

# Lecture 1

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## Regression

- Curve fitting
  - Given a set of points, try to learn a function to describe them
  - Given a value  $x$ , we can predict the corresponding value  $y$
  - Not just for straight line fitting

### Simple example

Let us consider a simple *linear* example with 1 independent & 1 dependent variable  $D = \{(x_1, y_1), \dots, (x_n, y_n)\} = \{(x_i, y_i)\}_{i=1}^N$ . Model the relationship between  $x$  and  $y$  with the function  $L(\mathbf{w}, x)$ , s.t.  $y \approx f(\mathbf{w}, x)$ . Measurements of  $y$ , subject to noise are defined by,  $y_i = f(\mathbf{w}, x_i) + \epsilon_i$ . Where  $\epsilon_i$  is a random number drawn from some continuous probability density function. Goal is to find some  $\mathbf{w}$  that solves the above equation

First, let us approach this as an optimisation problem in which the objective is to find the value of  $\mathbf{w}$  (denoted  $\mathbf{w}^*$ ) that minimises some *loss* or objective function  $L(\mathbf{w})$

$$\mathbf{w}^* = \underset{\mathbf{w}}{\operatorname{argmin}} L(\mathbf{w})$$

Intuitively,  $L(\mathbf{w})$  should be designed to capture the difference between the data and the predictions of the model, and seek to minimise this. One common choice for  $L(\mathbf{w})$  is *least-squares error*. Given our dataset  $D$  and modelling function  $f(\mathbf{w}, x)$ , we construct  $\forall d \in D$  a residual error defined as:  $r_i(\mathbf{w}) = y_i - f(\mathbf{w}, x_i)$

