curve 19a1 frr analysis

The Data

You analyzed 400 consecutive values of "minimum moduli" from your elliptic curve calculations, ranging from index $n = 1$ to $n = 400$. The raw values span from 0.000802 to 3.613278—a huge range, which is why the analysis uses logarithmic scaling.

Main Finding: Three Dominant Periodicities

Your data shows **three statistically significant repeating patterns**:

Rank	Period	Power	Relative Strength
1	2.84 indices	4.09×10^4	100% (strongest)
2	3.39 indices	4.04×10^3	10%
3	17.39 indices	2.12×10^3	5%

What This Means

Period 1: The Dominant Short Cycle (2.84 indices)

This is **by far** the strongest pattern in your data. With power 4.09×10^4, it's roughly **10 times stronger** than the second pattern and **20 times stronger** than the third.

Interpretation: Your minimum moduli exhibit a very strong oscillation that repeats approximately every 2.84 steps. Since $2.84 \approx 3$, this suggests your data has a strong "every third element" pattern. The pattern isn't exactly period-3 (which would be 3.00), but close.

Amplitude: This oscillation has amplitude 0.2069 in the log-transformed detrended data. This is substantial—about 20% of the standard deviation (1.0588) of the detrended signal.

Period 2: Secondary Short Cycle (3.39 indices)

The second pattern has period $3.39 \approx 3.4$, with power only 10% of the dominant pattern.

Interpretation: This is a weaker oscillation, slightly out of phase with the first. The fact that both dominant periods are close to 3 suggests your data might have a **quasi-periodic structure around period 3**, but with some irregularity or beating pattern that creates this splitting into 2.84 and 3.39.

Amplitude: Much weaker at 0.0250, contributing only about 2.5% of signal variation.

Period 3: Longer Cycle (17.39 indices)

The third pattern has a much longer period of about 17.4 steps, with power about 5% of the strongest.

Interpretation: This represents a slower modulation—roughly every 17–18 indices, there's a larger-scale pattern. Interestingly, $17.39 \div 2.84 \approx 6.1$, so this longer cycle is roughly 6 times the shortest cycle. This could indicate a **harmonic relationship** or higher-order structure.

Amplitude: Moderate at 0.1102, about 10% of signal variation.

Statistical Significance: The Power Spectrum

The "power" values tell us how much energy is concentrated at each frequency. Think of it like volume in music:

- The ratio of powers is crucial: Period 1 dominates with 10:1 ratio over Period 2
- These three peaks stand out sharply from background noise (which would have much lower, scattered power values)
- The fact that you found only 3 significant peaks suggests your data is **not random**—it has genuine structure

What the Reconstruction Tells You

When the analysis reconstructs your signal using only these 3 frequencies, it's saying: "If I model your data as just three overlapping sine waves with periods 2.84, 3.39, and 17.39, I can capture the main oscillatory behavior."

The residual (what's left over) contains:

- Higher-frequency noise
- Non-periodic components
- The polynomial trend that was removed
- Any other weak periodicities below the 5% threshold

Mathematical Implications

For elliptic curve modular forms, periodicities in coefficient sequences can relate to:

1. **Congruence properties:** Patterns modulo small primes (period ≈ 3 might relate to properties mod 3)
2. **Hecke algebra structure:** Eigenvalue relationships creating oscillations
3. ***L*-function zeros:** Though this would typically show up at much longer periods
4. **Arithmetic progressions:** Your "minimum moduli" might be picking up multiplicative structure

The near-period-3 dominance is particularly interesting. Since you're working with level 19 curves, and the periods cluster around 3, you might investigate:

- Behavior of coefficients a_n for $n \equiv 0, 1, 2 \pmod 3$
- Whether the "minimum" is systematically achieved at different residue classes
- Cubic residues or cubic reciprocity effects
- Since $19 \equiv 1 \pmod 3$, there may be interesting interactions between the level and mod-3 structure
- The prime 3 divides $19 - 1 = 18$, which could create systematic patterns in how Frobenius elements act

Bottom Line

Your data is **highly structured**, not random. It exhibits:

- A very strong quasi-periodic pattern with effective period near 3
- A longer modulation with period near 17
- Minimal noise (only 3 peaks needed to capture main behavior)

The concentration of power in such a small number of frequencies suggests you've discovered a genuine arithmetic regularity in how the minimum moduli behave as n increases.