Exercise Class 7
More Natural Deduction for Predicate Calculus

These questions will give you some practice on more challenging natural deduction proofs for predicate calculus. You should use the same set of inference rules as in Exercise 6 (these are shown overleaf).

Remember also the new rules of thumb for constructing a proof presented in the last lecture.

Construct a proof of validity for each of the arguments/theorems below. For arguments that are expressed in natural language, you first need to translate them into predicate calculus.

1. These are from the lecture. We already sketched the proofs.

   (a)
   $$\forall x[D(x) \vee B(x)],\ \forall x[D(x) \rightarrow B(x)] \ : \ \forall x[B(x)]$$

   (b)
   $$\forall y[G(y) \rightarrow H(y)] \ : \ \exists x[G(x)] \rightarrow \exists z[H(z)]$$

   (c)
   $$\exists x[\neg F(x)] \vee \exists y[G(y)] \ : \ \exists z[F(z) \rightarrow G(z)]$$

2. Quantified and unquantified
   $$\forall x[(P(x) \wedge Q(x)) \rightarrow R(x)], Q(a) \ : \ \exists x[P(x) \rightarrow R(x)]$$

3. Two choices or one
   $$\exists x[F(x)] \vee \exists x[G(x)] \ : \ \exists x[F(x) \vee G(x)]$$

4. A moral tale

   Some rich people are kind. All kind people are loved. Therefore, some rich people are loved.

5. Universally pessimistic
   $$\exists x[G(x)] \ : \ \neg\forall x[\neg G(x)]$$

Inference Rules

Conjunction ($\wedge$)	**Disjunction ($\vee$)**
$\dfrac{A \quad B}{A \wedge B}$ $\wedge$-introduction $\dfrac{A \wedge B}{A}$ $\wedge$-elimination $\quad \dfrac{A \wedge B}{B}$ $\wedge$-elimination	$\dfrac{A}{A \vee B}$ $\vee$-introduction $\quad \dfrac{A}{B \vee A}$ $\vee$-introduction $\dfrac{A \vee B \quad A \vdash C \quad B \vdash C}{C}$ $\vee$-elimination
Implication ($\rightarrow$)	**Negation ($\neg$)**
$\dfrac{A \vdash B}{A \rightarrow B}$ $\rightarrow$-introduction $\dfrac{A \rightarrow B \quad A}{B}$ $\rightarrow$-elimination	$\dfrac{A \vdash \bot}{\neg A}$ $\neg$-introduction $\dfrac{\neg\neg A}{A}$ $\neg\neg$-elimination
Universal quantification ($\forall$)	**Existential quantification ($\exists$)**
$\dfrac{\phi(a)}{\forall x[\phi(x)]}$ $\forall$-introduction* $\dfrac{\forall x[\phi(x)]}{\phi(a)}$ $\forall$-elimination * For $\forall$-introduction, a must not occur in any dependency of $\phi(a)$.	$\dfrac{\phi(a)}{\exists x[\phi(x)]}$ $\exists$-introduction $\dfrac{\exists x[\phi(x)] \quad \phi(a) \vdash C}{C}$ $\exists$-elimination* *For $\exists$-elimination, a must not occur in C or any dependency of C except $\phi(a)$.