

Exercise Class 9

Proof by Induction

These questions will give you some practice on *proof by induction*. Question 1 is on *mathematical induction* and the remainder are on *structural induction*. If time is short and your maths is rusty, skip straight to Question 2.

1. Prove that $\sum_{i=0}^n i^2 = n(n+1)(2n+1)/6$.
2. Recall the OCaml `append` function for list concatenation from the lecture:

```
let rec append l1 l2 = match l1 with
| [] -> l2
| hd::tl -> hd :: (append tl l2)
```

From the definition of the `append` function, it is clear that appending a list to an empty list returns the same list, i.e., `append [] l = l`. Prove that appending an empty list to the end has the same effect, i.e., `append l [] = l`.

3. Here are two more OCaml functions for lists. Function `mem` tests for membership of a list:

```
let rec mem x = function
| [] -> false
| hd::tl -> (hd = x) || (mem x tl)
```

Function `setify` removes all duplicates from a list, producing a list in which all elements occur exactly once:

```
let rec setify = function
| [] -> []
| hd::tl -> if mem hd tl then setify tl else hd::(setify tl)
```

Prove that the `setify` function correctly preserves which items are in the list, i.e., that `mem x l = mem x (setify l)`

4. Using the same functions as in the previous question, prove that applying `setify` for a second time has no effect, i.e., `setify (setify l) = setify l`.