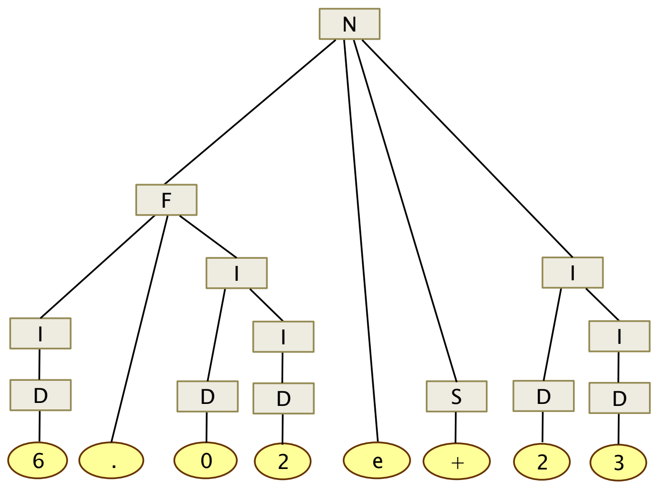
Assignment 1 – Solutions
Grammars & Truth Tables

1. (a) This *can* be derived from the grammar. The parse tree is:



   (b) This *cannot* be derived from the existing grammar. To make this possible, we can modify the grammar by extending the F non-terminal to allow numbers without a decimal point:

   F $\rightarrow$ I | I [.] I

2. In addition to the existing rules:

   F $\rightarrow$ Ap | $\neg$ F | F $\wedge$ F | F $\vee$ F | F $\rightarrow$ F
   AP $\rightarrow$ [P] | [Q] | [R]

   we add the new rules:

   Arg $\rightarrow$ Prems [:] F
   Prems $\rightarrow$ Prems0 | Prems1
   Prems0 $\rightarrow$
   Prems1 $\rightarrow$ F | F [,] Prems1

   and make Arg the start symbol.

   Prems1 denotes a list of 1 or more comma-separated formulas, and Prems0 an empty list.

3. We identify the following atomic propositions:

   - A = Alice studies logic
   - B = Bob studies logic

   So the argument comprises:

   - Premise 1: $A \rightarrow B$
     Premise 2: $(A \vee B) \rightarrow A$
     Conclusion: $A \wedge B$

   The truth table is:

		P1		P2	C
A	B	$A \rightarrow B$	$(A \vee B)$	$(A \vee B) \rightarrow A$	$A \wedge B$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	T	F	F
F	F	**T**	F	**T**	**F**

   There are two rows where both premises are true: the first and the last. Since the conclusion is false in the last row, the argument is *invalid*.

   So, a counterexample is the situation where A and B are both false, i.e., when neither Alice or Bob study logic. This makes the premises both true, but the conclusion false.

4. The argument is:

$$P \rightarrow Q,\ Q \vee R\ :\ \neg(P \wedge Q) \rightarrow R$$

   The truth table is:

			P1	P2			C
P	Q	R	$P \rightarrow Q$	$Q \vee R$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg(P \wedge Q) \rightarrow R$
T	T	T	T	T	T	F	T
T	T	F	T	T	T	F	T
T	F	T	F	T	F	T	T
T	F	F	F	F	F	T	F
F	T	T	T	T	F	T	T
F	T	F	**T**	**T**	F	T	**F**
F	F	T	T	T	F	T	T
F	F	F	T	F	F	T	F

   This shows that the argument is *invalid* because, in the 6th row (and only this row), both premises are true, but the conclusion is not (the counterexample is when P and R are false and Q is true).

   Adding, e.g. $P \wedge Q$ as an extra premise would make the argument valid, since this false for the problematic row (in which where P is are false and Q is true).