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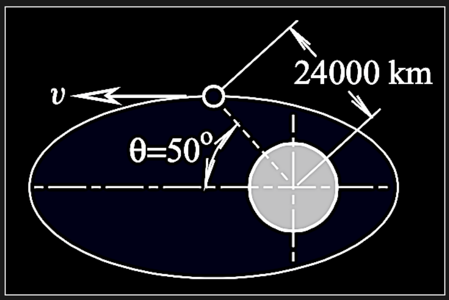
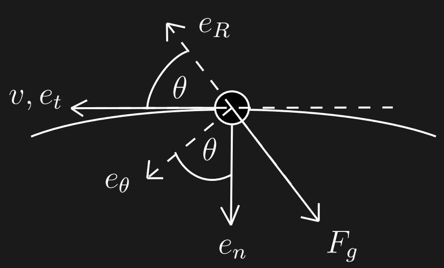
MODULE 8 — Midterm Exam

Problem 1

A satellite is in an orbit about the Earth. The magnitude of the acceleration of this body is $g(R_e/R)^2$, where R is the distance from the body to the center of the Earth, $R_e = 6370$ km is the radius of the Earth, and $g = 9.807$ m/s^2. At the position shown, the speed of the body is $v = 27\ 000$km/h.
(a) Determine the rate of change of the speed and the radius of curvature of the orbit at this position.
(b) Determine $\dot{R}$, $\ddot{R}$, $\dot{\theta}$, and $\ddot{\theta}$ at this position.



Sketching the following block diagram where, by inspection, the unit tangent vector e_t is parallel with the velocity vector, and the normal direction unit vector e_n extends toward the center of curvature (which is toward the center of the elliptical orbit.)



From this, we can define the unit tangent vector as:

$$\bar{e}_t = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \bar{e}_R \\ \sin(\theta) & \bar{e}_\theta \end{bmatrix}$$

Because $\dot{v}$ is the tangential component of acceleration, we find that:

$$\dot{v} = \bar{a} \cdot \bar{e}_t = g\left(\frac{R_e}{R}\right)^2 \begin{bmatrix} -1 & \bar{e}_R \\ 0 & \bar{e}_\theta \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta) & \bar{e}_R \\ \sin(\theta) & \bar{e}_\theta \end{bmatrix} = -9.807\left(\frac{6370 \times 10^3}{24000 \times 10^3}\right)^2 \cos(\theta)$$

This evaluates to:

$$\boldsymbol{\dot{v} = -0.4441 \ \mathrm{m/s^2}}$$

$\longrightarrow$ *Answer*

Looking at the equation for acceleration:

$$\bar{a} = \dot{v}\bar{e}_t + \frac{v^2}{\rho}\bar{e}_n$$

To solve for the radius of curvature:

$$\frac{v^2}{\rho}\bar{e}_n = \bar{a} - \dot{v}\bar{e}_t \ \rightarrow \ \rho = \frac{v^2}{|\bar{a} - \dot{v}\bar{e}_t|}$$

The radius of curvature is:

$$\rho = \frac{v^2}{|\bar{a} - \dot{v}\bar{e}_t|} = \frac{\left(27\,000 \times 10^3/3600\right)^2}{\left|-9.807\left(\frac{6370\times10^3}{24000\times10^3}\right)^2 \begin{bmatrix} -1 & \bar{e}_R \\ 0 & \bar{e}_\theta \end{bmatrix} - (-0.4441) \begin{bmatrix} \cos(\theta) & \bar{e}_R \\ \sin(\theta) & \bar{e}_\theta \end{bmatrix}\right|}$$

This evaluates to:

$$\boldsymbol{\rho = 1.0629 \times 10^8 \ \mathrm{m}}$$

$\longrightarrow$ *Answer*

Using the following velocity equation:

$$\bar{v} = \dot{R}\ \bar{e}_R + R\dot{\theta}\ \bar{e}_\theta$$

Looking at the sketch:

$$\bar{v} = \left(\frac{27\ 000 \times 10^3}{3600}\right)\begin{bmatrix} \cos(\theta)\ \ \bar{e}_R \\ \sin(\theta)\ \ \bar{e}_\theta \end{bmatrix}$$

From these two equations we get that:

$$\boldsymbol{\dot{R} = \left(\frac{27\ 000 \times 10^3}{3600}\right)\cos(\theta) = 4820.9\ \mathrm{m/s}}$$
$$\boldsymbol{\dot{\theta} = \left(\frac{27\ 000 \times 10^3}{3600}\right)\sin(\theta)/R = 0.0002393\ \mathrm{rad/s}}$$

$\longrightarrow$ $\mathcal{A}$nswer

Using the following acceleration equation:

$$\bar{a} = \left(\ddot{R} - R\dot{\theta}^2\right)\bar{e}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\bar{e}_\theta$$

Where:

$$\bar{a} = -9.807\left(\frac{6370 \times 10^3}{24000 \times 10^3}\right)^2\begin{bmatrix} -1\ \bar{e}_R \\ 0\ \bar{e}_\theta \end{bmatrix}$$

We get that:

$$\boldsymbol{\ddot{R} = \bar{a}\cdot\bar{e}_R + R\dot{\theta}^2 = 0.6845\ \mathrm{m/s^2}}$$
$$\boldsymbol{\ddot{\theta} = \left(\cancelto{0}{\bar{a}\cdot\bar{e}_\theta} - 2\dot{R}\dot{\theta}\right)/R = -9.617 \times 10^8\ \mathrm{rad/s^2}}$$

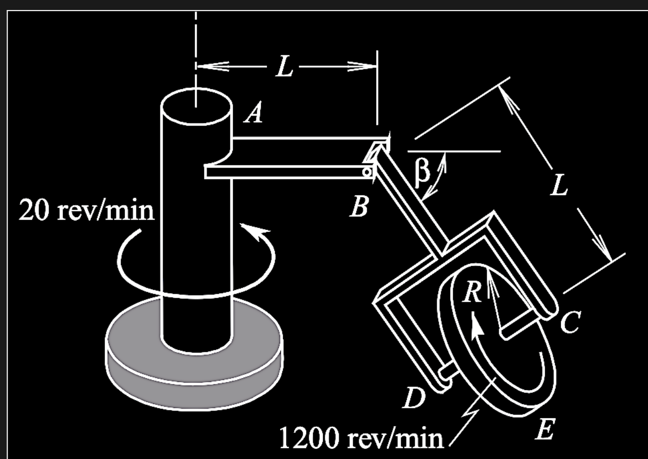
$\longrightarrow$ $\mathcal{A}$nswer

The following MATLAB script was used to solve this problem:

```matlab
%% EXERCISE 2.43
clc,clear
syms t
syms theta(t)
assume(t,{'real','positive'})

e_t = zRot(theta)*[1;0;0];
e_n = zRot(theta)*[0;0;0];

R = 24000E3;
Re = 6370E3;
g = 9.807;
a_bar = -g*(Re/R)^2*[1;0;0];

v_dot = subs(transpose(a_bar)*e_t,theta(t),50*pi/180); % [ans]
v_bar = double((27000E3/3600)*subs(e_t,theta(t),50*pi/180));
rho = ((27000E3/3600)^2)/... [ans] (2.1.10)
subs(norm(a_bar - v_dot*e_t),theta(t),50*pi/180); % m

R_dot = v_bar(1); % m/s^2 (2.3.11)
theta_dot = v_bar(2)/R; % rad/s^2 (2.3.11)

R_ddot = a_bar(1)+ R*theta_dot^2; % m/s^2 (2.3.13)
theta_ddot = (a_bar(2) - 2*R_dot*theta_dot)/R; % rad/s^2 (2.3.13)

function R = zRot(ang)
    R = [ cos(ang) -sin(ang) 0;
          sin(ang) cos(ang) 0;
                 0        0 1];
end
```

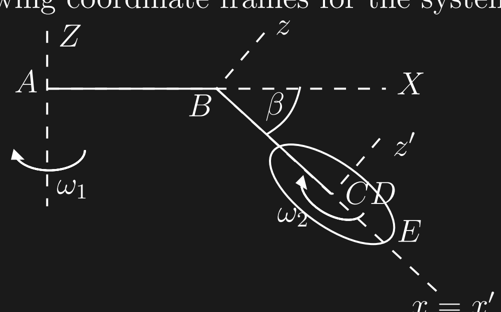
Problem 2

The disk spins about its axis C D at 1200 rev/min as the system rotates about the vertical axis at 20 rev/min. Both rates are constant. The angle of elevation of the arm supporting the disk is such that $\dot{\beta} = 10$ rad/s and $\ddot{\beta} = -500$ rad/s^2 when $\beta = 36.87°$. Determine the velocity and acceleration of point E, which is the lowest point on the perimeter of the disk. Note: the solution for velocity in the text is for a clockwise rotation about the vertical axis



We will solve this problem consistent with the note in the prompt.

We will define the following coordinate frames for the system:



You will note that the {xyz} frame and {x'y'z'} frame are identical for this problem, so I will use {xyz} to express the coordinate frame at both points from this point. Constructing the angular velocity vector $\bar{\omega}$ of $\{xyz\}$ by vectorially adding the simple rotation rates according to:

$$\bar{\omega} = \omega_1 \bar{e}_1 + \omega_2 \bar{e}_2 + \omega_3 \bar{e}_3$$

This gives us:

$$\bar{\omega} = -\omega_1 K + \dot{\beta} j - \omega_2 k$$

Using the following coordinate transformation:

$$R_{\text{rot}} = \begin{bmatrix} \cos(\beta) & 0 & -\sin(\beta) \\ 0 & 1 & 0 \\ \sin(\beta) & 0 & \cos(\beta) \end{bmatrix}$$

$$K = R_{\text{rot}} \begin{bmatrix} 0 & I \\ 0 & J \\ 1 & K \end{bmatrix} = \begin{bmatrix} -\sin(\beta) & i \\ 0 & j \\ \cos(\beta) & k \end{bmatrix} = -\sin(\beta)\, i + \cos(\beta)\, k$$

This gives us the angular velocity vector in the form:

$$\bar{\omega} = \begin{bmatrix} \omega_1 \sin(\beta) & i \\ \dot{\beta} & j \\ -\omega_2 - \omega_1 \cos(\beta) & k \end{bmatrix}$$

For the angular rotations, the angular velocity is:

$$\Omega_1 = -\omega_1 K = \begin{bmatrix} \omega_1 \sin(\beta) & i \\ 0 & j \\ -\omega_1 \cos(\beta) & k \end{bmatrix}$$

$$\Omega_2 = -\omega_1 K + \dot{\beta} j = \begin{bmatrix} \omega_1 \sin(\beta) & i \\ \dot{\beta} & j \\ -\omega_1 \cos(\beta) & k \end{bmatrix}$$

$$\Omega_3 = \bar{\omega}$$

Using the following relative positions:

$$\bar{r}_{B/A} = L \begin{pmatrix} \cos(\beta) \ i \\ 0 \ j \\ \sin(\beta) \ k \end{pmatrix}$$

$$\bar{r}_{CD/B} = L \ i$$

$$\bar{r}_{E/CD} = R \ i$$

Referencing the velocity equation:

$$\bar{v}_P = \bar{v}_O + \bar{\omega} \times \bar{r}_{P/O}$$

We solve for the velocities along the system from A to E:

$$\bar{v}_B = \Omega_1 \times \bar{r}_{B/A} = \begin{bmatrix} 0 \\ -2.0944\,L \\ 0 \end{bmatrix}$$

$$\bar{v}_{CD} = \bar{v}_B + \Omega_2 \times \bar{r}_{CD/B} = \begin{bmatrix} 0 \\ -3.7699\,L \\ -10\,L \end{bmatrix}$$

$$\bar{v}_E = \bar{v}_{CD} + \Omega_3 \times \bar{r}_{E/CD} = \begin{bmatrix} 0 \\ -3.7699\,L - 127.34\,R \\ -10\,L - 10\,R \end{bmatrix}$$

Therefore:

$$\boldsymbol{\bar{v}_E = -\left(3.7699\,L + 127.34\,R\right) j - 10\left(L + R\right) k}$$

$\longrightarrow \mathcal{A}$nswer

We can solve for the angular rotation at the rotation points using the equation for angular acceleration:

$$\bar{\alpha} = \sum_n \left(\dot{\omega}_n \bar{e}_n + \bar{\Omega}_n \times \omega_n \bar{e}_n\right)$$

The relative angular accelerations across the system for points A to E are:

$$\bar{\alpha}_1 = \Omega_1 \times \Omega_1 = 0$$
$$\bar{\alpha}_2 = \bar{\alpha}_1 + \ddot{\beta} j + \Omega_2 \times \dot{\beta} j$$
$$\bar{\alpha}_3 = \bar{\alpha}_2 + \Omega_3 \times \omega_2(-k) = \bar{\alpha}$$

Using the acceleration equation:

$$\bar{a}_P = \bar{a}_O + \bar{\alpha} \times \bar{r}_{P/O} + \bar{\omega} \times \left(\bar{\omega} \times \bar{r}_{P/O}\right)$$

Solving for the acceleration across the system for points A to E:

$$\bar{a}_B = 0 + \bar{\alpha}_1 \times \bar{r}_{B/A} + \Omega_1 \times \left(\Omega_1 \times \bar{r}_{B/A}\right) = \begin{bmatrix} -3.5092\,L \\ 0 \\ -2.6319\,L \end{bmatrix}$$

$$\bar{a}_{CD} = \bar{a}_B + \bar{\alpha}_2 \times \bar{r}_{CD/B} + \Omega_2 \times \left(\Omega_2 \times \bar{r}_{CD/B}\right) = \begin{bmatrix} -106.32\,L \\ 25.133\,L \\ 495.26\,L \end{bmatrix}$$

$$\bar{a}_E = \bar{a}_{CD} + \bar{\alpha}_3 \times \bar{r}_{E/CD} + \Omega_3 \times \left(\Omega_3 \times \bar{r}_{E/CD}\right) = \begin{bmatrix} -106.32\,L - 16315\,R \\ 25.133\,L + 25.133\,R \\ 495.26\,L + 182.07\,R \end{bmatrix}$$

Therefore:

$$\boldsymbol{a_E = -(106.32\,L + 16315\,R)\,i + 25.133(L + R)\,j + (495.26\,L + 182.07\,R)\,k}$$

$\longrightarrow$ *Answer*

If I solve this for the solution where the vertical axis rotation is counter clockwise, we get that:

$$\boldsymbol{v_E = (0.4189L - 127.3392R)\,j - 10\,(L + R)\,k}$$
$$\boldsymbol{a_E = -(106.32\,L + 16315\,R)\,i + 25.133(L + R)\,j + (495.26\,L + 182.07\,R)\,k}$$

$\longrightarrow$ *Answer*

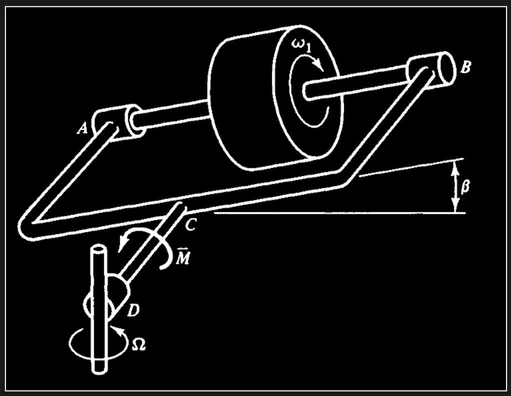
These solutions match the text.

The following MATLAB script was used to solve this problem:

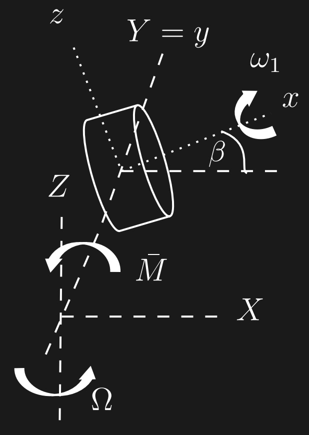
```
clear
syms L R

x = [1;0;0];
y = [0;1;0];
z = [0;0;1];

omega_1 = 20*2*pi/60; % rad/s
omega_2 = 1200*2*pi/60; % rad/s

beta = 36.87*pi/180; % rad
beta_dot = 10; % rad/s
beta_ddot = -500; % rad/s^2

R_rot = yRot(beta) ;
Z = R_rot*z;

Omega_1 = -omega_1*Z;
Omega_2 = -omega_1*Z+beta_dot*y;
Omega_3 = -omega_1*Z+beta_dot*y-omega_2*z; % omega_bar

r_B_A = L*[cos(beta);0;sin(beta)];
r_CD_B = L*x;
r_E_CD = R*x;

v_B = 0 + cross(Omega_1,r_B_A);
v_CD = v_B + cross(Omega_2,r_CD_B);
v_E = v_CD + cross(Omega_3,r_E_CD);

alpha_1 = cross(Omega_1,Omega_1);
alpha_2 = alpha_1 + beta_ddot*y + cross(Omega_2,beta_dot*y);
alpha_3 = alpha_2 + cross(Omega_3,-omega_2*z); % alpha_bar

a_B = 0 + cross(alpha_1,r_B_A) + ...
    cross(Omega_1,cross(Omega_1,r_B_A));
a_CD = a_B + cross(alpha_2,r_CD_B) + ...
    cross(Omega_2,cross(Omega_2,r_CD_B));
a_E = a_CD + cross(alpha_3,r_E_CD) + ...
    cross(Omega_3,cross(Omega_3,r_E_CD));

function R = yRot(ang)
R = [ cos(ang) 0 sin(ang);
0 1 0;
-sin(ang) 0 cos(ang)]';
end
```

Problem 3

The gyroscopic turn indicator consists of a 1-kg flywheel whose principal radii of gyration are $k = 50$ mm and $k = k = 40$ mm. The center of mass of the flywheel coincides with the intersection of axes AB and CD. The flywheel spins relative to the gimbal at the constant rate $\omega_1 = 10,000$ rev/min. A couple $\bar{M}$ acts about shaft CD, which supports the gimbal, in order to control the angle β between the gimbal and the horizontal. Determine $\bar{M}$ when the rotation rate about the vertical axis is $\Omega = 0.8$ rad/s.



We will define the following frames for the system:



For this problem the prompt seems to imply that the rotation rate about the vertical axis is constant.

Constructing the angular velocity vector $\bar{\omega}$ of $\{xyz\}$ by vectorially adding the simple rotation rates according to:

$$\bar{\omega} = \omega_1 \bar{e}_1 + \omega_2 \bar{e}_2$$

This gives us:

$$\bar{\omega} = \Omega K + \omega_1(-i)$$

Using the following coordinate transformation:

$$R_{\text{rot}} = \begin{bmatrix} \cos(-\beta) & 0 & -\sin(-\beta) \\ 0 & 1 & 0 \\ \sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix}$$

$$K = R_{\text{rot}} \begin{bmatrix} 0 & I \\ 0 & J \\ 1 & K \end{bmatrix} = \begin{bmatrix} \sin(\beta) \\ 0 \\ \cos(\beta) \end{bmatrix} = \sin(\beta)\, i + \cos(\beta)\, k$$

This gives us the angular velocity vector in the form:

$$\bar{\omega} = \begin{bmatrix} \big(\Omega \sin(\beta) - \omega_1\big) & i \\ 0 & j \\ \Omega \cos(\beta) & k \end{bmatrix} = \big(\Omega \sin(\beta) - \omega_1\big)\, i + \Omega \cos(\beta)\, k$$

Using the equation for angular acceleration:

$$\bar{\alpha} = \sum_n \left(\dot{\omega}_n \bar{e}_n + \bar{\Omega}_n \times \omega_n \bar{e}_n\right)$$

We see compute that the angular acceleration is:

$$\bar{\alpha} = \bar{\omega} \times \omega_1(-i) = -\Omega\, \omega_1 \cos(\beta)\, j$$

Since the torque $\bar{M}$ is acting in the j direction, we can refer to equation 6.1.6 in the textbook:

$$\sum \bar{M}\ j = I_{yy}\alpha_y + (I_{zz} - I_{xx})\, \omega_x \omega_z$$

Solving for the principal axis inertia quantities:

$$I_{xx} = mk_1^2 = (1 \text{ kg}) \left(\frac{50}{1000} \text{ m}\right)^2 = 0.0025 \text{ kg-m}^2$$
$$I_{yy} = I_{zz} = mk_2^2 = (1 \text{ kg}) \left(\frac{40}{1000} \text{ m}\right)^2 = 0.0016 \text{ kg-m}^2$$

We can solve for $\bar{M}$ as:

$$\begin{aligned}
\bar{M} &= 0.0016 \left(-\Omega\,\omega_1 \cos(\beta)\right) + (0.0016 - 0.0025) \left(\Omega \sin(\beta) - \omega_1\right) \left(\Omega \cos(\beta)\right) \\
&= 0.0016 \left(-0.8 \left(\frac{10000(2\pi)}{60}\right) \cos(\beta)\right) + \\
&\qquad (0.0016 - 0.0025) \left(0.8 \sin(\beta) - \left(\frac{10000(2\pi)}{60}\right)\right) \left(0.8 \cos(\beta)\right) \\
&= 0.000576 \cos(\beta) \sin(\beta) - 2.0944 \cos(\beta) \text{ N-m}
\end{aligned}$$

The couple $\boldsymbol{\bar{M}}$ when the rotation rate about the vertical axis is $\boldsymbol{\Omega = 0.8}$ rad/s is:

$$\boldsymbol{\bar{M} = 0.000576 \cos(\beta) \sin(\beta) - 2.0944 \cos(\beta)} \textbf{ N-m}$$

$\longrightarrow$ *Answer*

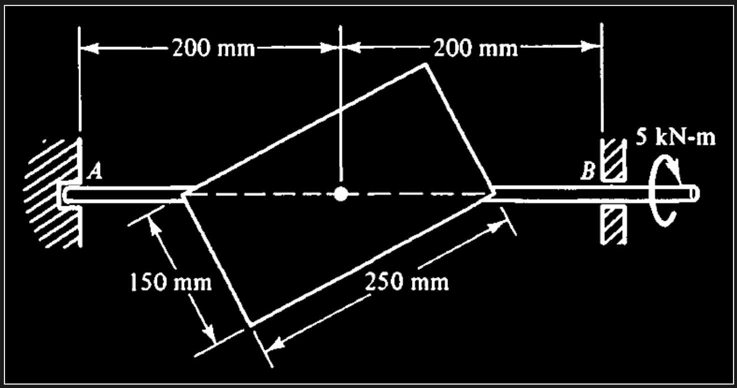
The following MATLAB script was used to solve this problem:

```
clc,clear
syms k t beta
syms omega_1 Omega M
sympref('FloatingPointOutput',true);

R = yRot(-beta);
omega_bar = Omega*R*[0;0;1] + omega_1*[-1;0;0];

alpha_bar = cross(omega_bar,omega_1*[-1;0;0]);

m = 1; % kg
k_1 = 50/1000; % m
k_2 = 40/1000; % m
I_1 = m*k_1^2;
```

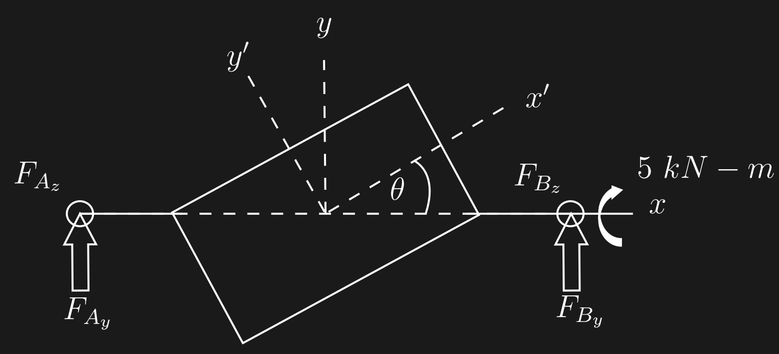
```
I_2 = m*k_2^2;

M = I_2*alpha_bar-(I_2-I_1)*omega_bar(1)*omega_bar(3);

M = subs(M,[omega_1 Omega],[10000*2*pi/60,0.8]);

M = expand(simplify(M,1000));

function R = yRot(ang)
    R = [ cos(ang) 0 -sin(ang); %% Neg body 2 in
                  0 1        0;
          sin(ang) 0  cos(ang)];
end
```

Problem 4

A 50-kg rectangular plate is mounted diagonally on a shaft whose mass is negligible. The system was initially at rest when a constant torque of 5 kN-m is applied to the shaft. Determine the reactions at bearings A and B four seconds after the application of the torque.



The following sketch details the body fixed frames $\{xyz\}$ and $\{x'y'z'\}$ and the reaction forces F_A and F_B.



The forces both consist of x and z components. The x-components are in the positive x direction which is up in the sketch, and the z-components are in the positive z direction which is out of the page in the sketch. The z-components are denoted by the circles in the sketch.

There is one rotation about the x-axis that I will denote as Ω. The angular velocity and angular rotation for the system is:

$$\bar{\omega} = \omega_1 \bar{e}_1 = -\Omega\ i$$

$$\bar{\alpha} = \dot{\omega}_1 \bar{e}_1 + \cancelto{0}{\bar{\Omega}_1 \times \omega_1 \bar{e}_1} = -\dot{\Omega}\ i$$

Using the centriodal inertia properties of a rectangular parallelepiped from the back of the textbook:

$$\begin{aligned}
I_{x'x'} &= \frac{1}{12} m \left(0.15\right)^2 = 0.09375 \text{ kg-m}^2 \\
I_{y'y'} &= \frac{1}{12} m \left(0.25\right)^2 = 0.26042 \text{ kg-m}^2 \\
I_{z'z'} &= \frac{1}{12} m \left(0.15^2 + 0.25^2\right) = 0.35417 \text{ kg-m}^2
\end{aligned}$$

This expressed as an inertia tensor is:

$$I_{\text{pa}} = \begin{bmatrix} 0.0938 & 0 & 0 \\ 0 & 0.2604 & 0 \\ 0 & 0 & 0.3542 \end{bmatrix}$$

Solving for θ, and the rotation matrix that expresses $\{x'y'z'\}$ in terms of $\{xyz\}$:

$$\theta = \tan^{-1}\left(\frac{75}{125}\right) = 0.5404 \text{ rads}$$

$$R = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.8575 & 0.5145 & 0 \\ -0.5145 & 0.8575 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

With this we can solve for the inertia tensor in the $\{xyz\}$ frame:

$$I = I_{\{xyz\}} = R^T I_{\text{pa}} R = \begin{bmatrix} 0.1379 & -0.0735 & 0 \\ -0.0735 & 0.2163 & 0 \\ 0 & 0 & 0.3542 \end{bmatrix}$$

The equation for the resultant moment is:

$$\sum \bar{M}_A = \frac{\partial \bar{H}_A}{\partial t} + \bar{\omega} \times \bar{H}_A$$

Computing $\bar{H}_A$ and $\frac{\partial \bar{H}_A}{\partial t}$ by:

$$\begin{aligned} \bar{H}_A &= \left(I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right) i + \left(I_{yy}\omega_y - I_{yx}\omega_x - I_{yz}\omega_z\right) j + \left(I_{zz}\omega_z - I_{zx}\omega_x - I_{zy}\omega_y\right) k \\ &= \begin{bmatrix} -0.1379\,\Omega \\ -0.0735\,\Omega \\ 0 \end{bmatrix} \\ \frac{\partial \bar{H}_A}{\partial t} &= \left(I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z\right) i + \left(I_{yy}\alpha_y - I_{yx}\alpha_x - I_{yz}\alpha_z\right) j + \left(I_{zz}\alpha_z - I_{zx}\alpha_x - I_{zy}\alpha_y\right) k \\ &= \begin{bmatrix} -0.1379\,\dot{\Omega} \\ -0.0735\,\dot{\Omega} \\ 0 \end{bmatrix} \end{aligned}$$

The equation for the resultant moment becomes:

$$\bar{M} = \frac{\partial \bar{H}_A}{\partial t} + \bar{\omega} \times \bar{H}_A = \begin{bmatrix} -0.1379\,\dot{\Omega} \\ -0.0735\,\dot{\Omega} \\ 0.0735\,\Omega^2 \end{bmatrix}$$

Equating this to the moments acting on the system:

$$\begin{bmatrix} 5000 \text{ N-m} \\ 0.2F_{A_z} - 0.2F_{B_z} \\ 0.2F_{B_y} - 0.2F_{A_y} \end{bmatrix} = \begin{bmatrix} -0.1379\,\dot{\Omega} \\ -0.0735\,\dot{\Omega} \\ 0.0735\,\Omega^2 \end{bmatrix}$$

Since we can see by inspection that $\sum F_y = F_{A_y} + F_{B_y} = 0$ and $\sum F_z = F_{A_z} + F_{B_z} = 0$, we can rewrite the equation as:

$$\begin{bmatrix} 5000 \\ 0.4F_{A_z} \\ -0.4F_{A_y} \end{bmatrix} = \begin{bmatrix} -0.1379\,\dot{\Omega} \\ -0.0735\,\dot{\Omega} \\ 0.0735\,\Omega^2 \end{bmatrix}$$

Given that $\Omega = \dot{\Omega}t$ we have four equations and four unknowns. Solving this system of equations, we get that:

$$\Omega = 145066.7 \text{ rad/s}$$
$$\dot{\Omega} = 36266.67 \text{rad/s}^2$$

$$\boldsymbol{F_{A_y} = -3.8684 \times 10^9} \text{ N}$$
$$\boldsymbol{F_{A_z} = -6666.7} \text{ N}$$
$$\boldsymbol{F_{B_y} = 3.8684 \times 10^9} \text{ N}$$
$$\boldsymbol{F_{B_z} = 6666.7} \text{ N}$$

$\longrightarrow$ *Answer*

The following MATLAB script was used to solve this problem:

```
clc,clear
syms omega Omega Omega_dot
% syms theta real
sympref('FloatingPointOutput',true);

theta = atan(75/125);

R = zRot(theta);

omega_bar = Omega*[-1;0;0];

alpha_bar = Omega_dot*[-1;0;0];

```

```
m = 50;

I_pa_xx = (1/12)*m*(0.15)^2;
I_pa_yy = (1/12)*m*(0.25)^2;
I_pa_zz = (1/12)*m*(0.15^2+0.25^2);

% Principal axi inertia tensor:
I_pa = diag([I_pa_xx,I_pa_yy,I_pa_zz]);

I = transpose(R)*I_pa*R;

[M_A,H_A,d_H_A] = computeResultantMoment(I,omega_bar,alpha_bar);

syms Fy Fz

soln = solve([-5000;0.4*Fz;-0.4*Fy;Omega] == [M_A;Omega_dot*4]);

function R = zRot(ang)
    R = [ cos(ang) -sin(ang) 0;
          sin(ang) cos(ang) 0;
                  0         0 1]';
end

function [M_A,H_A,d_H_A] = computeResultantMoment(I,omega,alpha)
    arguments
        I     (3,3)
        omega (3,1)
        alpha (3,1)
    end

    H_A = [ I(1,1)*omega(1) - I(1,2)*omega(2) - I(1,3)*omega(3);
            I(2,2)*omega(2) - I(2,1)*omega(1) - I(2,3)*omega(3);
            I(3,3)*omega(3) - I(3,2)*omega(1) - I(3,2)*omega(2)];

    d_H_A = [ I(1,1)*alpha(1) - I(1,2)*alpha(2) - I(1,3)*alpha(3);
              I(2,2)*alpha(2) - I(2,1)*alpha(1) - I(2,3)*alpha(3);
              I(3,3)*alpha(3) - I(3,2)*alpha(1) - I(3,2)*alpha(2)];

    M_A = d_H_A + cross(omega,H_A);

end
```

Submitted by Austin Barrilleaux on October 20, 2024.