

(Revised) Outline

01 Trend & Seasonality

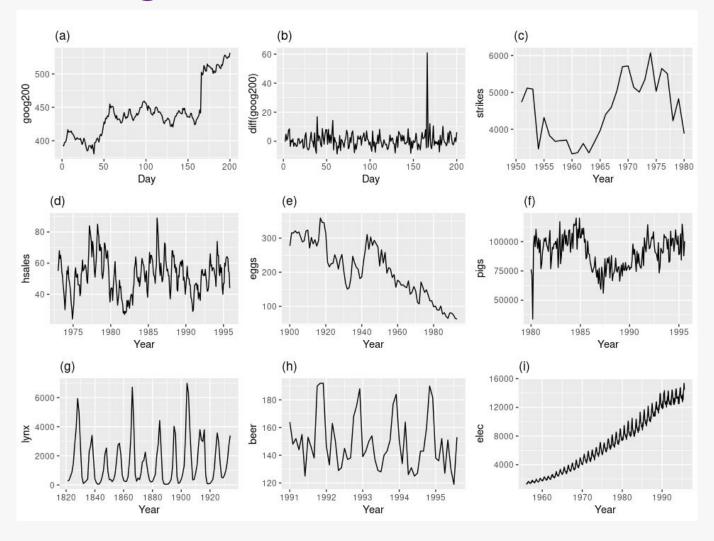
02 Forecasting

```
router.get('/register', function(req, res, next) {
return res.render('register', { title: '5ign Up' })
```



Recalling: Autocorrelation, Trend, and Seasonality

Stationarity

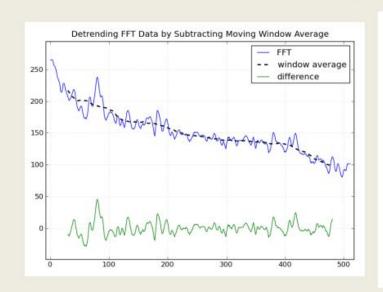


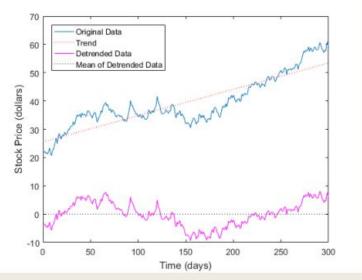
YOU NEED TO REMOVE THE TREND AND SEASONAL ELEMENTS BEFORE FORECASTING

- Most (interesting) data in the real world will show
 - Trends
 - Seasonality
- Most models require data that shows neither of these properties to say something interesting
- In particular, we need a stationary time series

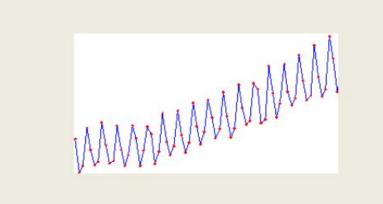
DE-TREND YOUR DATA

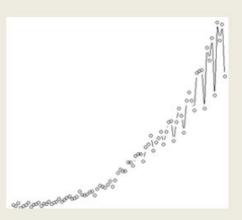
Use local smoothing or a linear regression





SEASONALITY





REMOVE SEASONALITY

- Simplest: average de-trended values for specific season
- More common: use 'loess' method ('locally weighted scatterplot smoothing')
 - Window of specified width is placed over the data
 - A weighted regression line or curve is fitted to the data, with points closest to center of curve having greatest weight
 - Weighting is reduced on points farthest from regression line/curve and calculation is rerun several times.
 - This yields one point on loess curve
 - Helps reduce impact of outlier points
 - Computationally taxing

SOME DETAILS (ONLY IF NEEDED)

DICKEY-FULLER TEST

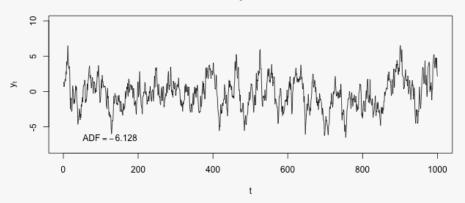
- Tests the null hypothesis of whether a unit root is present in an autoregressive model
- In plain English, tests whether $\rho = 1$ in

$$y_t = \rho y_{t-1} + u_t$$

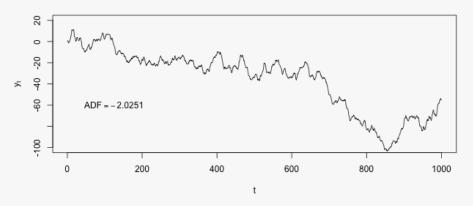
- The test gives back several values to help you assess significance with standard pvalue reasoning.
- Basic intuition: ρ should not have unit value

ADF - Stationarity Test

Stationary Time Series

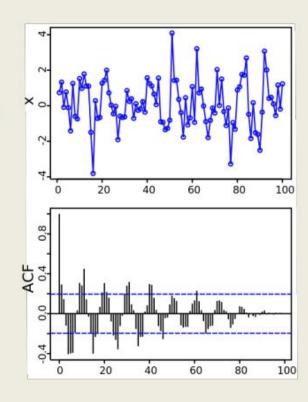


Non-stationary Time Series



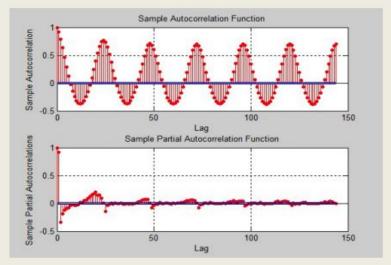
AUTOCORRELATION FUNCTION

- Used to help identify possible structures of time series data
- Gives a sense of how different points in time relate to each other in a way explained by temporal distance

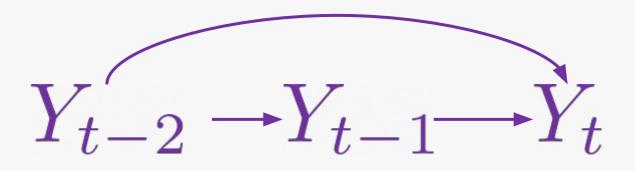


PARTIAL AUTOCORRELATION FUNCTION

- "gives the partial correlation of a time series with its own lagged values, controlling for the values of time series at all shorter lags"
- Why would this be useful?

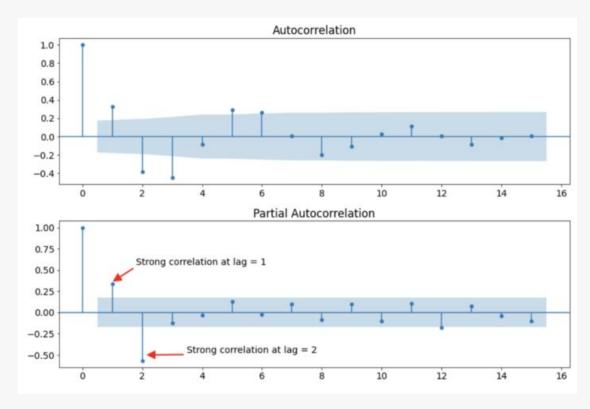


ACF and PACF

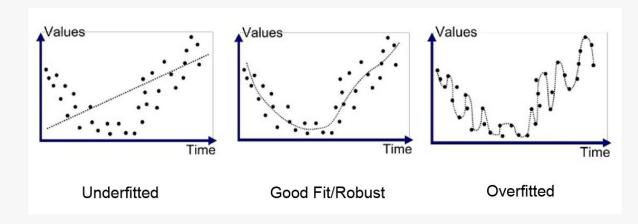


ACF is the correlation between a time series with a lagged version of itself.

PACF is the autocorrelation between X_t_t and X_(t-k) that is not accounted for by lags 1 through *k*-1.

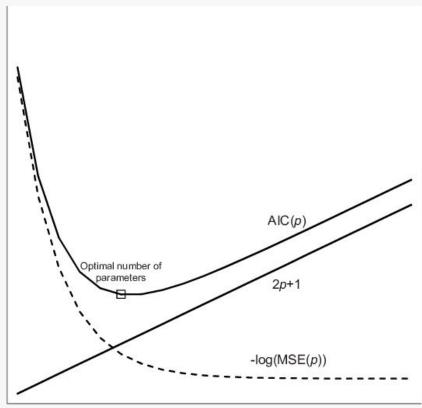


AIC and BIC



$$\mathrm{AIC} \,=\, 2k - 2\ln(\hat{L})$$

$$\mathrm{BIC} = k \ln(n) - 2 \ln(\widehat{L})$$



MOVING AVERAGE PROCESS (MA)

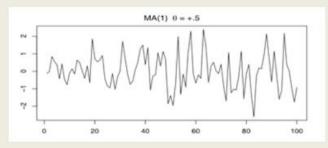
Defined as having the form:

$$X_{t} = \mu + \varepsilon_{t} + \theta_{1} \varepsilon_{t-1} + \dots + \theta_{q} \varepsilon_{t-q}$$

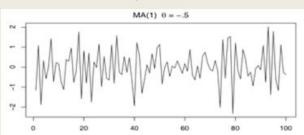
- μ is the mean of the series, θ are parameters, θ_q not 0
- This is a stationary process regardless of values of θ
- Consider an MA(I) process (centered at 0):

$$X_{t} = \varepsilon_{t} + \theta_{1} \varepsilon_{t-1}$$

$$\theta_1 = +.5$$



$$\theta_1 = -.5$$





Recalling: Forecasting Model

Autoregression (p)

$$\hat{Y}_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_p Y_{t-p} + \epsilon$$

Contoh AR(2)

Sensor ID	Timestamp	Value Yt	Value Yt-1	Value Yt-2
Sensor_1	01/01/2020	236	(to fill)	
Sensor_1	02/01/2020	133		
Sensor_1	03/01/2020	148		
Sensor_1	04/01/2020	152		
Sensor_1	05/01/2020	241		
Sensor_1	06/01/2020	?		

AUTOREGRESSIVE PROCESS (AR)

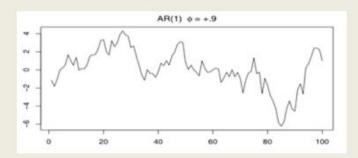
Defined as having the form:

$$X_{t} = \phi_{1}X_{t-1} + \dots + \phi_{p}X_{t-p} + \varepsilon_{t}$$

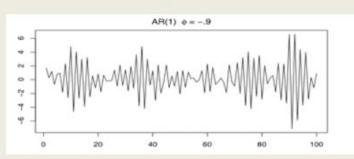
- This is a stationary process if abs(φ) < 1
- Consider an AR(I) process:

$$X_t = \phi_1 X_{t-1} + \varepsilon_t$$

$$\phi_1 = +.9$$



$$\phi_1 = -.9$$



Moving Average (q)

$$\widehat{Y}_t = \beta_0 + \beta_1 \phi_{t-1} + \beta_2 \phi_{t-2} + \dots + \beta_q \phi_{t-q} + \epsilon$$

Misconception Alert!!

Simple Moving Average is done simply by averaging Moving Average model is a regression model

MOVING AVERAGE PROCESS (MA)

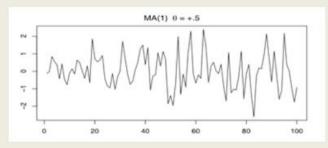
Defined as having the form:

$$X_{t} = \mu + \varepsilon_{t} + \theta_{1} \varepsilon_{t-1} + \dots + \theta_{q} \varepsilon_{t-q}$$

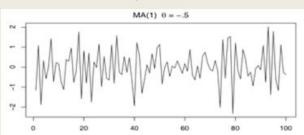
- μ is the mean of the series, θ are parameters, θ_q not 0
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$$X_{t} = \varepsilon_{t} + \theta_{1} \varepsilon_{t-1}$$

$$\theta_1 = +.5$$



$$\theta_1 = -.5$$



ARIMA (p,d,q)

Autoregressive Integrated Moving Average

AR(p)

differentiate

MA(q)

ARIMA MODEL (A.K.A. BOX-JENKINS)

- AR = autoregressive terms
- I = differencing
- MA = moving average
- Hence specified as (autoregressive terms, differencing terms, moving average terms)

ARIMA MODE: 'THE MOST GENERAL CLASS OF MODELS FOR FORECASTING A TIME SERIES WHICH CAN BE MADE TO BE STATIONARY

- Statistical properties (mean, variance) constant overt time
- 'its short-term random time patterns always look the same in a statistical sense'
- Autocorrelation function & power spectrum remain constant over time
- Ok to do non-linear transformations to get there
- ARIMA model can be viewed as a combination of signal ad noise
- Extrapolate the signal to obtain forecasts

APPLYING THE APPROPRIATE ARIMA MODEL

- Need to determine what ARIMA model to use
- Use plot of the data, the ACF, and the PACF
- With the plot of the data: look for trend (linear or otherwise) & determine whether to transform data
- Most software will use a maximum likelihood estimation to determine appropriate ARIMA parameters



Next?

Next

01 Fine-tuning parameters

02 Panel Data



THANK YOU!