1. $k(\vec{x},\vec{y})=(\vec{x}\cdot\vec{y})^n$

定义 $x_{j1}x_{j2}\cdots x_{jd}$ 为 x 的一个 d 阶多项式，其中 $j1, j2,\ldots\ldots, jd\in\{1,2,\ldots\ldots,n\}$。

考虑二维空间（ $x\in R^2$ ）的模式， $\vec{x}=(x_1,x_2)$ ，其所有的二阶单项式为 $x_1^2,x_2^2,x_1x_2,x_2x_1$ 为有序单项式。

$C_2(x)=(x_1^2,x_2^2,x_1x_2,x_2x_1)$

$C_d(x)=(x_{j1}x_{j2}\cdots x_{jd}\,|\,j1, j2,\ldots\ldots, jd\in\{1,2,\ldots\ldots,n\})$

$$\begin{aligned}
(\vec{x}\cdot\vec{y})^n &= (\sum_{j=1}^{n}x_iy_i)^n=\sum_{j1=1}^{n}x_{j1}y_{j1}\ldots\ldots\sum_{j1=1}^{n}x_{jd}y_{jd}\\
&=\sum_{j1=1}^{n}\ldots\sum_{j1=1}^{n}x_{j1}\ldots x_{jd}\cdot y_{j1}\ldots y_{jd}\\
&=(x_{j1}x_{j2}\cdots x_{jd}\,|\,j1, j2,\ldots\ldots, jd\in\{1,2,\ldots\ldots,n\})\cdot(y_{j1}y_{j2}\cdots y_{jd}\,|\,j1, j2,\ldots\ldots, jd\in\{1,2,\ldots\ldots,n\})\\
&=C_d(x)\cdot C_d(y)\\
&=\gamma(\vec{x})\cdot\gamma(\vec{y})
\end{aligned}$$

VCdim = n+1

2. 证明没证出来。。

VCdim = n+1

3.
$$\begin{aligned}
k(\vec{x},\vec{y}) &= e^{-\sigma(\|\vec{x}\|^2+\|\vec{y}\|^2)}\\
&= e^{-\sigma\|\vec{x}\|^2}\cdot e^{-\sigma\|\vec{y}\|^2}\\
&=\gamma(\vec{x})\cdot\gamma(\vec{y})
\end{aligned}$$

VCdim = 无穷大