Austin Barrilleaux
Whiting School of Engineering
Johns Hopkins University

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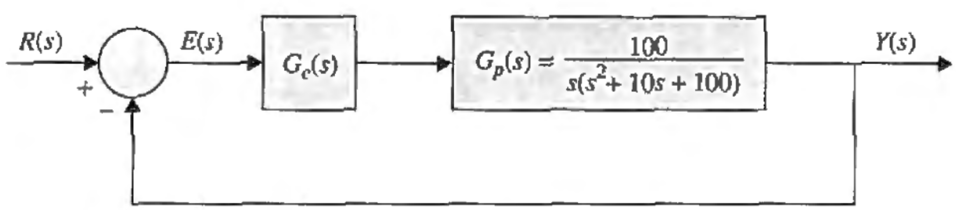
MODULE 11 — Practice Assignment

Problem 1

Solve the following 9th Edition textbook problems:

- **9-1**
- **9-43**

(9-1) The block diagram of a control system with a series controller below.



Find the transfer function of the controller $\mathbf{G_c(s)}$ so that the following specifications are satisfied:

- **The ramp error constant $\mathbf{K_v}$ is 5.**
- **The closed-loop transfer function is of the form:**

$$\mathbf{M(s) = \frac{Y(s)}{R(s)} = \frac{K}{(s^2 + 20s + 200)(s + a)}}$$

where K and a are real constants. Find the values of K and a.

We know that the closed-loop transfer function is:

$$M(s) = \frac{G(s)}{1 + G(s)} = \frac{K}{(s^2 + 20s + 200)(s + a)}$$

Solving for $G(s)$:

$$G(s) = \frac{M(s)}{1 - M(s)} = \frac{K}{s^3 + (20 + a)s^2 + (200 + 20a)s + 200a - K}$$

For this system to have a constant ramp error, the system must be type 1, so in order to have a zero pole:

$$200a - K = 0 \quad \rightarrow \quad K = 200a$$

So $G(s)$ becomes:

$$G(s) = \frac{M(s)}{1 - M(s)} = \frac{200a}{s^3 + (20 + a)s^2 + (200 + 20a)s}$$

The ramp error constant is computed as:

$$K_v = \lim_{s \to 0} sG(s) = \frac{s \cdot 200a}{s^3 + (20 + a)s^2 + (200 + 20a)s} = \frac{200a}{s^2 + (20 + a)s + (200 + 20a)}$$

Evaluating this:

$$K_v = \frac{200a}{\cancelto{0}{s^2} + \cancelto{0}{(20 + a)s} + (200 + 20a)} = \frac{200a}{200 + 20a} = 5$$

Solving for $\boldsymbol{a}$, $\boldsymbol{a = 10}$ and $\boldsymbol{K = 2000}$.

$\longrightarrow$ *Answer*

From this we can solve for the transfer function of the controller, $G_c(s)$:

$$G(s) = G_c(s)G_p(s) \quad \rightarrow \quad G_c(s) = \frac{G(s)}{G_p(s)}$$

So:

$$G_c(s) = \frac{2000}{s(s^2 + 20s + 400)} \left(\frac{s(s^2 + 10s + 100)}{100} \right)$$

$G_c(s)$ reduces to:

$$\boldsymbol{G_c(s) = \frac{20(s^2 + 10s + 100)}{(s^2 + 20s + 400)}}$$

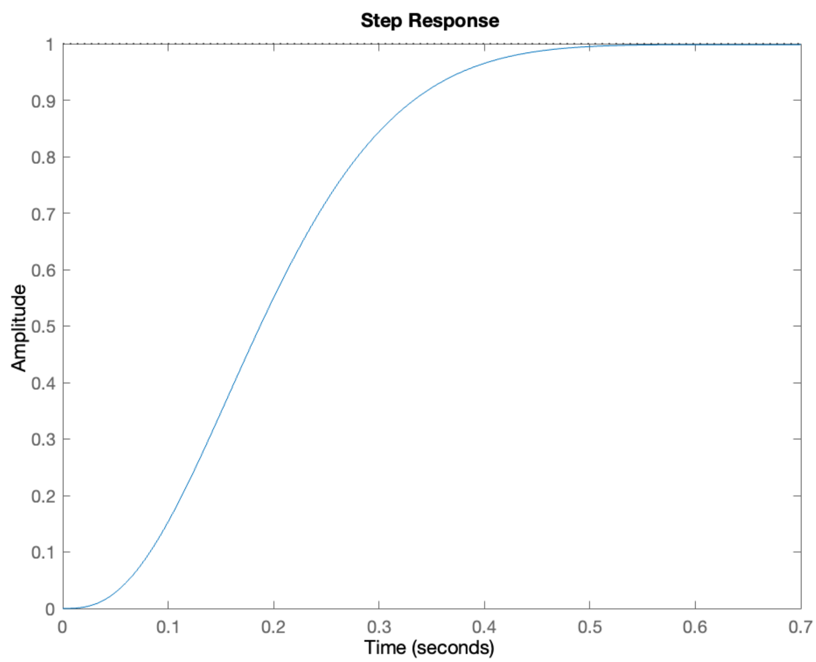
$\longrightarrow$ *Answer*

The design strategy is to place the closed-loop poles at $-10 + j10$ and $-10 - j10$, and then adjust the values of K and a to satisfy the steady-state requirement. The value of a is large so that it will not affect the transient response appreciably. Find the maximum overshoot of the designed system.

We can determine the maximum overshoot of the system by using the `stepinfo()` function in MATLAB. Doing so, gives us a **maximum overshoot** of **0**.

$\longrightarrow$ *Answer*

If we plot the response we can see that there is no overshoot:



(9-43) Consider that the process of a unity-feedback control system is:

$$\mathbf{G_p(s) = \frac{1000}{s(s+10)}}$$

Let the series controller be a single-stage phase-lead controller:

$$\mathbf{G_c(s) = \frac{1 + aTs}{1 + Ts}, \quad a > 1}$$

(a) Determine the values of a and T so that the zero of $\mathbf{G_c(s)}$ cancels the pole of Gp(s) at $\mathbf{s = -10}$. The damping ratio of the designed system should be unity. Find the attributes of the unit-step response of the designed system.

(b) Carry out the design in the frequency domain using the Bode plot. The design specifications are as follows:

- **Phase margin $\mathbf{> 75^\circ}$**
- **$\mathbf{M_r < 1.1}$**

Find the attributes of the unit-step response of the designed system.

Submitted by Austin Barrilleaux on November 17, 2023.