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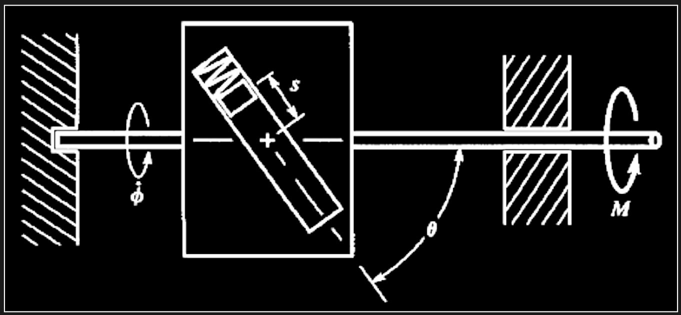
MODULE 10 — Assignment

Problem 1: Multibody System

The slider, whose mass is m_1 oscillates within the groove in the housing. The moment of inertia of the housing about the axis of rotation is I. The spring restraining the slider is unstretched when $s = 0$. Derive differential equations for the distance s and spin angle ϕ resulting from application of a torque $M(t)$ to the shaft.



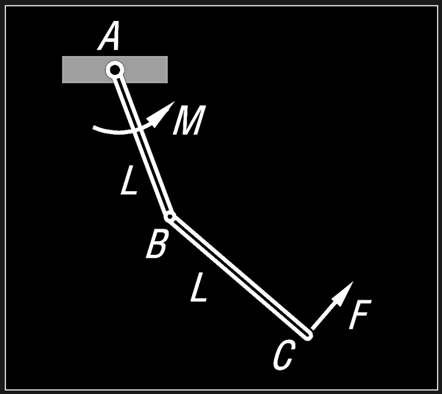
The following MATLAB function was used to solve this problem:

```
clc,clear
syms t m g theta phi(t)
syms s(t) I k
%[text] Position
p = s*[-cos(theta);
        sin(theta)*cos(phi);
        sin(theta)*sin(phi)];
%[text] Velocity
p_dot = diff(p,t) %[output:552a80dc]
%[text] Kinetic Energy
```

```
T = (1/2)*I*diff(phi,t)^2 ... %[output:group:940b71c8] %[output:04ea3874]
    + (1/2)*m*simplify(... %[output:04ea3874]
       transpose(p_dot)*... %[output:04ea3874]
       p_dot,1000) %[output:group:940b71c8] %[output:04ea3874]
%[text] Potential Energy
V = (1/2)*k*s^2 + m*g*s*sin(theta)*cos(phi) %[output:3372388c]
%[text] Lagrange Function
L = T - V;
%[text] Derivative of s and phi:
s_dot = diff(s,t);
phi_dot = diff(phi,t);
%[text]  Coordinate Lagrange Function derivatives for s:
dL_ds        = diff(L,s) %[output:3c912068]
dL_ds_dot    = diff(L,s_dot) %[output:818bf61d]
dL_ds_dot_dt = diff(dL_ds_dot,t) %[output:2a416b8d]
%[text]  Coordinate Lagrange Function derivatives for phi:
dL_dphi        = diff(L,phi) %[output:2be53048]
dL_dphi_dot    = diff(L,phi_dot) %[output:4f128612]
dL_dphi_dot_dt = diff(dL_dphi_dot,t) %[output:8741427c]
%[text] Resulting equations of motion:
EOM_s   = simplify(dL_ds_dot_dt-dL_ds) %[output:2b62797e]
EOM_phi = simplify(dL_dphi_dot_dt-dL_dphi) %[output:7796cc0c]
```

Problem 2: Nonholonomic System

A known couple $M(t)$ is applied to the upper bar. Force F, which is applied perpendicularly to the lower bar, acts to make the velocity of end C always be parallel to the line from joint A to end B. The bars have equal mass m, and the system lies in the vertical plane. Use the method of Lagrange multipliers to derive the equations of motion.



Submitted by Austin Barrilleaux on November 1, 2024.