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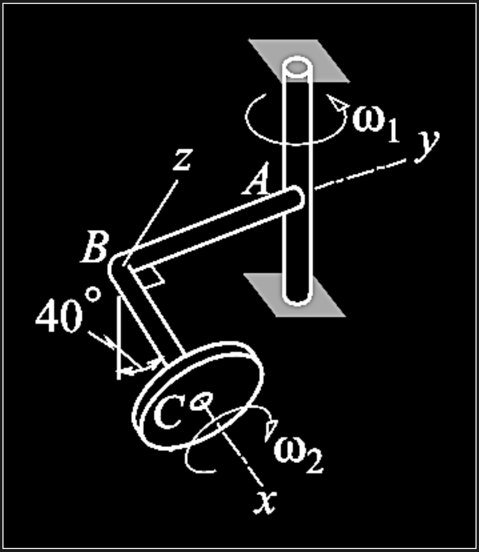
September 18, 2024

MODULE 3 — Assignment

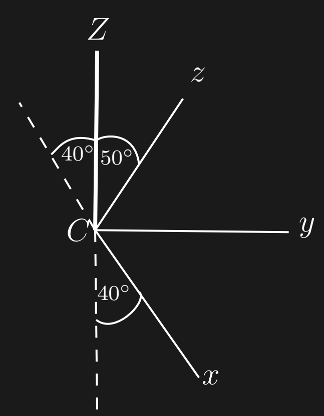
EXERCISE 3.28

The entire system rotates about the vertical axis at constant angular speed ω_1, and the rotation rate ω_2 of the rotor relative to bar BC also is constant.



(a) Describe the angular velocity of the rotor in terms of a superposition of simple rotations.

Sketching out the frames of rotation relative to each other:



Constructing the angular velocity vector $\bar{\omega}$ of xyz by vectorially adding the simple rotation rates according to:

$$\bar{\omega} = \omega_1 \bar{e}_1 + \omega_2 \bar{e}_2$$

This gives us:

$$\bar{\omega} = \omega_1 \bar{K} + \omega_2 (-\bar{i})$$

Or:

$$\boldsymbol{\bar{\omega} = \omega_1 \bar{K} + \omega_2 \bar{i}}$$

$\longrightarrow$ *Answer*

(b) Solely from an examination of the description in Part (a), predict the direction(s) in which the angular acceleration of the rotor will be situated relative to the xyz axes defined in the sketch. Briefly explain your answer.

In terms of the reference frame xyz, the angular acceleration is only affected by the constant rotations of ω_1 and ω_2. The angular acceleration component of ω_2 is in the negative in the i direction. The component of the rotation ω_1 that is perpendicular to the rotor axis is $\omega_1 \cos(50)$. The vector cross of these two vectors results in an acceleration of in the $-j$ direction. All together the acceleration is:

$$\boldsymbol{\bar{\alpha} = \omega_1 \omega_2 \cos(50°) \left(-\bar{j}\right)}$$

$\longrightarrow$ *Answer*

(c) Describe the angular velocity and angular acceleration of the rotor in terms of components relative to xyz.

Because the first auxiliary reference frame is stationary, and the second one is xyz, we have:

$$\bar{\Omega}_1 = \bar{0}, \quad \bar{\Omega}_2 = \bar{\omega}$$

We find the global components of the unit vectors by inspection of the sketch, which leads to

$$\begin{aligned}\bar{K} &= \cos(40^\circ)(-\bar{i}) + \cos(50^\circ)\bar{k} \\ &= -\cos(40^\circ)\bar{i} + \cos(50^\circ)\bar{k} \\ \bar{i} &= \bar{i}\end{aligned}$$

This makes the velocity:

$$\boldsymbol{\bar{\omega} = \left(\omega_2 - \cos(40^\circ)\omega_1\right)\bar{i} + \omega_1 \cos(50^\circ)\bar{k}}$$

$\longrightarrow$ *Answer*

Next solve for angular acceleration:

$$\bar{\alpha} = \sum_n \left(\dot{\omega}_n \bar{e}_n + \bar{\Omega}_n \times \omega_n \bar{e}_n\right)$$

Since $n = 2$:

$$\bar{\alpha} = \dot{\omega}_1 \bar{e}_1 + \bar{\Omega}_1 \times \omega_1 \bar{e}_1 + \dot{\omega}_2 \bar{e}_2 + \bar{\Omega}_2 \times \omega_2 \bar{e}_2$$

And since $\dot{\omega}_1 = \dot{\omega}_2 = 0$:

$$\bar{\alpha} = \bar{\Omega}_1 \times \omega_1 \bar{e}_1 + \bar{\Omega}_2 \times \omega_2 \bar{e}_2$$

Futher, as stated above that $\bar{\Omega}_1 = \bar{0}$ and $\bar{\Omega}_2 = \bar{\omega}$:

$$\begin{aligned}\bar{\alpha} &= \bar{\Omega}_2 \times \omega_2 \bar{e}_2 \\ &= \left(\left(\omega_2 - \cos(40^\circ)\omega_1\right)\bar{i} + \omega_1 \cos(50^\circ)\bar{k}\right) \times \omega_2(-\bar{i}) \\ &= -\omega_2 \left(\left(\omega_2 - \cos(40^\circ)\omega_1\right)\bar{i} + \omega_1 \cos(50^\circ)\bar{k}\right) \times \bar{i}\end{aligned}$$

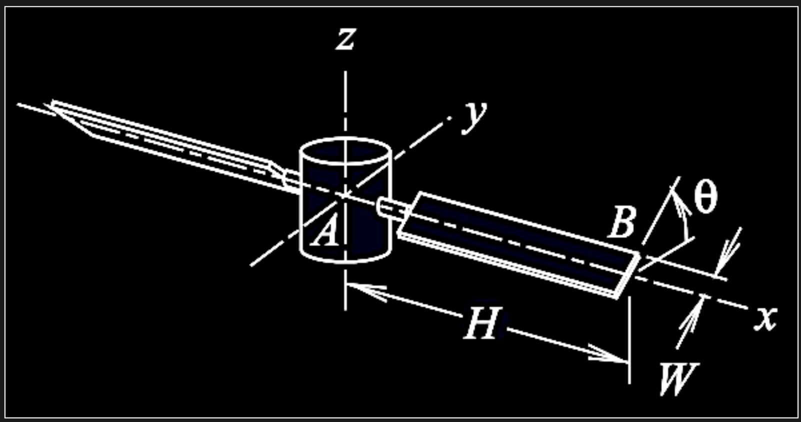
Which solves to:

$$\bar{\alpha} = -\omega_1\omega_2\cos(50°)\bar{j}$$

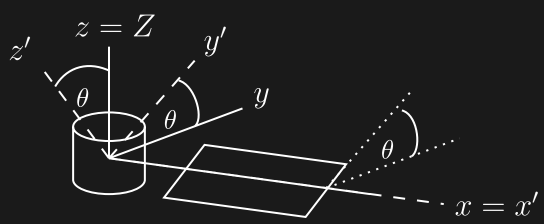
⟶ Answer

EXERCISE 3.37

The angle θ describing the rotation of a reconnaissance satellite's solar panels about the body-fixed x axis is an arbitrary function of time. The satellite spins about the z axis at the constant rate Ω. Derive expressions for the absolute velocity and acceleration of point B relative to the origin of xyz.



This problem can be defined in terms of three reference frames: the inertially fixed $\{XYZ\}$ frame, the satellite body fixed frame $\{xyz\}$, and a frame defining the rotation of the solar panel relative to the satellite body frame $\{x'y'z'\}$. These are all detailed in the following diagram:



Construct the angular velocity vector $\bar{\omega}$ of xyz by vectorially adding the simple rotation rates according to:

$$\bar{\omega} = \omega_1 \bar{e}_1 + \omega_2 \bar{e}_2$$

This gives us:

$$\bar{\omega} = \Omega \ \bar{K} + \dot{\theta} \ \bar{i}'$$

We find the global components of the unit vectors by inspection of the sketch, which leads to

$$\begin{aligned} \bar{i}' &= \bar{i} \\ \bar{K} &= \bar{k} \end{aligned}$$

This gives us:

$$\bar{\omega} = \dot{\theta} \ \bar{i} + \Omega \bar{k}$$

Next solve for angular acceleration:

$$\bar{\alpha} = \sum_n \left(\dot{\omega}_n \bar{e}_n + \bar{\Omega}_n \times \omega_n \bar{e}_n \right)$$

Since $n = 2$:

$$\bar{\alpha} = \dot{\omega}_1 \bar{e}_1 + \bar{\Omega}_1 \times \omega_1 \bar{e}_1 + \dot{\omega}_2 \bar{e}_2 + \bar{\Omega}_2 \times \omega_2 \bar{e}_2$$

And since $\dot{\omega}_1 = 0$:

$$\bar{\alpha} = \bar{\Omega}_1 \times \omega_1 \bar{e}_1 + \dot{\omega}_2 \bar{e}_2 + \bar{\Omega}_2 \times \omega_2 \bar{e}_2$$

Because the there is only a z rotation for the frame xyz, and the frame $x'y'z'$ experience the full system rotation:

$$\bar{\Omega}_1 = \Omega, \quad \bar{\Omega}_2 = \bar{\omega}$$

Therefore,

$$\begin{aligned} \bar{\alpha} &= \cancelto{0}{\Omega \bar{k} \times \Omega \bar{k}} + \dot{\theta} \bar{i} + \bar{\omega} \times \dot{\theta} \bar{i} \\ &= \ddot{\theta} \bar{i} + \Omega \, \dot{\theta} \bar{j} \end{aligned}$$

The relative position to B from A is computed as:

$$
\bar{r}_{B/A} = \begin{bmatrix} H\ \bar{i} \\ W\ \cos(\theta)\ \bar{j} \\ W\ \sin(\theta)\ \bar{k} \end{bmatrix}
$$

Given this, we can solve for the absolute velocity of B relative to the origin as:

$$
\begin{aligned}
\bar{v}_B &= \cancelto{0}{\bar{v}_A} + \bar{\omega} \times \bar{r}_{B/A} \\
&= \begin{bmatrix} -\Omega\, W\ \cos(\theta)\, \bar{i} \\ H\, \Omega - W\ \sin(\theta)\ \dot{\theta}\bar{j} \\ W\ \cos(\theta)\ \dot{\theta}\bar{k} \end{bmatrix}
\end{aligned}
$$

So,

$$
\boldsymbol{\bar{v}_B} = \begin{bmatrix} \boldsymbol{-\Omega\, W\ \cos(\theta)\, \bar{i}} \\ \boldsymbol{H\, \Omega - W\ \sin(\theta)\ \dot{\theta}\bar{j}} \\ \boldsymbol{W\ \cos(\theta)\ \dot{\theta}\bar{k}} \end{bmatrix}
$$

$\longrightarrow \mathcal{A}\text{nswer}$

We can also compute the absolute acceleration of B relative to the origin as:

$$
\begin{aligned}
\bar{a}_B &= \cancelto{0}{\bar{a}_A} + \bar{\alpha} \times \bar{r}_{B/A} + \bar{\omega} \times \left(\bar{\omega} \times \bar{r}_{B/A}\right) \\
&= \begin{bmatrix} \left\{2\, \Omega\, \dot{\theta}\ \sin(\theta)\ W - H\, \Omega^2\right\} \bar{i} \\ \left\{-\left(\dot{\theta}^2 + \Omega^2\right)\ \cos(\theta)\ W - \ddot{\theta}\ \sin(\theta)\ W\right\} \bar{j} \\ \left\{\ddot{\theta}\ \cos(\theta)\ W - \dot{\theta}^2\ \sin(\theta)\ W\right\} \bar{k} \end{bmatrix}
\end{aligned}
$$

So,

$$\bar{a}_B = \begin{bmatrix} \left\{2\,\Omega\,\dot{\theta}\,\sin(\theta)\,W - H\,\Omega^2\right\}\bar{i} \\ \left\{-\left(\dot{\theta}^2 + \Omega^2\right)\cos(\theta)\,W - \ddot{\theta}\,\sin(\theta)\,W\right\}\bar{j} \\ \left\{\ddot{\theta}\,\cos(\theta)\,W - \dot{\theta}^2\,\sin(\theta)\,W\right\}\bar{k} \end{bmatrix}$$

$\longrightarrow \mathcal{A}$nswer

Submitted by Austin Barrilleaux on September 18, 2024.