

Inference Rules

Conjunction (\wedge)	Disjunction (\vee)
$\frac{A \quad B}{A \wedge B} \wedge\text{-introduction}$ $\frac{A \wedge B}{A} \wedge\text{-elimination} \quad \frac{A \wedge B}{B} \wedge\text{-elimination}$	$\frac{A}{A \vee B} \vee\text{-introduction} \quad \frac{A}{B \vee A} \vee\text{-introduction}$ $\frac{A \vee B \quad A \vdash C \quad B \vdash C}{C} \vee\text{-elimination}$
Implication (\rightarrow)	Negation (\neg)
$\frac{A \vdash B}{A \rightarrow B} \rightarrow\text{-introduction}$ $\frac{A \rightarrow B \quad A}{B} \rightarrow\text{-elimination}$	$\frac{A \vdash \perp}{\neg A} \neg\text{-introduction}$ $\frac{\neg\neg A}{A} \neg\neg\text{-elimination}$
Universal quantification (\forall)	Existential quantification (\exists)
$\frac{\phi(a)}{\forall x[\phi(x)]} \forall\text{-introduction}^*$ $\frac{\forall x[\phi(x)]}{\phi(a)} \forall\text{-elimination}$ <p>* For \forall-introduction, a must not occur in any dependency of $\phi(a)$.</p>	$\frac{\phi(a)}{\exists x[\phi(x)]} \exists\text{-introduction}$ $\frac{\exists x[\phi(x)] \quad \phi(a) \vdash C}{C} \exists\text{-elimination}^*$ <p>*For \exists-elimination, a must not occur in C or any dependency of C except $\phi(a)$.</p>
Identity ($=$)	
$\frac{}{a = a} =\text{-introduction}$ $\frac{a = b \quad \phi(a)}{\phi(b)} =\text{-elimination}$	