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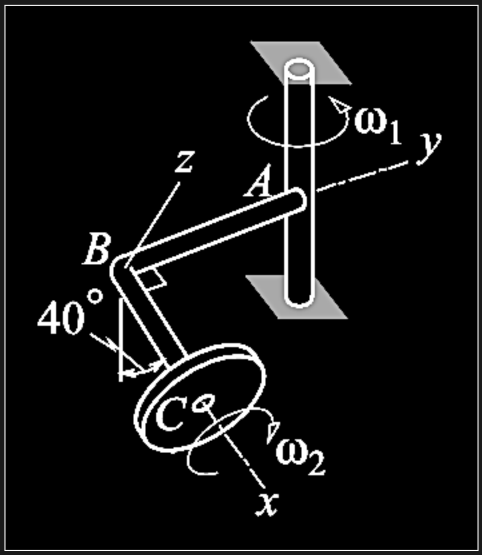
MODULE 3 — Assignment

EXERCISE3.28

The entire system rotates about the vertical axis at constant angular speed ω_1, and the rotation rate ω_2 of the rotor relative to bar BC also is constant.



(a) Describe the angular velocity of the rotor in terms of a superposition of simple rotations.

Construct the angular velocity vector $\bar{\omega}$ of xyz by vectorially adding the simple rotation rates according to:

$$\bar{\omega} = \omega_1 \bar{e}_1 + \omega_2 \bar{e}_2$$

This gives us:

$$\bar{\omega} = \omega_1 \bar{K} + \omega_2(-\bar{i})$$

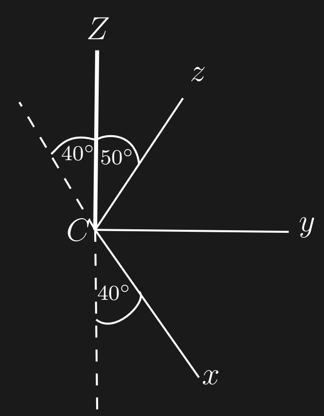
Or:

$$\boldsymbol{\bar{\omega} = \omega_1 \bar{K} + \omega_2 \bar{i}}$$

$\longrightarrow$ $\mathcal{A}$nswer

(b) Solely from an examination of the description in Part (a), predict the direction(s) in which the angular acceleration of the rotor will be situated relative to the xyz axes defined in the sketch. Briefly explain your answer.

Sketching this out the frames relative to each other:



(c) Describe the angular velocity and angular acceleration of the rotor in terms of components relative to xyz.

Because the first auxiliary reference frame is stationary, and the second one is xyz, we have:

$$\bar{\Omega}_1 = \bar{0}, \quad \bar{\Omega}_2 = \bar{\omega}$$

We find the global components of the unit vectors by inspection of the sketch, which leads to

$$\begin{aligned} \bar{K} &= \cos(40°)(-\hat{i}) + \cos(50°)\hat{k} \\ &= -\cos(40°)\hat{i} + \cos(50°)\hat{k} \\ \bar{i} &= \hat{i} \end{aligned}$$

This makes the velocity:

$$\boldsymbol{\bar{\omega} = \left(\omega_2 - \cos(40^\circ)\omega_1\right)\hat{i} + \omega_1 \cos(50^\circ)\hat{k}}$$

$\longrightarrow \mathcal{A}$nswer

Next solve for angular acceleration:

$$\bar{\alpha} = \sum_n \left(\dot{\omega}_n \bar{e}_n + \bar{\Omega}_n \times \omega_n \bar{e}_n\right)$$

Since $n = 2$:

$$\bar{\alpha} = \dot{\omega}_1 \bar{e}_1 + \bar{\Omega}_1 \times \omega_1 \bar{e}_1 + \dot{\omega}_2 \bar{e}_2 + \bar{\Omega}_2 \times \omega_2 \bar{e}_2$$

And since $\dot{\omega}_1 = \dot{\omega}_2 = 0$:

$$\bar{\alpha} = \bar{\Omega}_1 \times \omega_1 \bar{e}_1 + \bar{\Omega}_2 \times \omega_2 \bar{e}_2$$

Futher, as stated above that $\bar{\Omega}_1 = \bar{0}$ and $\bar{\Omega}_2 = \bar{\omega}$:

$$\begin{aligned}
\bar{\alpha} &= \bar{\Omega}_2 \times \omega_2 \bar{e}_2 \\
&= \left(\left(\omega_2 - \cos(40^\circ)\omega_1\right)\hat{i} + \omega_1 \cos(50^\circ)\hat{k}\right) \times \omega_2(-i) \\
&= -\omega_2 \left(\left(\omega_2 - \cos(40^\circ)\omega_1\right)\hat{i} + \omega_1 \cos(50^\circ)\hat{k}\right) \times i
\end{aligned}$$

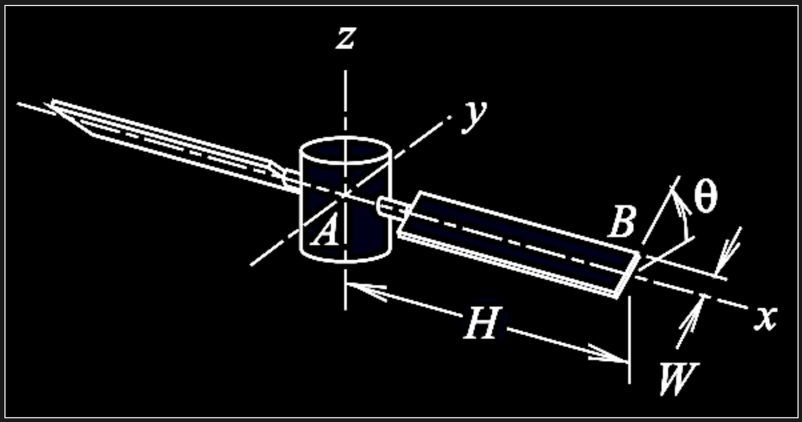
Which solves to:

$$\boldsymbol{\bar{\alpha} = -\omega_1 \omega_2 \cos(50^\circ)\hat{j}}$$

$\longrightarrow \mathcal{A}$nswer

EXERCISE 3.37

The angle $\boldsymbol{\theta}$ describing the rotation of a reconnaissance satellite's solar panels about the body-fixed $\boldsymbol{x}$ axis is an arbitrary function of time. The satellite spins about the $\boldsymbol{z}$ axis at the constant rate $\boldsymbol{\Omega}$. Derive expressions for the absolute velocity and acceleration of point $\boldsymbol{B}$ relative to the origin of $\boldsymbol{xyz}$.



Construct the angular velocity vector $\bar{\omega}$ of xyz by vectorially adding the simple rotation rates according to:

$$\bar{\omega} = \omega_1 \bar{e}_1 + \omega_2 \bar{e}_2$$

This gives us:

$$\bar{\omega} = \Omega \ \bar{k}' + \dot{\theta} \ \bar{i}'$$

Next solve for angular acceleration:

$$\bar{\alpha} = \sum_n \left(\dot{\omega}_n \bar{e}_n + \bar{\Omega}_n \times \omega_n \bar{e}_n \right)$$

Since $n = 2$:

$$\bar{\alpha} = \dot{\omega}_1 \bar{e}_1 + \bar{\Omega}_1 \times \omega_1 \bar{e}_1 + \dot{\omega}_2 \bar{e}_2 + \bar{\Omega}_2 \times \omega_2 \bar{e}_2$$

And since $\dot{\omega}_1 = \dot{\omega}_2 = 0$:

$$\bar{\alpha} = \bar{\Omega}_1 \times \omega_1 \bar{e}_1 + \bar{\Omega}_2 \times \omega_2 \bar{e}_2$$

Since, as stated above that $\bar{\Omega}_1 = \Omega \bar{k}'$ and $\bar{\Omega}_2 = \bar{\omega}$:

$$\begin{aligned} \bar{\alpha} &= \Omega \hat{k} \times \Omega \hat{k} + \left(\Omega \ \hat{k} + \dot{\theta} \ \hat{i} \right) \times \theta \hat{i} \\ &= \Omega \theta \hat{j} \end{aligned}$$

Submitted by Austin Barrilleaux on September 13, 2024.