Exercise Class 3
Natural Deduction for Propositional Logic

For this exercise class, we'll be writing natural deduction proofs using the set of inference rules covered in Lecture 5 (these are shown overleaf). We have not yet discussed proof strategies in much detail, but the titles of some of the questions will give you a hint as to which rules(s) to apply.

Construct a proof of validity for each of the propositional logic arguments below:

1. A simple problem

$$P,\ P \rightarrow Q\ :\ P \land (Q \lor R)$$

2. Implications

$$P \rightarrow (Q \land R)\ :\ P \rightarrow Q$$

3. Nested implications

$$P \rightarrow (Q \rightarrow R)\ :\ Q \rightarrow (P \rightarrow R)$$

4. Or what?

$$P \lor (Q \land R),\ S\ :\ (S \land P) \lor Q$$

5. Suppose the contrary

$$Q \rightarrow \neg P\ :\ \neg(P \land Q)$$

6. An absurd problem

$$\neg P \rightarrow Q,\ \neg Q\ :\ P$$

7. A tough problem[1]

$$P \lor Q,\ P \rightarrow R,\ \neg S \rightarrow \neg Q\ :\ R \lor S$$

[1] Don't forget that modus tollens is <u>not</u> on the list of inference rules.

Inference Rules

$\dfrac{A \quad B}{A \wedge B}$ $\wedge$-introduction $\dfrac{A \wedge B}{A}$ $\wedge$-elimination $\quad$ $\dfrac{A \wedge B}{B}$ $\wedge$-elimination	$\dfrac{A}{A \vee B}$ $\vee$-introduction $\quad$ $\dfrac{A}{B \vee A}$ $\vee$-introduction $\dfrac{A \vee B \quad A \vdash C \quad B \vdash C}{C}$ $\vee$-elimination
$\dfrac{A \vdash B}{A \to B}$ $\to$-introduction $\dfrac{A \to B \quad A}{B}$ $\to$-elimination	$\dfrac{A \vdash \bot}{\neg A}$ $\neg$-introduction $\dfrac{\neg\neg A}{A}$ $\neg\neg$-elimination