Assignment 2 – Solutions
Natural Deduction for Propositional Logic

Model solutions for the required proofs are given below, although these are not unique (and not necessarily the best or the shortest proofs possible!).

1. The argument is:

$$(R \wedge Q) \rightarrow S \ : \ (P \wedge R) \rightarrow (Q \rightarrow (S \wedge P))$$

A proof of validity is:

1.	$P \wedge R$	Hypothesis	{1}
2.	Q	Hypothesis	{2}
3.	R	$\wedge$-elimination$_1$	{1}
4.	$R \wedge Q$	$\wedge$-introduction$_{3,2}$	{1,2}
5.	$(R \wedge Q) \rightarrow S$	Premise	{5}
6.	S	$\rightarrow$-elimination$_{5,4}$	{1,2,5}
7.	P	$\wedge$-elimination$_1$	{1}
8.	$S \wedge P$	$\wedge$-introduction$_{6,7}$	{1,2,5}
9.	$Q \rightarrow (S \wedge P)$	$\rightarrow$-introduction$_{2,8}$	{1,5}
10.	$(P \wedge R) \rightarrow (Q \rightarrow (S \wedge P))$	$\rightarrow$-introduction$_{1,9}$	{5}

2. The argument is:

$$R \rightarrow (S \wedge P),\ \neg S \rightarrow P,\ Q,\ \neg P \vee (Q \rightarrow R) \ : \ S$$

A proof of validity is:

1.	$\neg P \vee (Q \rightarrow R)$	Premise	{1}
2.	$\neg P$	Hypothesis	{2}
3.	$\neg S \rightarrow P$	Premise	{3}
4.	$\neg S$	Hypothesis	{4}
5.	P	$\rightarrow$-elimination$_{3,4}$	{3,4}
6.	$\bot$	$\wedge$-introduction$_{5,2}$	{2,3,4}
7.	$\neg\neg S$	$\neg$-introduction$_{4,6}$	{2,3}
8.	S	$\neg\neg$-elimination$_7$	{2,3}
9.	$Q \rightarrow R$	Hypothesis	{9}
10.	Q	Premise	{10}
11.	R	$\rightarrow$-elimination$_{9,10}$	{9,10}
12.	$R \rightarrow (S \wedge P)$	Premise	{12}
13.	$S \wedge P$	$\rightarrow$-elimination$_{12,11}$	{9,10,12}
14.	S	$\wedge$-elimination$_{13}$	{9,10,12}
15.	S	$\vee$-elimination$_{1,2,8,9,14}$	{1,3,10,12}

3. The argument is:

$$: \ (Q \wedge (P \rightarrow (Q \rightarrow R))) \rightarrow (\neg P \vee R)$$

A proof of validity is:

1.	$Q \wedge (P \rightarrow (Q \rightarrow R))$	Hypothesis	{1}
2.	Q	$\wedge$-elimination$_1$	{1}
3.	$P \rightarrow (Q \rightarrow R)$	$\wedge$-elimination$_1$	{1}
4.	$\neg(\neg P \vee R)$	Hypothesis	{4}
5.	P	Hypothesis	{5}
6.	$Q \rightarrow R$	$\rightarrow$-elimination$_{3,5}$	{1,5}
7.	R	$\rightarrow$-elimination$_{6,2}$	{1,5}
8.	$\neg P \vee R$	$\vee$-introduction$_7$	{1,5}
9.	$\bot$	$\wedge$-introduction$_{8,4}$	{1,4,5}
10.	$\neg P$	$\neg$-introduction$_{5,9}$	{1,4}
11.	$\neg P \vee R$	$\vee$-introduction$_{10}$	{1,4}
12.	$\bot$	$\wedge$-introduction$_{11,4}$	{1,4}
13.	$\neg\neg(\neg P \vee R)$	$\neg$-introduction$_{4,12}$	{1}
14.	$\neg P \vee R$	$\neg\neg$-elimination$_{13}$	{1}
15.	$(Q \wedge (P \rightarrow (Q \rightarrow R))) \rightarrow (\neg P \vee R)$	$\rightarrow$-elimination$_{1,14}$	{}

4. The argument is:

$$\neg(P \wedge (Q \vee R)) \ : \ \neg P \vee (\neg Q \wedge \neg R)$$

A proof of validity is:

1.	$\neg(P \wedge (Q \vee R))$	Premise	{1}
2.	$\neg(\neg P \vee (\neg Q \wedge \neg R))$	Hypothesis	{2}
3.	$\neg P$	Hypothesis	{3}
4.	$\neg P \vee (\neg Q \wedge \neg R)$	$\vee$-introduction$_3$	{3}
5.	$\bot$	$\wedge$-introduction$_{4,2}$	{2,3}
6.	$\neg\neg P$	$\neg$-introduction$_{3,5}$	{2}
7.	P	$\neg\neg$-elimination$_6$	{2}
8.	$\neg(Q \vee R)$	Hypothesis	{8}
9.	Q	Hypothesis	{9}
10.	$Q \vee R$	$\vee$-introduction$_9$	{9}
11.	$\bot$	$\wedge$-introduction$_{10,8}$	{8,9}
12.	$\neg Q$	$\neg$-introduction$_{9,11}$	{8}
13.	R	Hypothesis	{13}
14.	$Q \vee R$	$\vee$-introduction$_{13}$	{13}
15.	$\bot$	$\wedge$-introduction$_{14,8}$	{8,13}
16.	$\neg R$	$\neg$-introduction$_{13,15}$	{8}
17.	$\neg Q \wedge \neg R$	$\wedge$-introduction$_{12,16}$	{8}
18.	$\neg P \vee (\neg Q \wedge \neg R)$	$\vee$-introduction$_{17}$	{8}
19.	$\bot$	$\wedge$-introduction$_{18,2}$	{2,8}
20.	$\neg\neg(Q \vee R)$	$\neg$-introduction$_{8,19}$	{2}
21.	$Q \vee R$	$\neg\neg$-elimination$_{20}$	{2}
22.	$P \wedge (Q \vee R)$	$\wedge$-introduction$_{7,21}$	{2}
23.	$\bot$	$\wedge$-introduction$_{22,1}$	{1,2}
24.	$\neg\neg(\neg P \vee (\neg Q \wedge \neg R))$	$\neg$-introduction$_{2,23}$	{1}
25.	$\neg P \vee (\neg Q \wedge \neg R)$	$\neg\neg$-elimination$_{24}$	{1}