(2)

HYPOTHESIS TEST FOR TWO POPULATIONS' MEANS

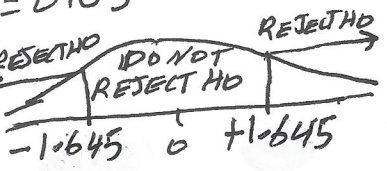
EXAMPLE 5-3 P236-237 σ_1 AND σ_2 OR σ_1^2 AND σ_2^2 ARE KNOWN

$\bar{X}_1 = 87.6$, $\bar{X}_2 = 74.5$, $\sigma_1 = 1$, $\sigma_2 = 1.5$, $n_1 = 10$, $n_2 = 12$

IF WE ARE ASKED IF THE MEANS OF TWO DIFFERENT GRADES OF ALUMINUM SPARS HAVE THE SAME TENSILE STRENGHT THIS IS A TWO SIDED TEST

(1) $H_0: \mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$ $\quad \alpha = 0.1$, $\alpha/2 = 0.05$

$H_1: \mu_1 \neq \mu_2$ or $\mu_1 - \mu_2 \neq 0$ $\quad Z_{\alpha/2} = 1.645$



(2) IF $Z_{OBT} > 1.645$ OR $Z_{OBT} < -1.645$ REJECT H_0

(3) $$Z_{OBT} = \frac{\bar{X}_1 - \bar{X}_2 - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{87.6 - 74.5 - 0}{\sqrt{\frac{1}{10} + \frac{1.5^2}{12}}} = 24.43$$

(4) AS $Z_{OBT} > Z_{\alpha/2}$ OR $24.43 > 1.645$ REJECT H_0

(5) WE ARE 90% CONFIDENT THAT THE MEAN TENSILE STRENGTH OF ALUMINUM GRADE 1 IS NOT EQUAL THE MEAN TENSILE STRENGTH OF ALL ALUMINUM GRADE 2.

USING THE CONFIDENCE INTERVAL

(2A) IF 0 IS INSIDE CI, DO NOT REJECT H_0

(3A) $$CI: \bar{X}_1 - \bar{X}_2 - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{X}_1 - \bar{X}_2 + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$CI: 87.6 - 74.5 - 1.645\sqrt{\frac{1}{10} + \frac{1.5^2}{12}} \leq \mu_1 - \mu_2 \leq 87.6 - 74.5 + 1.645\sqrt{\frac{1}{10} + \frac{1.5^2}{12}}$$

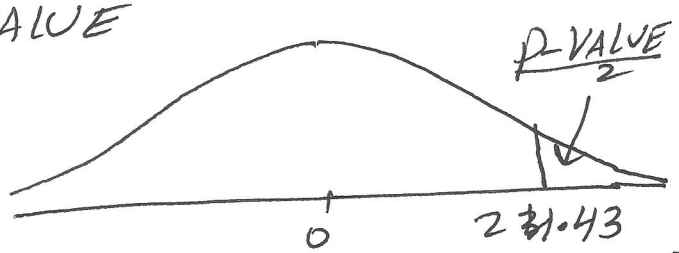
$$12.22 \leq \mu_1 - \mu_2 \leq 13.98 \; ; \; \therefore \mu_1 > \mu_2 \text{ HERE}$$

(4A) AS 0 IS OUTSIDE THE CI, REJECT H_0

USING THE P-VALUE

(2B) IF P-VALUE $< \alpha$ REJECT H_0

(3B) $$Z_{OBT} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = 24.43$$



$$\frac{\text{P-VALUE}}{2} = 1 - \Phi(24.43) = 1 - \Phi(3.99) = 1 - 0.999967 = 0.000033$$

$$\text{P-VALUE} = 2(0.000033) = 0.000066$$

(4B) AS P-VALUE $< \alpha$ OR $0.000066 < 0.1$ REJECT H_0