Chapter 1: Introduction to ECON 1630

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Jonathan Roth

Mathematical Econometrics I
Brown University
Fall 2023

Outline

Course Preliminaries I - Introducing Ourselves

Welcome to ECON 1630! I'm looking forward to teaching you all

I'm Professor Jonathan Roth

- Individual OH: Typically Mon 415-445 and Tue 330-4; Sign up on my website: here, 8 Fones Alley 014 (or Zoom by request)
- Group OH on Zoom (link): Thu 330-4

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Our Grad TAs are Tommaso Coen and Eddie Wu
Our undergrad TAs are Henry Powers and Preetish Juneja (star students from last semster!)

- All TAs will hold OHs
- Grad TAs will hold weekly sessions: TBA shortly
- Times/locations for TA OHs will be announced shortly on Canvas

Course preliminaries II

Course materials and communications will be posted on Canvas:
https://canvas.brown.edu/courses/1093393

Course Preliminaries III

Meeting times:

- Lectures: Tues/Thurs 9-1020 (S02) or 1-220pm (S01). Recordings will be posted online after class.
- TA Sessions: Time/location TBD
  - Sessions will review concepts from class and help with Stata
- Attendance is not required but is **highly encouraged**

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Prerequisites:

- Multivariate calculus, probability/statistics, and linear algebra
- Some familiarity with reading/writing proofs and code

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Software:

- Stata for statistical analyses (to be covered in TA sessions)
  - You're welcome to use R/Python instead, but we can't promise any help with coding issues
- LaTeX for typing up problem sets (optional, for extra credit)
- Ask us for help if you're having any problems accessing software

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- 6 problem sets due approximately every 2 weeks
- 1 midterm exam (October 19, in class)
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- 40% problem sets, 30% on each exam.
- I'll drop your lowest PSet grade. Use your drop wisely!
- Psets are due at 4PM on Fridays; late submissions won't be graded. Collaboration is OK (please list collaborators)
- The exams will be in-class, closed-book, 1-page "cheat-sheet" allowed
- 5 points extra credit if you use LaTeX for assignments. Please attach your Stata code + output as a single PDF regardless

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Course materials:

- Main material: Lectures and lecture slides, which will be posted on Canvas

Course Preliminaries IV

- Lecture recordings will be posted on Canvas (but you cannot join live)

- Office hours will be in person, unless Zoom is requested
- Please submit all problem sets using Gradescope
- You are encouraged to use EdDiscussion for questions

Course Preliminaries IV

- Lecture recordings will be posted on Canvas (but you cannot join live)

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Any questions on logistics?

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1. Course Preliminaries ✓
2. What is Econometrics?
3. Parameters, Estimands, and Estimators
4. Course Roadmap

What is Econometrics?

$\rightarrow$ The statistical toolkit that economists use to answer economic questions with data

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In this course, we will focus mainly on descriptive and causal questions, with an emphasis on causal questions

Why is answering these questions hard?

- For descriptive Qs: we only observe data for a **sample** of individuals, not for the full **population**
  - Example: we want to know how the distribution of income in the US has changed. But we only observe income for a survey of workers

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  Our sample is **randomly** selected from the population
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- Worst case scenario: our sample is *not representative* of the population that we care about
  - E.g., workers with certain characteristics were more likely to respond to the survey

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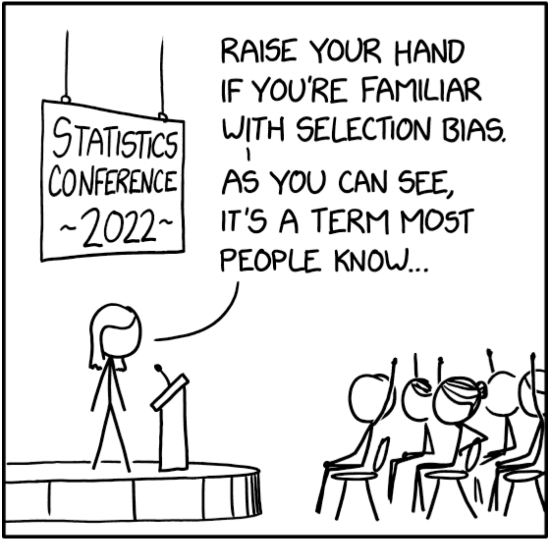
- But their survey was conducted by phone. In 1948, only rich people had phones: sample $\neq$ population $\rightarrow$ misleading results!

Selection bias referes to settings like Dewey-Truman where the sample is not drawn randomly from the population of interest



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- Counterfactual Qs can't ever be answered with data alone. Need additional assumptions to learn about them!

Splitting up the problem

- When thinking about causal Qs, it's often easier to split the problem in two
- **Identification:** what could we learn about the parameters we care about (causal effects) if we had the observable data for the entire population
  - Need to make assumptions about how observed outcomes relate to outcomes that would have been realized under different treatments
- **Statistics**: what can we learn about the full population that we care about from the finite sample that we have?
  - Need to understand the process by which our data is generated from the full population

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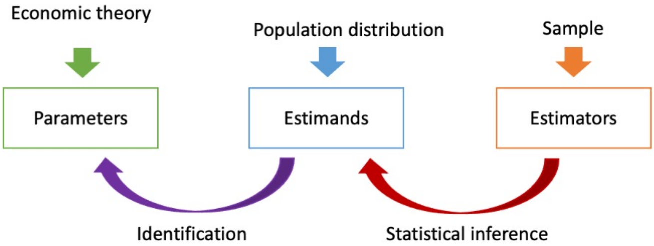
- The process of learning about the *parameter* from the *estimand* is called **identification**.


Economic theory
Population distribution
Sample
Parameters
Estimands
Estimators
Identification
Statistical inference

Let's add some math...

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  - Super useful framework for thinking about causality!
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- We can write the observed outcome as $Y_i = D_i Y_i(1) + (1 - D_i) Y_i(0)$

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  - Difference in sample mean of earnings for people who went to Brown and people who went to URI:

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- Example estimand:
  - Difference in population mean of earnings for people went to Brown and people who went to URI:

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- Example target parameter:
  - Causal effect of Brown for Brown students:

$$\underbrace{E[Y_i(1)|D_i = 1]}_{\text{Earnings at Brown for Brown students in pop}} - \underbrace{E[Y_i(0)|D_i = 1]}_{\text{Earnings at URI for Brown students in pop}} .$$

Why is causal identification hard?

- Thought experiment: suppose we had data on earnings for *every* Brown and URI graduate
- We can learn from the data:

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- The data doesn't tell us $\underbrace{E[Y_i(0)|D_i = 1]}_{\text{Earnings at URI for Brown Students}}$. Why not?
  - Because we never see Brown students going to URI!

- One idea to solve this problem would be to assume that:

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- Why might this give us the wrong answer?
- Because Brown students may be different from URI students in other ways that would affect their earnings (regardless of where they went to college)
  - Academic ability, family background, career goals, etc.
- These differences are referred to as *omitted variables* or *confounding factors*

What about experiments?

- The gold standard for learning about causal effects is a randomized controlled trial (RCT), aka experiment
- Suppose that the Brown and URI administration randomized who got into which college (assume these are the only 2 colleges for simplicity)
- Since college is randomly assigned, the only thing that differs between Brown and URI students is the college they went to
- Hence,

$$\underbrace{E[Y_i(0)|D_i = 1]}_{\text{Earnings at URI for Brown Students}} = \underbrace{E[Y_i(0)|D_i = 0]}_{\text{Earnings at URI for URI Students}}$$

since we've eliminated any confounding factors

But running experiments is often hard/impossible

- Unfortunately, Brown/URI have not let us randomize who gets into which college
  - At least not yet! If you could convince them to do this, it'd make for a cool senior thesis!
- Likewise, it is difficult to convince states to randomize their minimum wages, or other policies
- In some cases, randomization is not just difficult but would be immoral
  - "What is the causal effect of spousal death on labor supply?"

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  - "What is the causal effect of spousal death on labor supply?"
- In this course, we'll discuss tools economists try to use when running experiments is not possible.

Course Roadmap – Where we're going

- **Part I ($\sim$ 7 lectures): Review of probability/statistics**. This will give us a mathematical language to talk about:
  1. *Statistical estimation/inference:* how does the sample we observe relate to the population of interest
  2. *Identification:* how do observable features of the population relate to (causal) parameters we care about

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- **Part III ($\sim$ 7 lectures:) Other "quasi-experimental" strategies**: We'll discuss other strategies for "mimicking" an experiment when it's not available, including instrumental variables (IV) and regression discontinuity (RD)