Exercise Class 1 – Solutions
Truth Tables

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1. The argument is:

   - If Alice goes to work early then his wife Bob doesn't prepare breakfast.
     Alice didn't go to work early.
     Therefore Bob made breakfast.

   We identify the following atomic propositions:

   - A = Alice goes to work early
   - B = Bob makes breakfast

   So the argument comprises:

   - Premise 1: $A \rightarrow \neg B$ ("if A then not B")
     Premise 2: $\neg A$ ("not A")
     Conclusion: B

   The truth table is:

			P1	P2	C
A	B	$\neg B$	$A \rightarrow \neg B$	$\neg A$	B
T	T	F	F	F	T
T	F	T	T	F	F
F	T	F	T	T	T
F	F	T	T	T	F

   The bottom line indicates that this argument is *invalid* since the two premises are true, but the conclusion is false.

   For a counterexample, consider the case where A and B are both false, i.e., Alice does not go to work early and Bob does not make breakfast. This makes both premises of the argument true, but the conclusion is clearly false.

2. The argument is:

   - If England beat France, then Wales finish third.
     If Wales finish third, then Wales are better than New Zealand.
     Therefore, if England beat France, then Wales are better than New Zealand.

   We identify the following atomic propositions:

   - E = England beat France
   - W = Wales finish third
   - N = Wales are better than New Zealand

   So the argument comprises:

   - Premise 1: $E \to W$
     Premise 2: $W \to N$
     Conclusion: $E \to N$

   The truth table is:

			P1	P2	C
E	W	N	$E \to W$	$W \to N$	$E \to N$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

   The argument is *valid* since, for all 4 rows where the premises are true, the conclusion is also true.

3. The argument is:

   - If Thomas is not a goldsmith then John is not a merchant. Therefore Thomas is in fact a goldsmith because John is certainly a merchant.

   We identify the following atomic propositions:

   - G = Thomas is a goldsmith
   - M = John is a merchant.

   Notice that we require a little thought here to identify the premises and conclusions, since they are not arranged neatly in rows as in the first two questions. The "Therefore" indicates that what follow is the conclusion, and the "because" suggests that the second half of the same sentence is a premise. So the argument comprises:

   - Premise 1: $\neg G \rightarrow \neg M$
     Premise 2: M
     Conclusion: G

   The truth table is:

				P1	P2	C
G	M	$\neg G$	$\neg M$	$\neg G \rightarrow \neg M$	M	G
T	T	F	F	T	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	F
F	F	T	T	T	F	F

   The argument is *valid* since, for the only row where both the premises are true (row 1), the conclusion is also true.

4. The argument is:

   - If arachnids have eight legs then crabs spin webs and scorpions live underwater. Crabs do not spin webs. So arachnids do not have eight legs.

   We identify the following atomic propositions:

   - A = Arachnids have eight legs
   - C = Crabs spin webs
   - S = Scorpions live underwater

   So the argument comprises:

   - Premise 1: $A \rightarrow (C \wedge S)$
     Premise 2: $\neg C$
     Conclusion: $\neg A$

   The truth table is:

				P1	P2	C
A	C	S	$C \wedge S$	$A \rightarrow (C \wedge S)$	$\neg C$	$\neg A$
T	T	T	T	T	F	F
T	T	F	F	F	F	F
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

   The argument is *valid* since, for all rows where the premises are true (the last two), the conclusion is also true.

   [This question also illustrates the potential for ambiguity in natural language descriptions of arguments. Using the same atomic propositions, it would also have been reasonable to represent the first premise as: $(A \rightarrow C) \wedge S$. In fact, in this instance, the argument remains valid]