Austin Barrilleaux
Whiting School of Engineering
Johns Hopkins University

September 29, 2023

MODULE 5 — Practice Assignment

Problem 1

Solve the following practice problems in the 9th edition textbook.

- **Chapter 2:**
  - **2-33** (a-f)
  - **2-39**

2-33. Without using the Routh-Hurwitz criterion, determine if the following systems are asymp- totically stable. marginally stable, or unstable. In each case, the closed-loop system transfer function is given.

(a) $\mathbf{M(s) = \dfrac{10(s+2)}{s^3 + 3s^2 + 5s}}$

Using the `roots()` function MATLAB, we get that the roots are:

$$\texttt{roots([1,3,4,0])} = \begin{pmatrix} 0 \\ -1.5000 + 1.6583\mathrm{i} \\ -1.5000 - 1.6583\mathrm{i} \end{pmatrix}$$

There is a real pole at $s = 0$.

The system is **marginally stable**.

$\longrightarrow \mathcal{A}$nswer

(b) $\mathbf{M(s) = \dfrac{(s-1)}{(s+5)(s^2+2)}}$

Using the `roots()` function MATLAB, we get that the roots are:

$$\texttt{roots(conv([1,5],[1,0,2]))} = \begin{pmatrix} -5 \\ +1.4142\mathrm{i} \\ -1.4142\mathrm{i} \end{pmatrix}$$

There are complex conjugate poles on the imaginary axis (real parts of s equal to zero).

The system is **marginally stable**.

$\longrightarrow$ *Answer*

(c) $\mathbf{M(s) = \dfrac{K}{s^3+5s+5}}$

Using the `roots()` function MATLAB, we get that the roots are:

$$\texttt{roots([1,0,5,5])} = \begin{pmatrix} 0.4344 + 2.3593\mathrm{i} \\ 0.4344 - 2.3593\mathrm{i} \\ -0.8688 \end{pmatrix}$$

There are complex conjugate poles in the right-hand plane (RHP).

The system is **unstable**.

$\longrightarrow$ *Answer*

(d) $\mathbf{M(s) = \dfrac{100(s-1)}{(s+5)(s^2+2s+2)}}$

Using the `roots()` function MATLAB, we get that the roots are:

$$\texttt{roots(conv([1,5],[1,2,2]))} = \begin{pmatrix} -5 \\ -1 + 1\mathrm{i} \\ -1 - 1\mathrm{i} \end{pmatrix}$$

All poles exist in the left-hand plane (LHP).

The system is **stable**.

$\longrightarrow$ *Answer*

(e) $\mathbf{M(s) = \dfrac{100}{s^3 - 2s^2 + 3s + 10}}$

Using the `roots()` function MATLAB, we get that the roots are:

$$\texttt{roots([1,-2,3,10])} = \begin{pmatrix} 1.6694 + 2.1640i \\ 1.6694 - 2.1640i \\ -1.3387 \end{pmatrix}$$

There are complex conjugate poles in the right-hand plane (RHP).

The system is **unstable**.

$\longrightarrow$ $\mathcal{A}$nswer

(f) $\mathbf{M(s) = \dfrac{10(s + 12.5)}{s^4 + s^3 + 50s^2 + s + 10^6}}$

Using the `roots()` function MATLAB, we get that the roots are:

$$\texttt{roots([1,3,50,1,10\^{}6])} = \begin{pmatrix} -22.8487 + 22.6376i \\ -22.8487 - 22.6376i \\ 21.3487 + 22.6023i \\ 21.3487 - 22.6023i \end{pmatrix}$$

There are complex conjugate poles in the right-hand plane (RHP).

The system is **unstable**.

$\longrightarrow$ $\mathcal{A}$nswer

2-39. The loop transfer function of a single-loop feedback control system is given as

$$\mathbf{G(s)H(s) = \frac{K(s + 5)}{s(s + 2)(1 + Ts)}}$$

The parameters K and T may be represented in a plane with K as the horizontal axis and T as the vertical axis. Determine the regions in the T-versus-K parameter plane where the closed-loop system is asymptotically stable and where it is unstable. Indicate the boundary on which the system is marginally stable.

The closed loop transfer function of the above loop transfer function is:

$$\frac{\mathbf{G(s)H(s)}}{\mathbf{1+G(s)H(s)}} = \frac{\frac{\mathbf{K(s+5)}}{\mathbf{s(s+2)(1+Ts)}}}{\mathbf{1}+\frac{\mathbf{K(s+5)}}{\mathbf{s(s+2)(1+Ts)}}} = \frac{K(s+5)}{s(s+2)(1+Ts)+K(s+5)}$$

This makes the characteristic equation:

$$s(s+2)(1+Ts)+K(s+5)=0$$

Which can be written as:

$$Ts^3+(1+2T)s^2+(2+K)s+5K=0$$

$\mathbf{s^3}$	T	(2 + K)
$\mathbf{s^2}$	(1 + 2T)	5K
$\mathbf{s^1}$	$\dfrac{(2+K)(1+2T)-5TK}{(1+2T)}$	0
$\mathbf{s^0}$	5K	0

The third left-most row is calculated as:

$$-\frac{\begin{vmatrix} T & (2+K) \\ (1+2T) & 5K \end{vmatrix}}{(1+2T)} = \frac{(2+K)(1+2T)-5TK}{(1+2T)}$$

The fourth left-most row is simply equal to the coefficient of s^0.

Given this Routh array, the following must be true so that no signs change occours in the left-most column.

$$T>0$$

$$(T+\frac{1}{2})>0$$

$$(2+K)(1+2T)-5TK>0$$

$$K>0$$

Taking $(2+K)(1+2T)-5TK>0$, we can write this as:

$$2 + K + 4T + 2TK - 5TK > 0$$

Which simplifies to:

$$2 + K + 4T + -3TK > 0$$

Solving for K:

$$2 + 4T + K(1 - 3T) > 0$$

$$2 + 4T + > K(3T - 1)$$

$$\frac{(2 + 4T)}{(3T - 1)} > K$$

The condition of stability exists when $T > 0$, $K > 0$ and $K < \frac{(2+4T)}{(3T-1)}$
The boundaries of marginal stability are along $T = 0$, $K = 0$ and $K = \frac{(2+4T)}{(3T-1)}$

Problem 2

Consider the following transfer function and do the following

$$\mathbf{G}(\mathbf{s}) = \frac{\mathbf{Y}(\mathbf{s})}{\mathbf{R}(\mathbf{s})} = \frac{\mathbf{3s + 2}}{\mathbf{2s^3 + 4s^2 + 5s + 1}}$$

(a) Employ the Routh-Hurwitz criterion to determine if this system stable or unstable?

I wrote the following MATLAB function to solve for a Routh-Array given the coefficients of a characteristic equation:

```
function table = routhHurwitz(CE)

    arguments
        CE (1,:) double % Characteristic Equation
    end

    % Get number of rows and columns
```

```
    num_row = size(CE,2);
    num_col = ceil(num_row/2); % Rounds up

    % Initialize table as all zeros:
    table = zeros(num_row,num_col);

    % Assign the first two rows:
    row = 1;
    col = 1;
    for k = 1:num_row
        % Assign coeffcient to table
        table(row,col) = CE(k);
        % Assign rows and  for next iteration
        if row == 1
            row = 2;
        elseif row == 2
            row = 1;
            col = col+1;
        end
    end

    % Compute rest ob table
    for iterRow = 3:num_row-1
        for iterCol = 1:num_col-1
            % Intialize matrix for determinant
            matrix = zeros(2,2);
            % Populate matrix for determinant
            matrix(1,1) = table(iterRow-2,1);
            matrix(2,1) = table(iterRow-1,1);
            matrix(1,2) = table(iterRow-2,iterCol+1);
            matrix(2,2) = table(iterRow-1,iterCol+1);
            % Compute table value
            table(iterRow,iterCol) = ...
                -det(matrix)/table(iterRow-1,1);
        end
    end
    % Last table value is equal to coefficient of
    % s^0
    table(end,1) = CE(1,end);

end
```

If I run it using the command `routhHurwitz([2,4,5,1])`, I get the following result:

$\mathbf{s^3}$	2	5
$\mathbf{s^2}$	4	1
$\mathbf{s^1}$	4.5	0
$\mathbf{s^0}$	1	0

Since there are no sign changes in the left-most column, the system is **stable**.

$\longrightarrow$ $\mathcal{A}$nswer

If I evaluate the command `roots([2,4,5,1])` in MATLAB, I get the following:

$$\texttt{roots([2,4,5,1])} = \begin{pmatrix} -0.8796 + 1.1414i \\ -0.8796 - 1.1414i \\ -0.2408 \end{pmatrix}$$

All poles exist in the left-hand plane (LHP).

The system is **stable**.

$\longrightarrow$ $\mathcal{A}$nswer

(b) Define the numerator and denominator in MATLAB, and use the STEP command to plot the system unit step response

I evaluated the following in MATLAB:

```
% Problem 2 (b)

num = [3,2];
den = [2,4,5,1];

sys = tf(num,den);
step(sys)
```

This produced the following plot:

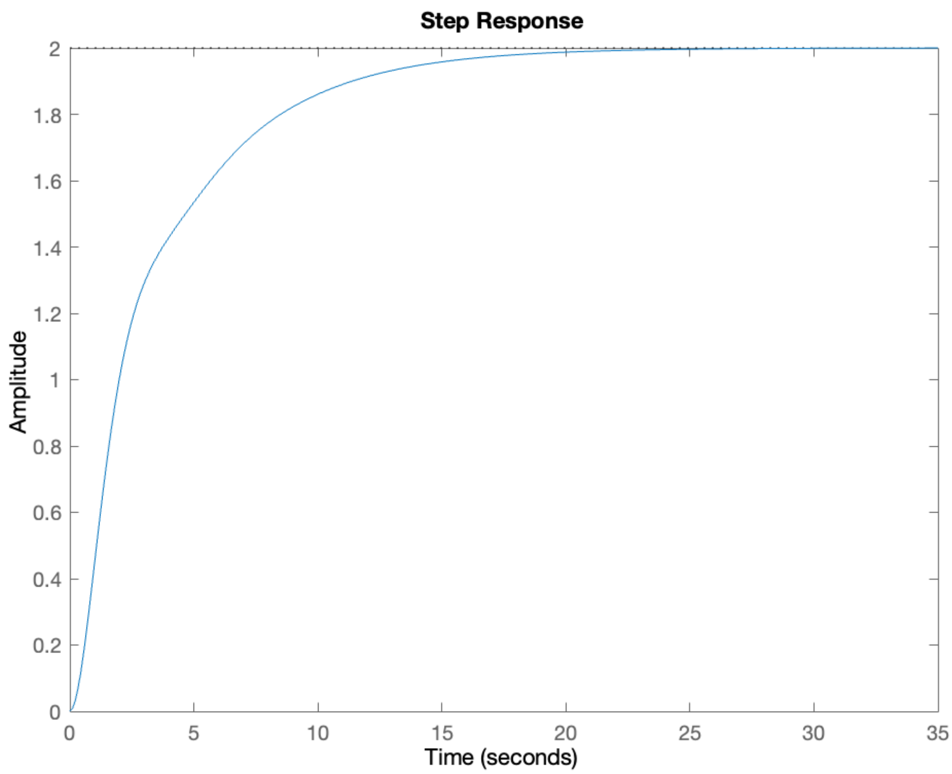

Figure 1: Step Response

(c) Use the final value theorem to determine the steady state value of the system - does this agree with the step response?

From the final value theorem we know that:

$$\lim_{t\to+\infty} f(t) = \lim_{s\to 0} sF(s)$$

Therefore:

$$\lim_{t\to+\infty} y(t) = \lim_{s\to 0} sY(s)$$

For the open loop case, given $r(t) = u_s(t)Y$:

$$R(s) = \frac{1}{s}$$

If we substitute this into the closed loop transfer function, we get:

$$sY(s) = \frac{3s+2}{2s^3 + 4s^2 + 5s + 1}$$

Therefore:

$$\boldsymbol{\lim_{t \to +\infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \frac{3s+2}{2s^3 + 4s^2 + 5s + 1} = \frac{2}{1} = 2}$$

$\longrightarrow$ Answer

The steady state value of the system is **2**, and this **agrees** with the plot that I produced in part (c).

$\longrightarrow$ Answer

Submitted by Austin Barrilleaux on September 29, 2023.