PHY 485 KLBF Essentials

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$

Taylor Series $\dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$

$e^x = e^a\left[1 + (x-a) + \frac{(x-a)^2}{2} + \dots\right]$ (about 0)

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

$h = 6.63 \times 10^{-34}\ m^2 kg/s$

O.D.E.

$\frac{dy}{dx} = ay \Rightarrow \ln y = ax + C$

$\frac{dy}{dx} = \frac{g(x)}{h(y)} \Rightarrow \int h(y)dy = \int g(x)dx + C$

$\frac{dy}{dx} + h(x)y = g(x)$

$\mu(x) y = \int \mu(x) g(x)\, dx + C$

where $\mu(x) = e^{\int h(x)dx}$

HH Eqn w/ outgoing wave is solved by $G(x) = C_1 \frac{e^{ikr}}{4\pi r} - C_2 \frac{e^{ikr}}{4\pi r}$

$E = h\nu \quad k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T} = 2\pi\nu$

$p = \frac{h}{\lambda} = \frac{h\nu}{c} \quad \nu\lambda = v = \lambda f$

$\gamma \approx 10^{+10}\ s^{-1} \quad \omega \approx 10^{+15}\ s^{-1}$

$e^{i\omega} = \cos(\omega) + i\sin(\omega) \quad \frac{N_2}{N_1} = e^{-h\nu/k_BT}$

$\tilde{H}(\omega) = \frac{1}{2}\delta(\omega) - \frac{1}{2\pi i\omega}$

$A_{21} = \frac{2\pi\nu^2 e^2}{\varepsilon_0 m_e c^3}\frac{g_1}{g_2} f_{12} \quad \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$

$B_{21} = \frac{e^2}{4\varepsilon_0 m_e h\nu}\frac{g_1}{g_2} f_{12} \quad \frac{B_{21}}{B_{12}} = \frac{g_1}{g_2}$

$B_{12} = \frac{e^2}{4\varepsilon_0 m_e h\nu} f_{12}$

① Absorption, Scattering, etc.

$M\frac{d^2R}{dt^2} = e(\vec{x}\cdot\nabla_{\vec{R}})\vec{E}(\vec{R},t)$

↳ Force on a dipole by external field

$\frac{d^2x}{dt^2} = \frac{e}{m}\vec{E}(\vec{R},t) + \frac{1}{m}\vec{F}_{en}$

↳ effect of external field on spring

Damping effect of decay

$\ddot{x}(t) + \gamma\dot{x}(t) + \omega^2 x(t) = \frac{e}{m}E_0(0,t)$

$\vec{x} = \vec{A}e^{-\gamma t/2}\cos(\omega_1 t + \phi)$

↳ $\omega_1^2 = \omega_0^2 - \left(\frac{\gamma}{2}\right)^2$

Energy $\mathcal{E} \simeq \frac{mA^2}{2}e^{-\gamma t} = \frac{m}{2}(\dot{x}^2 + \omega_0^2 x^2)$

Power radiated by decay

$P = \frac{1}{4\pi\varepsilon_0}\frac{2}{3}\frac{e^2}{c^2}(\ddot{x})^2$

$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2}{3}\right)\left(\frac{e^2}{c^3}\right)\left(\frac{\omega_1^2}{m}\right)E$

Response due to light (spring)

$X(t) = \Sigma \frac{e}{m}E_0 \operatorname{Re}\left\{\frac{e^{-i(\omega_L t + \phi)}}{-\omega_L^2 - i\omega_L\gamma + \omega_0^2}\right\}$

PLANCK SPECTRUM

$\rho(\nu) = \frac{8\pi h\nu^3/c^3}{\exp\left(\frac{h\nu}{k_BT}\right) - 1}$

$\beta_\nu = \frac{2\nu^3}{c}\frac{h\nu}{e^{h\nu/k_BT} - 1}$

$\frac{W}{m^2\, sr\, Hz}$

$I = \frac{1}{2}c\varepsilon_0 E_0^2$

Energy Absorbed:

$\frac{d\mathcal{E}}{dt} = \frac{(eE_0)^2}{m}\frac{\omega_L^2\frac{\gamma}{2}}{4(\omega_0 - \omega_L)^2 + (\omega_L\gamma)^2}$

$= \frac{e^2}{4mc\varepsilon_0} I \left\{\frac{\gamma/2\pi}{(\omega_0 - \omega_L)^2 + (\gamma/2)^2}\right\}$

equal to radiative broadening Lorentzian.

Normalized Spectra, $S(\nu)$

Rate of losses, decay

$\frac{dN_1}{dt} \approx -\frac{A_{21}N_1 c^2}{8\pi h\nu_0^3}\rho(\nu_0)$ ↳ planck

PLANCK RATE EQN

$\frac{dN_1}{dt} = -\frac{dN_2}{dt} = A_{21}N_2 - \frac{A_{21}c^3}{8\pi h\nu_0^3}N_1\rho(\nu_0)$

$g_1 = g_2$ (equal levels) $+ \frac{A_{21}c^3}{8\pi h\nu_0^3}N_2\rho(\nu_0)$

$= \frac{\sigma(\nu)}{h(\nu)}\left(N_2 - \frac{g_2}{g_1}N_1\right)I + A_{21}N_2$

where $\sigma(\nu) = \frac{\lambda^2}{8\pi}A_{21}S(\nu)$

& $I = c\rho(\nu_0)$ ↑ $g_1 \neq g_2$!

$\left(\frac{g_2}{g_1}N_1 \text{ for both}\right)$

BROADENING

→ Collision ↳ $\gamma \to \beta = \gamma + \nu$

→ Doppler

↳ $S(\nu) = \frac{1}{\sqrt{\pi}\,\delta\nu_D}e^{-\frac{(\nu-\nu_0)^2}{2\delta\nu_D}}$

↳ $\delta\nu_D = \sqrt{\frac{k_BT}{M}}\frac{\nu_0}{c}$

→ VOIGHT

↳ $S_{eff}(\nu) \; \frac{1}{\pi^{3/2}}\frac{b^2}{\delta\nu_D}\int_{-\infty}^{\infty}\frac{dy\, e^{-y^2}}{(y-x)^2 + b^2}$

$b = \sqrt{\frac{M}{2k_B}}\frac{c\,\delta\nu_0}{\nu_0} \quad x = \frac{\nu_0 - \nu}{\delta\nu} \quad y = \sqrt{\frac{M}{2kT}}u$

REFRACTIVE INDEX

$n(\omega) = \sqrt{1 + 2\pi\tilde{\chi}(\omega)} = n_1(\omega) + i n_2(\omega)$

Response of material

$P(t) = \varepsilon_0\int \chi(t)\vec{E}(t - \tau)\,dt$

$\chi(t < 0) = 0$ {causality}

effect on material

$\exp\left[\frac{i\omega}{c}n(\omega)\vec{u}\,r\right]$ ↙ Plane wave

↳ causes decay if $n_2 \neq 0$

Phase speed:

$v_\phi = \frac{c}{n_1(\omega)}$

② Gain: $g = \sigma(\nu)\left(N_2 - \frac{g_2}{g_1}N_1\right)$ ★

↑ cross section

For a beam in $\hat{z}$ direction.

$\left(\frac{d}{dz} + \frac{1}{c}\frac{d}{dt}\right)I = gI$

SS. → $I = e^{gz}$

★ for SS operation $r_1 r_2 e^{2gl} = 1$

$g_{th} \simeq \frac{1 - r_1 r_2}{2l}$

gain med.

2 level R.E.

$\dot{N}_2 + \alpha N_2 = \sigma\Phi N \qquad \alpha = A_{21} + 2\sigma\Phi$

From $\dot{N}_1 = A_{21}N_2 + g\Phi$

$\dot{N}_2 = -A_{21}N_2 - g\Phi$

$N_2(t) = \left[N_2(0) + \frac{\sigma\Phi N}{\alpha}\right]e^{-\alpha t} + \frac{N\sigma\Phi}{\alpha}$

$N_2(\infty) = \frac{N\sigma\Phi}{A_{21} + 2\sigma\Phi} \quad N_1^{(\infty)} = \frac{N(A_{21} + \sigma\Phi)}{A_{21} + 2\sigma\Phi}$

$A_{21} \gg \sigma\Phi$ (decay ≫ absorption) $- N_2 \ll N$

$A_{21} \ll \sigma\Phi \quad N_2 = \frac{1}{2}N$ } no gain $N_2 - N_1 < 0$

$\boxed{\text{Small signal gain} \quad g_0 = -N\sigma \quad g = \frac{g_0}{1 + \Phi/\Phi_s} \quad \Phi_{sat} = A_{21}/2\sigma \;\left(\text{when } g = \frac{-1}{2}\frac{y}{}\right)}$

Broadening during SS op: due to Φ.

$N_2(\infty) = \frac{N\sigma_0\,\beta\Phi/(\pi A_{21})}{(\nu - \nu_0)^2 + \beta^2 + \frac{2\sigma_0\beta\Phi}{\pi A_{21}}}$ ↙ term

3 level System

$\dot{N}_1 = A_{21}N_2 + \sigma(N_2 - N_1)\Phi + P(N_3 - N_1)$

$\dot{N}_2 = -A_{21}N_2 - \sigma(N_2 - N_1)\Phi + A_{32}N_3$

$\dot{N}_3 = -P(N_3 - N_1) - A_{32}N_3$

with $\mathcal{P} = \frac{A_{32}}{A_{32} + P}P \approx P$

$g_0 = N\frac{(\mathcal{P} - A_{21})}{\mathcal{P} + A_{21}}\sigma(\nu) \quad \Phi_{sat} = \left(\frac{A_{21} + \mathcal{P}}{2\sigma(\nu)}\right)$

Resonance effects.

↳ $g_0(\nu) = \frac{g_0(\nu_0)}{1 + \left(\frac{\nu - \nu_0}{\delta\nu_0}\right)^2} \quad \Phi_{sat} = \Phi_{sat}(\nu_0)\{-\}$

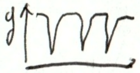
$g(\nu) = \frac{g(\nu_0)}{1 + \left(\frac{\nu - \nu_0}{\delta\nu_0}\right)^2 + \Phi/\Phi_{sat}}$

difficult to threshold if $\nu \neq \nu_0$

[high Φ widens line.

$= \ln(r_1 r_2)\, x\, \frac{1}{2L}$

Spatial Hole

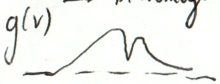


$$g = \frac{g_0(\nu)}{1+\frac{2\Phi_0}{\Phi_{sat}}\sin^2(kz)} \rightarrow I \approx 2\Phi_0 \sin^2(kz)$$

args to usual.

Spectral Hole

↳ In homogeneous broadening



some atoms saturate easily (on resonance)

Gain clamping

$$\frac{d\Phi}{dt}(\text{in cavity}) = \frac{cl}{L} g(\nu)\Phi - \frac{c}{2L}(1-r_1r_2)\Phi$$

$\sigma(\nu)(N_2-N_1)$

{gain medium} {output}

in SS: $g_{th}(\nu) = \frac{1}{2}(1-r_1r_2)$

$$\Phi = \Phi_{sat}\left(\frac{g_0(\nu)}{g_{th}} - 1\right)$$

Output Power

$r_1 \approx 1$, $t_2 \approx 1$ (small ...)

$$\Phi_{out} = t_1 \Phi(-)\, t t_2 \Phi(t)$$

$$= \frac{t}{2}\Phi_{sat}\left(\frac{g_0(\nu)}{g_{th}} - 1\right)$$

$g_{th} = \frac{t}{2l}$

$$= l\Phi_{sat}(g_0(\nu) - g_{th})$$

drops out

$$= \frac{lN(\beta - A_{21})}{2}$$

$$N_{cavity} = \frac{\Phi}{(1-r_1r_2)c/2L}$$

$$t_{opt} = \sqrt{2g_0(\nu)/S} - S$$

$$\rightarrow g_{opt} = \sqrt{\frac{g_0 S}{2l}}$$

$$r + t + S = 1$$

$$g_{th} = \frac{1}{2l}(t+s)$$

Cavities & Resonators : Fabry Perot

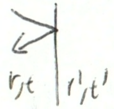
FSR: $\Delta\nu = \frac{c}{2Ln\cos\theta'}$ (FPE)

angle in FPE

$\delta = 2\pi \nu/\Delta\nu$

delay / two bounces

Stokes Relations



$$r^2 + tt' = 1$$

$$t(r + r') = 0$$

$$r_e = \frac{-r_1' + (r_1'^2 r_2' + r_2' t_1 t_2) e^{i\delta}}{1 - r_1'}$$

if $r_1' \approx r_2' \approx R$

$$|r_e|^2 = \frac{4R\sin^2(\frac{\delta}{2}) + (r_1' - r_2')^2}{(1-R)^2 + 4R\sin^2(\frac{\delta}{2})}$$

$$T_e = 1 - |r_e|^2 = 1 - R_e$$

$$= \frac{T_{max}}{1 + F\sin^2(\frac{\pi\nu}{\Delta\nu})}$$

where $F = \frac{4R}{(1-R)^2}$ $\quad T_{max} = 1 - \frac{(r_1' - r_2')}{(1 - r_1'r_2')}$

$$\text{HWHM } \delta\nu = \frac{\Delta\nu}{\pi\sqrt{F}}$$

$$\mathcal{F} = \frac{\pi\sqrt{F}}{2} = \frac{\pi\sqrt{R}}{1-R}$$

Inside the Etalon,

$$I(z) = I_0 \left\{\frac{1 - \frac{4r_2'}{(1+r_2')^2}\sin^2(\frac{\pi\nu}{\Delta\nu}\frac{z}{L})}{1 + F\sin^2(\frac{\pi\nu}{\Delta\nu})}\right\}$$

$r_2' = 1 - 3$

$$= I_0\left\{\frac{1 - (1 - \frac{3^2}{9})\sin^2(\frac{\pi\nu}{\Delta\nu}\frac{z}{L})}{1 + F\sin^2(\frac{\pi\nu}{\Delta\nu})}\right\}$$

The allowed modes: $\nu_m = \Delta\nu\, m$, $m \in \mathbb{Z}$

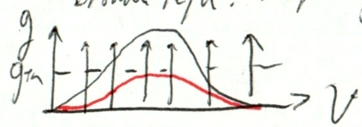
Single Mode Operation

↳ 1. limit the bandwidth of the $g(\nu)$

$\frac{c}{2L} \gg \Delta\nu_g : L \ll \frac{c}{2\Delta\nu_g}$

{ain't happening: low ...}

↳ w/ gain clamping, we can just broaden the fn: resize to g_{th}.



→ to make this more robust use another cavity tuned to only 1 mode (e.g. FP)

Bandwidth

$$\Delta\nu \geq \frac{\bar{N_2}}{\Delta N t} \frac{h\nu_1 (4\pi\Delta\nu_c)^2}{P_{out}}$$

Avg N in level 2

Avg Pop^n inv. Term dominated by decay.

$$\Delta\nu_{FPE} = \frac{1}{2\pi}\frac{c l g_{th}}{2L}$$

(ain't the main)

Stabilization

Use a Piezo Electric transducer

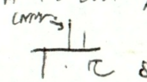
PDH to stabilize.

↳ Phase modulate & measure.

↳ PM: $E_{out} = e^{i\phi(t)} E_{in}$

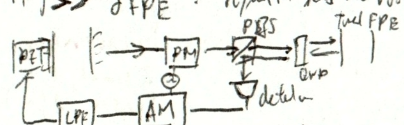
AM: $E_{out} = A(t) E_{in}$

Creates carrier & sidebands



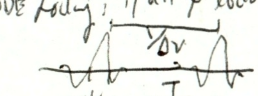
e.g. $e^{i\beta\sin\omega t} \approx \phi \ldots \beta\sin(2\pi ft)$

if $f \gg \delta_{FPE}$: reflection has no V_{out}



Pulsed Lasers: Q switching : low = charge, high = release

Mode Locking: if all ϕ locked.



Wien Displacement Law $\quad \lambda_{max} = \frac{b}{T} = 2.8978 \times 10^6\ \text{nm K}$

4π steradians in a sphere

Stephan-Boltzman law

$$\hookrightarrow \frac{I}{A} = \sigma T^4$$

$$\hookrightarrow 5.67 \times 10^{-8} \frac{W}{m^2} K^{-4}$$

Directionality

$$\Delta\Omega = \frac{\lambda^2}{A}$$

Spectral Brightness:

$$\frac{c\rho(\nu)}{4\pi} : \frac{W}{m^2\,Hz}$$

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-(E_2 - E_1)/k_B T}$$

Frequency Shift

$$\nu' = \nu\left(1 - \frac{|\vec{v}|}{c}\right), \ |\vec{v}| \text{ away from source}$$

$$\nu_{absorb} = \nu_0\left(1 + \frac{u}{c}\right)$$