Austin Barrilleaux
Whiting School of Engineering
Johns Hopkins University

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MODULE 14 — Practice Assignment

Problem 1

Solve the following 9th Edition textbook problems:

- **10-18 (a,c,f)**
- **10-59 (a)**

(10-18) Check the contro1ihi1ity of the following systems:

(a) $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} u$$

The controllability matrix (determined using MATLAB) is:

$$\begin{aligned} \mathbf{Q}_c &= \begin{bmatrix} \mathbf{b} & \mathbf{Ab} \end{bmatrix} \\ &= \begin{bmatrix} 2 & -2 \\ 5 & -10 \end{bmatrix} \end{aligned}$$

The rank of $\mathbf{Q}_c$ determined using the `rank()` function in MATLAB is:

$$\texttt{rank}(\mathbf{Q}_c) = 2$$

Since the $n \times n$ controllability matrix has a rank of n, **the system is controllable**.

$\longrightarrow$ Answer

(c)
$$\begin{bmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{-1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-2} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \boldsymbol{x}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{4} & \mathbf{2} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{3} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \end{bmatrix}$$

The controllability matrix (determined using MATLAB) is:

$$\begin{aligned} \mathbf{Q}_c &= \begin{bmatrix} \mathbf{b} & \mathbf{Ab} & \mathbf{A}^2\mathbf{b} \end{bmatrix} \\ &= \begin{bmatrix} 4 & 2 & -4 & -2 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & -6 & 0 & 12 & 0 \end{bmatrix} \end{aligned}$$

We know that, for controllability, the controllability matrix $\mathbf{Q}_c$ **must be square**. Since it is not, we know **the system is not controllable**.

$\longrightarrow$ Answer

Regardless, we can still check the rank of $\mathbf{Q}_c$ using the `rank()` function in MATLAB:

$$\texttt{rank}(\mathbf{Q}_c) = 2$$

This confirms the above conclusion.

(f)
$$\begin{bmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \\ \dot{\boldsymbol{x}}_3 \\ \dot{\boldsymbol{x}}_4 \\ \dot{\boldsymbol{x}}_5 \end{bmatrix} = \begin{bmatrix} \mathbf{-2} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{-2} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{-5} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{-5} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \boldsymbol{x}_3 \\ \boldsymbol{x}_4 \\ \boldsymbol{x}_5 \end{bmatrix} + \begin{bmatrix} \mathbf{4} \\ \mathbf{2} \\ \mathbf{1} \\ \mathbf{3} \\ \mathbf{0} \end{bmatrix} \boldsymbol{u}$$

The controllability matrix (determined using MATLAB) is:

$$\begin{aligned} \mathbf{Q}_c &= \begin{bmatrix} \mathbf{b} & \mathbf{Ab} & \mathbf{A}^2\mathbf{b} & \mathbf{A}^3\mathbf{b} & \mathbf{A}^4\mathbf{b} \end{bmatrix} \\ &= \begin{bmatrix} 4 & -6 & 9 & -14 & 24 \\ 2 & -3 & 4 & -4 & 0 \\ 1 & -2 & 4 & -8 & 16 \\ 3 & -15 & 75 & -375 & 1875 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

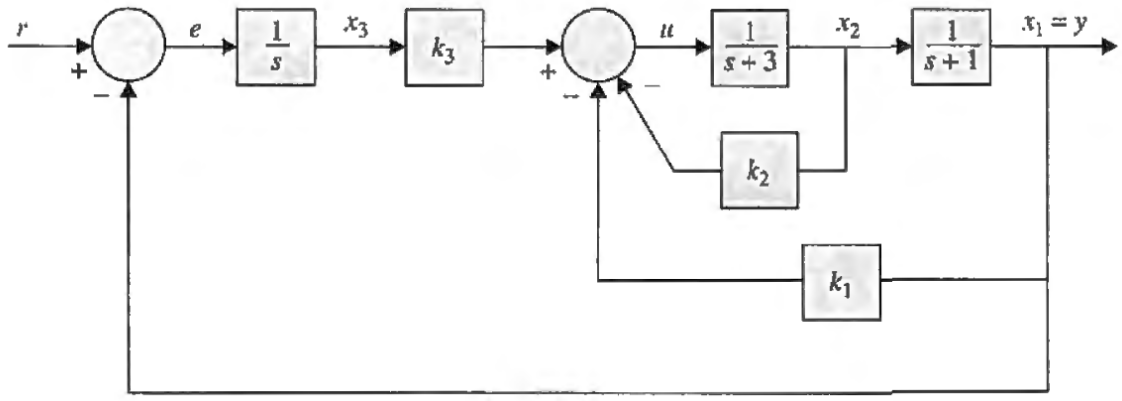
The rank of $\mathbf{Q}_c$ determined using the `rank()` function in MATLAB is:

$$\texttt{rank}(\mathbf{Q}_c) = 4$$

Since the $\boldsymbol{n} \times \boldsymbol{n}$ controllability matrix does not have a rank of $\boldsymbol{n}$ ($\boldsymbol{n = 5, n \neq 4}$), **the system is not controllable**.

$\longrightarrow$ Answer

(10-59) The block diagram of a control system with state feedback is shown in the following figure. The feedback gains k_1, k_2, and k_3 are real constants.



Submitted by Austin Barrilleaux on December 4, 2023.