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**MODULE 13 — Assignment**

## Problem 1

**Compute the angular velocity for the rotation parameterized with the $\boldsymbol{ZXZ}$ Euler angles. Compute both the body and the spatial angular velocity. Note that the rotation matrix is**

$$\boldsymbol{R_{ZXZ}(\psi,\theta,\phi) = R_3(\psi)R_1(\theta)R_3(\phi)}$$

$$\begin{aligned} R_{ZXZ} &= R_Z(\psi)R_X(\theta)R_Z(\phi) \\ &= \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos(\phi)\cos(\psi) - \cos(\theta)\sin(\phi)\sin(\psi) & -\cos(\psi)\sin(\phi) - \cos(\phi)\cos(\theta)\sin(\psi) & \sin(\psi)\sin(\theta) \\ \cos(\phi)\sin(\psi) + \cos(\psi)\cos(\theta)\sin(\phi) & \cos(\phi)\cos(\psi)\cos(\theta) - \sin(\phi)\sin(\psi) & -\cos(\psi)\sin(\theta) \\ \sin(\phi)\sin(\theta) & \cos(\phi)\sin(\theta) & \cos(\theta) \end{bmatrix} \end{aligned}$$

We can compute the spatial angular velocity by:

$$\boldsymbol{\omega_s} = \textbf{vect}\left(\boldsymbol{\dot{R}R^T}\right) = \left\langle \begin{matrix} \cos(\psi)\ \dot{\theta} + \sin(\psi)\ \sin(\theta)\ \dot{\phi} \\ \sin(\psi)\ \dot{\theta} - \cos(\psi)\ \sin(\theta)\ \dot{\phi} \\ \cos(\theta)\ \dot{\phi} + \dot{\psi} \end{matrix} \right\rangle$$

$\longrightarrow$ *Answer*

We can compute the body angular velocity by:

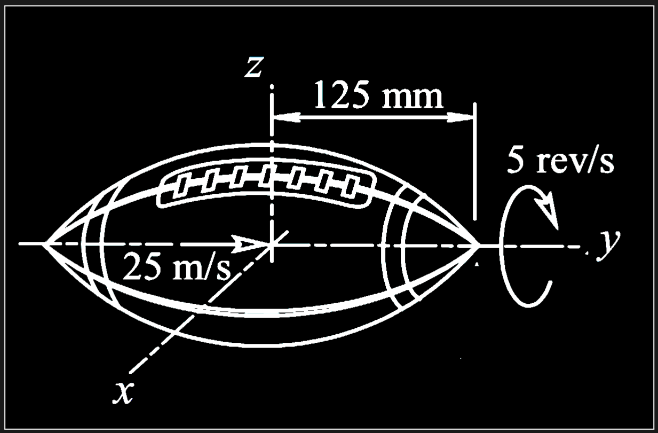
$$\boldsymbol{\omega_b} = \textbf{vect}\left(\boldsymbol{R^T\dot{R}}\right) = \left\langle \begin{matrix} \cos(\phi)\ \dot{\theta} + \sin(\phi)\ \sin(\theta)\ \dot{\psi} \\ \cos(\phi)\ \sin(\theta)\ \dot{\psi} - \sin(\phi)\ \dot{\theta} \\ \cos(\theta)\ \dot{\psi} + \dot{\phi} \end{matrix} \right\rangle$$

$\longrightarrow$ *Answer*

The following MATLAB script was used to solve this problem:

```
syms t psi(t) theta(t) phi(t)

R = simplify(zRot(psi)*xRot(theta)*zRot(phi)) %[output:5d5dec6e]
R_dot = simplify(diff(R),1000) %[output:7e3fbfdb]
Omega_b = simplify(transpose(R)*R_dot,1000);
Omega_b = Omega_b(t) %[output:08922e58]
omega_b = vect(Omega_b) %[output:02d3fe7a]

Omega_s = simplify(R_dot*transpose(R),1000);
Omega_s = Omega_s(t) %[output:05052463]
omega_s = vect(Omega_s) %[output:4731dd26]



function R = xRot(ang)
R = [ 1 0 0;
0 cos(ang) -sin(ang);
0 sin(ang) cos(ang)];
end

function R = yRot(ang)
R = [ cos(ang) 0 sin(ang);
0 1 0;
-sin(ang) 0 cos(ang)];
end

function R = zRot(ang)
R = [ cos(ang) -sin(ang) 0;
sin(ang) cos(ang) 0;
0 0 1];
end

function omega = vect(Omega)
omega = [Omega(3,2);Omega(1,3);Omega(2,1)];
end
```

## Problem 2




**A football of mass m is flying at a velocity of 25 m/s along the y-axis of the body fixed frame. The radii of gyration about the axes of the body-fixed frame are 40 mm and 70 mm along the $x$-/$z$-and the $y$-axis, respectively.**

According to the appendix of the textbook, the centriodal inertia properties of the football are:

$$I = \begin{bmatrix} \frac{1}{5}m(r_y^2 + r_z^2) & 0 & 0 \\ 0 & \frac{1}{5}m(r_x^2 + r_z^2) & 0 \\ 0 & 0 & \frac{1}{5}m(r_x^2 + r_y^2) \end{bmatrix}$$
$$= \begin{bmatrix} 0.0041\,m & 0 & 0 \\ 0 & 0.0013\,m & 0 \\ 0 & 0 & 0.0034\,m \end{bmatrix}$$

**(a) Compute the kinetic energy of the ball. Use the $ZYZ$ Euler angles to represent the orientation of the ball.**

**(b) If the ball is spinning about the y-axis of the body-fixed frame at 5 rev/s, as shown in the figure, what is the kinetic energy?**

*Submitted by Austin Barrilleaux on November 21, 2024.*