1910SU **Summer**

Group Discussion: Computing Limits

Solutions

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1. Compute the following limits.

1.1. $\lim_{x\to\infty} \frac{x^2+1}{2+\cos(4x)}$

Note that as $2+\cos(4x) \leq 3$,

$$\frac{x^2+1}{3} \leq \frac{x^2+1}{2+\cos(4x)}.$$

The lower bound tends to infinity as $x \to \infty$. Hence, $\lim_{x\to\infty} \frac{x^2+1}{2+\cos(4x)} = \infty$ by the Squeeze Theorem.

1.2. $\lim_{x\to\infty} \frac{2x+\sin(x)}{x+4}$

Consider,

$$\lim_{x\to\infty} \frac{2x+\sin(x)}{x+4} = \lim_{x\to\infty} \frac{2+\frac{\sin x}{x}}{1+\frac{4}{x}} = 2.$$

We justify $\lim_{x\to\infty} \frac{\sin(x)}{x} = 0$ by using the Squeeze Theorem again, nothing that

$$-\frac{1}{x} \leq \frac{\sin(x)}{x} \leq \frac{1}{x}$$

and both $-\frac{1}{x}$ and $\frac{1}{x}$ tend to 0 as $x \to \infty$.

1.3. $\lim_{x\to\pi} \frac{\sin(x)}{x-\pi}$

We can use L'Höpital's rule as this is a $\frac{0}{0}$ indeterminate. So,

$$\lim_{x\to\pi} \frac{\sin x}{x-\pi} = \lim_{x\to\pi} \cos x = -1.$$

1.4. $\lim_{x\to\infty} \frac{3x}{x^2-1}$

We can do this by using L'Höpital's rule or just dividing the numerator and denominator by x^2 as follows:

$$\lim_{x\to\infty} \frac{3x}{x^2-1} = \lim_{x\to\infty} \frac{\frac{3}{x}}{1-\frac{1}{x^2}} = \frac{0}{1} = 0.$$

2. More Limits

2.1. $\lim_{x\to 0^+, 1^-, 1^+} \frac{1}{x\ln x}$

We consider each limit seperately. First, we just apply L'Höpital's rule to evaluate the limit for $x \to 0^+$.

$$\lim_{x\to 0^+} \frac{1}{x\ln x} = \lim_{x\to 0^+} \frac{1/x}{\ln x} = \lim_{x\to 0^+} \frac{-\frac{1}{x^2}}{\frac{1}{x}} = \lim_{x\to 0^+} -\frac{1}{x} = -\infty$$

.

Next, we consider the limit as $x \to 1^-$. We start by noting that

$$\lim_{x\to 1^-} \frac{1}{\ln x} = -\infty$$

as $\ln x \to 0$ acquiring just negative values as $x \to 0$. Thus,

$$\lim_{x\to 1^-} \frac{1}{x \ln x} = \lim_{x\to 1^-} \frac{1}{x} \lim_{x\to 1^-} \frac{1}{\ln x} = -\infty.$$

Finally, consider the limit as $x \to 1^-$. Again, as $\ln x \to 0$ acquiring just positive values as $x \to 0$. Then,

$$\lim_{x\to 1^+} \frac{1}{\ln x} = \infty.$$

Thus,

$$\lim_{x\to 1^+} \frac{1}{x \ln x} = \lim_{x\to 1^+} \frac{1}{x} \lim_{x\to 1^+} \frac{1}{\ln x} = \infty.$$

A slight remark, we can justify $\lim_{x\to 1^+} \frac{1}{\ln x} = \infty$ and $\lim_{x\to 1^-} \frac{1}{\ln x} = -\infty$ by using the Squeeze Theorem on the fact that

$$\ln x \leq x - 1 \Rightarrow \frac{1}{x-1} \leq \frac{1}{\ln x}.$$

2.2. $\lim_{x\to 0} \frac{x^{40}+2\ln x}{\sqrt{x^{90}+1}+x^{20}}$

$$\lim_{x\to 0} \frac{x^{40} + 2\ln x}{\sqrt{x^{90}+1} + x^{20}} = \frac{\lim_{x\to 0} x^{40} + 2\ln x}{\lim_{x\to 0} \sqrt{x^{90}+1} + x^{20}} = \lim_{x\to 0} x^{40} + 2\ln x = -\infty.$$

2.3. $\lim_{x\to\infty} \frac{4e^x-1}{4-e^x}$

Note,

$$\lim_{x\to\infty} \frac{4e^x - 1}{4 - e^x} = \lim_{x\to\infty} \frac{4 - \frac{1}{e^x}}{\frac{4}{e^x} - 1} = \frac{4 - \lim_{x\to\infty} \frac{1}{e^x}}{-1 + \lim_{x\to\infty} \frac{4}{e^x}} = -4.$$

2.4. $\lim_{x\to -1} \frac{\ln(x+2)}{e^{-x}-e}$

We use L'Höpital's rule

$$\lim_{x\to -1} \frac{\ln(x+2)}{e^{-x} - e} = \lim_{x\to -1} \frac{\frac{1}{x+2}}{-e^{-x}} = -\frac{1}{e}.$$