1. See Figure 1 for labelled transition system

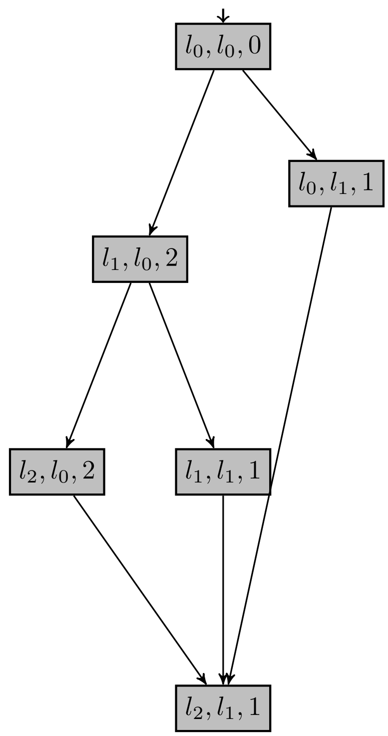

Figure 1: Question 1 Transition System

2. (a) whenever a is true, b is also true.

$$\Box(a \to b) \tag{1}$$

This is an example of a invariant. This is the case as must be true in every state and it can be checked separately in each state.

In Figure 2 you can see a counterexample trace, if there exists a state which satisfies $a \wedge \neg b$ then the property is not satisfied by the trace.

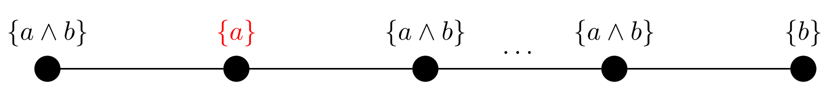

Figure 2: 2 a, counterexample trace

(b) a is eventually true

$$\Diamond a \tag{2}$$

This is an example of a liveness property as any finite word can be extended to satisfy this property. Any trace where a never appears can be extended with a single case where a is true & it will now satisfy the property.

In Figure 3 a counterexample example trace is given. If, over an infinite trace, there does not exist a state where a is true: The property is not satisfied by the trace.

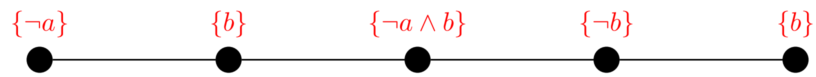

Figure 3: 2 b, counterexample trace

(c) a eventually comes true but is then subsequently false again.

$$\Diamond a \to (a \wedge \bigcirc \neg a) \tag{3}$$

This is a safety property as any violation produces a trace that cannot be extended into a satisfactory trace.

Figure 4 shows that if a occurs in two consecutive states then the trace does not satisfy the property. I found this easier to think about in the form $\Box \neg a \vee (a \wedge \bigcirc \neg a)$ Where a satisfactory trace can either be infinitely $\neg a$ or if a occurs, the following state must satisfy $\neg a$.

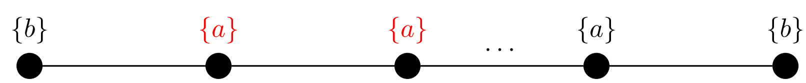

Figure 4: 2 c, counterexample trace

(d) a and b are both true infinitely often, but never simultaneously.

$$\Box \Diamond a \wedge \Box \Diamond b \wedge \Box \neg (a \wedge b) \tag{4}$$

This is a safety property as if the final clause of the property ($\Box \neg (a \wedge b)$) is unsatisfied in one state, the entire trace is invalidated and cannot be extended into a valid trace.

Figure 5 shows that if a state exists that satisfies $a \wedge b$ then the overall property is not held by the trace.

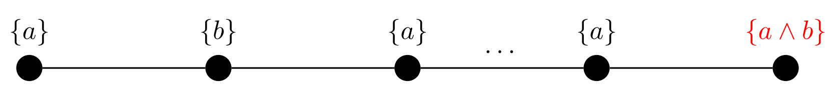

Figure 5: 2 d, counterexample trace

(e) the first occurrence (if any) of a is immediately followed by b

$$a \vee b \cup a \to \bigcirc b \tag{5}$$

This is an example of a safety property as any finite word σ' that does not satisfy the property cannot be extended by any infinite word such that it now satisfies the property.

Figure 6 illustrates that if the first state satisfying a is not immediately followed by a state satisfying b then the trace does not satisfy the property.

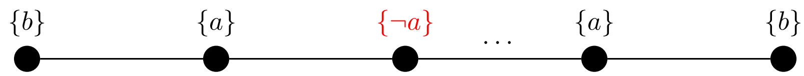

Figure 6: 2 e, counterexample trace

3. We first negate the safety property, Ψ, giving us $\neg\Psi$ in Equation 7. Constructing an NFA of this property gives us $\mathcal{A}_{\neg\Psi}$ shown in Figure 8. The product of M and $\mathcal{A}_{\neg\Psi}$, $M \otimes \mathcal{A}_{\neg\Psi}$ is shown in Figure 9. This system has a reachable accept state so therefore this property does not hold for all possible traces over the LTS M, $M \not\models \Box(a \rightarrow \bigcirc\Box b)$.

$$\Psi = \Box(a \rightarrow \bigcirc\Box b) \tag{6}$$

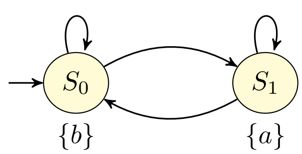

Figure 7: LTS-M

$$\begin{aligned}
\neg\Psi &= \neg\Box(a \rightarrow \bigcirc\Box b) \\
&\equiv \Diamond\neg(a \rightarrow \bigcirc\Box b) \\
&\equiv \Diamond\neg(\neg a \vee \bigcirc\Box b) \\
&\equiv \Diamond(a \wedge \neg \bigcirc \Box b) \\
&\equiv \Diamond(a \wedge \bigcirc\neg\Box b) \\
&\equiv \Diamond(a \wedge \bigcirc\Diamond\neg b)
\end{aligned} \tag{7}$$

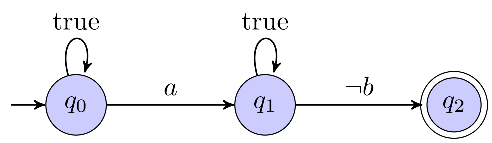

Figure 8: NFA-$\mathcal{A}_{\neg\Psi}$

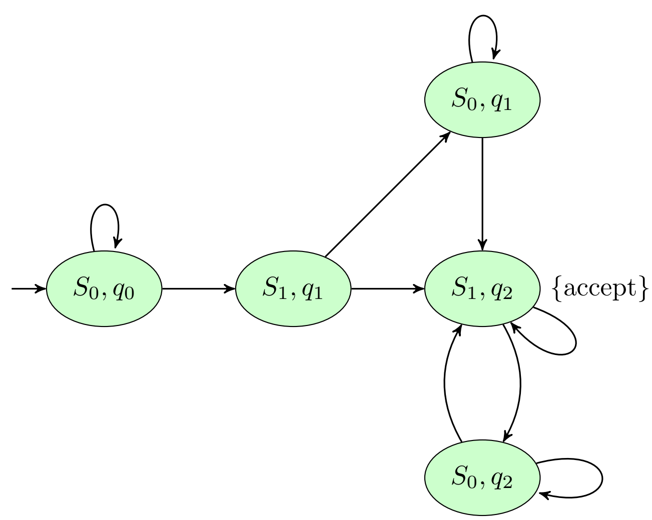

Figure 9: $M \otimes \mathcal{A}_{\neg\Psi}$