Exercise Class 2 – Solutions
Simple Proofs in Propositional Logic

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1. We identify the following atomic propositions:

   - P = Phillip watches football
   - Q = Quinton watches football
   - R = Roger watches hockey

   So the argument is:

   $$P \wedge Q, Q \rightarrow R \ : \ R$$

   A proof of validity is:

1.	$P \wedge Q$	Premise	{1}
2.	Q	$\wedge$-Elimination$_1$	{1}
3.	$Q \rightarrow R$	Premise	{3}
4.	R	Modus Ponens$_{3,2}$	{1,3}

2. The argument is:

   $$P, Q, R \ : \ P \wedge (Q \wedge R)$$

   A proof of validity is:

1.	Q	Premise	{1}
2.	R	Premise	{2}
3.	$Q \wedge R$	$\wedge$-Introduction$_{1,2}$	{1,2}
4.	P	Premise	{4}
5.	$P \wedge (Q \wedge R)$	$\wedge$-Introduction$_{4,3}$	{1,2,4}

3. The argument is:

   $$P \wedge \neg Q, R \rightarrow Q, \neg R \rightarrow Z \ : \ Z$$

   A proof of validity is:

1.	$P \wedge \neg Q$	Premise	{1}
2.	$\neg Q$	$\wedge$-Elimination$_1$	{1}
3.	$R \rightarrow Q$	Premise	{3}
4.	$\neg R$	Modus Tollens$_{3,2}$	{1,3}
5.	$\neg R \rightarrow Z$	Premise	{5}
6.	Z	Modus Ponens$_{5,4}$	{1,3,5}

4. The argument is:

$$R, (P \to Q) \wedge (Q \to P), Q \to Z, R \to P \ : \ Z$$

A proof of validity is:

1.	R	Premise	{1}
2.	$R \to P$	Premise	{2}
3.	P	Modus Ponens$_{2,1}$	{1,2}
4.	$(P \to Q) \wedge (Q \to P)$	Premise	{4}
5.	$P \to Q$	$\wedge$-Elimination$_4$	{4}
6.	Q	Modus Ponens$_{5,3}$	{1,2,4}
7.	$Q \to Z$	Premise	{7}
8.	Z	Modus Ponens$_{7,6}$	{1,2,4,7}

5. The argument is:

$$\neg P \to \neg Q, P \to Z, \neg\neg Q \ : \ Z$$

A proof of validity is:

1.	$\neg P \to \neg Q$	Premise	{1}
2.	$\neg\neg Q$	Premise	{2}
3.	$\neg\neg P$	Modus Tollens$_{1,2}$	{1,2}
4.	P	Double Negative Elimination$_3$	{1,2}
5.	$P \to Z$	Premise	{5}
6.	Z	Modus Ponens$_{5,4}$	{1,2,5}