Pattern Recognition and Machine Learning: Homework 6, Zhengzuo Liu

Problem 1

Answer:

Program for this problem see "PRML_H6_1.py".

(1) The parameters obtained from training are as follows:

Initial Probabilites: [0. 1.]

Transmission Probabilites: [[0.882 0.118],[0.201 0.799]]

Emission Probabilites:[[0.16 0.168 0.174 0.183 0.18 0.134],[0.098 0.089 0.091 0.082 0.093 0.547]]

(2)

Forward Algorithm:

$$\alpha_1(1) = e_1(6) \times \pi_1 = 0.134 \times 0 = 0$$

$$\alpha_1(2) = e_2(6) \times \pi_2 = 0.547 \times 1 = 0.547$$

$$\alpha_2(1) = e_1(6) \times (\alpha_1(1) \times a_{11} + \alpha_1(2) \times a_{21}) = 0.134 \times (0 \times 0.882 + 0.547 \times 0.201) = 0.0147.$$

$$\alpha_2(2) = e_2(6) \times (\alpha_1(1) \times a_{12} + \alpha_1(2) \times a_{22}) = 0.547 \times (0 \times 0.118 + 0.547 \times 0.799) = 0.239.$$

$$\alpha_3(1) = e_1(6) \times (\alpha_2(1) \times a_{11} + \alpha_2(2) \times a_{21}) = 0.134 \times (0.0147 \times 0.882 + 0.239 \times 0.201) = 0.00817$$

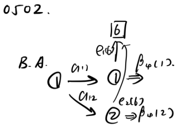
$$\alpha_3(2) = e_2(6) \times (\alpha_2(1) \times a_{12} + \alpha_2(2) \times a_{22}) = 0.547 \times (0.0147 \times 0.118 + 0.239 \times 0.799) = 0.105$$

$$\alpha_4(1) = e_1(6) \times (\alpha_3(1) \times a_{11} + \alpha_3(2) \times a_{21}) = 0.134 \times (0.00817 \times 0.882 + 0.105 \times 0.201) = 0.00379$$

$$\alpha_4(2) = e_2(6) \times (\alpha_3(1) \times a_{12} + \alpha_3(2) \times a_{22}) = 0.547 \times (0.00817 \times 0.118 + 0.105 \times 0.799) = 0.0464$$

Therefore $P(6666) = \alpha_4(1) + \alpha_4(2) = 0.0502.$

Backward Algorithm:



$$\beta_4(1) = 1.$$

$$\beta_4(2) = 1.$$

$$\beta_3(1) = a_{11}e_1(6)\beta_4(1) + a_{12}e_2(6)\beta_4(2) = 0.882 \times 0.134 \times 1 + 0.118 \times 0.547 \times 1 = 0.183.$$

$$\beta_3(2) = a_{21}e_1(6)\beta_4(1) + a_{22}e_2(6)\beta_4(2) = 0.201 \times 0.134 \times 1 + 0.799 \times 0.547 \times 1 = 0.464.$$

$$\beta_2(1) = a_{11}e_1(6)\beta_3(1) + a_{12}e_2(6)\beta_3(2) = 0.882 \times 0.134 \times 0.183 + 0.118 \times 0.547 \times 0.464 = 0.0516$$

$$\beta_2(2) = a_{21}e_1(6)\beta_3(1) + a_{22}e_2(6)\beta_3(2) = 0.201 \times 0.134 \times 0.183 + 0.799 \times 0.547 \times 0.464 = 0.208.$$

$$\beta_1(1) = a_{11}e_1(6)\beta_2(1) + a_{12}e_2(6)\beta_2(2) = 0.882 \times 0.134 \times 0.0516 + 0.118 \times 0.547 \times 0.208 = 0.0185.$$

$$\beta_1(2) = a_{21}e_1(6)\beta_2(1) + a_{22}e_2(6)\beta_2(2) = 0.201 \times 0.134 \times 0.0516 + 0.799 \times 0.547 \times 0.208 = 0.0923$$

Therefore. $p(6666) = \pi_1 e_1(6)\beta_1(1) + \pi_2 e_2(6)\beta_1(2) = 0 + 1 \times 0.547 \times 0.0923 = 0.0505$

(3)The hidden states are: [0 0 0 0 0 0 0 0 0 0 0 1 1 1 1]. So the player is cheating, and he switched his dice to the loaded one on the 12th roll.

Problem 2

Answer:

See code ”sk_all_algorithms.py”.