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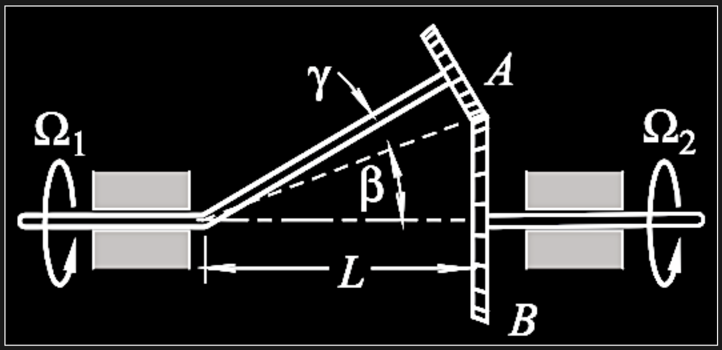
MODULE 4 — Assignment

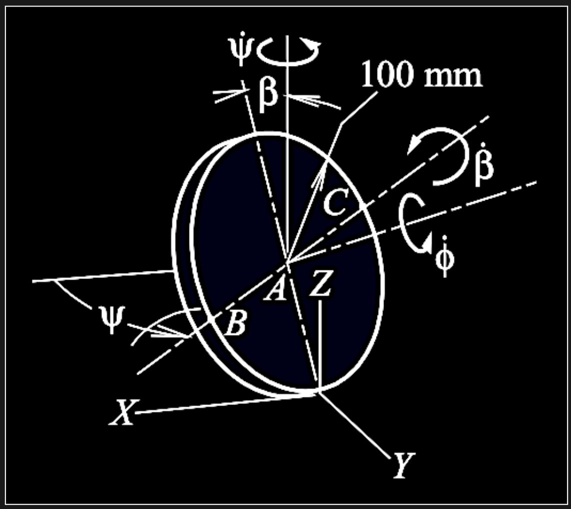
EXERCISE 4.39

Gear A spins relative to its shaft, which rotates at variable rate Ω_1 about the horizontal axis. Gear B rotates at the variable rate Ω_2. Determine the angular velocity and angular acceleration of gear A.

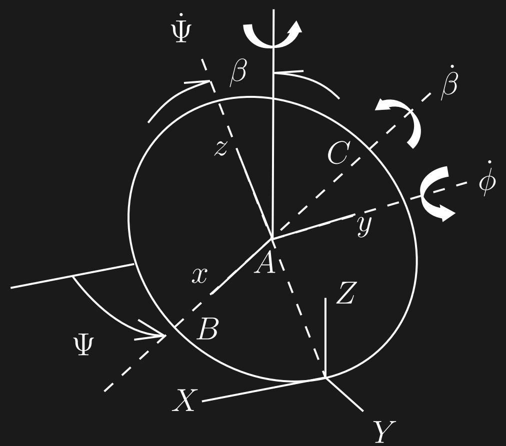


EXERCISE 4.44

The disk rolls without slipping over the horizontal XY plane. At the instant when $\beta = 36.87°$, the X and Y components of the velocity of point B on the horizontal diameter of the disk are 8 m/s and -4 m/s, respectively, and the corresponding velocity components of center A at this instant are 4 m/s and 2 m/s. Determine the precession angle Ψ between the horizontal diameter BAC and the X axis, and also evaluate the precession, nutation, and spin rates.



In the following sketch, we will define two coordinate frames, $\{XYZ\}$ and $\{xyz\}$:



The transformation to convert $\{XYZ\}$ to $\{xyz\}$ is:

$$R = \begin{bmatrix} \cos(\Psi) & -\sin(\Psi) & 0 \\ \sin(\Psi) & \cos(\Psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{bmatrix}$$
$$= \begin{bmatrix} \cos(\Psi) & -\sin(\Psi)\cos(\beta) & \sin(\Psi)\sin(\beta) \\ \sin(\Psi) & \cos(\Psi)\cos(\beta) & -\cos(\Psi)\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{bmatrix}$$

From this we see that:

$$\bar{i} = \begin{bmatrix} \cos(\Psi)\,\bar{I} \\ \sin(\Psi)\,\bar{J} \end{bmatrix}$$
$$\bar{j} = \begin{bmatrix} -\sin(\Psi)\cos(\beta)\,\bar{I} \\ \cos(\Psi)\cos(\beta)\,\bar{J} \\ \sin(\beta)\,\bar{K} \end{bmatrix}$$
$$\bar{k} = \begin{bmatrix} \sin(\Psi)\sin(\beta)\,\bar{I} \\ -\cos(\Psi)\sin(\beta)\,\bar{J} \\ \cos(\beta)\,\bar{K} \end{bmatrix}$$

Submitted by Austin Barrilleaux on September 22, 2024.