Exercise Class 2
Simple Proofs in Propositional Logic

For this exercise class, we'll construct some simple natural deduction proofs for arguments in propositional logic. For now, we'll use the following (incomplete) set of inference rules:

- Modus Ponens: $P \to Q, P \vdash Q$
- Modus Tollens: $P \to Q, \neg Q \vdash \neg P$
- Double Negative Elimination: $\neg\neg P \vdash P$
- $\wedge$-Introduction: $P, Q \vdash P \wedge Q$
- $\wedge$-Elimination: $P \wedge Q \vdash P$

1. Translate the following into an argument expressed in propositional logic. Then construct a proof to show its validity.

   Phillip and Quinton watch football.
   If Quinton watches football then Roger watches hockey.
   Therefore Roger watches hockey.

2. Construct a proof of validity for:

$$P, Q, R \ : \ P \wedge (Q \wedge R)$$

3. Construct a proof of validity for:

$$P \wedge \neg Q, R \to Q, \neg R \to Z \ : \ Z$$

4. Construct a proof of validity for:

$$R, (P \to Q) \wedge (Q \to P), Q \to Z, R \to P \ : \ Z$$

5. Construct a proof of validity for:

$$\neg P \to \neg Q, P \to Z, \neg\neg Q \ : \ Z$$