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HOMEWORK 1 — Assignment

Problem 1

Solve the following practice problems in the 9-th edition textbook.

- **Chapter 2: problems 2-16(a) and 2-22(a).**

2-16(a): Find the Laplace transform of the following function:

$$g(t) = 5te^{-5t}u_s(t)$$

The unit-step function that is defined as:

$$u_s(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

The Laplace transform is:

$$G(s) = \mathcal{L}\{5te^{-5t}u_s(t)\} = \int_{t=0}^{\infty} \left(5te^{-5t}\right) e^{-st} dt$$

This can be written as:

$$G(s) = 5 \int_{t=0}^{\infty} te^{-(s+5)t} dt$$

Using integration by parts:

$$G(s) = \frac{5te^{-(s+5)t}}{-(s+5)}\bigg|_0^{\infty} - \frac{5}{-(s+5)} \int_{t=0}^{\infty} e^{-(s+5)t} dt$$

Which further evaluates to:

$$G(s) = \frac{5te^{-(s+5)t}}{-(s+5)}\bigg|_0^\infty - \frac{5}{-(s+5)}\frac{e^{-(s+5)t}}{-(s+5)}\bigg|_0^\infty$$

Or:

$$G(s) = \frac{5te^{-(s+5)t}}{-(s+5)}\bigg|_0^\infty - \frac{5e^{-(s+5)t}}{(s+5)^2}\bigg|_0^\infty$$

Evaluating the limits:

$$G(s) = \left[\frac{\cancelto{0}{5\infty e^{-(s+5)\infty}}}{-(s+5)} - \frac{\cancelto{0}{50e^{-(s+5)0}}}{-(s+5)}\right] - \left[\frac{\cancelto{0}{5e^{-(s+5)\infty}}}{(s+5)^2} - \frac{\cancelto{1}{5e^{-(s+5)0}}}{(s+5)^2}\right]$$

This results in the solution:

$$\boldsymbol{G(s) = \frac{5}{(s+5)^2}}$$

$\longrightarrow$ $\mathcal{A}$nswer

Using the following Laplace Transform from the table in the textbook, we can get the same answer:

$$te^{-\alpha t} \Leftrightarrow \frac{1}{(s+\alpha)^2}$$

2-22(a): Solve the following differential equations by means of the Laplace transform. Assume zero initial conditions.

$$\frac{d^2f(t)}{dt^2} + 5\frac{df(t)}{dt} + 4f(t) = e^{-2t}u_s(t)$$

Take the Laplace transform of both sides:

$$s^2F(s) - sf(0) - f'(0) + 5sF(s) - f(0) + 4F(s) = \frac{1}{s+2}$$

Evaluate initial conditions:

$$s^2F(s) \cancelto{0}{- sf(0)} - \cancelto{0}{f'(0)} + 5sF(s) - \cancelto{0}{f(0)} + 4F(s) = \frac{1}{s+2}$$

Which gives us:

$$s^2F(s) + 5sF(s) + 4F(s) = \frac{1}{s+2}$$

Solving for $F(s)$, the solution is:

$$\boldsymbol{F(s) = \frac{s^2 + 5s + 4}{s + 2}}$$

$\longrightarrow$ $\mathcal{A}$nswer

Problem 2

Consider the nonlinear system, below, and linearize the system about the nominal point x=2

$$\dot{x} = 2x^2 - x$$

Linearizing via Taylor Series expansion:

$$\dot{x} = f(x) \approx f(x_n) + \left.\frac{df(x)}{dx}\right|_{x=x_n} (x - x_n)$$

where:

$$f(x) = 2x^2 - x$$

and:

$$\frac{df(x)}{dx} = 4x - 1$$

Using this, we can solve that:

$$\dot{x} \approx (2(2)^2 - 2) + (4(2) - 1)(x - 2)$$

This simplifies to following linear equation:

$$\boldsymbol{\dot{x} \approx 7x - 8}$$

$\longrightarrow$ $\mathcal{A}$nswer

Problem 3

Compute the Laplace Transform of $f(t) = \sin(\omega t)$ using the one-sided transform integral

Submitted by Austin Barrilleaux on September 3, 2023.