Language & Logic

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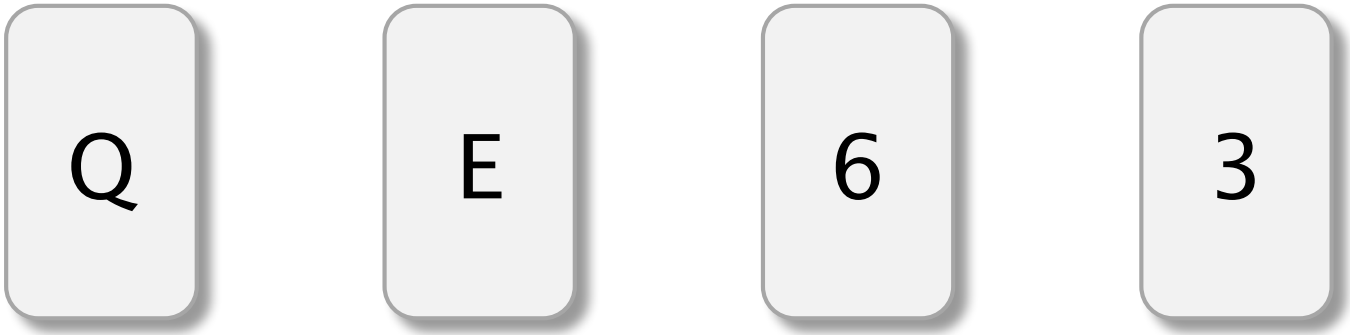
A puzzle

- There are 4 cards, each with a letter on one side and a number on the other
- Here is a rule: "every card with a vowel has an even number on the other side"

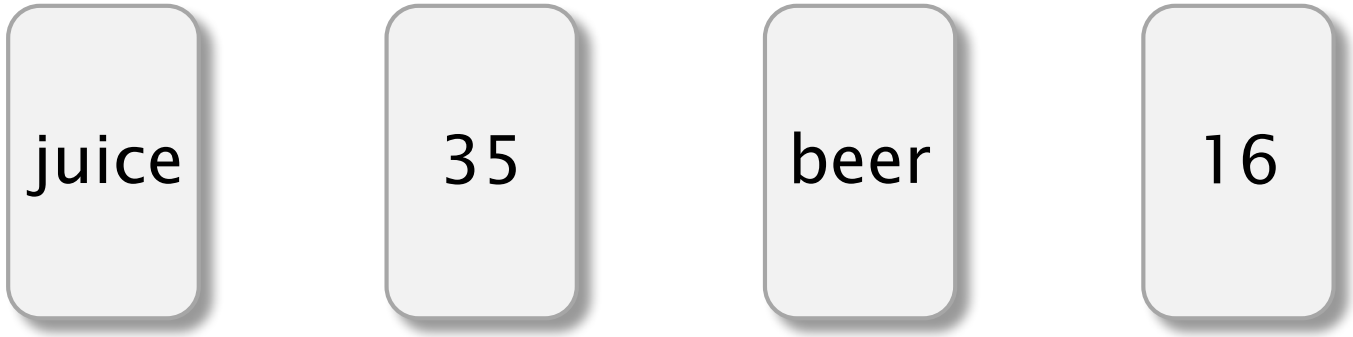


- Which card(s) must you turn over in order to test whether this rule holds?

- (1) E 6 (2) E 6 3 (3) E 3 (4) E

Another puzzle

- There are 4 cards, each with a drink on one side and an age on the other
- Here is a rule: "if the age is under 18, then the drink on the other side of the card is non-alcoholic"



- Which card(s) must you turn over in order to test whether this rule holds?

What is logic?

- Logic is about formalising arguments and reasoning

1. All men are mortal.
2. Socrates is a man.
3. Therefore, Socrates is mortal.

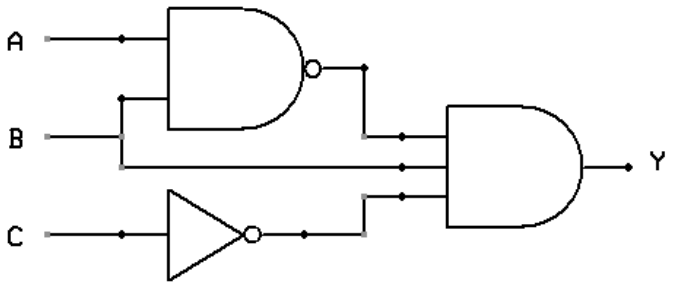
What is logic?

- Which of these 3 statements are equivalent?

1. You will get a good job if you learn logic
2. You will get a good job unless you do not learn logic
3. If you have not got a good job, you did not learn logic

Why study logic?

- Logic is fundamental in computer science
  - also philosophy, mathematics, linguistics, psychology, ...
- Logic in computer science:
  - understanding, formalisation/rigour, correctness/proof, computation/automation, …
- Logic plays a key role in many areas of computer science
  - one simple example….



A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Why study logic?

- Plays a key role in many areas of computer science…
- Theoretical computer science
  - historic roots of computer science (Church, Turing): theory of computation, complexity, incompleteness, …
  - Curry–Howard correspondence (between proofs and programs)
- Correctness & verification
  - how do we guarantee that an algorithm/program/… is correct?
- Artificial intelligence
  - how do we represent the goals and state of a rational agent?
- Natural language processing, databases, ...

This module

Learning outcomes

- By the end of this module, you should be able to:

1. Analyse the information content and structure of statements in both natural and formal languages
2. Demonstrate an ability to prove statements and arguments expressed in symbolic logic
3. Encode natural language statements and arguments in symbolic logic and make simple logical inferences
4. Demonstrate basic skills in proving correctness properties of programs

Syllabus

- Syntax of formal & natural languages
  - grammars, parsing
- Propositional logic
  - proofs via natural deduction, truth tables, semantics
- Predicate calculus
  - proofs via natural deduction
- Program correctness
  - structural induction

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Lectures and classes

- Two sessions per week (usually)
- In 3 slots:
  - Mon 4–5, Tue 11–12, Thu 10–11 (all in Arts Main Lec Theatre)

- Usually (but not always):
  - one lecture on new material on Monday
  - one class for (non-assessed) exercises/discussion:
    - on Tuesday, if your surname is in the range A–J (by default)
    - on Thursday, if your surname is in the range K–Z (by default)

- This week:
  - 2 lectures: today and Thursday 10am
  - i.e., no lecture tomorrow

Assessment

- 20% continuous assessment
- 80% exam (1.5hrs, in the Summer term)

- Continuous assessment assignments
  - 3 exercises, in weeks 3, 6 and 10
  - worth 6%, 6% and 8%, respectively

- Submission
  - electronic, via Canvas
  - typeset (Word/Latex/…) or scanned (phone/tablet/…)

Resources

- Canvas page
  - https://canvas.bham.ac.uk/courses/27268
  - lecture recordings (Panopto) and slides (see “Modules”)
  - assignments, quizzes, announcements, …
- Facebook group
  - https://www.facebook.com/groups/bham.logic.1718
- Office hours
  - currently: Tue 1–2; Thur 2–3 (see my door/webpage)
- Books
  - *Logic,* Paul Tomassi (1999)
  - *Logic,* Wilfred Hodges (1997)
  - *Logic in Computer Science*, Michael Huth and Mark Ryan (2004)

Tips

- How to pass this module…
- Work hard
  - practice is essential
  - attempt all assessed/non-assessed questions
- Get feedback
  - assignment feedback on Canvas
  - exercise classes, office hours
- Don’t plagiarise

Logic

Propositions

- A proposition is a sentence which states a fact
  - i.e. a statement that can (in principle) be true or false

- Example sentences
  - Birmingham is north of London
  - $8 \times 7 = 42$
  - Please mind the gap
  - Every even natural number > 2 is the sum of two primes
  - Is black the opposite of white?
  - The program will get stuck in an infinite loop
  - At least one of these examples is not a proposition

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Arguments

- An argument is a collection of propositions
  - comprising 0 or more premises and 1 conclusion

- Example
  1. If there is smoke, there is a fire
  2. There is no fire
  3. Therefore, there is no smoke

Validity & soundness

- An argument is valid if (and only if), whenever the premises are true, then so is the conclusion

- Example
  1. If there is smoke, there is a fire
  2. There is no fire
  3. Therefore, there is no smoke

- If an argument is not valid, then it is invalid

- An argument is sound if (and only if) it is valid and, in addition, the premises are all true

Example arguments (see Quiz)

- If a cat has no tail, then it is a Manx cat.
  Whiskers is a cat and has no tail.
  Therefore, Whiskers is a Manx cat.

- If John is at home, then his television is on.
  His television is not on.
  Therefore, John is not at home.

- If England beat France, then Fiji finish third.
  If Fiji finish third, then Fiji are better than New Zealand.
  Therefore, if England beat France, then Fiji are better than New Zealand.

- If the control software crashes, then the car's brakes will fail.
  The car's brakes failed.
  Therefore, the control software crashed.