1. See Figure 1 for labelled transition system

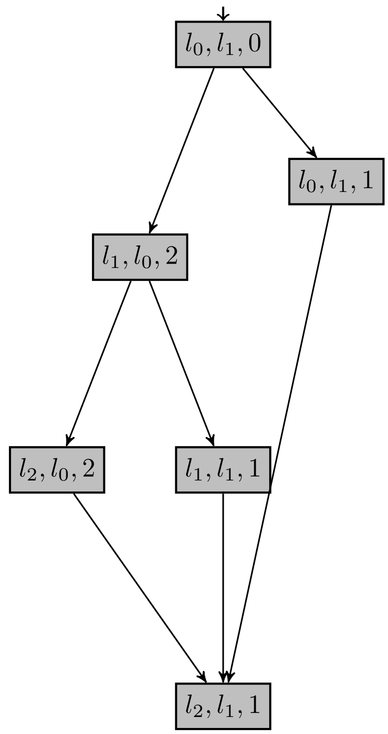

Figure 1: Question 1 Transition System

2. (a) whenever a is true, b is also true.

$$\Box(a \to b) \tag{1}$$

This is an example of a invariant. This is the case as it must always be true and it can be checked separately in each state.

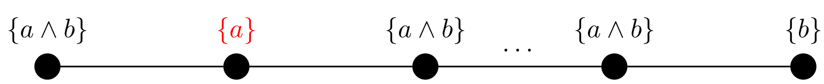

Figure 2: 2 a, counterexample trace

(b) a is eventually true

$$\Diamond a \tag{2}$$

This is an example of a liveness property as any finite word can be extended to satisfy this property. Any trace where a never appears can be extended with a single case where a is true & it will now satisfy the property.

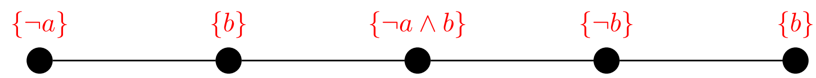

Figure 3: 2 b, counterexample trace

(c) a eventually comes true but is then subsequently false again.

$$\Diamond a \rightarrow (a \wedge \bigcirc \neg a) \tag{3}$$

This is a safety property as any violation produces a trace that cannot be extended into a satisfactory trace.

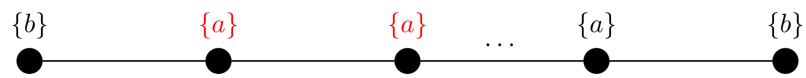

Figure 4: 2 c, counterexample trace

(d) a and b are both true infinitely often, but never simultaneously.

$$\Box\Diamond a \wedge \Box\Diamond b \wedge \Box\neg(a \wedge b) \tag{4}$$

This is a safety property as if the final clause of the property ($\Box\neg(a \wedge b)$) is unsatisfied in one state, the entire trace is invalidated and cannot be extended into a valid trace.

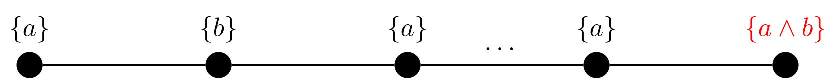

Figure 5: 2 d, counterexample trace

(e) the first occurrence (if any) of a is immediately followed by b

$$a \vee b \cup a \rightarrow \bigcirc b \tag{5}$$

This is an example of a safety property as any finite word σ' that does not satisfy the property cannot be extended by any infinite word such that it now satisfies the property.

3. We first negate the safety property, Ψ, giving us $\neg\Psi$ in Equation 7. Constructing a NFA of this property gives us $\mathcal{A}_{\neg\Psi}$ shown in Figure 8. The product of M and $\mathcal{A}_{\neg\Psi}$, $M \otimes \mathcal{A}_{\neg\Psi}$ is shown in Figure 9. This graph has an accept state so therefore this property does not hold for the LTS M.

$$\Psi = \Box\,(a \rightarrow \bigcirc\Box b) \tag{6}$$

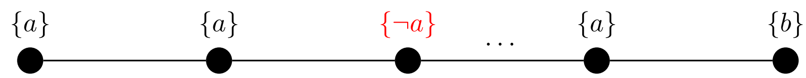

Figure 6: 2 e, counterexample trace

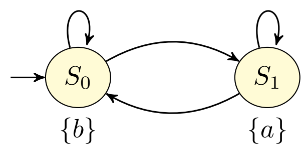

Figure 7: LTS-M

$$\begin{aligned}
\neg\Psi &= \neg\Box(a \rightarrow \bigcirc\Box b) \\
&\equiv \Diamond\neg(a \rightarrow \bigcirc\Box b) \\
&\equiv \Diamond\neg(\neg a \vee \bigcirc\Box b) \\
&\equiv \Diamond(a \wedge \neg \bigcirc \Box b) \\
&\equiv \Diamond(a \wedge \bigcirc\neg\Box b) \\
&\equiv \Diamond(a \wedge \bigcirc\Diamond\neg b)
\end{aligned} \tag{7}$$

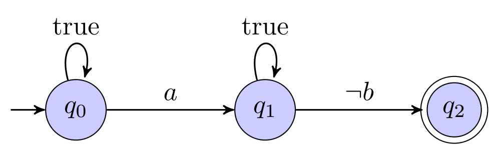

Figure 8: NFA-$\mathcal{A}_{\neg\Psi}$

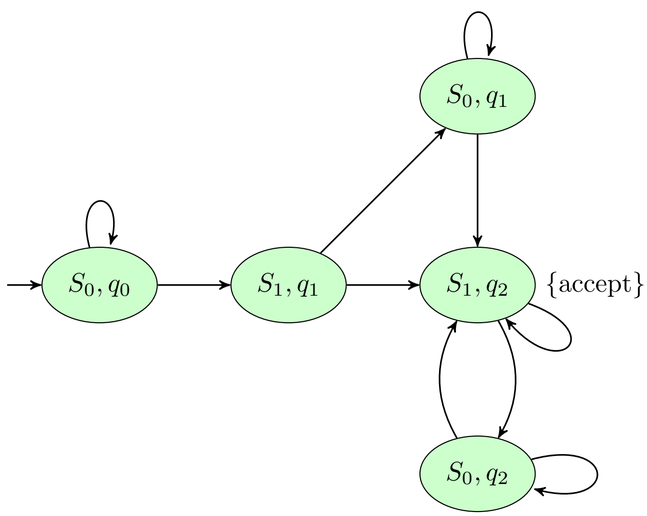

Figure 9: $M \otimes \mathcal{A}_{\neg\Psi}$