1910SU **Summer**

Group Discussion: Trigonometry

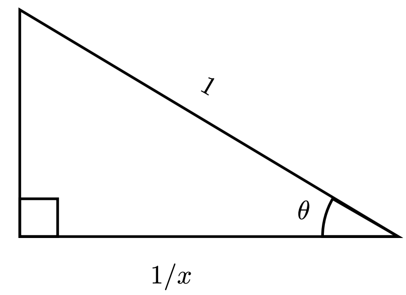
Solutions

Authors: Shaleen Baral

1. Composition of Trigonometric and Inverse Trigonometric Functions.

1.1. $\tan(\sec^{-1}(x))$

Let $\theta = \sec^{-1}(x) \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$. Consider the right-angled triangle with unit hypotenuse and adjacent side of length $\frac{1}{x}$. One of the angles in this right-angled triangle must be θ. Pictorally,



The perpendicular side is then given by,

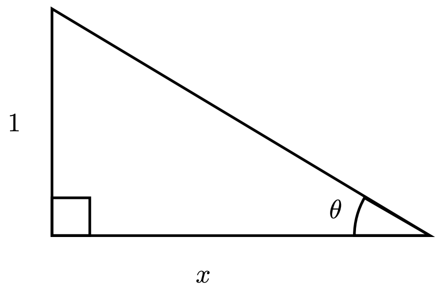
$$\sin\theta = \sqrt{1 - \frac{1}{x^2}}$$

Then,

$$\tan(\sec^{-1}(x)) = \tan(\theta) = \frac{\sin\theta}{\cos\theta} = x \cdot \sqrt{1 - \frac{1}{x^2}} = \frac{x}{|x|} \cdot \sqrt{x^2 - 1}.$$

1.2. $\csc(\cot^{-1}(x))$

Let $\theta = \cot^{-1}(x) \in [0, \pi]$. Consider the right-angled triangle with adjacent side of length x and perpendicular side of unit length. One of the angles in this right-angled triangle must be θ. Pictorally,

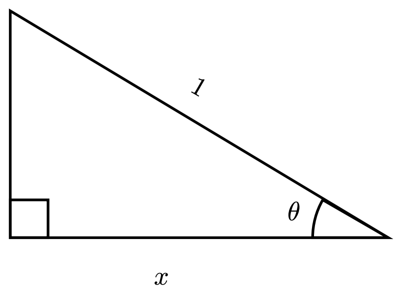


Then the hypotenuse has length $\sqrt{1 + x^2}$. Finally,

$$\sin(\cot^{-1}(x)) = \frac{1}{\sin(\theta)} = \sqrt{1 + x^2}.$$

1.3. $\sin(\cos^{-1}(x))$

Let $\theta = \cos^{-1}(x) \in [0, \pi]$. Consider the right-angled triangle with unit length hypotenuse and adjacent side of length x. One of the angles in this right-angled triangle must be θ. Pictorally,

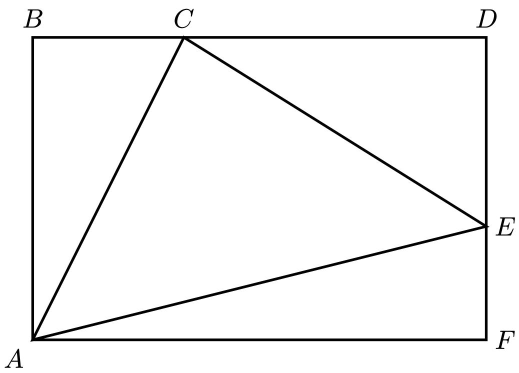


Then,

$$\sin(\cos^{-1}(x)) = \sin\theta = \sqrt{1 - x^2}.$$

2. Sum and Difference Formulas.

Label the points on the provided diagram as follows.



In $\triangle ABC$, we have

$$AB = \cos\beta\cos\alpha,$$

$$BC = \sin\beta\cos\alpha.$$

In $\triangle CDE$, we have

$$CD = \cos\beta\sin\alpha,$$

$$DE = \sin\beta\sin\alpha.$$

In $\triangle AEF$, we have

$$EF = \cos(\alpha + \beta),$$

$$AF = \sin(\alpha + \beta).$$

Note further that,

$$EF = AB - DE \Longrightarrow \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\beta\sin\alpha,$$

$$AF = BC + CD \Longrightarrow \sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta.$$