Exercise Class 4
More Natural Deduction for Propositional Logic

For this exercise class, we'll be working with some more challenging natural deduction proofs. You should use the same set of inference rules as in Exercise 3 (these are shown overleaf).

Remember the rules of thumb for constructing a proof presented in the last lecture.

Construct a proof of validity for each of the arguments/theorems below:

1. Warm up
$$P \to Q, Q \to R \; : \; P \to (Q \land R)$$

2. Follow the rules!
$$: \; ((P \to Q) \land \neg Q) \to \neg P$$

3. Anything goes
$$\neg R, P \to Q, R \to \neg Q, P \lor R \; : \; Q$$

4. Ands and ors
$$: \; ((P \lor Q) \land (P \lor R)) \to (P \lor (Q \land R))$$

5. Excluded middle
$$: P \lor \neg P$$

6. Seems inevitable
$$P \to Q, \neg P \to Q \; : \; Q$$

   [Hint: You may want to re-use a result you've already proved (via theorem introduction)]

Inference Rules

Conjunction ($\land$)	Disjunction ($\lor$)
$\dfrac{A \quad B}{A \land B}$ $\land$-introduction $\dfrac{A \land B}{A}$ $\land$-elimination $\quad$ $\dfrac{A \land B}{B}$ $\land$-elimination	$\dfrac{A}{A \lor B}$ $\lor$-introduction $\quad$ $\dfrac{A}{B \lor A}$ $\lor$-introduction $\dfrac{A \lor B \quad A \vdash C \quad B \vdash C}{C}$ $\lor$-elimination
Implication ($\rightarrow$)	**Negation ($\neg$)**
$\dfrac{A \vdash B}{A \rightarrow B}$ $\rightarrow$-introduction $\dfrac{A \rightarrow B \quad A}{B}$ $\rightarrow$-elimination	$\dfrac{A \vdash \perp}{\neg A}$ $\neg$-introduction $\dfrac{\neg\neg A}{A}$ $\neg\neg$-elimination