Exam for Programming Language Principals, Design and Implementation (Extended)

ID 1803086

After inserting your student ID and the module title in the preamble, write your answers below.

Question 1

(a)

$$M = \lambda f : \mathbb{B} \to \mathbb{B}.\lambda g : \mathbb{B} \to \mathbb{B}.\lambda x : \mathbb{B}.\lambda y : \mathbb{B}.\texttt{if } x \texttt{ then } fy \texttt{ else } gy$$

(i)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : \mathbb{B}}\,\texttt{VAR} \qquad \dfrac{\dfrac{}{\Gamma \vdash f : \mathbb{B} \to \mathbb{B}}\,\texttt{VAR} \quad \dfrac{}{\Gamma \vdash y : \mathbb{B}}\,\texttt{VAR}}{\Gamma \vdash fy : \mathbb{B}}\,\texttt{APP} \qquad \Pi_1}{\Gamma, y : \mathbb{B} \vdash \texttt{if } x \texttt{ then } fy \texttt{ else } gy : \mathbb{B}}}{\Gamma, x : \mathbb{B} \vdash \lambda y : \mathbb{B}.\texttt{if } x \texttt{ then } fy \texttt{ else } gy : \mathbb{B} \to \mathbb{B}}\,\texttt{ABS}}{\Gamma, g : \mathbb{B} \to \mathbb{B} \vdash \lambda x : \mathbb{B}.\lambda y : \mathbb{B}.\texttt{if } x \texttt{ then } fy \texttt{ else } gy : \mathbb{B} \to \mathbb{B} \to \mathbb{B}}\,\texttt{ABS}}{f : \mathbb{B} \to \mathbb{B} \vdash M_1 : (\mathbb{B} \to \mathbb{B}) \to \mathbb{B} \to \mathbb{B} \to \mathbb{B}}\,\texttt{ABS}}{\{\} \vdash M : (\mathbb{B} \to \mathbb{B}) \to (\mathbb{B} \to \mathbb{B}) \to \mathbb{B} \to \mathbb{B} \to \mathbb{B}}\,\texttt{ABS}$$

Where:

- $M_1 = \lambda g : \mathbb{B} \to \mathbb{B}.\lambda x : \mathbb{B}.\lambda y : \mathbb{B}.\texttt{if } x \texttt{ then } fy \texttt{ else } gy$
- $\Pi_1 =$

$$\dfrac{\dfrac{}{\Gamma \vdash g : \mathbb{B} \to \mathbb{B}}\,\texttt{VAR} \qquad \dfrac{}{\Gamma \vdash y : \mathbb{B}}\,\texttt{VAR}}{\Gamma \vdash gy : \mathbb{B}}\,\texttt{APP}$$

(ii) To produce the exclusive function from M we can define the first order parameters F and G as follows:

$$F = \lambda y : \mathbb{B}.\texttt{if } y \texttt{ then false else true} : \mathbb{B} \to \mathbb{B}$$

$$G = \lambda y : \mathbb{B}.\texttt{if } y \texttt{ then true else false} : \mathbb{B} \to \mathbb{B}$$

Alternatively, G can simply be defined as the boolean identity function $\lambda y : \mathbb{B}.y : \mathbb{B} \to \mathbb{B}$. This is the definition I will use in later parts of the question.

(iii) The fact that this expression is well-typed under the Simply-typed λ-Calculus means

Finish this question

(iv)

$$\dfrac{\dfrac{}{\lambda f:\mathbb{B}\to\mathbb{B}.M_1 F \to_v \lambda g:\mathbb{B}\to\mathbb{B}\lambda x:\mathbb{B}\texttt{if}\ x\ \texttt{then}\ Fy\ \texttt{else}\ gy}\,\beta}{\dfrac{\lambda f:\mathbb{B}\to\mathbb{B}.M_1 FG \to_v \\ (\lambda g:\mathbb{B}\to\mathbb{B}\lambda x:\mathbb{B}\texttt{if}\ x\ \texttt{then}\ Fy\ \texttt{else}\ gy)G}{(MFG)\texttt{false true} \to_v \\ ((\lambda g:\mathbb{B}\to\mathbb{B}\lambda x:\mathbb{B}\texttt{if}\ x\ \texttt{then}\ Fy\ \texttt{else}\ gy)G)\texttt{false true}}\ \texttt{CTX}_{(\bullet)\texttt{false true}}}\ \texttt{CTX}_{\bullet G} \tag{1}$$

$$\dfrac{\dfrac{}{(\lambda g:\mathbb{B}\to\mathbb{B}\lambda x:\mathbb{B}\texttt{if}\ x\ \texttt{then}\ Fy\ \texttt{else}\ gy)G \to_v \\ \lambda x:\mathbb{B}.\lambda y:\mathbb{B}.\texttt{if}\ x\ \texttt{then}\ Fy\ \texttt{else}\ Gy}\,\beta}{((\lambda g:\mathbb{B}\to\mathbb{B}\lambda x:\mathbb{B}\texttt{if}\ x\ \texttt{then}\ Fy\ \texttt{else}\ gy)G)\texttt{false true} \to_v \\ (\lambda x:\mathbb{B}.\lambda y:\mathbb{B}.\texttt{if}\ x\ \texttt{then}\ Fy\ \texttt{else}\ Gy)\texttt{false true}}\ \texttt{CTX}_{(\bullet)\texttt{false true}} \tag{2}$$

$$\dfrac{\dfrac{}{(\lambda x:\mathbb{B}.\lambda y:\mathbb{B}.\texttt{if}\ x\ \texttt{then}\ Fy\ \texttt{else}\ Gy)\texttt{false} \to_v \\ \lambda y:\mathbb{B}\texttt{if false then}\ Fy\ \texttt{else}\ Gy}\,\beta}{(\lambda x:\mathbb{B}.\lambda y:\mathbb{B}.\texttt{if}\ x\ \texttt{then}\ Fy\ \texttt{else}\ gy)\texttt{false true} \to_v \\ (\lambda y:\mathbb{B}\texttt{if false then}\ Fy\ \texttt{else}\ Gy)\texttt{true}}\ \texttt{CTX}_{\bullet\texttt{true}} \tag{3}$$

$$\dfrac{}{(\lambda y:\mathbb{B}\texttt{if false then}\ Fy\ \texttt{else}\ Gy)\texttt{true} \to_v \\ \texttt{if false then}\ F\texttt{true}\ \texttt{else}\ G\texttt{true}}\,\beta \tag{4}$$

$$\dfrac{}{\texttt{if false then}\ F\texttt{true}\ \texttt{else}\ G\texttt{true} \to_v G\texttt{true}}\ \texttt{IteF} \tag{5}$$

$$\dfrac{}{(\lambda y:\mathbb{B}.y)\texttt{true} \to_v \texttt{true}}\,\beta \tag{6}$$

Where:

- M_1 is as defined the same as above.

Question 2

Question 3

Add more questions if necessary.

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