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LOWER BOUNDS FOR THE STABLE MARRIAGE PROBLEM AND ITS VARIANTS*

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Abstract. In an instance of the stable marriage problem of size n, n men and n women, each participant ranks members of the opposite sex in order of preference. A *stable marriage* is a complete matching $M=\{(m_1, w_{i_1}), (m_2, w_{i_2}), \cdots, (m_n, w_{i_n})\}$ such that no unmatched man and woman prefer each other to their partners in M. There exists an efficient algorithm, due to Gale and Shapley, that finds a stable marriage for any given problem instance.

A pair (m_i, w_j) is *stable* if it is contained in some stable marriage. In this paper, the problem of determining whether an arbitrary pair is stable in a given problem instance is studied. It is shown that the problem has a lower bound of $\Omega(n^2)$ in the worst case. Hence, a previous known algorithm for the problem is asymptotically optimal.

As corollaries of these results, the lower bound of $\Omega(n^2)$ is established for several stable marriage related problems. Knuth, in his treatise on stable marriage, asks if there is an algorithm that finds a stable marriage in less than $\Theta(n^2)$ time. The results in this paper show that such an algorithm does not exist.

Key words. stable marriage problem, stable pair, analysis of algorithms, lower bounds

AMS(MOS) subject classifications. 68Q25, 90B99

Introduction. An instance of the stable marriage problem involves two disjoint sets of equal cardinality n, the men denoted by m_i's and women denoted by w_i's. Each individual ranks all members of the opposite sex in order of decreasing preference. A matching $M=\{(m_1, w_{i_1}), (m_2, w_{i_2}), \cdots, (m_n, w_{i_n})\}$ is a *stable marriage* if there does not exist an unmatched man-woman pair (m_i, w_j) such that both prefer each other to their partners in M. At least one stable marriage exists for any given problem instance. In most problem instances, there exists more than one stable marriage. Moreover, there are problem instances of size n where the number of stable marriages are exponential in n [IL86] [Kn76].

Gale and Shapley [GS62] first demonstrated that stable marriages exist for all problem instances and gave an algorithm that finds a stable marriage for any problem instance. The stable marriage obtained with the Gale-Shapley algorithm is *male-optimal*; that is, no man can receive a better match in any other stable marriage for the same problem instance. Moreover, by reversing the roles of men and women, the algorithm also finds the *female-optimal* stable marriage.

There are numerous expositions and analyses of the Gale-Shapley algorithm available in the literature [It78], [MW71], [Kn76]. The algorithm's worst-case asymptotic time complexity, $\Theta(n^2)$, is optimal for the stable marriage problem in the following sense. To input the description of a problem instance, which includes all preference rankings, requires $\Omega(n^2)$ time. However, the "computational" component (omitting time required for input) of the Gale-Shapley algorithm requires $O(n \log n)$ operations on the average [Wi72], despite its $\Theta(n^2)$ worst-case complexity.

It is interesting to investigate if there exists a faster algorithm that solves the problem under a model that ignores the input requirement. We shall elaborate on this model in the next section. In 1976, Knuth posed this question as one of twelve research problems in his treatise on stable marriage [Kn76]. Our main contribution in this paper

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is to show that such an algorithm does not exist; that the computational component of the stable marriage problem has a worst-case complexity of $\Omega(n^2)$. In a related problem, Gusfield [Gu87] asks if it is possible to determine in $o(n^2)$ time if an arbitrary complete matching is stable. We also answer this question in the negative by showing the lower bound of $\Omega(n^2)$ for this problem.

We have noted earlier that it is possible to have multiple stable marriages in a problem instance. We define a man-woman pair (m_i, w_j) *stable* if it is contained in some stable marriage. Consider the problem of determining whether an arbitrary pair is stable in a given problem instance. Gusfield [Gu87] provides an $O(n^2)$ algorithm that finds all stable pairs, and hence also solves the above problem. Our approach in this paper is first to show the $\Omega(n^2)$ lower bound for this problem. The other results follow as corollary.

1. Model of computation. In the introduction, we noted that our lower bound results must not depend on the time required to read the input for a problem instance. Hence, our model assumes that all participants' preferences are available in memory. It is useful to organize these preferences into two $n \times n$ integer matrices MP and WP such that the ith row of MP (WP) gives the preferences of m_i (w_i). For example, $MP[i, j] = k$ if m_i's jth preference is w_k.

For maximum generality, we also assume that two ranking matrices, denoted MR and WR, are available in memory. An entry in the men's ranking matrix, $MR[i, j]$, gives the ranking (position of preference) of w_j by m_i. Entries in WR, the women's ranking matrix, have similar interpretations.

The preference and ranking matrices are inverses of each other; for example, $MP[i, MR[i, j]] = j$ and $MR[i, MP[i, j]] = j$. Hence, the ranking matrices can be completely constructed from the preference matrices in $O(n^2)$ time. However, an algorithm may rely on the ranking matrices to determine quickly the ranking assigned to a participant by another of the opposite sex. Using the preference matrices to obtain this information can be slower because the algorithm has to search an entire row in the worst case.

We will use the notations MP, MR, WP, WR only when the problem instance associated with these matrices can be clearly determined from context. When there is a possibility of ambiguity, we use the notations MP_S, MR_S, WP_S, and WR_S, where S denotes a specific instance of the stable marriage problem.

Our lower bound is established by counting the number of times an algorithm must obtain information about the problem instance. In our model, such information is obtained with two types of queries. Given the identity of a participant and an integer i, the first type of query obtains the identity of his/her ith preference. Given two participants of opposite sex, the second type of query finds the ranking of the first participant in the second's preference. Each query can be accomplished in $O(1)$ time via a simple lookup of one of the four matrices.

2. The canonical instance. For every size n, our proofs are centered on a special instance of the stable marriage problem that we call the *canonical instance* and denote by C. An important characteristic of C is that the pair (m_n, w_n) is stable in it. However, there exists a large family of problem instances that differ only slightly yet sufficiently from C such that (m_n, w_n) is not stable in them. Later we will show how to construct such a problem instance which we call a *minimally noncanonical instance* and denote by $\sim C$.

We will show that before any algorithm can correctly determine that (m_n, w_n) is stable in C, it must make a certain minimum number of queries on the preference and

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ranking matrices. Otherwise, it is possible to complete these matrices by giving appropriate values to the remaining entries that are not queried, and obtain a $\sim C$ that refutes the algorithm's correctness. This is due to the large number of possible $\sim C$'s, each derivable with only minor changes to C. Hence, the algorithm must make a large number of queries to eliminate all potential $\sim C$'s, supporting our lower bound claim.

We now define the women's preference matrix, WP_C. Entries in WP_C are defined by the function $WP_C[i, j] = j$, as illustrated in Fig. 1. Lemmas 1 and 2 give two properties of WP_C.

$$\begin{pmatrix} 1 & 2 & \dots & n \\ 1 & 2 & \dots & n \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & \dots & n \end{pmatrix}$$

FIG. 1. *Women's preference matrix, WP_C.*

LEMMA 1. *In a problem instance where WP_C is the women's preference matrix, the matching S constructed by the following rules is a stable marriage.*

(i) *When a woman receives a match, she is removed from the preference list of all remaining men.*

(ii) *Match m_1 with his highest preference.*

(iii) *After $m_1, m_2, \cdots, m_{i-1}$ are matched, m_i is matched with the highest preference remaining on his list.*

Proof. Rule (i) ensures that each woman is matched only once. Hence, S is a proper matching.

If m_i prefers w_j to his match in S, then by rule (iii), w_j is matched with m_k such that $k < i$. However, the preferences in WP_C show that w_j prefers m_k to m_i. Hence, m_i and w_j cannot destabilize S. □

LEMMA 2. *Regardless of the men's preferences, any problem instance that has WP_C as the women's preference matrix yields exactly one stable marriage.*

Proof. Any stable marriage is represented in WP_C by exactly one entry in each column. In particular, this is true of the male-optimal stable marriage S obtained by the Gale–Shapley algorithm.

Suppose there exists another stable marriage S'. For every matched pair (m_i, w_j) in S that has changed partners in S', m_i receives a less preferable partner in S' because S is male-optimal. Therefore, w_j receives a more preferable partner in S'. Otherwise, (m_i, w_j) is an unstable couple in S'.

Hence, every woman either has the same partner in both S and S', or she has a more preferable partner in S' than in S. According to WP_C, the subscript of each woman's partner decreases or stays the same. However, this requires that some column in WP_C be represented in S' by more than one entry. We conclude that S' does not exist. □

Lemma 1 gives us an algorithm that we will use in the proofs of Lemmas 3 and 4. The algorithm finds a stable marriage; it is shown by Lemma 2 to be the only one available.

We now describe the men's preference matrix, MP_C. Entries in MP_C fall into three groups. The first group, underlined in Fig. 2, includes the first row, last row, and tridiagonal entries of the remaining rows. The first and last rows consist of the integers 1 to n in increasing order. The tridiagonal entries of row i are the integers i, n, and

$$
\begin{array}{c|ccccccccccccccccc|}
 & 1 & 2 & 3 & 4 & 5 & \dots & i-2 & i-1 & i & i+1 & i+2 & \dots & n-4 & n-3 & n-2 & n-1 & n \\
\hline
1 & \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{\dots} & & & & & & & & & & \underline{n-2} & \underline{n-1} & \underline{n} \\
2 & \underline{2} & \underline{n} & \underline{1} & 3 & 4 & \dots & & & & & & & & & & & n-1 \\
3 & 1 & \underline{3} & \underline{n} & \underline{2} & 4 & \dots & & & & & & & & & & & n-1 \\
\vdots & \vdots & \vdots & & & & \ddots & & & & & & & & & & & \vdots \\
i & 1 & 2 & \dots & & & & i-2 & \underline{i} & \underline{n} & \underline{i-1} & i+1 & \dots & & & & & n-1 \\
\vdots & \vdots & \vdots & & & & & & & & & & \ddots & & & & & \vdots \\
n-2 & 1 & 2 & \dots & & & & & & & & & & n-4 & \underline{n-2} & \underline{n} & \underline{n-3} & n-1 \\
n-1 & 1 & 2 & \dots & & & & & & & & & & & n-3 & \underline{n-1} & \underline{n} & \underline{n-2} \\
n & \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{\dots} & & & & & & & & & & \underline{n-2} & \underline{n-1} & \underline{n}
\end{array}
$$

FIG. 2. *Men's preference matrix*, MP_C.

$i-1$ in that order. Of the remaining entries, the group left of the tridiagonals consists of integers 1 to $i-2$ in increasing order. The group right of the tridiagonals consi of integers $i+1$ to $n-1$ also arranged in increasing order.

LEMMA 3. (m_n, w_n) *is a stable pair in* C.

Proof. Apply the algorithm of Lemma 1 to C. Scanning the ith row of MP_C, note that all entries to the left of i have values less than i. These entries represent those women matched in previous rows. Hence, m_i's stable partner is w_i and $\{(m_1, w_1), (m_2, w_2), \cdots, (m_n, w_n)\}$ is a stable marriage in C. □

3. Obtaining noncanonical instances. Starting with C, we obtain a $\sim C$ by selecting a row i of MP_C such that $3 \leqq i \leqq n-2$, and exchanging two special entries, l and r, in that row. All entries left of the tridiagonal are candidates for l, but only those right of the tridiagonal with *values that differ from i by odd numbers* are candidates for r. Note that l is equal to its column number, and r's column number is $r+1$.

To formalize the above construction, we define, for each i, two sets of integers

$$L_i = \{x \mid 1 \leqq x \leqq i-2\}, \quad \text{and}$$

$$R_i = \{x \mid i+1 \leqq x \leqq n-1 \quad \text{and} \quad x \not\equiv i \pmod 2\}.$$

Then, for any i, l, and r satisfying $3 \leqq i \leqq n-2$, $l \in L_i$, and $r \in R_i$; we define $MP_{\sim C}[i, l] = r$ and $MP_{\sim C}[i, r+1] = l$. All other entries of $MP_{\sim C}$ and $WP_{\sim C}$ are equal to their corresponding entries in MP_C and WP_C.

LEMMA 4. (m_n, w_n) *is not a stable pair in* $\sim C$.

Proof. Apply the algorithm of Lemma 1 to $\sim C$. Figure 3 illustrates the stable marriage that results.

- m_k is matched with w_k for $1 \leqq k \leqq i-1$ because these rows are unchanged from C.
- m_i is matched with w_r. Note that $r \not\equiv i \pmod 2$, which guarantees that there is an even number of rows between row $i+1$ and row $r-1$ inclusive.
- For $i+1 \leqq k \leqq r-1$, m_k is matched with w_k if $k \not\equiv i \pmod 2$ and m_k is matched with w_{k-2} if $k \equiv i \pmod 2$. Note that m_{r-2} is matched with w_{r-2} and m_{r-1} is matched with w_{r-3}.

The above discussion shows that $w_1, w_2, \cdots, w_{i-1}$ are matched in rows 1 to $i-1$; $w_i, w_{i+1}, \cdots, w_{r-2}$ are matched in rows $i+1$ to $r-1$; and w_r is matched in row i. The subscripts of these women account for every entry left of the diagonal entry n in row r. Hence, m_r's partner is w_n.

$\sim C$ has only one stable marriage by Lemma 2. Since w_n is married to m_r and not m_n in this marriage, (m_n, w_n) is not a stable pair. □

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$$
\begin{array}{cc}
 & \begin{array}{cccccccccccccccccc} 1 & 2 & 3 & \cdots & l & \cdots & i-2 & i-1 & i & i+1 & i+2 & \cdots & & r-3 & r-2 & r-1 & r & \cdots \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ i-1 \\ i \\ i+1 \\ i+2 \\ i+3 \\ i+4 \\ \vdots \\ r-2 \\ r-1 \\ r \\ \vdots \end{array} &
\left(\begin{array}{cccccccccccccccccc}
\boxed{1} & 2 & 3 & \cdots & & & & & & & & & & & & & & \\
\boxed{2} & n & 1 & \cdots & & & & & & & & & & & & & & \\
1 & \boxed{3} & n & \cdots & & & & & & & & & & & & & & \\
\vdots & & & \ddots & & & & & & & & & & & & & & \\
1 & 2 & \cdots & & & \cdots & \boxed{i-1} & n & i-2 & \cdots & & & & & & & & \\
1 & 2 & \cdots & & \boxed{r} & & \cdots & i & n & i-1 & \cdots & & & & & & & \\
1 & 2 & \cdots & & & & \cdots & i-1 & \boxed{i+1} & n & i & \cdots & & & & & & \\
1 & 2 & \cdots & & & & \cdots & i-1 & \boxed{i} & i+2 & n & i+1 & \cdots & & & & & \\
1 & 2 & \cdots & & & & & & \cdots & i+1 & \boxed{i+3} & n & \cdots & & & & & \\
1 & 2 & \cdots & & & & & & \cdots & i+1 & \boxed{i+2} & i+4 & \cdots & & & & & \\
\vdots & \vdots & & & & & & & & & & \vdots & \ddots & & & & & \\
1 & 2 & \cdots & & & & & & & & & & \cdots & \boxed{r-2} & n & r-3 & \cdots & \\
1 & 2 & \cdots & & & & & & & & & & \cdots & \boxed{r-3} & r-1 & n & r-2 & \cdots \\
1 & 2 & \cdots & & & & & & & & & & \cdots & r-3 & r-2 & r & \boxed{n} & \cdots \\
\vdots & \vdots & & & & & & & & & & & & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
\end{array}
$$

FIG. 3. *Stable marriage in* $\sim C$.

4. A counting argument. The construction of $\sim C$ is made possible by the exchange of appropriate l and r values. Until an algorithm has eliminated all possibilities of such exchanges, it cannot conclude correctly that it is dealing with the problem instance C. However, the large number of valid choices of i, l, and r gives us the following bound.

LEMMA 5. *If $n=3k+4$ for some integer $k\geqq 1$, the minimum number of queries needed to eliminate all possible constructions of $\sim C$'s is $\frac{3}{2}k(k+1)$.*

Proof. To eliminate row i from participating in the construction of a $\sim C$, the algorithm must query either all of L_i or all of R_i. To eliminate all possible constructions of $\sim C$'s, all rows must be eliminated.

$$|L_i|=i-2\leqq k<\lceil(2k+1)/2\rceil\leqq\lceil(n-i-1)/2\rceil=|R_i|$$
$$\text{for } 3\leqq i\leqq k+2, \qquad \text{and}$$
$$|L_i|=i-2\geqq k+1>\lceil 2k/2\rceil\geqq\lceil(n-i-1)/2\rceil=|R_i|$$
$$\text{for } k+3\leqq i\leqq n-2.$$

Therefore the minimum number of queries needed

$$
\begin{aligned}
&=\sum_{i=3}^{n-2}\min(|L_i|,|R_i|)\\
&=\sum_{i=3}^{k+2}|L_i|+\sum_{i=k+3}^{n-2}|R_i|\\
&=\sum_{i=3}^{k+2}(i-2)+\sum_{i=k+3}^{n-2}\lceil(n-i-1)/2\rceil\\
&=\sum_{i=3}^{k+2}(i-2)+\sum_{i=k+3}^{3k+2}\lceil(3k+3-i)/2\rceil\\
&=\sum_{j=1}^{k}j+\sum_{j=1}^{k}2j\\
&=\tfrac{3}{2}k(k+1). \qquad \square
\end{aligned}
$$

5. Lower bounds results. We are now ready to state our main result.

THEOREM. *Determining if an arbitrary pair is stable in a problem instance of size n requires $\Omega(n^2)$ time in the worst case.*

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Proof. Without loss of generality, we may assume that $n = 3k + 4$ for some integer $k \geqq 1$; otherwise, we extend the problem instance by adding the appropriate number of men and women.

By Lemmas 3 and 4, it is necessary to distinguish between C and $\sim C$ in order to determine if (m_n, w_n) is stable. By Lemma 5, any algorithm that distinguishes between C and $\sim C$ must make at least $\frac{3}{2}k(k+1) = \frac{3}{2}((n-4)/3)(((n-4)/3)+1)$ queries. Hence, the number of queries necessary is $\Omega(n^2)$. □

COROLLARY 1. *The asymptotic time complexity for determining if an arbitrary pair is stable in a problem instance of size n is* $\Theta(n^2)$.

Proof. The theorem provides an $\Omega(n^2)$ lower bound. Gusfield's algorithm provides an $O(n^2)$ upper bound. □

COROLLARY 2. *The asymptotic time complexity for finding a stable marriage in a given problem instance of size n is* $\Theta(n^2)$.

Proof. We noted earlier that the Gale–Shapley algorithm runs in $O(n^2)$ time. The only stable marriage in C is different from the only stable marriage in $\sim C$, and $\Omega(n^2)$ queries are required to distinguish between them. □

COROLLARY 3. *The asymptotic time complexity for determining if an arbitrary complete matching is a stable marriage in a given problem instance of size n is* $\Theta(n^2)$.

Proof. An obvious algorithm that solves this problem in $O(n^2)$ time exists. The matching $\{(m_1, w_1), (m_2, w_2), \cdots, (m_n, w_n)\}$ is a stable marriage in C but not in $\sim C$, and $\Omega(n^2)$ queries are required to distinguish between them. □

Historical note. The problem in Corollary 3 was raised by Gusfield [Gu87, p. 127]. He gives an algorithm that requires only $\frac{1}{2}n(n-1)+2n$ queries. By Lemma 5, we show that at least $\frac{1}{6}(n-4)(n-1)$ queries are needed.

6. Conclusions. We have shown that the lower bound of $\Omega(n^2)$ holds for three stable marriage-related problems. This lower bound is fundamental to stable marriage and holds for other variants of the stable marriage problem, including the following class of optimization problems. Given an instance of the stable marriage problem X, we define a real-valued function V, whose domain is the set of stable marriages in X. The problem of finding a stable marriage M that maximizes (or minimizes) $V(M)$ has a lower bound of $\Omega(n^2)$, by an argument similar to that of Corollary 2. By varying the definition of V, we can formulate different variants of the stable marriage problem. We give three such problems that have been studied in the literature.

Suppose (m_i, w_j) is a pair in a marriage. The *regret* of m_i is the ranking he gives to w_j, which equals $MR[i, j]$. Similarly, the regret of w_j equals $WR[j, i]$. The regret of a marriage M is the maximum regret among all the participants. The *minimum regret stable marriage problem* is to find a stable marriage with the minimum regret. Gusfield [Gu87] gives an algorithm that solves this problem in $O(n^2)$ time, which is asymptotically optimal.

The Gale–Shapley algorithm favors one set of participants heavily over the other. It is often desirable to obtain a stable marriage that treats both sexes more equitably. The *egalitarian stable marriage problem* is to find such a marriage M, one that minimizes $\sum_{(m_i,w_j)\in M} MR[i,j] + WR[j,i]$. An algorithm that solves the egalitarian problem in $O(n^3 \log n)$ time is given in [Fe89].

A weighted version of the egalitarian problem is the *optimal stable marriage problem.* In this problem, the rankings are replaced by general "unhappiness" functions $um(i, j)$ and $uw(j, i)$ for every possible pair (m_i, w_j). The goal is to find a marriage M that minimizes $\sum_{(m_i,w_j)\in M} um(i, j) + uw(j, i)$. An algorithm that solves the optimal problem in $O(n^4 \log n)$ time is given in [ILG87].

An important generalization of the stable marriage problem that has received substantial attention is the *stable roommate problem.* This problem involves only one set of participants. Using a similar definition for stability, the goal is to find an assignment (a partition of the participants into pairs) that is stable. For every variant of the stable marriage problem described in this paper, there is a corresponding stable roommate variant that is similarly defined. Moreover, the $\Omega(n^2)$ lower bound applies to these variants. This claim is supported by the observation that every instance of the stable marriage problem is also an instance of the stable roommate problem having the same solution structure. We refer readers to [Gu88, p. 767] for a general discussion of this relation.

Variants of the stable roommate problem that have $O(n^2)$ algorithms include the following: determining whether an arbitrary pair is stable [Gu89]; determining whether an assignment is stable; finding a stable assignment [Ir85]; and the minimum regret problem [Ir86]. Obviously, no asymptotic improvement is possible with these problems. Feder has shown that the egalitarian stable roommate problem—and by implication, the optimal stable roommate problem—is NP-complete [Fe89].

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REFERENCES

[Fe89] T. FEDER, *A new fixed point approach for stable networks and stable marriages*, in Proc. 21st Symposium on Theory of Computing, Seattle, WA, 1989, pp. 513-522.

[GS62] D. GALE AND L. SHAPLEY, *College admissions and the stability of marriage*, Amer. Math. Monthly, 69 (1962), pp. 9-15.

[Gu87] D. GUSFIELD, *Three fast algorithms for four problems in stable marriage*, SIAM J. Comput., 16 (1987), pp. 111-128.

[Gu88] ———, *The structure of the stable roommate problem: Efficient representation and enumeration of all stable assignments*, SIAM J. Comput., 17 (1988), pp. 742-769.

[Gu89] ———, *Personal communications*, 1989.

[IL86] R. W. IRVING AND P. LEATHER, *The complexity of counting stable marriages*, SIAM J.Comput., 15 (1986), pp. 655-667.

[ILG87] R. W. IRVING, P. LEATHER, AND D. GUSFIELD, *An efficient algorithm for the "optimal" stable marriage*, J. Assoc. Comput. Mach., 34 (1987), pp. 532-543.

[Ir85] R. W. IRVING, *An efficient algorithm for the "stable roommates" problem*, J. Algorithms, 6 (1985), pp. 577-595.

[Ir86] ———, *On the stable room-mates problem*, Tech. Report CSC/86/R5, University of Glasgow, Glasgow, UK, 1986.

[It78] S. Y. ITOGA, *The upper bound for the stable marriage problem*, Oper. Res., 29 (1978), pp. 811-814.

[Kn76] D. E. KNUTH, *Mariages Stables*, Les Presses de L'Université de Montréal, Montréal, 1976.

[MW71] D. G. MCVITIE AND L. B. WILSON, *The stable marriage problem*, Comm. ACM, 14 (1971), pp. 486-492.

[Wi72] L. B. WILSON, *An analysis of the stable marriage assignment problem*, BIT, 12 (1972), pp. 569-575.

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