3. Truth Tables

Language & Logic

Dave Parker

University of Birmingham

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Timetable

- Mon 4pm: lecture (logic & truth tables)
- Tue 11am / Thur 10am: class (exercises/discussion)
  - on Tuesday, if your surname is in the range A–J (by default)
  - on Thursday, if your surname is in the range K–Z (by default)
- Reminder: practice quizzes online (basic logic + grammars)
  - solutions online; questions? see me in office hours (Tue 1pm, Thur 2pm) or classes/lectures, or ask on the Facebook group
- First continuous assessment next week

Syllabus

- Syntax of formal & natural languages
  - grammars, parsing
- Propositional logic
  - truth tables, semantics, proofs via natural deduction
- Predicate calculus
  - proofs via natural deduction
- Program correctness
  - structural induction

Recap (arguments)

- A proposition is a sentence which states a fact
  - i.e. a statement that can (in principle) be true or false

- An argument is a collection of propositions
  - comprising 0 or more premises and 1 conclusion

- An argument is valid if (and only if), whenever the premises are true, then so is the conclusion

Example arguments

- Examples

  - If John is at home, then his television is on
    His television is not on
    Therefore, John is not at home [valid]

  - If the control software crashes, then the car's brakes will fail
    The car's brakes failed
    Therefore, the control software crashed [invalid]

Today

- Propositional logic
  - atomic propositions
  - conjunction, disjunction, negation, material implication
- Truth tables
  - representing semantics of logical connectives
  - deduction: checking validity of arguments

Atomic propositions

- Propositions
  - (recall: check whether you ask “is it true that X?”)

- Atomic propositions:
  - propositions that cannot be broken into smaller parts
  - combined into more complex propositions with connectives

- Examples
  - If $\boxed{\text{the control software crashes,}}$ then $\boxed{\text{the car's brakes will fail}}$
  - $\boxed{\text{The car's brakes failed}}$
  - $\boxed{\text{The control software crashed}}$ and $\boxed{\text{the car's brakes failed}}$

Conjunction

- Conjunction
  - logical connective corresponding to “and”
  - denoted by symbol $\wedge$
  - true if both individual propositions are true

- Example propositions
  1. There is smoke and there is a fire
  2. It is late and I want to go home
  3. The car is small and red
  4. Alice and Bob are married

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  1. There is smoke and there is a fire
  2. It is late and I want to go home
  3. The car is small and ~~red~~ the car is red
  4. Alice and Bob are married

Semantics of conjunction

- We can define the semantics of a logical connective using a truth table. For example, for $P \wedge Q$ (i.e. "P and Q"):

P	Q	$P \wedge Q$
T	T	
T	F	
F	T	
F	F	

- Format
  - one row for each possible logical combination
  - i.e. $2^2 = 4$ rows for 2 atomic propositions, P and Q
  - abbreviate true to T and false to F

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Disjunction

- Disjunction ("or")
  - e.g., Either the control software crashes or there is a power failure
  - connective denoted by symbol $\vee$

P	Q	$P \vee Q$
T	T	
T	F	
F	T	
F	F	

Disjunction

- Disjunction ("or")
  - e.g., Either the control software crashes or there is a power failure
  - connective denoted by symbol $\vee$

P	Q	P $\vee$ Q
T	T	T
T	F	T
F	T	T
F	F	F

- Note: English sentences will often translate to "exclusive or"
  - e.g., Your mark will either be "pass" or "fail"
- But logical disjunction is always defined as above

Negation

- Negation ("not")
  - e.g., This connective is not difficult to understand
  - unary connective denoted by symbol ¬

P	¬P
T	F
F	T

Material implication

- Material implication ("if… then...")
  - e.g., If there is smoke then there is a fire
  - denoted by symbol →
  - we say: "if P then Q", or "P implies Q"

P	Q	P → Q
T	T	
T	F	
F	T	
F	F	

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P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

- Sometimes confusing at first sight
  - just remember the truth table!

Another example

- If there is an exam and you do not study, you will not pass

- How do we represent this proposition as a truth table?

Checking validity

- How do we tell if a given argument is valid?
  - (this is one of the key topic studied in this module)
- Example
  - If John is at home, then his television is on
    His television is not on
    Therefore, John is not at home
- One way: use a truth table…
  1. identify and abbreviate atomic propositions
  2. translate premises/conclusion into propositional logic
  3. construct a truth table for premises and conclusion
  4. check the definition of validity
     - when all premises are true, the conclusion must also be true
     - or, in other words: there is <u>no</u> situation
       where the premises are true but the conclusion is false

Example 1

- Argument:

  P1: If John is at home, then his television is on → $H \rightarrow O$
  P2: His television is not on → $\neg O$
  C: Therefore, John is not at home → $\neg H$

- Atomic propositions
  - H = "John is at home"
  - O = "his television is on"

argument

valid

H	O	$H \rightarrow O$ (P1)	$\neg O$ (P2)	$\neg H$ (C)
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Example 2

- Argument:

P1: If the control software crashes, then the car's brakes will fail $C \rightarrow B$
P2: The car's brakes failed B
C: Therefore, the control software crashed C

- Atomic propositions
  - C = "control software crashes"
  - B = "brakes fail"

argument

invalid

		P1	P2	C
C	B	C → B	B	C
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Summary

- Propositions, arguments, validity

- Propositional logic
  - atomic propositions
  - logical connectives
  - conjunction (and), disjunction (or), negation (not), material implication (if … then)

- Truth tables
  - defining connective semantics
  - deduction: checking argument validity

Next session

- Tue 11am / Thur 10am: class (exercises/discussion)
  - on Tuesday, if your surname is in the range A–J (by default)
  - on Thursday, if your surname is in the range K–Z (by default)

- Topic: truth tables