Exercise Class 5 – Solutions
Predicate Calculus

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1. We will use the following predicates:

   - $W(x) = x$ is white; $M(x) = x$ is a mouse; $T(x) = x$ has a tail; $P(x) = x$ is pink

   and constant symbols

   - $b =$ Basil; $c =$ Charlie

   (a) All white animals are mice
   $\forall x[W(x) \rightarrow M(x)]$

   (b) Basil is a white mouse
   $M(b) \wedge W(b)$

   (c) All white mice have tails
   $\forall x[(W(x) \wedge M(x)) \rightarrow T(x)]$

   (d) There are no pink mice
   $\neg\exists x[P(x) \wedge M(x)]$

   (e) At least one of Basil and Charlie has a tail
   $T(b) \vee T(c)$

2. We will use the following predicates:

   - $L(x, y) = x$ loves y; $H(x, y) = x$ hates y

   and constant symbols

   - $j =$ John; $m =$ Mary; $c =$ Chris

   (a) John loves Mary
   $L(j, m)$

   (b) Everybody hates Chris
   $\forall x[H(x, c)]$

   (c) Somebody loves Chris
   $\exists x[L(x, c)]$

   (d) John loves everybody
   $\forall x[L(j, x)]$

   (e) Nobody loves John
   $\neg\exists x[L(x, j)]$
   or: $\forall x[\neg L(x, j)]$

   (f) Mary doesn't love anybody and John loves Mary
   $\neg\exists x[L(m, x)] \wedge L(j, m)$

   (g) Mary hates Chris but Chris loves Mary
   $H(m, c) \wedge L(c, m)$
   Note: the 'but' conveys some additional context, but nothing we need to capture logically.

(h) Mary doesn't love everybody or somebody doesn't love Mary
$\neg\forall x[L(m,x)] \vee \exists x[\neg L(x,m)]$

(i) If Mary loves everybody then somebody doesn't love Mary and Mary loves somebody
$\forall x[L(m,x)] \rightarrow (\exists x[\neg L(x,m)] \wedge \exists x[L(m,x)])$
Note: technically this is ambiguous and $\wedge$ could be the main connective too

(j) Everyone who loves Mary also loves either Chris or John
$\forall x[L(x,m) \rightarrow (L(x,c) \vee L(x,j))]$

(k) Everyone who loves Chris loves someone who loves John
$\forall x[L(x,c) \rightarrow \exists y[(L(x,y) \wedge L(y,j))]]$

3. We will use the following predicates:

- $L(x,y) = x$ loves y; $R(x) = x$ has red hair
- $V(x) = x$ is a Virgo; $C(x) = x$ is a Capricorn; $S(x) = x$ is a Scorpio

(a) Everybody loves everybody
$\forall x[\forall y[L(x,y)]]$

(b) Everybody loves somebody
$\forall x[\exists y[L(x,y)]]$

(c) Everyone loves themselves
$\forall x[L(x,x)]$

(d) Everybody loves anybody with red hair
$\forall x[\forall y[R(y) \rightarrow L(x,y)]]$

(e) All Virgos love Scorpios
$\forall x[V(x) \rightarrow \forall y[Le(y) \rightarrow S(x,y)]]]$

(f) All Virgos love a Capricorn
$\forall x[V(x) \rightarrow \exists y[C(y) \wedge L(x,y)]]]$

(g) No Scorpio loves a Capricorn
$\neg\exists x\exists y[S(x) \wedge C(y) \wedge L(x,y)]$

4. (a) Everybody has a mother therefore somebody is the mother of everyone.

We use prediate $M(x,y) =$ "x is the mother of y". Then the argument is:
$\forall x[\exists y[M(y,x)]] : \exists x[\forall y[M(x,y)]]$

(b) Everybody who loves everybody loves themself.

We again use prediate $L(x,y) =$ "x loves y". Then the argument is:
$: \forall x[\forall y[L(x,y)] \rightarrow L(x,x)]$

The first of these is invalid, the second is valid. We'll look at ways of formally proving this in the coming weeks.