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MODULE 8 — Practice Assignment

Problem 1

Solve the following practice problems in the 9th edition textbook.

- **Chapter 5:**
  - **7-1** (a)
  - **7-6** (a)

7-1. Find the angles of the asymptotes and the intersect of the asymptotes of the root loci of the following equations when K varies from $-\infty$ to ∞.

(a) $\mathbf{s^4 + 4s^3 + 4s^2 + (K+8)s + K = 0}$

Putting the above equation into the form:

$$1 + \frac{KQ(s)}{P(s)} = 0$$

We get:

$$Q(s) = s + 1$$

And:

$$P(s) = s^4 + 4s^3 + 4s^2 + 8s$$

The poles are, (using the `roots()` function in MATLAB):

$$\texttt{roots([1,4,4,8,0])} = \begin{bmatrix} 0.0000 & +0.0000j \\ -3.5098 & +0.0000j \\ -0.2451 & +1.4897j \\ -0.2451 & -1.4897j \end{bmatrix}$$

The zero is $s = -1$.

Factoring $P(s)$ results in 4 poles, and the equation has 1 zero:

For large values of s, the root locus for $K > 0$ are asymptotic to asymptotes with angles given by:

$$\theta_i = \frac{(2i+1)}{|n-m|} \times 180^\circ, \quad n \neq m, \quad i = 0, 1, 2... \; |n-m| - 1$$

For this case:

$$|n-m| - 1 = |4-1| - 1 = 2 \rightarrow i = 0, 1, 2$$

Therefore, when $K > 0$:

$$\boldsymbol{\theta_0 = \frac{(2(0)+1)}{3} \times 180^\circ = 60^\circ, \quad [K > 0]}$$

$\longrightarrow$ *A*nswer

$$\boldsymbol{\theta_1 = \frac{(2(1)+1)}{3} \times 180^\circ = 180^\circ, \quad [K > 0]}$$

$\longrightarrow$ *A*nswer

$$\boldsymbol{\theta_2 = \frac{(2(2)+1)}{3} \times 180^\circ = 300^\circ, \quad [K > 0]}$$

$\longrightarrow$ *A*nswer

For large values of s, the root locus for $K < 0$ are asymptotic to asymptotes with angles given by:

$$\theta_i = \frac{(2i)}{|n-m|} \times 180^\circ, \quad n \neq m, \quad i = 0, 1, 2... \; |n-m| - 1$$

Where again:

$$|n-m| - 1 = |4-1| - 1 = 2 \rightarrow i = 0, 1, 2$$

Therefore, when $K < 0$:

$$\boldsymbol{\theta_0 = \frac{(2(0))}{3} \times 180^\circ = 0^\circ, \quad [K < 0]}$$

$\longrightarrow$ *Answer*

$$\boldsymbol{\theta_1 = \frac{(2(1))}{3} \times 180^\circ = 120^\circ, \quad [K < 0]}$$

$\longrightarrow$ *Answer*

$$\boldsymbol{\theta_2 = \frac{(2(2))}{3} \times 180^\circ = 240^\circ, \quad [K < 0]}$$

$\longrightarrow$ *Answer*

The point of intersection of the asymptotes is given by:

$$\sigma_1 = \frac{\sum \text{real parts of poles of } G(s)H(s) - \sum \text{real parts of zeros of } G(s)H(s)}{n - m}$$

Where:

$$G(s)H(s) = \frac{KQ(s)}{P(s)}$$

Evaluating the sum of the real parts of the poles in MATLAB:

`sum(real(roots([1,4,4,8,0])))` $= -4$

Therefore:

$$\boldsymbol{\sigma_1 = \frac{(-4) - (-1)}{4 - 1} = -1}$$

$\longrightarrow$ *Answer*

7-6. For the loop transfer functions that follow, find the angle of departure or arrival of the root loci at the designated pole or zero.

(a) $\mathbf{G(s)H(s) = \dfrac{Ks}{(s+1)(s^2+1)}}$

Angle of arrival $(\mathbf{K < 0})$ and angle of departure $(\mathbf{K > 0})$ at $\mathbf{s = j}$.

This equation can be rewritten as:

$$G(s)H(s) = \frac{Ks}{(s+1)(s+j)(s-j)}$$

Submitted by Austin Barrilleaux on October 20, 2023.