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MODULE 12 — Practice Assignment

Problem 1

Solve the following 9th Edition textbook problems:

- **9-49 (a)**
- **9-50 (c,d)**

(9-49) Consider that the controller in the liquid-level control system shown in Fig. 9P-10 is a single-stage phase-lag controller:

$$\mathbf{G_c(s) = \frac{1 + aTs}{1 + Ts}, \quad a < 1}$$

$$\mathbf{G_p(s) = \frac{10N}{s(s+1)(s+10)}}$$

(a) For N = 20, select the values of a and T so that the two complex roots of the characteristic equation correspond to a relative damping ratio of approximately 0.707. Plot the unit-step response of the output y(t). Find the attributes of the unit-step response. Plot the Bode plot of $\mathbf{G_c(s)G_p(s)}$ and determine the phase margin of the designed system.

This makes the process:

$$G_p(s) = \frac{200}{s(s+1)(s+10)}$$

The compensated system is:

$$G_c(s)G_p(s) = \frac{200(1 + aTs)}{s(s+1)(s+10)(1+Ts)}$$

Which can also be written as:

$$G_c(s)G_p(s) = \frac{200a(s+\frac{1}{aT})}{s(s+1)(s+10)(s+\frac{1}{T})}$$

Looking at a zoomed in view of the root locus plot for the uncompensated system made using the `rlocus()` function in MATLAB, we can see what the complex conjugate roots look like when the damping ratio is roughly equal to $\cos(45°) \approx 0707$:

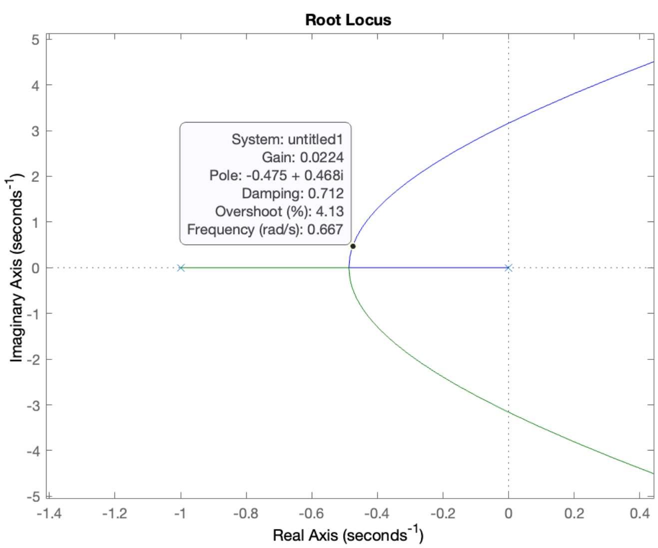

Figure 1: Root Locus

If we make the assumption that $\frac{1}{aT} = \frac{1}{T}$, we can write $G(s)$ as:

$$G_c(s)G_p(s) = \frac{200a}{s(s+1)(s+10)}$$

Using this equation we can solve for a using the following MATLAB code:

```
bounds = 0:0.001:0.1;
idx = 2; d_min = bounds;

while abs(d_min(idx)-cosd(45))>eps
```

```
    del = (bounds(end)-bounds(1))/100;
    k = 0;
    a_vals = bounds(1):del:bounds(end);

    for a = bounds(1):del:bounds(end)
        k = k+1;
        [~,d] = damp(tf(200*a,[1 11 10 200*a]));
        d_min(k) = abs(min(d));
    end

    [~,idx] = min(abs(d_min-cosd(45)));
    bounds = [a_vals(idx-2),a_vals(idx+2)];

end

a = a_vals(idx);
zeta = d_min(idx);
```

Not that the above algorithm only works given good initial boundaries.
This solves for a value for a of $a = 0.02268$, where $\zeta - \cos(45°) = -2.22 \cdot 10^{-16}$.

Submitted by Austin Barrilleaux on November 29, 2023.