Pattern Recognition and Machine Learning: Homework 3, Zhengzuo Liu

Problem 1

Answer:

Solve:

$$P(x,y|w_1)P(w_1) = \frac{1}{3\pi}e^{-\frac{x^2+y^2}{2}}$$

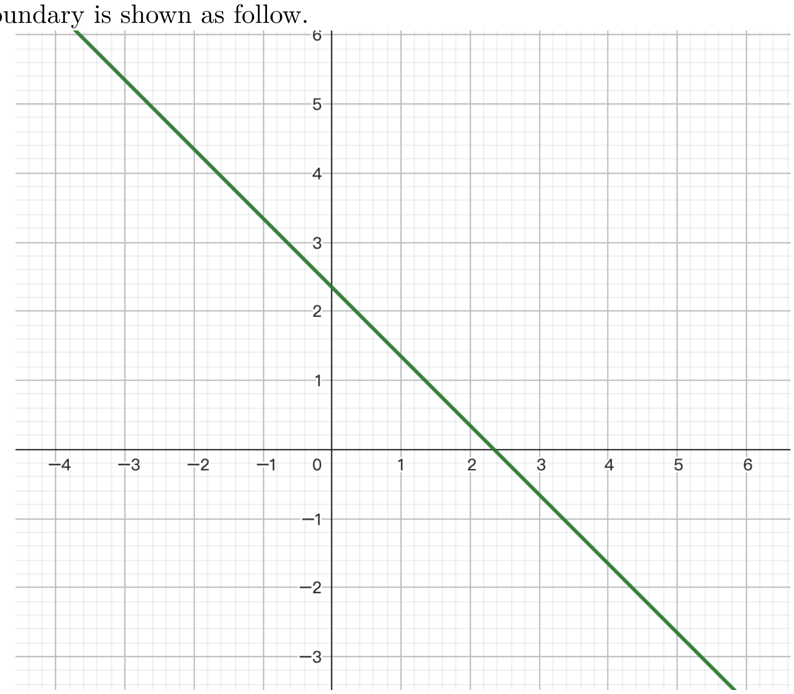
$$P(x,y|w_2)P(w_2) = \frac{1}{6\pi}e^{-\frac{(x-2)^2+(y-2)^2}{2}}$$

$$\frac{P(w_1|x,y)}{P(w_2|x,y)} = \frac{P(x,y|w_1)P(w_1)}{P(x,y|w_2)P(w_2)} = 2e^{-2x-2y+4}$$

Let $\frac{P(w_1|x,y)}{P(w_2|x,y)} > 1$, get

$$x + y < 2 + \frac{ln2}{2}ln2$$

The decision boundary is shown as follow.



Problem 2

Answer:

Prove:

For each x that is categorized into w_1, i.e. $f^*(x) = w_1$,

The single data loss function is

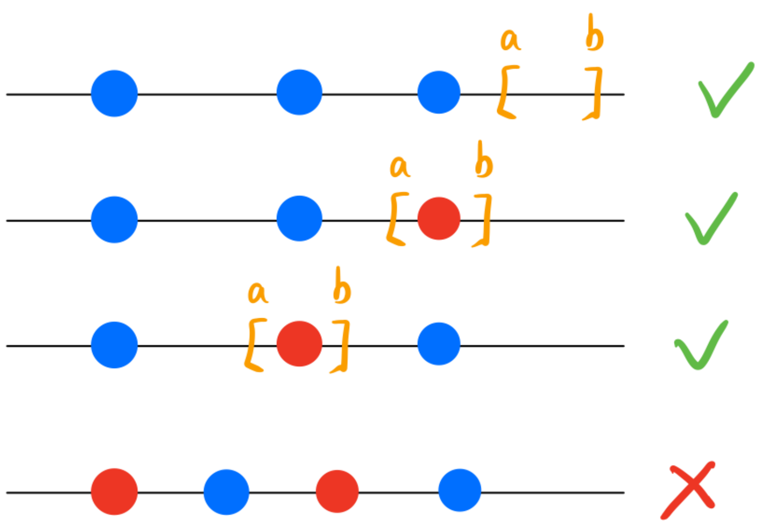
$$L^* = \lambda_1 \mathbb{I}(f(x) \neq w_1) P(w_1, x) + \lambda_2 \mathbb{I}(f(x) \neq w_2) P(w_2, x)$$

If $\frac{\lambda_1 P(w_1,x)}{\lambda_2 P(w_2,x)} \leq$, then $L*$ is minimized only when $f(x) = w_1$. If else, then $L*$ is minimized only when $f(x) = w_2$. So $f(x) = w_1$ if and only if $\frac{P(w_1,x)}{P(w_2,x)} \leq \frac{\lambda_2}{\lambda_1}$

Problem 3

Answer:

Prove:



Problem 4

Answer: