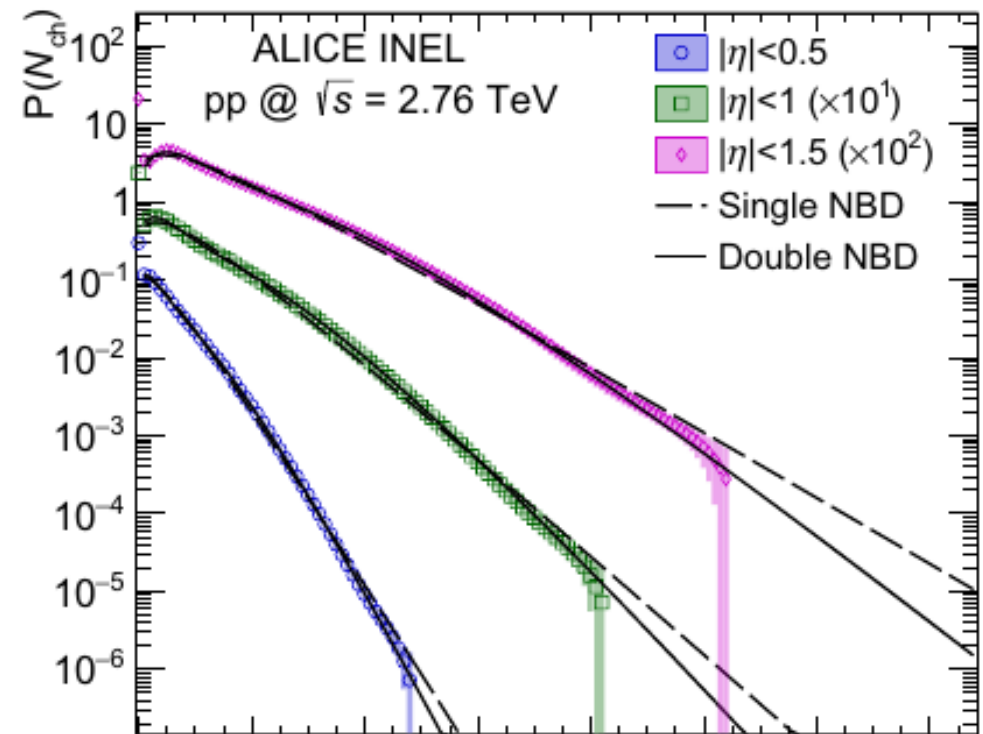


Modern Particle Physics Experiments

Many aspects of particle physics are of stochastic nature:

- final state properties in particle collisions:

- **number of particles**
- type of particles
- particle four-momenta



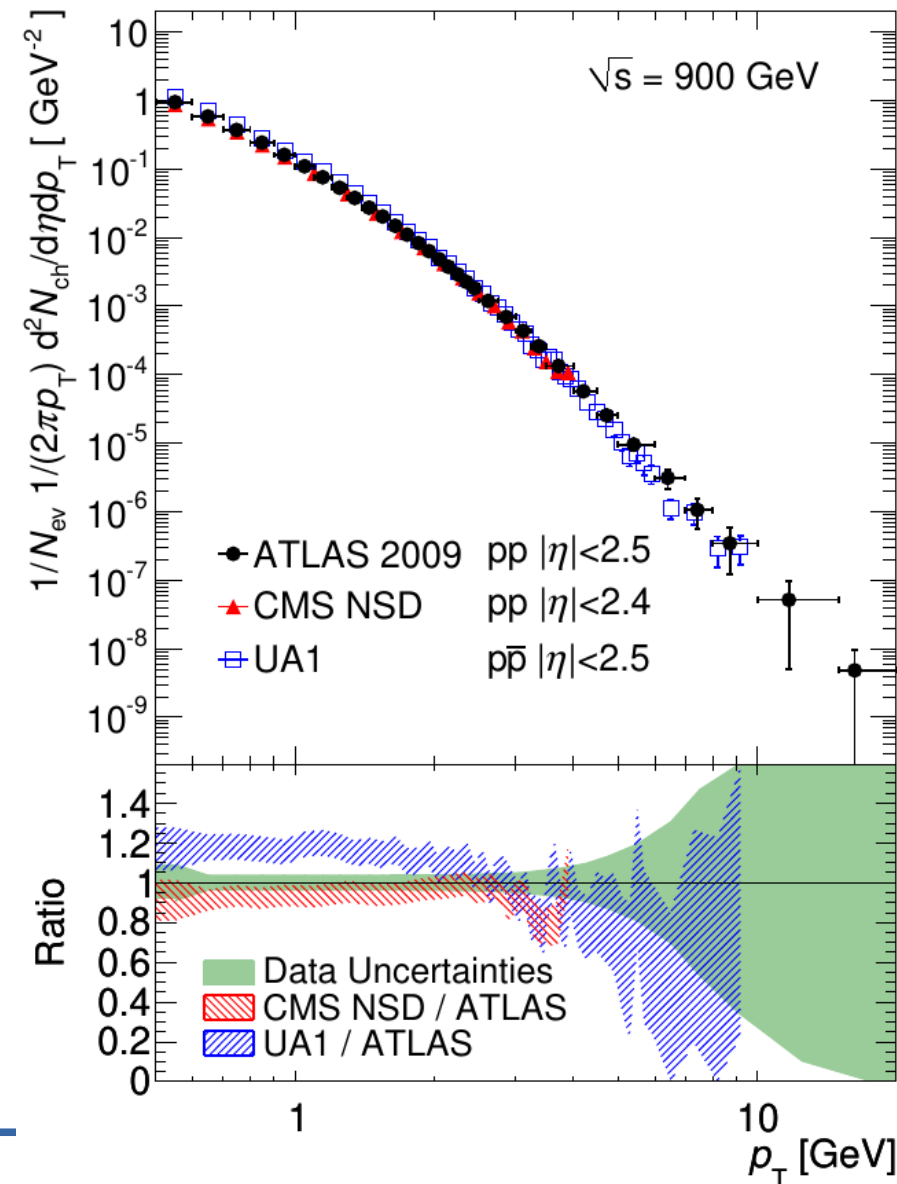
<https://arxiv.org/abs/1509.07541>

Many aspects of particle physics are of stochastic nature:

- final state properties in particle collisions:

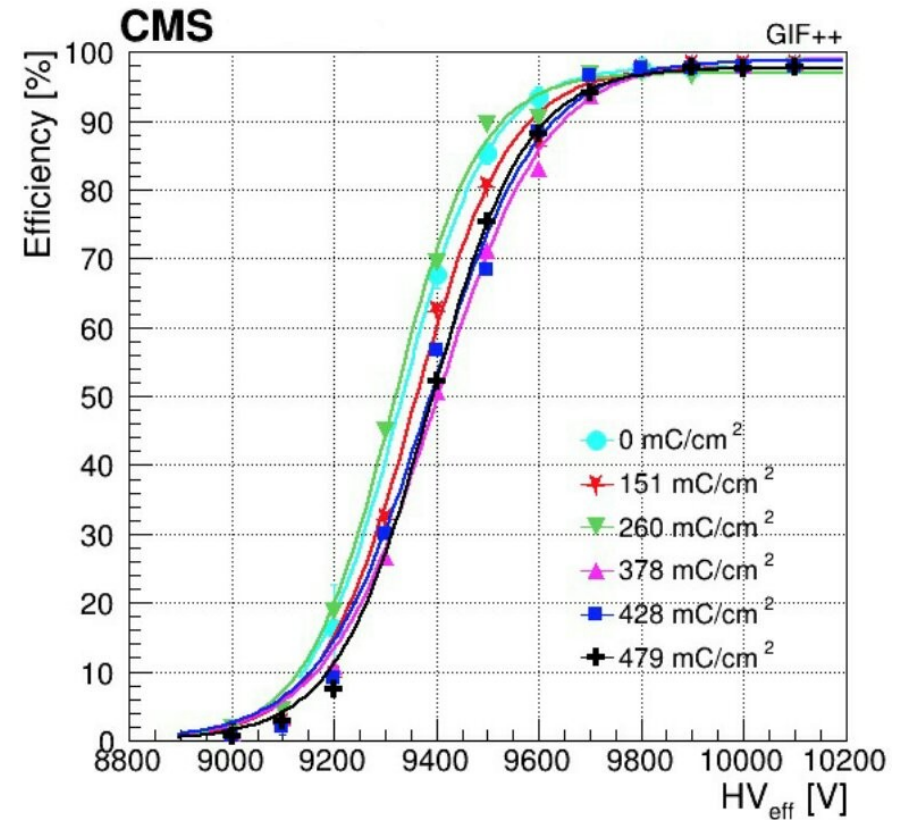
<https://arxiv.org/abs/1003.3124>

- number of particles
- type of particles
- **particle four-momenta**



Many aspects of particle physics are of stochastic nature:

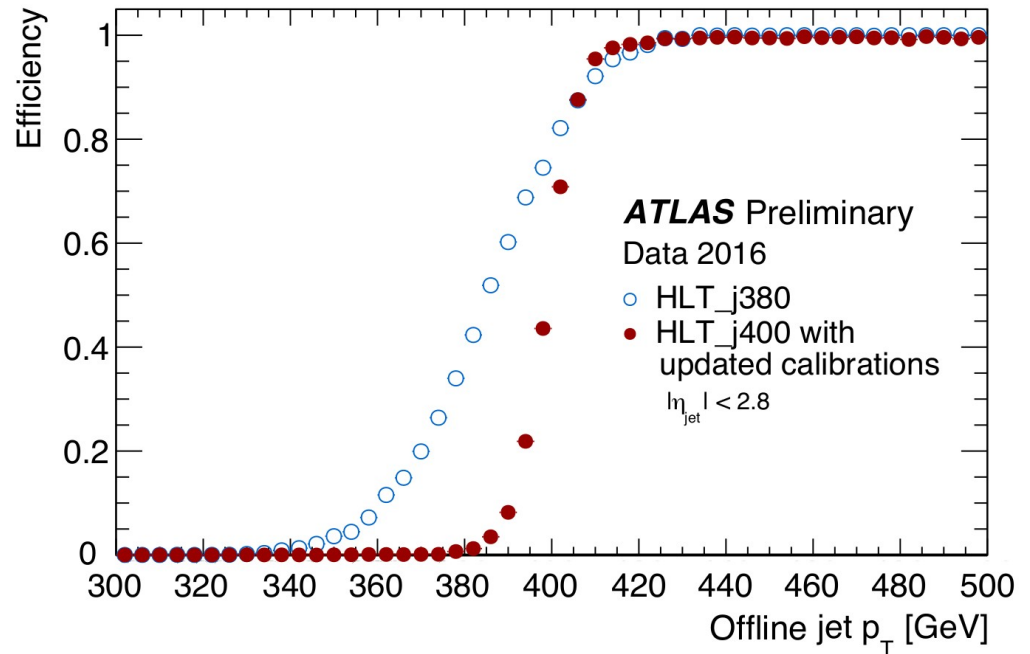
- response of the detector:
 - efficiency of
 - **detector elements**
 - trigger
 - reconstruction algorithms
 - difference of measured values from the true values (resolution)



<https://arxiv.org/abs/2005.11397>

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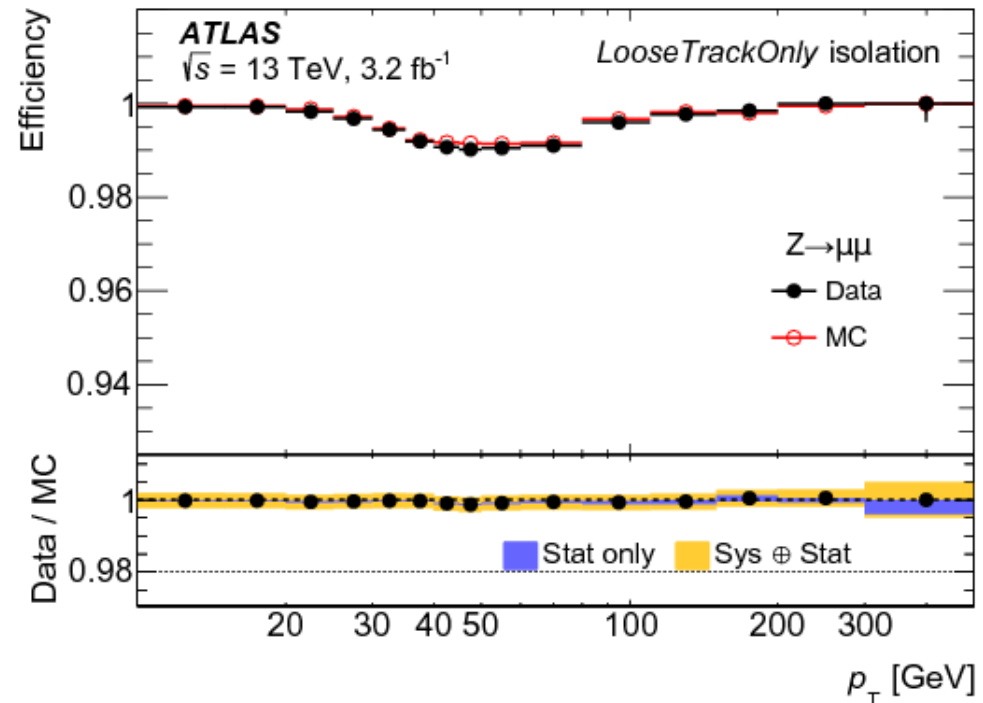
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<https://arxiv.org/abs/1711.02946>

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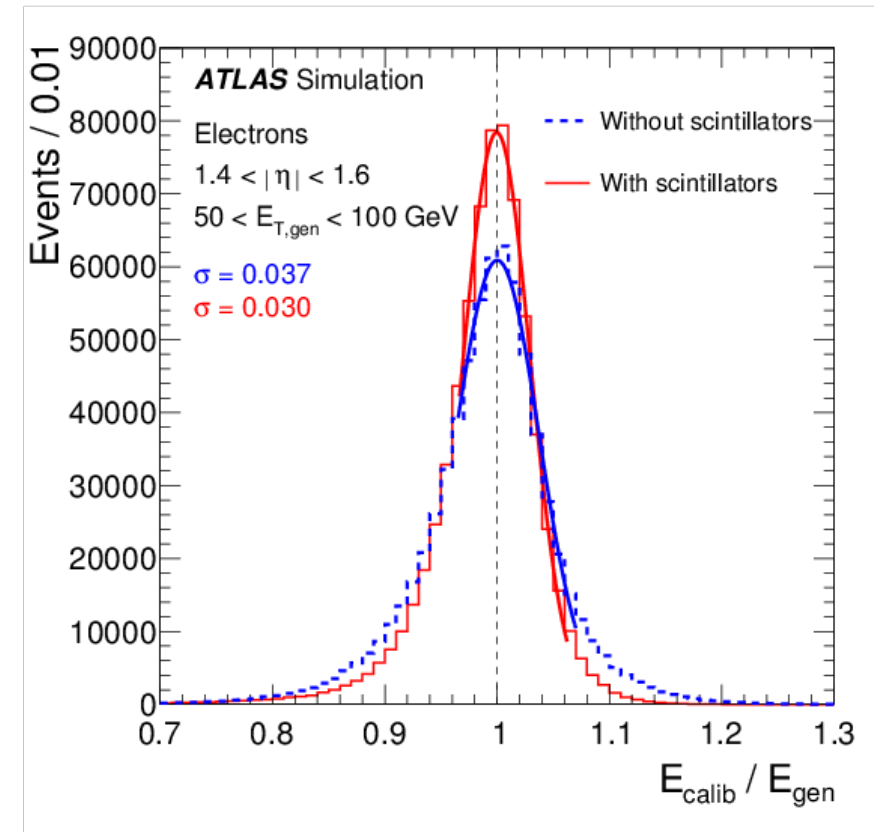
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<https://arxiv.org/abs/1603.05598>

Many aspects of particle physics are of stochastic nature:

- response of the detector:
 - efficiency of
 - detector elements
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 - reconstruction algorithms
 - **difference of measured values from the true values (resolution)**



<https://arxiv.org/abs/1812.03848>

Probability distribution, or probability density for results of particle interactions are proportional to differential cross sections:

$$\frac{d\sigma}{dN} = \dots, \quad \frac{d\sigma}{dp_T} = \dots, \quad \frac{d\sigma}{d\eta} = \dots$$

Probability distribution has to be correctly normalized:

$$p(\eta) = \frac{1}{\sigma_{tot}} \frac{d\sigma}{d\eta}$$

total cross section
– normalization
constant

differential cross
section – the
distribution shape

What is the probability distribution for the electron energy for electrons from muon decay?

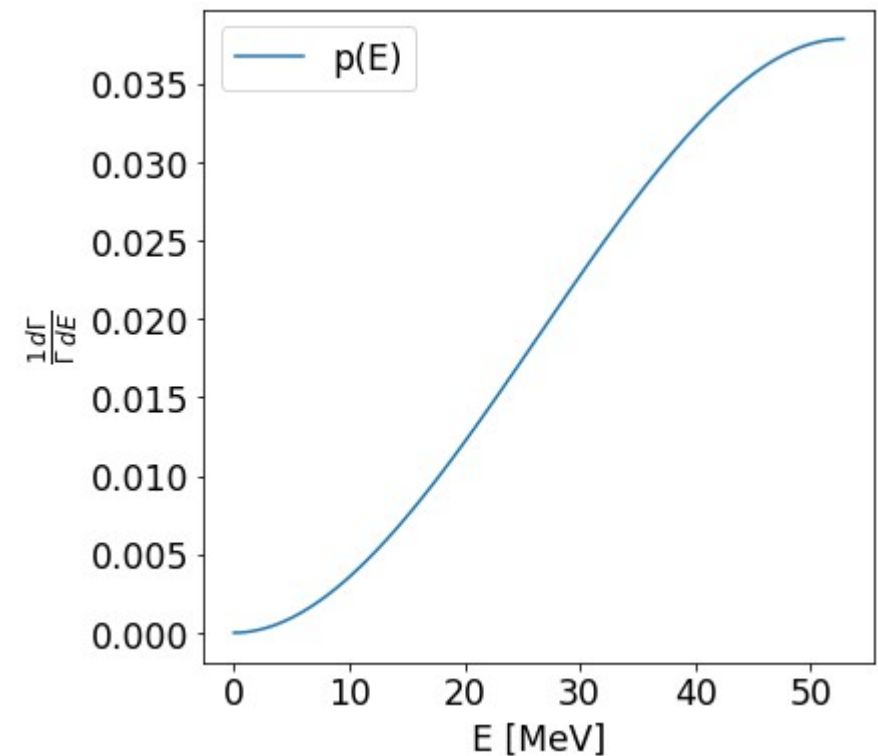
- the differential decay width section is as follows:

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{4\pi^3} m_\mu^2 E_e^2 \left(1 - \frac{4E_e}{3m_\mu}\right)$$

- the total decay width is as follows:

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

- the probability is: $p(E)dE = \frac{1}{\Gamma} \frac{d\Gamma}{dE_e} dE$



How to generate fake dataset (energy values only for now) of electrons from μ decay?

- draw numbers (energy values) from correct probability distribution.

How? Let's look at cumulative distribution function (CDF):

$$F(x) = \int_{-\infty}^x p(x) dx, \quad 0 \leq F(x) \leq 1$$

$$p(x) = \frac{dF(x)}{dx}$$

- what is probability distribution for $F(x)$?

$$P(x_0 < x < x_1) = \int_{x_0}^{x_1} p(x) dx = \int_{F(x_0)}^{F(x_1)} p(F) dF$$

$$\parallel$$

$$\int_{x_0}^{x_1} \frac{dF(x)}{dx} dx =$$

$$\int_{F(x_0)}^{F(x_1)} \frac{dF}{dx} \frac{dx}{dF} dF = \int_{F(x_0)}^{F(x_1)} \frac{dF}{dx} \left(\frac{dF}{dx} \right)^{-1} dF =$$

$$\int_{F(x_0)}^{F(x_1)} 1 dF$$

property of inverse
function derivative

change of integration
variable $x \rightarrow F$

$$p(F) = \frac{dF}{dx} \left(\frac{dF}{dx} \right)^{-1} = 1$$

← CDF has a flat distribution over
[0,1] range.

How to generate a number from arbitrary distribution?

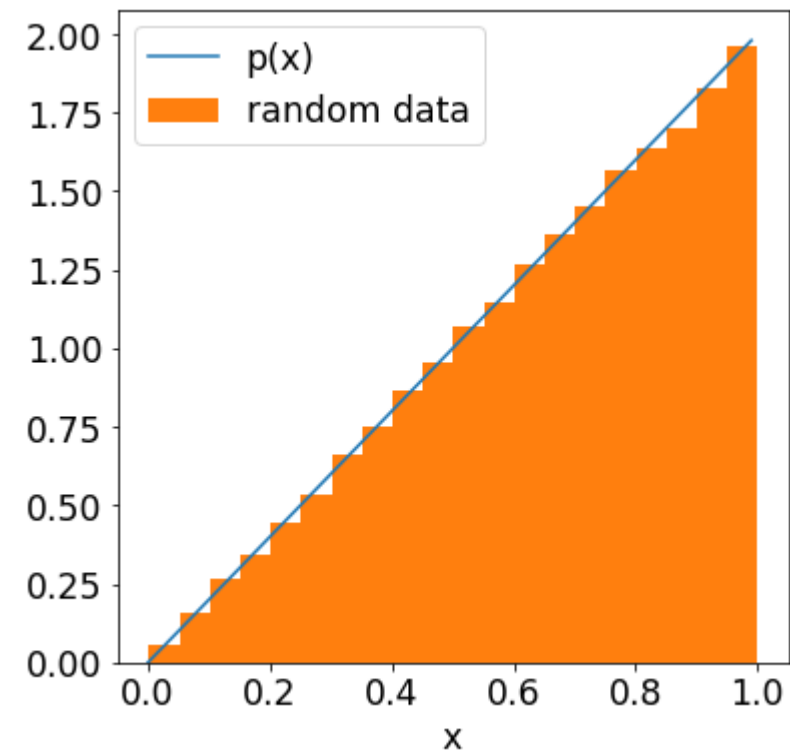
- 1) draw a random number z , from flat distribution in $[0,1]$
- 2) pretend this is a CDF value for your distribution at some x :

$$z = \int_0^x p(x) dx$$

- 3) calculate x using functional form of $F^{-1}(z)$:

$$x = F^{-1}(z)$$

Problem: $F^{-1}(z)$ is often not easy (or possible) to calculate.



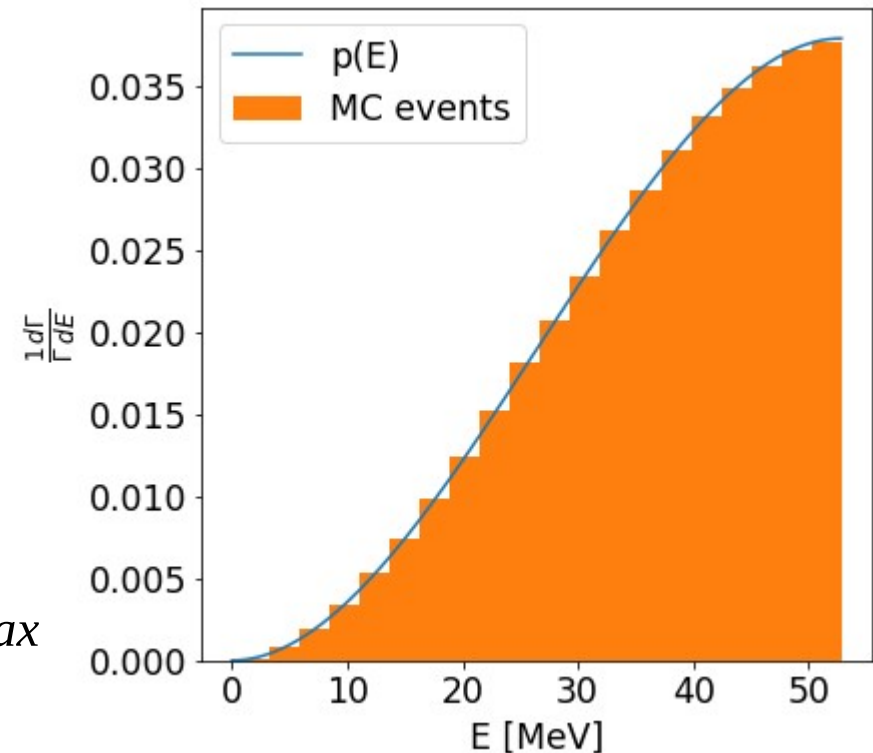
1) draw the value of number of interest (e.g. energy, E) from flat distribution in desired range: $[E_{\min}, E_{\max}]$

2) draw a second number, z , from flat distribution in $[0,1]$

3) calculate $p(E)$

4) accept the “event” if: $x < p(x)$

or more effectively if: $x < p(x) / p_{\max}$



Assume we want to generate random result: PASS/FAIL according to some probability: p

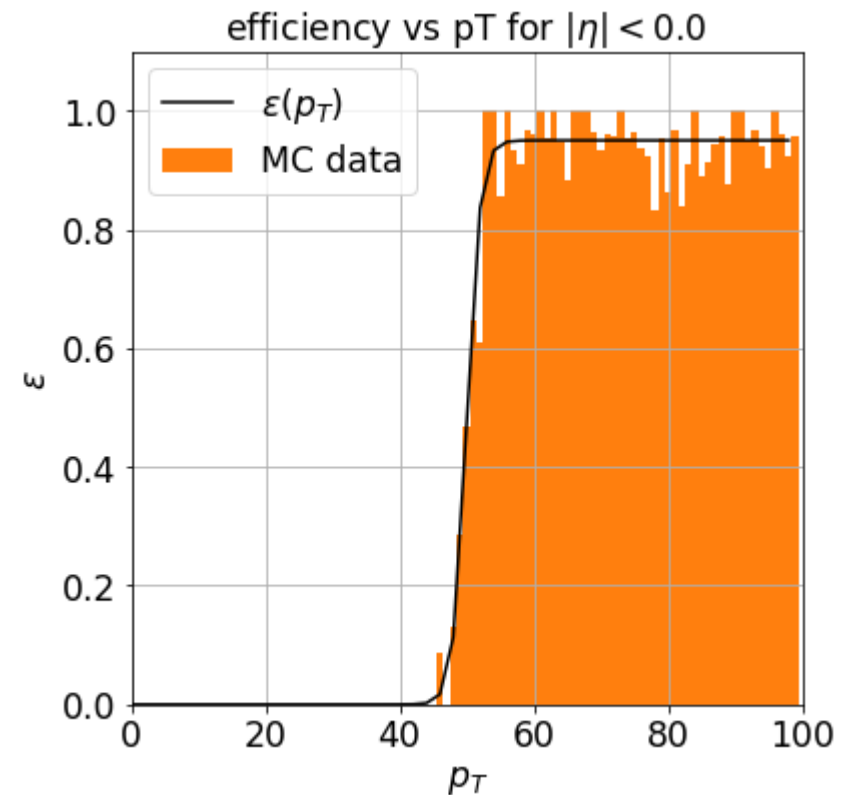
1) calculate the probability, e.g. some efficiency: $\varepsilon(p_T) = x$

2) draw second number, z , from flat distribution in $[0,1]$

3) assign “PASS” if: $z < x$

One can calculate the efficiency from the MC data plotting a histogram:

$$\epsilon^{estimate} = \frac{N^{MC \text{ events with PASS}}}{N^{total MC \text{ events}}}$$



The difference between the measured and true values is described by some probability function:

$$p(\Delta = x^{\text{measured}} - x^{\text{true}})$$

$p(\Delta)$ is usually a Gaussian distribution:

$$p(\Delta) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-\Delta^2}{2\sigma^2}}$$

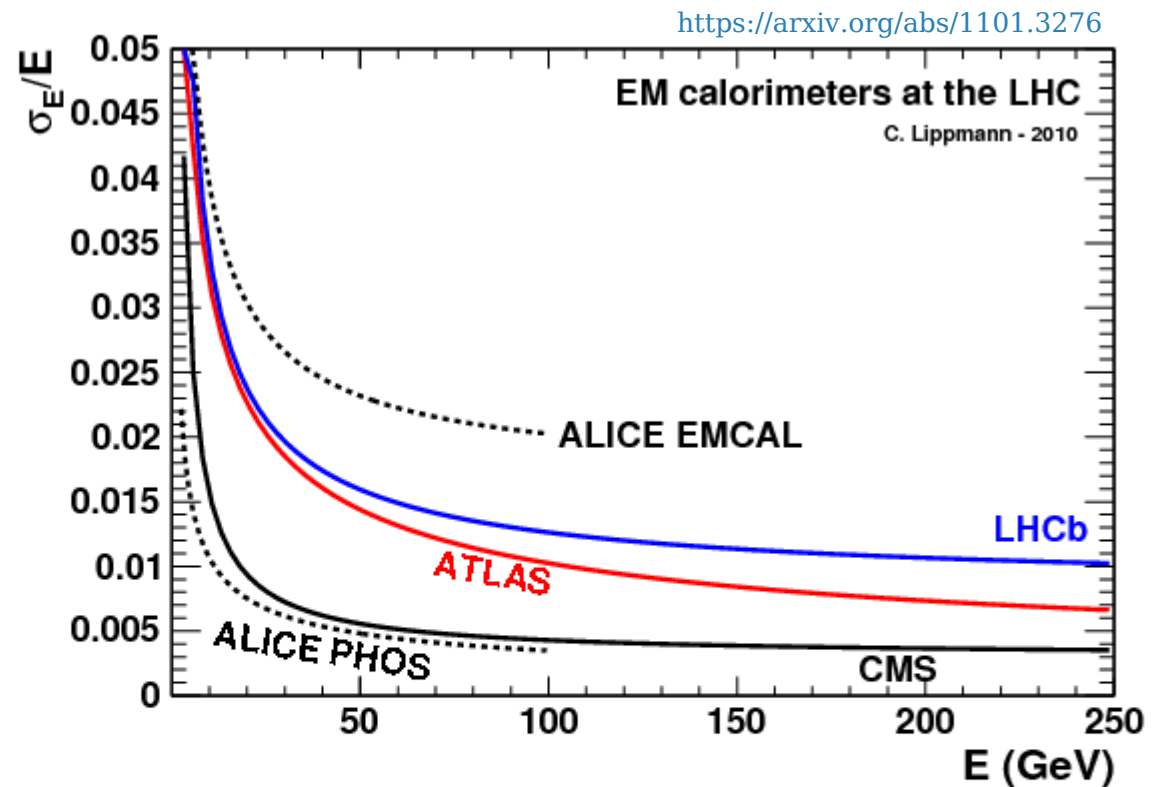
Often is it more convenient to use a ratio measured/true:

$$z = \frac{X^{\text{measured}} - X^{\text{true}}}{X^{\text{true}}} - 1 = \frac{X^{\text{measured}}}{X^{\text{true}}}$$

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma_z} e^{\frac{-(z-1)^2}{2\sigma_z^2}}$$

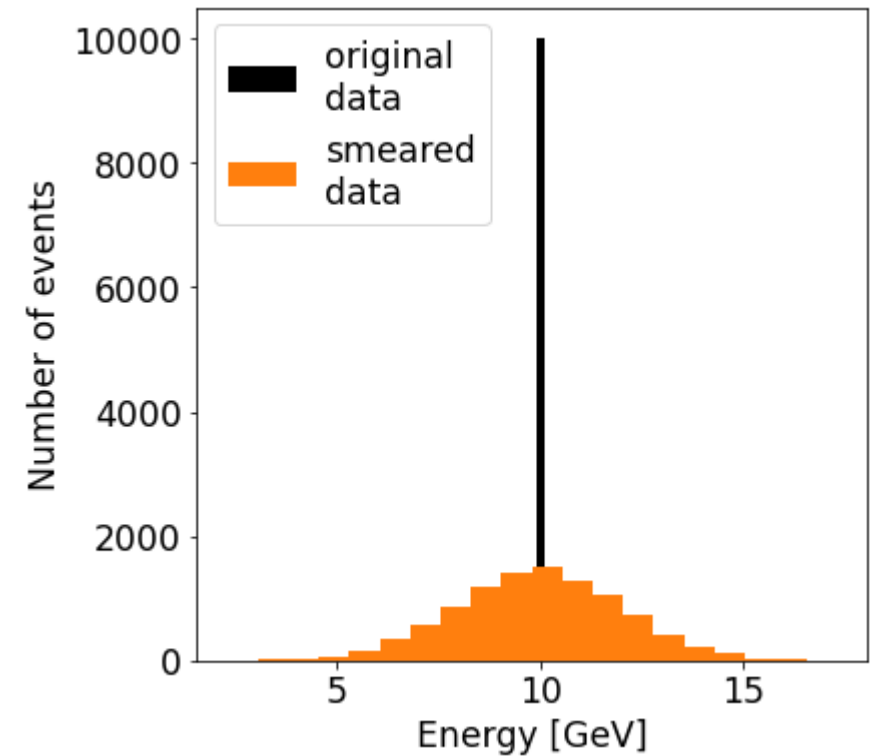
$$\sigma_z = \frac{\sigma}{X^{\text{true}}}$$

Here the σ_z is the relative resolution.



How to generate values for x^{measured} ?

- 1) generate a set of true values, e.g. E^{true}
- 2) multiply each element set by a “smearing” factor z , draw from the z distribution



Review articles of probability and statistics on PDG portal:

- *The Review of Particle Physics*

Python packages:

- *Scikit-HEP Project*