

October 13, 2023

MODULE 7 — Practice Assignment

Problem 1

Solve the following practice problems in the 9th edition textbook.

- Chapter 5:
 - 5-3 (b,d,f)
 - 5-4 (c,d)

5-3. Determine the step, ramp, and parabolic error constants of the following unity-feedback control systems. The forward-path transfer functions are given.

(b) $G(s) = \frac{100}{s(s^2 + 10s + 100)}$

The step error constant K_p , is calculated as:

$$K_p = \lim_{s \rightarrow 0} [G(s)] = \lim_{s \rightarrow 0} \left[\frac{100}{s(s^2 + 10s + 100)} \right] = \frac{100}{0} = \infty$$

The ramp error constant K_v , is calculated as:

$$K_v = \lim_{s \rightarrow 0} [sG(s)] = \lim_{s \rightarrow 0} \left[s \frac{100}{s(s^2 + 10s + 100)} \right] = \frac{100}{100} = 1$$

The parabolic error constant K_a , is calculated as:

$$K_a = \lim_{s \rightarrow 0} [s^2 G(s)] = \lim_{s \rightarrow 0} \left[s^2 \cdot \frac{100}{s(s^2 + 10s + 100)} \right] = 0 \left(\frac{100}{100} \right) = 0$$

Therefore:

$$K_p = \infty, \quad K_v = 1, \quad K_a = 0$$

→ Answer

$$(d) \quad G(s) = \frac{100}{s^2(s^2 + 10s + 100)}$$

The step error constant K_p , is calculated as:

$$K_p = \lim_{s \rightarrow 0} [G(s)] = \lim_{s \rightarrow 0} \left[\frac{100}{s^2(s^2 + 10s + 100)} \right] = \frac{100}{0} = \infty$$

The ramp error constant K_v , is calculated as:

$$K_v = \lim_{s \rightarrow 0} [sG(s)] = \lim_{s \rightarrow 0} \left[s \frac{100}{s^2(s^2 + 10s + 100)} \right] = \frac{100}{0} = \infty$$

The parabolic error constant K_a , is calculated as:

$$K_a = \lim_{s \rightarrow 0} [s^2 G(s)] = \lim_{s \rightarrow 0} \left[s^2 \frac{100}{s^2(s^2 + 10s + 100)} \right] = \frac{100}{100} = 1$$

Therefore:

$$K_p = \infty, \quad K_v = \infty, \quad K_a = 1$$

→ Answer

$$(d) \quad G(s) = \frac{K(1 + 2s)(1 + 4s)}{s^2(s^2 + s + 1)}$$

The step error constant K_p , is calculated as:

$$K_p = \lim_{s \rightarrow 0} [G(s)] = \lim_{s \rightarrow 0} \left[\frac{K(1 + 2s)(1 + 4s)}{s^2(s^2 + s + 1)} \right] = \frac{K(1)(1)}{0} = \infty$$

The ramp error constant K_v , is calculated as:

$$K_v = \lim_{s \rightarrow 0} [sG(s)] = \lim_{s \rightarrow 0} \left[s \frac{K(1 + 2s)(1 + 4s)}{s^2(s^2 + s + 1)} \right] = \frac{K(1)(1)}{0} = \infty$$

The parabolic error constant K_a , is calculated as:

$$K_a = \lim_{s \rightarrow 0} [s^2 G(s)] = \lim_{s \rightarrow 0} \left[s^2 \frac{K(1+2s)(1+4s)}{s^2(s^2+s+1)} \right] = \frac{K(1)(1)}{1} = K$$

Therefore:

$$K_p = \infty, \quad K_v = \infty, \quad K_a = K$$

→ Answer

5-4. For the unity-feedback control systems described in Problem 5-2, determine the steady-state error for a unit-step input, a unit-ramp input, and a parabolic input, $(t^2/2)u_s(t)$. Check the stability of the system before applying the final-value theorem.

$$(c) \quad G(s) = \frac{10(s+1)}{s(s+5)(s+6)}$$

The unity feedback transfer function is:

$$M(s) = \frac{G(s)}{1+G(s)} = \frac{10s+10}{s^3+11s^2+40s+10}$$

Using the `roots()` function in MATLAB, we can see that the poles are:

$$\text{roots}([1, 11, 40, 10]) = \begin{bmatrix} -5.3653 & +2.8848i \\ -5.3653 & -2.8848i \\ -0.2695 & +0.0000i \end{bmatrix}$$

All poles are in the left-hand plane (LHP). Therefore, the system is **stable**.

→ Answer

The step error constant K_p , is calculated as:

$$K_p = \lim_{s \rightarrow 0} [G(s)] = \lim_{s \rightarrow 0} \left[\frac{10(s+1)}{s(s+5)(s+6)} \right] = \frac{10}{0} = \infty$$

The steady state error for a unit-step input, e_{ss_p} , is calculated as:

$$e_{ss_p} = \frac{1}{1 + K_p} = \frac{1}{1 + \infty} = 0$$

→ Answer

The ramp error constant K_v , is calculated as:

$$K_v = \lim_{s \rightarrow 0} [sG(s)] = \lim_{s \rightarrow 0} \left[\frac{10s(s+1)}{s(s+5)(s+6)} \right] = \frac{10}{30} = \frac{1}{3}$$

The steady state error for a unit-ramp input, e_{ss_v} , is calculated as:

$$e_{ss_v} = \frac{1}{K_v} = \frac{1}{1/3} = 3$$

→ Answer

The parabolic error constant K_a , is calculated as:

$$K_a = \lim_{s \rightarrow 0} [s^2G(s)] = \lim_{s \rightarrow 0} \left[\frac{10s^2(s+1)}{s(s+5)(s+6)} \right] = \frac{0}{30} = 0$$

The steady state error for a parabolic input, e_{ss_a} , is calculated as:

$$e_{ss_a} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

→ Answer

Overall the unit-step, unit-ramp, and parabolic input steady state error for the system are:

$$e_{ss_p} = 0, \quad e_{ss_v} = 3, \quad e_{ss_a} = \infty$$

→ Answer

(c) $G(s) = \frac{100(s-1)}{s^2(s+5)(s+6)^2}$

The unity feedback transfer function is:

$$M(s) = \frac{100s - 100}{s^5 + 17s^4 + 96s^3 + 180s^2 + 100s - 100}$$

Using the `roots()` function in MATLAB, we can see that the poles are:

$$\text{roots}([1, 17, 96, 180, 100, -100]) = \begin{bmatrix} -7.1437 & +1.9796i \\ -7.1437 & -1.9796i \\ -1.5948 & +1.1275i \\ -1.5948 & -1.1275i \\ 0.4771 & +0.0000i \end{bmatrix}$$

There is a single pole in the right-hand plane (RHP). Therefore, the system is **unstable**, so **an error analysis would be meaningless**. There is no need to perform the analysis. **The steady state error is infinite.**

→ Answer

Problem 2

Consider the block diagram of the following missile attitude control system:

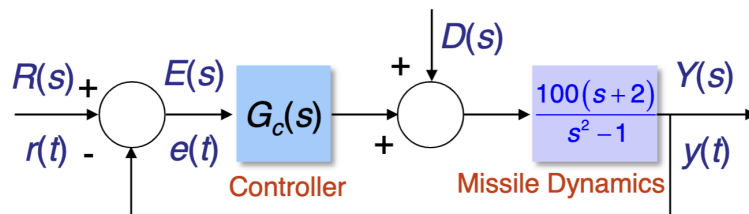


Figure 1: Missile Attitude Control System

(a) Let $G_c(s) = 1$, and find the SSE when $r(t)$ is a unit-step function.

For this system:

$$G(s) = \frac{100(s+2)}{(s^2-1)}$$

The transfer function for the system is:

$$\frac{Y(s)}{R(s)} = \frac{100s + 200}{s^2 + 100s + 199}$$

Using the `roots()` function in MATLAB, we can see that the poles are:

$$\text{roots}([1, 17, 96, 180, 100, -100]) = \begin{bmatrix} -97.9687 & +0.0000i \\ -2.0313 & +0.0000i \end{bmatrix}$$

All poles are in the left-hand plane (LHP). Therefore, the system is **stable**.

The step error constant K_p , is calculated as:

$$K_p = \lim_{s \rightarrow 0} [G(s)] = \lim_{s \rightarrow 0} \left[\frac{100(s+2)}{(s^2-1)} \right] = \frac{100(2)}{-1} = -200$$

The steady state error for a unit-step input, e_{ss_p} , is calculated as:

$$e_{ss_p} = \frac{1}{1 + K_p} = \frac{1}{1 + (-200)} = \frac{1}{-199} = -0.005$$

→ Answer

We can validate this by plotting the step response in MATLAB using the `step()` function, which produces the following figure:

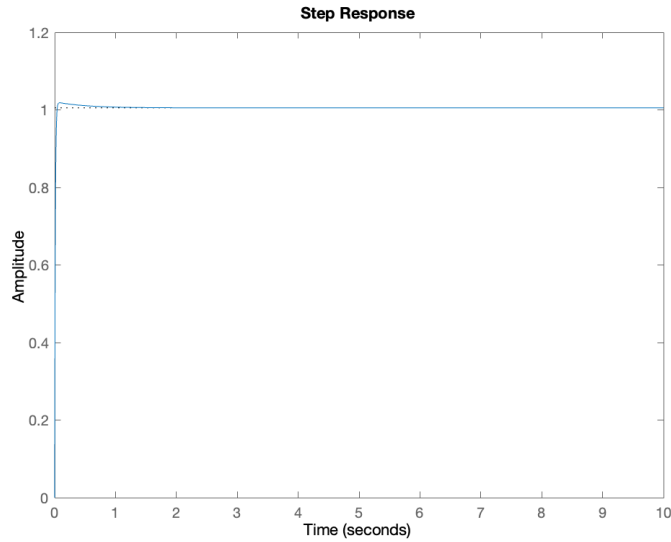


Figure 2: $G_c(s) = 1$, Step Response

The last value of $y(t)$ at the end of the plot is $y(t = 10) = 1.0050$, and the reference signal is 1. Since:

$$e(t) = \text{reference signal} - y(t)$$

We can see that our above analysis resulted in the correct steady state error:

$$e(t) = 1 - 1.0050 = -0.005$$

(b) Let $G_c(s) = \frac{(s+\alpha)}{s}$, and find the SSE when $r(t)$ is a unit-step function

For this system:

$$G(s) = \frac{100(s+2)(s+\alpha)}{s(s^2-1)}$$

$$\frac{Y(s)}{R(s)} = \frac{100s^2 + (200 + 100\alpha)s + 200\alpha}{s^3 + 100s^2 + (199 + 100\alpha)s + 200\alpha}$$

The stability of the system depends on the value of α .

Evaluating the characteristic equation using Routh Hurwitz, we get the following table:

s^3	1	$100\alpha + 199$
s^2	100	200α
s^1	$98\alpha + 199$	0
s^0	200α	0

In order for there to be no sign changes in the left most column, for a stable system, $\alpha \geq 0$.

When $\alpha \geq 0$, the step error constant K_p , is calculated as:

$$K_p = \lim_{s \rightarrow 0} [G(s)] = \lim_{s \rightarrow 0} \left[\frac{100(s+2)(s+\alpha)}{s(s^2-1)} \right] = \begin{cases} -200 & \text{if } \alpha = 0 \\ \frac{200\alpha}{0} = \infty & \text{if } \alpha > 0 \end{cases}$$

The steady state error for a unit-step input, e_{ssp} , is calculated as:

$$e_{ssp}[\alpha = 0] = \frac{1}{1 + K_p} = \frac{1}{1 + (-200)} = \frac{1}{-199} = -0.005$$

→ Answer

$$e_{ssp}[\alpha > 0] = \frac{1}{1 + K_p} = \frac{1}{1 + \infty} = 0$$

→ Answer

Problem 3

Consider the following non-unity feedback system:

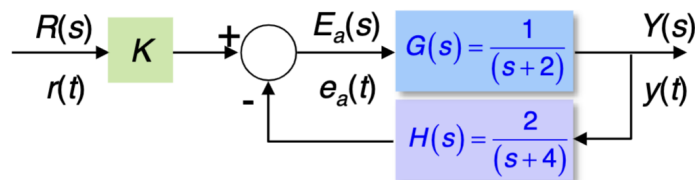


Figure 3: Non-Unity System

(a) Derive an equivalent unity-feedback forward path transfer function

Submitted by Austin Barrilleaux on October 13, 2023.