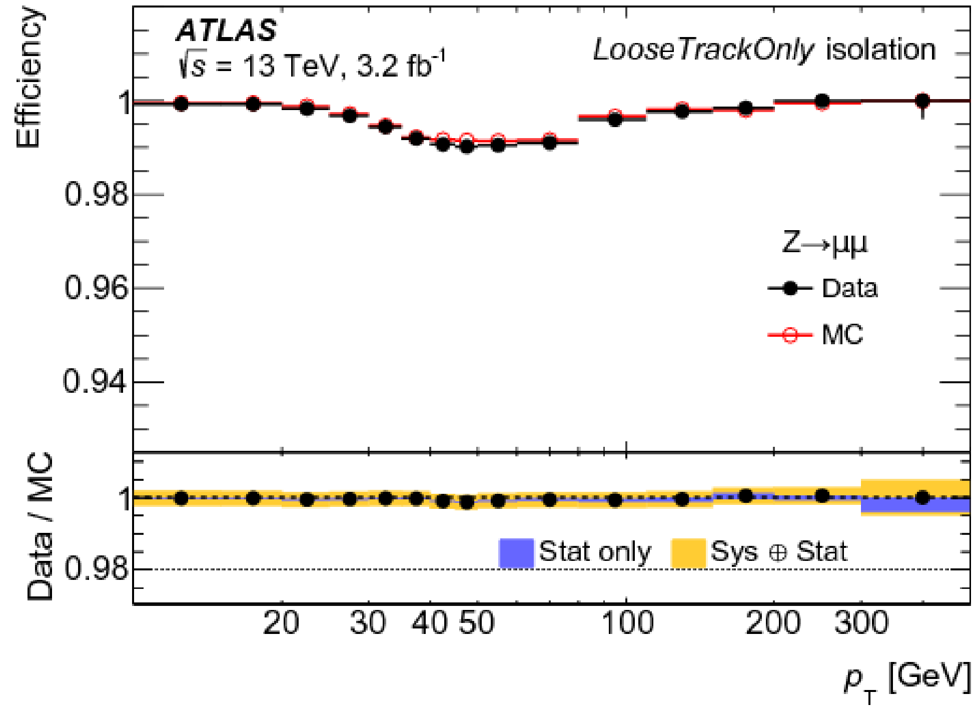

https://arxiv.org/abs/1603.05598

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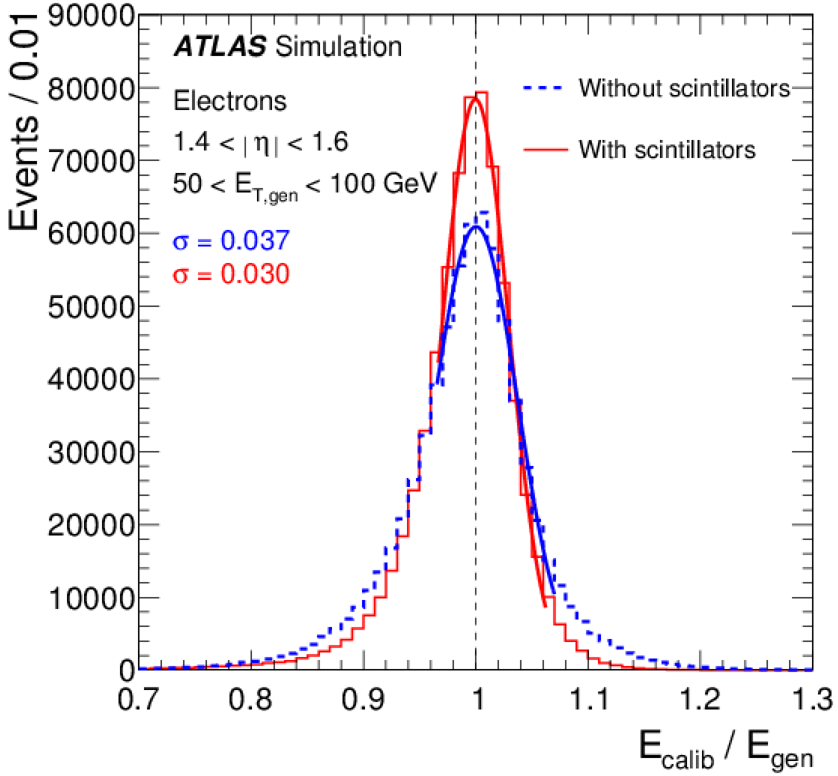


https://arxiv.org/abs/1812.03848

The probability

Probability distribution, or probability density for results of particle interactions are proportional to differential cross sections:

$$\frac{d\sigma}{dN} = \dots, \quad \frac{d\sigma}{dp_T} = \dots, \quad \frac{d\sigma}{d\eta} = \dots$$

Probability distribution has to correctly normalized:

$$p(\eta) = \frac{1}{\sigma_{tot}} \frac{d\sigma}{d\eta}$$

total cross section – normalization constant

differential cross section – the distribution shape

Electron spectrum from μ decay

What is the probability distribution for the electron energy for electrons from muon decay?

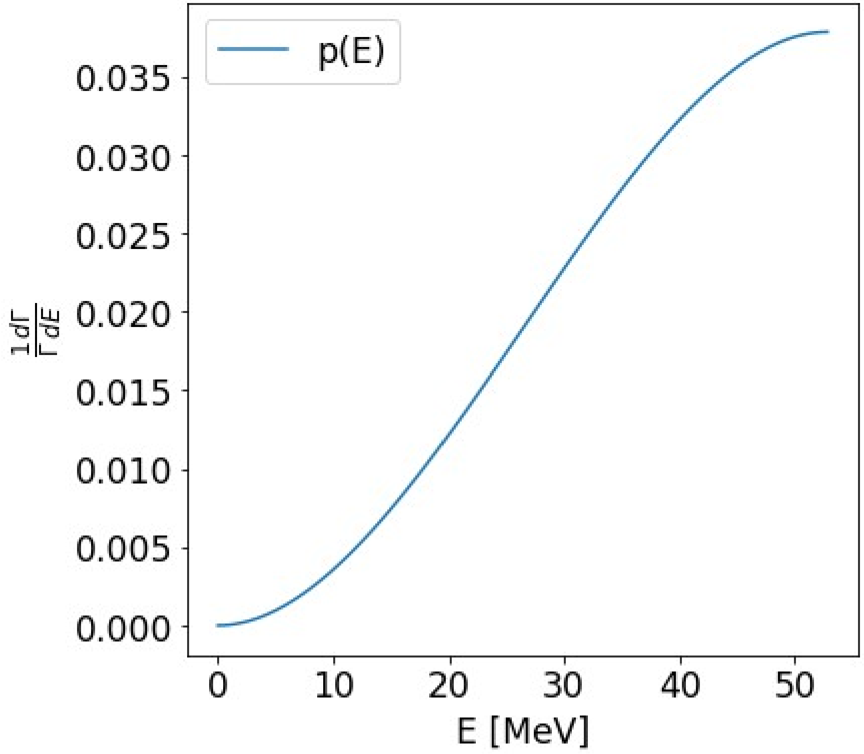
- the differential decay width section is as follows:

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{4\pi^3} m_\mu^2 E_e^2 \left(1 - \frac{4E_e}{3m_\mu}\right)$$

- the total decay width is as follows:

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

- the probability is: $p(E)\,dE = \frac{1}{\Gamma}\frac{d\Gamma}{dE_e}dE$



Event generation

How to generate fake dataset (energy values only for now) of electrons from μ decay?

- draw numbers (energy values) from correct probability distribution.

How? Let's look at cumulative distribution function (CDF):

$$F(x)=\int_{-\infty}^{x} p(x)\,dx\,,\quad 0\leq F(x)\leq 1$$

$$p(x)=\frac{dF(x)}{dx}$$

- what is probability distribution for F(x)?

Probability distribution for CDF

$$P(x_0 < x < x_1) = \int_{x_0}^{x_1} p(x)\,dx = \int_{F(x_0)}^{F(x_1)} p(F)\,dF$$

$$\parallel$$

$$\int_{x_0}^{x_1} \frac{dF(x)}{dx}\,dx =$$

property of inverse function derivative

$$\int_{F(x_0)}^{F(x_1)} \frac{dF}{dx}\frac{dx}{dF}\,dF = \int_{F(x_0)}^{F(x_1)} \frac{dF}{dx}\left(\frac{dF}{dx}\right)^{-1} dF =$$

change of integration variable x→ F

$$\int_{F(x_0)}^{F(x_1)} 1\,dF$$

$$\boxed{p(F) = \frac{dF}{dx}\left(\frac{dF}{dx}\right)^{-1} = 1}$$

CDF has a flat distribution over [0,1] range.

How to generate a number from arbitrary distribution?

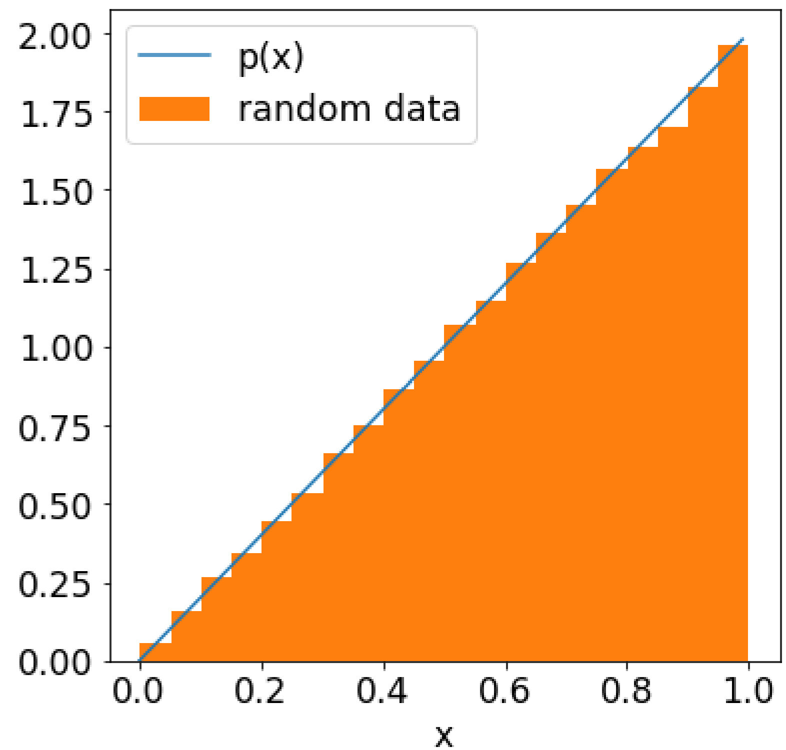
1) draw a random number z, from flat distribution in [0,1]

2) pretend this is a CDF value for your distribution at some x:

$$z = \int_0^x p(x)\,dx$$

3) calculate x using functional form of $F^{-1}(z)$:

$$x = F^{-1}(z)$$



Problem: $F^{-1}(z)$ is often not easy (or possible) to calculate.

Hit-or-miss method I

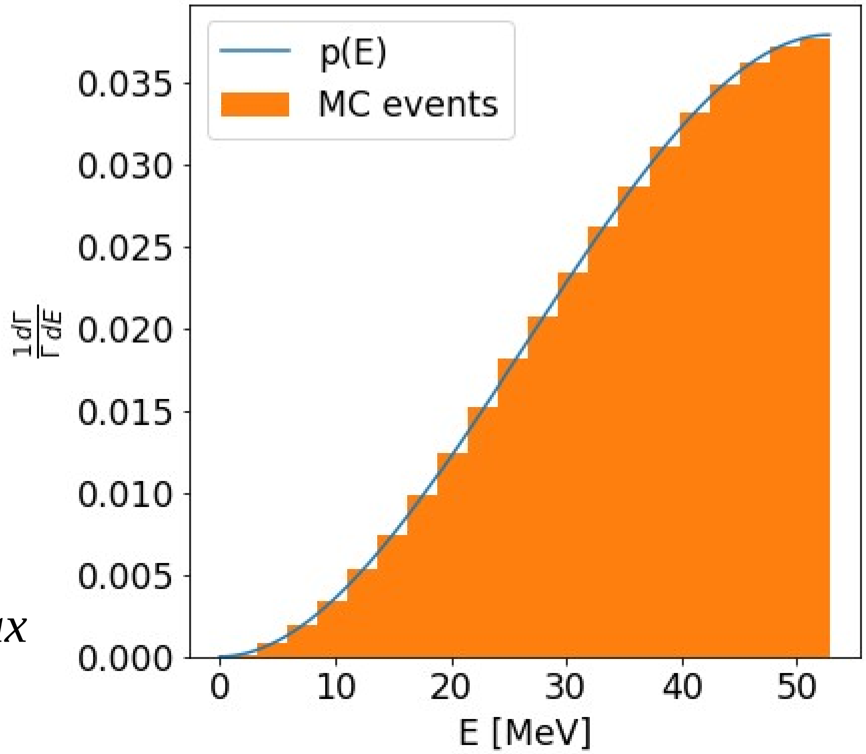
1) draw the value of number of interest (e.g. energy, E) from flat distribution in desired range: [E_{min}, E_{max}]

2) draw a second number, z, from flat distribution in [0,1]

3) calculate p(E)

4) accept the "event" if: $x < p(x)$

or more effectively if: $x < p(x)/p_{max}$



Hit-or-miss method II

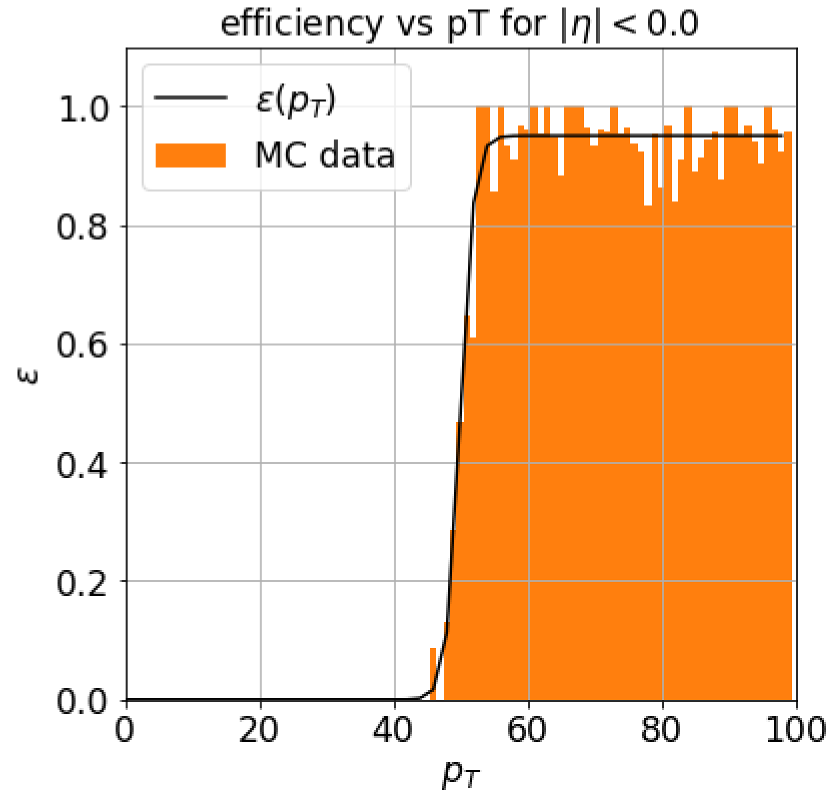
Assume we want to generate random result: PASS/FAIL according to some probability: p

1) calculate the probability, e.g. some efficiency: $\varepsilon(p_T)=x$

2) draw second number, z, from flat distribution in [0,1]

3) assign “PASS” if: $z < x$

One can calculate the efficiency from the MC data plotting a histogram:



$$\epsilon^{estimate} = \frac{N^{MC\,events\,with\,PASS}}{N^{totam\,MC\,events}}$$

Value smearing

The difference between the measured an true values is described by some probability function:

$$p\left(\Delta = x^{measured} - x^{true}\right)$$

p(*Δ*) is usually a Gaussian distribution:

$$p(\Delta) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-\Delta^2}{2\sigma^2}}$$

Value smearing

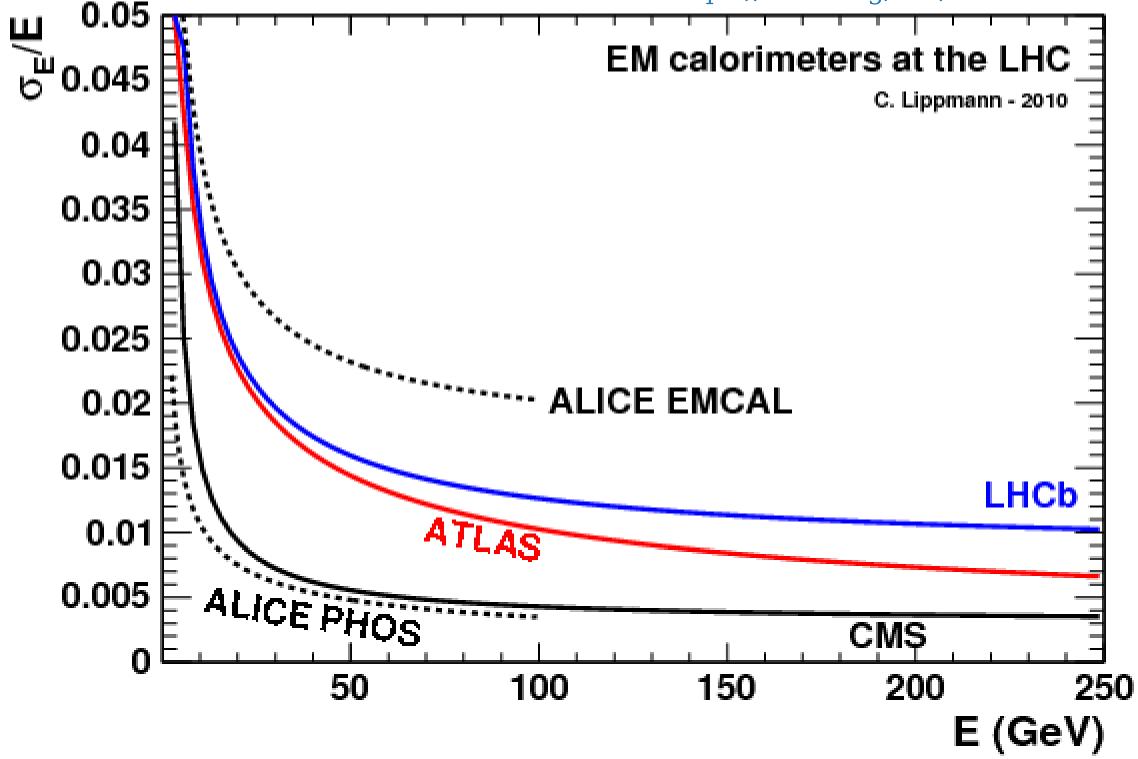
Often is it more convenient to use a ratio measured/true:

$$z=\frac{x^{measured}-x^{true}}{x^{true}}-1=\frac{x^{measured}}{x^{true}}$$

$$p(z)=\frac{1}{\sqrt{2\pi\sigma_z}}e^{\frac{-(z-1)^2}{2\sigma_z^2}}$$

$$\sigma_z=\frac{\sigma}{x^{true}}$$

https://arxiv.org/abs/1101.3276



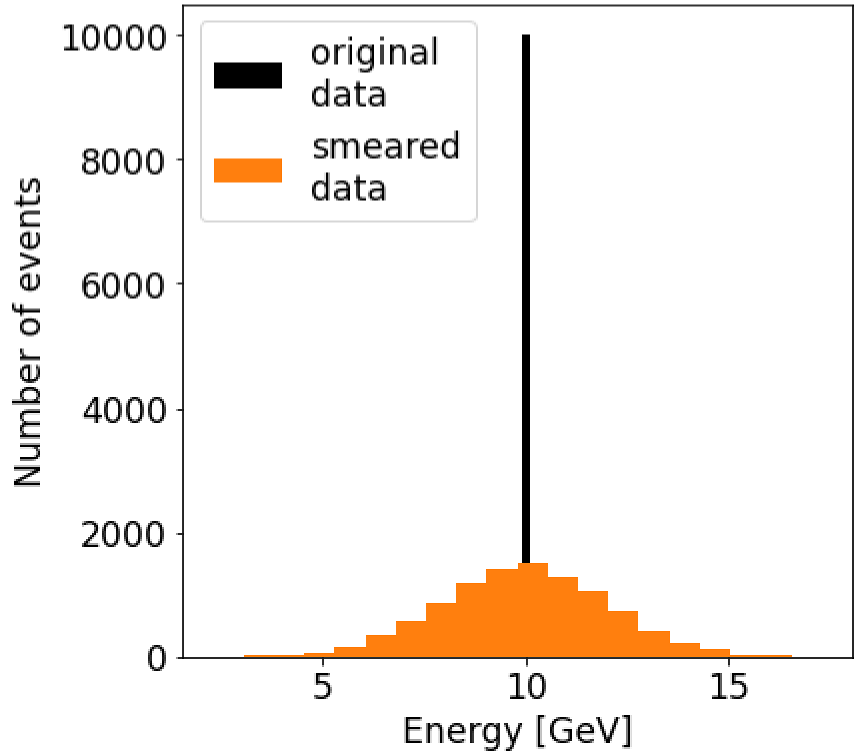
Here the σ_z is the relative resolution.

Value smearing

How to generate values for $x^{measured}$?

1) generate a set of true values, e.g. E^{true}

2) multiply each element set by a "smearing" factor z, draw from the z distribution



Materials

Review articles of probability and statistics on PDG portal:

- *The Review of Particle Physics*

Python packages:

- *Scikit-HEP Project*