ISA 365 Final Project Report

Title: We're on a Boat

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Problem Introduction:

Boats can only hold a certain amount of weight, which drew interest into testing which dimensions of boats would lead to highest level of buoyancy. Due to very limited budget constraints, the material chosen to create and test these boats was aluminum foil. In order to test the buoyancy, pennies, grouped in denominations of five, were added to test how much weight the aluminum foil boats were able to hold without dropping a quarter of an inch below water (the threshold for buoyancy). A quarter inch mark was chosen in order to avoid the height of the boat affecting water from pouring into the vessel from the outside environment. This experiment was carried out in an average sized popcorn bowl.

Factors Investigated:

The factors chosen for investigation were as follows:

- Boat Shape
- Boat Length
- Boat Width

The factors were chosen based on the assumption that they would be significant in determining the buoyancy. Additionally, the lack of experience in boat making lead to the factors chosen being fairly simple and manipulable.

Levels of Factors:

The levels of the factors for boat shape were as follows:

- Bowed: Coded as 1
- Square: Coded as -1

These levels were chosen under the assumption that a bowed boat would cause the weight to be more evenly distributed inside the boat and thus affect the buoyancy positively. A square boat was chosen to test how a uniform boat on all sides would handle the same amount of weight.

The levels of the factors for boat length were as follows:

- 4 inches: Coded as 1
- 3 inches: Coded as -1
- 4.2 inches: Coded as $\sqrt{2}$
- 2.75 inches: Coded as $-\sqrt{2}$

These levels were chosen in correlation to the size of the popcorn bowl that the experiment was carried out in. The given levels allowed for both believed variation in the amount of pennies a boat could hold as well as still being able to fit inside our bowl used to test buoyancy. The alpha level allowed the experiment to be carried out as a 2^2 rotatable and spherical central composite design.

The levels of the factors for boat width were as follows:
- 3 inches: Coded as 1
- 2 inches: Coded as -1
- 3.2 inches: Coded as √2
- 1.75 inches: Coded as -√2

The levels chosen for the boat width follow the logic of the levels for the boat length. Without having prior information, these were believed to be reasonable widths of a boat in correlation to the boat length. These levels again allowed for both the boat to fit inside the bowl as well as provide sufficient variation in the amount of pennies a boat could hold. The alpha level allowed the experiment to be carried out as a 2^2 rotatable and spherical central composite design.

Having the length differ than the width allowed for the analysis of both rectangular and square boats depending on the given levels of a run. While there were more runs of rectangular shaped boats, this still allowed for different shapes to be analyzed in the model.

Response:
This was measured by counting the amount of pennies, in groups of five, that an aluminum foil boat could hold without the bottom of the boat dropping a quarter of an inch below the surface of the water. Groupings of five were chosen due to an assumption that the relative weight of one penny being added at a time would not have had a significant enough impact to outweigh the time the runs would have taken otherwise.

The results in the model described in the report will give an idea of the dimensions of a boat that lead to the highest buoyancy.

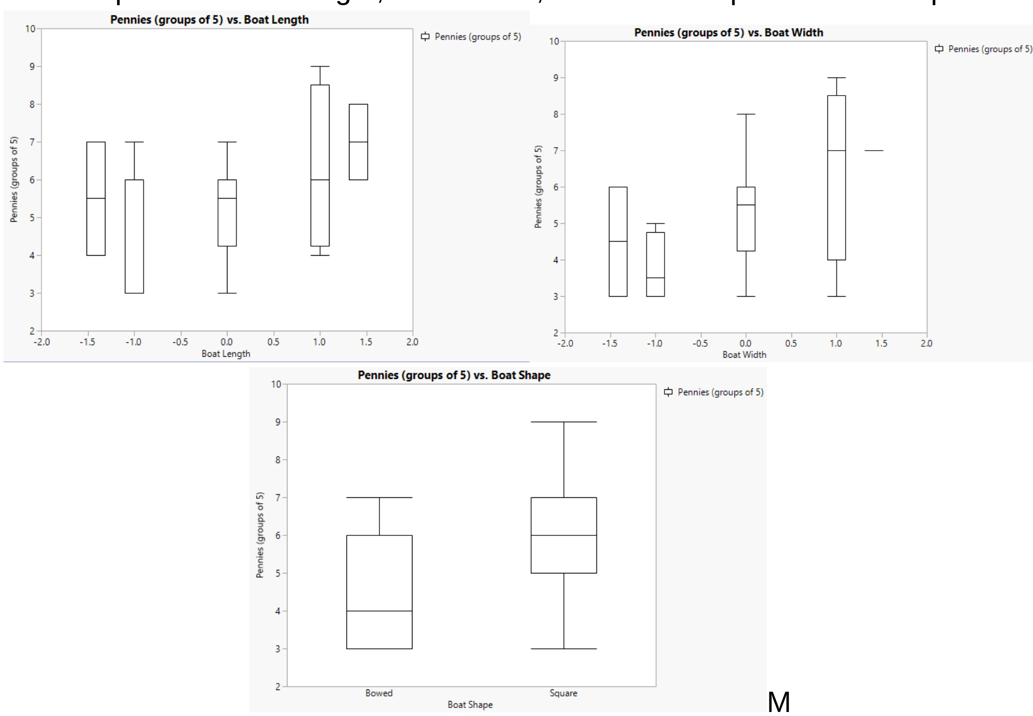
<u>How the Design was Chosen:</u>
The design that was selected for this experiment, as previously stated, is a 2^2 rotatable and spherical central composite design. This was chosen so that analysis could be done in order to determine whether there were significant quadratic effects in addition to determining whether there were any significant two factor interactions in the model. However, because one of the main effects (Boat Shape) was not continuous, it did not make sense to attempt to determine a quadratic effect for Boat Shape. This is because the effect can only be one shape and not anything in between. To accommodate this, the choice was made to run two identical 2^2 rotatable and spherical Central Composite Design (CCD) experiments. One experiment would be run using the squared boat shape and one experiment would be run using the bowed boat shape. Then, by analyzing the two experiments together, software allowed for the comparison between shapes while still allowing a full exploration into the other main effects, potential two-

factor interactions, and potential quadratic effects. The choice was also made to replicate the center point four times in each experiment, allowing for an average estimation of the quadratic effects in the model.

<u>Initial Exploration:</u>

Graphs were made in order to attempt to analyze each effect individually. This was done prior to the creation of the models in order to form expectations on what could be significant. These graphs can be seen here:

Figure 1: Boxplots of Boat Length, Boat Width, and Boat Shape versus Groups of Pennies



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With only a few runs at each level (the design was not replicated), it was difficult to make expectations of each effect. When looking at the box plot of Boat Length vs. Groups of Pennies, it appears, by looking at the means, that the longer the length of the boat the more pennies that it can hold. It is also worth noting that the maximums tend to increase in the same manner. When looking at the box plot of Boat Width vs. Groups of Pennies similar effects are evident. The means and maximums tend to increase except for when the width moved from the negative alpha value to the value of negative one. It is also odd that each level in boat width had a minimum of 3 groups of pennies except for the positive alpha level. This suggests that no matter how wide the boat is, usually it will hold at least three groups of five pennies. Finally, it is important to note that the values across the x axes in each plot are coded units and correspond to the natural units provided in the above section titled, “Levels of Factors.” The Box Plot of Boat Shape vs. Groups of Pennies indicates that the maximum and the median for the square design are greater than the bowed design, implying that a square shape may be more buoyant than a bowed shape.

Initial Model:

The first model to be tested contained each main effect, two-factor interactions, and quadratic effects except the quadratic term pertaining to boat shape, which allowed for each effect to be analyzed to determine statistical significance. According to the analysis of variance provided in Figure 2 below, the whole model F test has a significant P value at an alpha level of 10%, which suggests that at least one effect in the model is a significant predictor of buoyancy. The summary of fit, included in the same figure, indicates that the full model has an adjusted R^2 value of .5706, meaning it explains 57.06% of the variation of buoyancy after accounting for multiple terms being in the model.

Figure 2: Summary of Fit and Analysis of Variance of Initial Model

Summary of Fit

RSquare	0.719928
RSquare Adj	0.570556
Root Mean Square Error	1.140176
Mean of Response	5.375
Observations (or Sum Wgts)	24

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	8	50.124982	6.26562	4.8197
Error	15	19.500018	1.30000	Prob > F
C. Total	23	69.625000		0.0043*

By observing the effects test provided in Figure 3 below, each effect was evaluated on whether it should be included in the final model based on its P value. Several models were tested where the effect with the highest insignificant p value was removed until an efficient model with significant effects at an alpha level of 10% was reached. Based on the p-values in the Effects Test, it appears that only the main effects are significant predictors of a boat's buoyancy.This would indicate that a main effects only model should be run to ensure that they are significant.

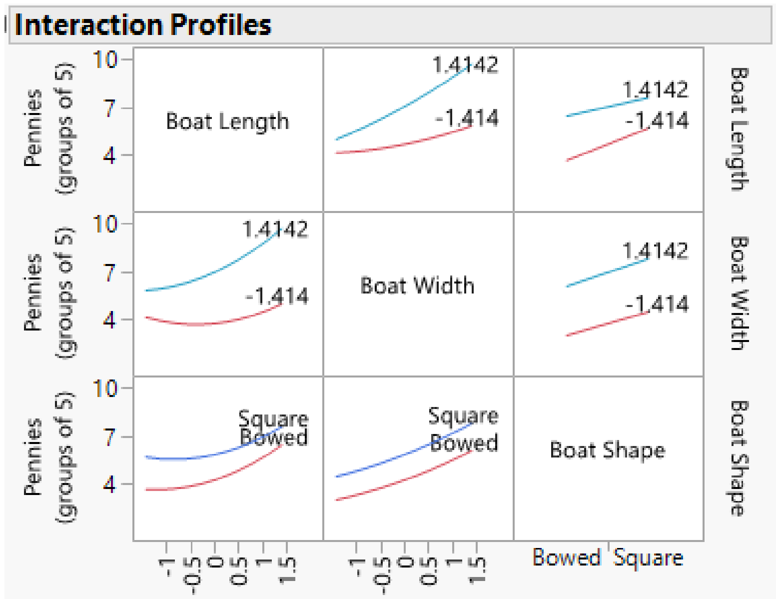
Figure 3: Effects Test of Initial Model

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Boat Length	1	1	10.960471	8.4311	0.0109*
Boat Width	1	1	20.410218	15.7002	0.0013*
Boat Shape	1	1	15.041667	11.5705	0.0039*
Boat Length*Boat Length	1	1	2.112500	1.6250	0.2218
Boat Length*Boat Width	1	1	1.125000	0.8654	0.3670
Boat Width*Boat Width	1	1	0.312500	0.2404	0.6310
Boat Length*Boat Shape	1	1	0.364277	0.2802	0.6043
Boat Width*Boat Shape	1	1	0.035850	0.0276	0.8703

The interaction plot of the initial model was used to verify that the two-factor interactions were insignificant. This plot can be seen below in Figure 4. Because none of the charts in the plot below have lines that intersect, it appears that there are no significant two-factor interactions and none of the factors depend on others when determining the number of pennies, in groups of five, that a boat can hold.

Figure 4: Interaction Plot of Initial Model



Final Model:

The final model that was tested was a main effects only model due to the results from the initial model's Effects Test, which is in Figure 3. The Summary of Fit and ANOVA tables can be seen below in Figure 5.

Figure 5: Summary of Fit and ANOVA tables for Final Model

Summary of Fit

RSquare	0.666605
RSquare Adj	0.616595
Root Mean Square Error	1.077326
Mean of Response	5.375
Observations (or Sum Wgts)	24

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	46.412356	15.4708	13.3296
Error	20	23.212644	1.1606	Prob > F
C. Total	23	69.625000		<.0001*

By looking at the Summary of Fit table, it can be seen that the Adjusted R^2 value is 0.6166, which indicates that the final main effects only model explains 61.66% of the variation in buoyancy after accounting for the model using additional terms. In the analysis of the ANOVA table, the p-value related to the F-Statistic indicates that at least one term in the model is a significant predictor of buoyancy. This is in line with the initial model's conclusions, but the Effects Test of the final model will verify these conclusions. The Effects Test of the final model can be seen in Figure 6 below.

Figure 6: Effects Test of the Final Model

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Boat Length	1	1	10.960471	9.4435	0.0060*
Boat Width	1	1	20.410218	17.5854	0.0004*
Boat Shape	1	1	15.041667	12.9599	0.0018*

The p-values in the Effects Test all suggest significance at an alpha level of 10%. Given the results of the Effects Test, it can be concluded that each of the main effects are significant predictors of buoyancy in boats. In order to create the final model, the coefficients must be obtained from the Parameter Estimates table, which can be seen in Figure 7 below.

Figure 7: Parameter Estimates of Final Model

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	5.375	0.219908	24.44	<.0001*
Boat Length	0.827665	0.269332	3.07	0.0060*
Boat Width	1.1294417	0.269332	4.19	0.0004*
Boat Shape[Bowed]	-0.791667	0.219908	-3.60	0.0018*

Using the equation parameters given in the Parameter Estimates table, the following equation can be derived for the model:

$$\hat{y} = 5.375 + .828(\text{boat length}) + 1.129(\text{boat width}) - .792(\text{boat shape})$$

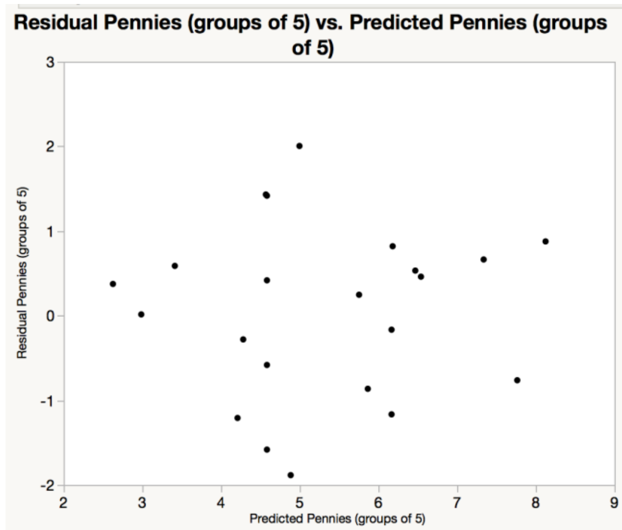
<u>Assumptions for the Final Model:</u>

After arriving at the final model, the final step in determining its validity was to verify the assumptions that the residuals have a constant variance, the residuals are normally distributed, and the residuals are independent of each other.

Assumption that Residuals have Constant Variance:

In the plot shown in Figure 8, anything outside of a two-sigma boundary line of the mean, zero, would indicate a non constant variance. For this model sigma equals 1.08, which was taken from the Summary of Fit table in Figure 5. This means that the two-sigma cut offs are negative and positive 2.16. According to the plot, none of the points appear to have a greater residual value than these cutoffs, demonstrating the variance of the residuals is constant. It is worth noting that the residuals near predicted values of 5 appear to have a larger variance than the rest, but fall within the two-sigma limits.

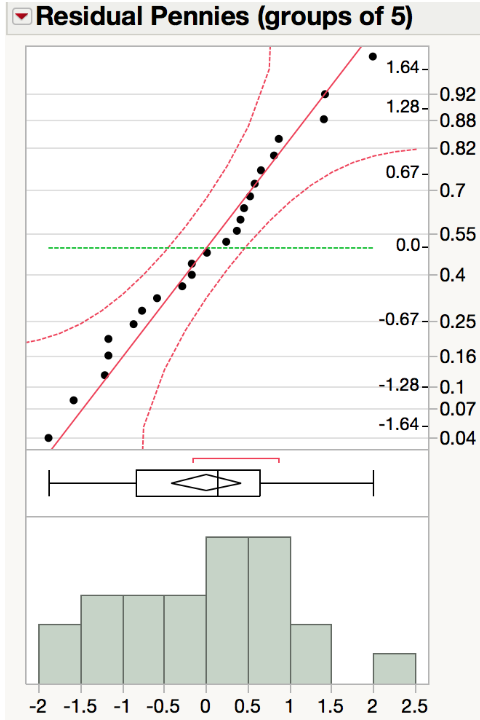
Figure 8: Residual vs. Predicted Plot



Assumption that Residuals are Normally Distributed:

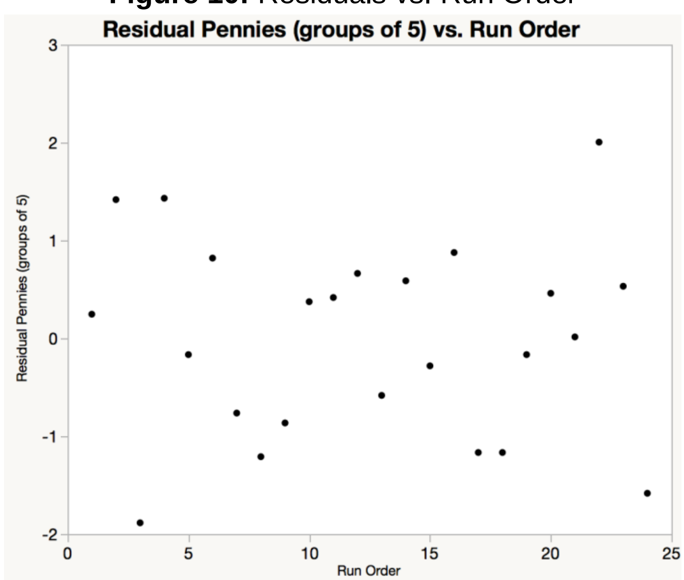
As shown in Figure 9 on the next page, the assumption that the residuals are normally distributed is met. The residual data points fall in a relatively straight line on the normal quantile plot, and while there are deviations off the line, all points fall within the confidence bands. Also, the histogram looks relatively normally distributed, given the small sample size.

Figure 9: Normal Quantile Plot and Distribution of Residuals



Assumption that Residuals are Independent:
As shown in Figure 10, the assumption that the residuals are independent of each other is safely met. There is no pattern present in the graph of residual vs. run order, and the variation in the results appear to be caused by white noise.

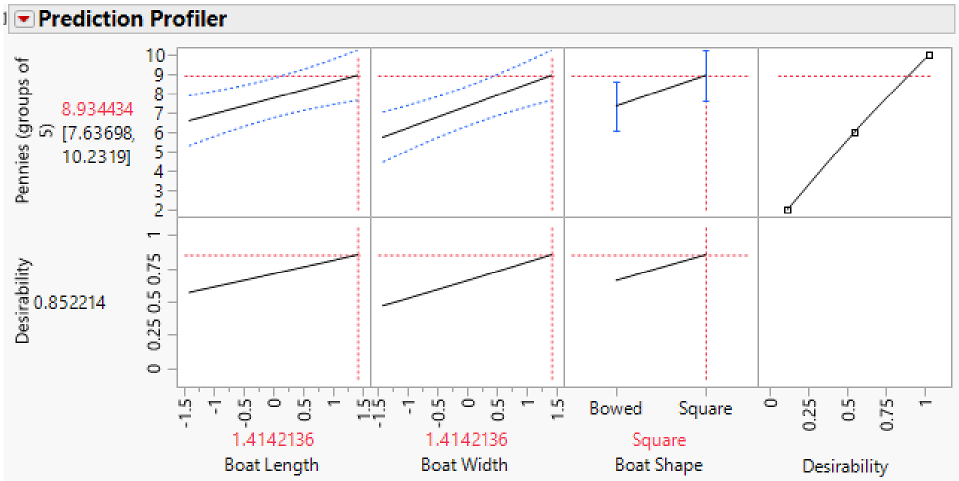
Figure 10: Residuals vs. Run Order



<u>Maximization Predictions and Confirmation Runs:</u>

In order to determine which levels of each effect would produce the boat with the most buoyancy, the prediction profiler was used to maximize the desirability of the response. The prediction profiler that was used can be seen in Figure 11 below.

Figure 11: Prediction Profiler



The suggested levels from the prediction profiler appear to have both Boat Length and Boat Width at a positive alpha value, and suggests the shape of the boat should be squared. When plugging these coded values into the prediction equation, we get the results seen here:

$$\hat{y} = 5.375 + .828(1.414) + 1.129(1.414) - .792(-1) = 8.934$$

The above equation implies that the optimal boat in the bounds of the experiment should hold almost nine stacks of five pennies. In order to confirm this, five confirmation runs needed to be run at the predicted maximizing values and the average response of the five runs should be relatively close to 8.934. The results of the five confirmation runs can be seen in Figure 12 below.

Figure 12: Results of Confirmation Runs

Run Number	Pennies (groups of 5)
Run 1	9
Run 2	8
Run 3	9
Run 4	8
Run 5	10

The 90% confidence interval for the predicted value given in Figure 8 is (7.08, 10.79), meaning that there is 90% confidence that the sample mean of the confirmation runs will fall between the interval values. Because the sample mean of the confirmation runs is 8.8, it's falling within the interval verifies that a boat of the size given in the prediction profiler would have a buoyancy that is suggested from the final model.

Analysis Summary:

In conclusion, the experiment and its design was successful in creating a model that could identify the relevant effects and optimize buoyancy in a boat. The 2^2 CCD design allowed the experiment to account for possible quadratic effects for boat length and width, and by including shape as a factor the model was able to suggest a more effective boat structure. By fitting multiple models it became clear that there were not any quadratic effects within the CCD or two factor interactions in the model. The final model included only the main effects, with boat width as the most significant effect on buoyancy at a p value of 0.0004. The other two main effects, boat shape and boat length, had p values of 0.0018 and 0.006, respectively. Given the results of the experiment, one can conclude, logically, that larger boats tend to be more buoyant when it comes to carrying weight.

Several issues were recognized throughout the experiment. First, aluminum foil appeared to be a flawed material for this kind of research. When the pennies were distributed unevenly the boat's structure would bend, making the depth sunk by the boat difficult to identify. Additionally, the bending effect would occasionally allow for water to enter the vessel and would add an unidentifiable amount of weight to the boat. A recommendation would be to use a sturdier material to construct the boats that could still be molded to the specifications. Secondly, the individual pennies used in the experiment potentially varied in weight, possibly adding an unrecorded effect in our model. Because of this, another recommendation would be to use a device that can add weight without a high amount of variation. The last major flaw was how the response was recorded. A piece of tape on the outside of the curved bowl marked where the boat was considered to be "sunk". This method made it difficult to establish when the boat sank below the marked depth level due to the curvature of the container. One way to avoid this problem would be to use a square container with flat sides and have the marked depth level be the bottom of the bowl.

Next Steps:

The next step after obtaining these results would be to design another experiment as experimentation is a sequential process. Given the results from the experiment, the next experimental design would be a 2^k CCD with four center point runs, using at least two factors. The two factors that would definitely be included in the model would be Boat Length and Boat Width, as both have proven to be significant. The new levels of these factors can be seen in Figure 13 on the next page.

Figure 13: Levels of Factors in Ensuing Experiment

Factor	Level (coded value)

Boat Length	5 inches (-1)
	8 inches (1)
Boat Width	4 inches (-1)
	6 inches (1)

Given that the Adjusted R^2 value of the final model was only 61.66%, it might be a good idea to talk to someone with more boat building experience in order to find out if there are potentially additional factors that were not originally considered. For this reason, an alpha level was not chosen for the next experiment. Without definitely knowing the number of factors, one cannot select an alpha level that will make the experiment either rotatable, spherical, or both. It is also important to note that because Square boats were proven to be better than Bowed boats, Boat Shape would no longer be a factor as the entire experiment would be carried out with Square boats. This could potentially change, however, if a more experienced builder suggests a different shape. In that case, it could be advisable to test the new shape against Square boats.

The new design would also be a CCD instead of a full factorial design. There was one main idea behind this. It was noted that potentially the levels in the original experiment were too close together. For this reason, the new levels have been given a greater range in order to once again attempt to discover any potential quadratic effects. It was thought that if the levels were too close together, there would not have been enough of a curve within the design space to prove a quadratic effect was evident. However, if there once again proves to be no quadratic effect, the next design after that should change to a full factorial. This is because the solution to the problem is very logical in that larger boats should be able to carry more weight and be more buoyant. Assuming there is no additional boat shape that should be tested, this experiment should run with lower costs and take less time as only half the runs of the initial experiment would be needed.