

SECTION I. DISTRIBUTIONS WITHIN A PLAYER'S 13 CARD HAND**SECTION I.A. Distributions of Suits****SECTION I.A.i. Simulated Distributions of 4 Suits within a Player's 13 Card Hand**

Simulated Histogram by Raw Counts (100 sets of 335 random hands)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
c	4.29	26.31	69.22	96.24	80.65	41.07	13.78	2.99	0.42	0.03	0	0	0	0
d	4.88	27.38	68.8	94.85	80.88	41.71	13.22	2.65	0.61	0.01	0.01	0	0	0
h	3.97	26.53	69.21	96.53	80.55	41.03	13.67	3.09	0.4	0.02	0	0	0	0
s	4.22	26.21	68.34	96.21	80.38	41.94	14.28	2.93	0.46	0.03	0	0	0	0

Simulated Histogram by Percentages (100 sets of 335 random hands)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
theor %	0.013	0.08	0.206	0.286	0.239	0.125	0.042	0.009	0.001	0	0	0	0	0
c %	0.013	0.079	0.207	0.287	0.241	0.123	0.041	0.009	0.001	0	0	0	0	0
d %	0.015	0.082	0.205	0.283	0.241	0.125	0.039	0.008	0.002	0	0	0	0	0
h %	0.012	0.079	0.207	0.288	0.24	0.122	0.041	0.009	0.001	0	0	0	0	0
s %	0.013	0.078	0.204	0.287	0.24	0.125	0.043	0.009	0.001	0	0	0	0	0

(Theoretical % Taken from Table 4 of <https://gregstoll.com/~gregstoll/bridge/math.html>)

Theoretical (Obtained by Simulation) Standard Deviation of Each Histogram Bin (100 sets of 335 random hands)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
c	2.007	4.83	7.243	7.92	7.302	5.765	3.871	1.795	0.669	0.171	0	0	0	0
d	2.324	4.792	6.877	7.576	7.862	6.629	4.069	1.635	0.815	0.1	0.1	0	0	0
h	2.289	5.149	7.512	9.129	7.526	6.122	3.96	1.7	0.651	0.141	0	0	0	0
s	1.993	4.562	7.209	8.339	7.954	6.518	3.548	1.794	0.717	0.171	0	0	0	0

SECTION I.A.ii. Observed Distributions of 4 Suits within a Player's 13 Cards

Observed Hearts.vex Histogram by Raw Counts (335 hearts.vex hands)

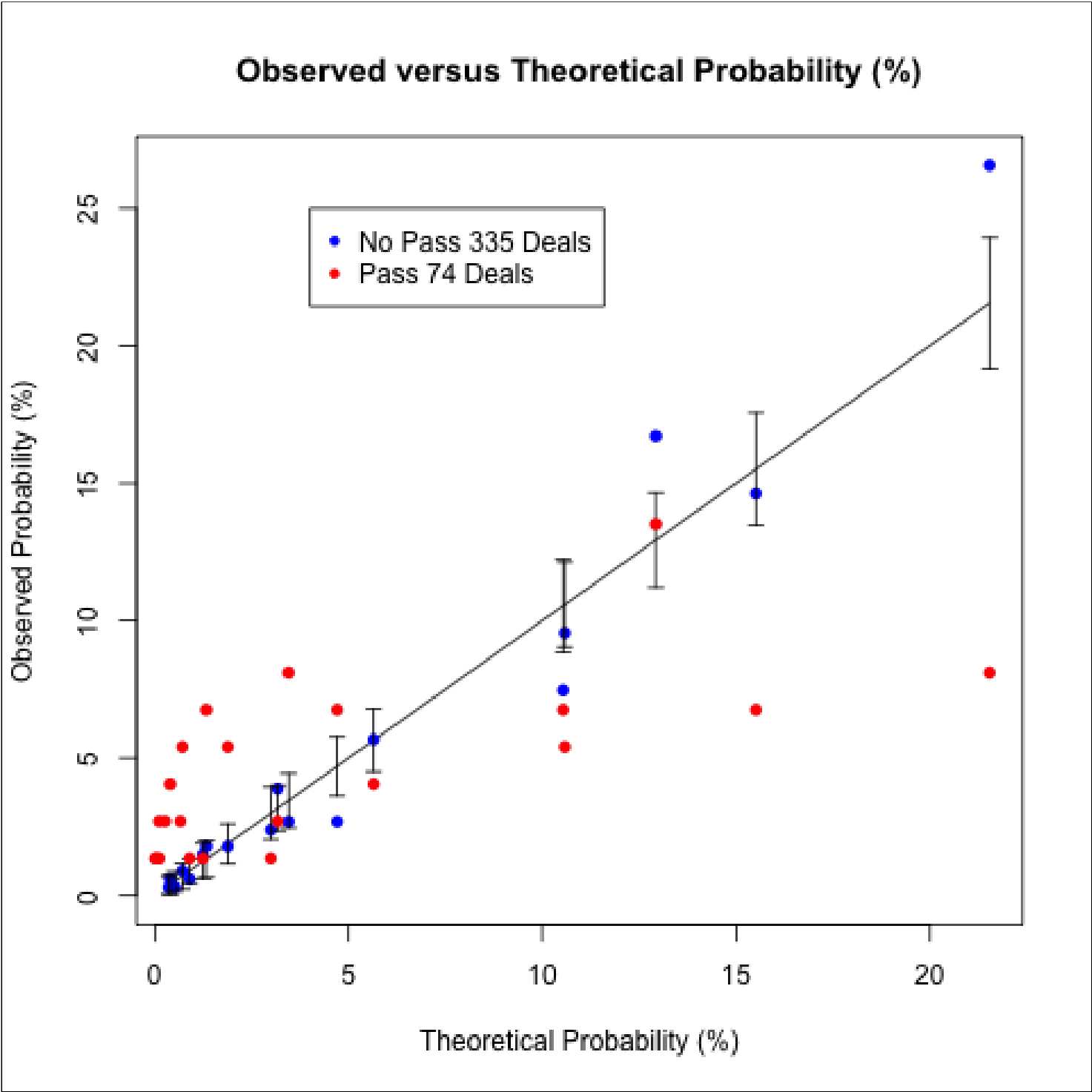
	0	1	2	3	4	5	6	7	8	9	10	11	12	13
c	3	21	64	80	88	62	12	5	0	0	0	0	0	0
d	5	25	76	103	87	30	8	1	0	0	0	0	0	0
h	4	34	62	97	82	42	12	2	0	0	0	0	0	0
s	2	31	70	89	86	41	14	2	0	0	0	0	0	0

Observed Hearts.vex Histogram by Percentages (335 hearts.vex hands)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
theor %	0.013	0.08	0.206	0.286	0.239	0.125	0.042	0.009	0.001	0	0	0	0	0
c %	0.009	0.063	0.191	0.239	0.263	0.185	0.036	0.015	0	0	0	0	0	0
d %	0.015	0.075	0.227	0.307	0.26	0.09	0.024	0.003	0	0	0	0	0	0
h %	0.012	0.101	0.185	0.29	0.245	0.125	0.036	0.006	0	0	0	0	0	0
s %	0.006	0.093	0.209	0.266	0.257	0.122	0.042	0.006	0	0	0	0	0	0

SECTION I.B. Probabilities for the Distribution of 4 Suits within a Player's 13 Card Hand**SECTION I.B.i. Theoretical Probabilities for Distributions of 4 Suits within One Player's 13 Card Hand**(Taken from Table 3 of <https://gregstoll.com/~gregstoll/bridge/math.html>)

SECTION I.B.ii. Observed versus Theoretical Probabilities for Distributions of 4 Suits within One Player's 13 Card Hand



SECTION I.C. Distributions of Ranks

SECTION I.C.i. Simulated Cumulative Statistical Frequency of Cards of a Given Rank per Number of Cards in a Suit

Simulated for Clubs

Frequency of Cards of a Given Rank (Columns) per Number of Cards (Rows) in a Suit (100 sets of 335 random hands)

	2	3	4	5	6	7	8	9	T	J	Q	K	A	standard_dev	mean
1	2.04	1.92	1.97	1.97	1.85	2.1	2.01	2.11	1.92	2.19	2.11	1.95	2.17	1.407	1.98
2	10.34	10.52	10.65	11.44	10.9	10.43	11.05	10.42	10.2	10.83	10.48	10.7	10.48	3.176	10.115

	2	3	4	5	6	7	8	9	T	J	Q	K	A	standard_dev	mean
3	22.01	22.15	22.45	21.49	22.73	21.67	22.17	22.48	21.83	22.28	22.39	22.74	22.33	4.196	20.923
4	24.83	24.39	25.18	24.24	24.38	23.84	24.56	25.09	24.97	25.21	24.91	25.65	25.35	4.121	23.337
5	15.7	15.57	15.79	16.1	16.41	15.53	15.91	15.97	15.72	15.29	15.68	15.97	15.71	3.297	14.903
6	6.31	6.21	6.54	6.28	6.34	6.31	6.49	6.36	6.43	6.37	6.52	6.11	6.41	1.84	6.037
7	1.65	1.57	1.51	1.78	1.62	1.5	1.74	1.66	1.54	1.5	1.64	1.51	1.71	0.831	1.554
8	0.22	0.28	0.19	0.26	0.22	0.21	0.27	0.26	0.25	0.34	0.29	0.24	0.33	0.188	0.253
9	0.02	0.01	0.02	0.03	0.02	0.02	0.02	0.01	0.03	0.02	0.02	0.03	0.02	0.014	0.02
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Simulated for Diamonds

Frequency of Cards of a Given Rank (Columns) per Number of Cards (Rows) in a Suit (100 sets of 335 random hands)

	2	3	4	5	6	7	8	9	T	J	Q	K	A	standard_dev	mean
1	2.25	2.09	2.1	2.03	2.25	2	2.24	1.89	2.29	1.89	2	2.33	2.02	1.443	2.059
2	10.52	10.16	10.79	10.6	10.64	10.97	10.6	10.72	10.61	10.45	10.52	10.69	10.33	3.15	10.054
3	22.17	21.91	21.75	22.18	21.83	21.98	22.29	20.86	21.51	22.29	22.43	21.83	21.52	4.191	20.624
4	25.01	24.93	24.73	24.78	24.76	25.29	25.35	24.26	25.04	25.56	24.84	24.13	24.84	4.234	23.411
5	16.42	16.15	16.49	15.69	15.79	15.96	15.66	15.7	15.98	16.54	16.28	16.29	15.6	3.16	15.122
6	6.05	5.98	6.41	5.99	5.98	6.27	6.18	6.12	6.05	5.99	6.26	6.16	5.88	1.828	5.796
7	1.39	1.47	1.38	1.51	1.38	1.5	1.3	1.31	1.49	1.46	1.48	1.46	1.42	0.768	1.38
8	0.35	0.39	0.37	0.37	0.37	0.34	0.42	0.36	0.44	0.39	0.37	0.4	0.31	0.26	0.367
9	0.01	0.01	0	0	0.01	0	0.01	0	0.01	0.01	0.01	0.01	0.01	0.005	0.007
10	0.01	0	0.01	0	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0	0.004	0.007
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Simulated for Hearts

Frequency of Cards of a Given Rank (Columns) per Number of Cards (Rows) in a Suit (100 sets of 335 random hands)

	2	3	4	5	6	7	8	9	T	J	Q	K	A	standard_dev	mean
1	2.39	2.19	2.06	2.13	1.91	2.01	1.95	1.95	1.79	1.93	2.04	2.28	1.9	1.433	1.997
2	10.44	10.82	10.27	10.93	10.71	10.31	10.78	10.67	10.63	10.59	10.74	10.53	11	2.962	10.099
3	22.15	22.51	21.72	22.55	22.72	22.36	22.82	22.3	22.01	22.24	21.81	21.94	22.46	4.108	20.978
4	25.08	24.81	25.29	24.81	24.52	24.93	25.53	24.66	24.37	24.08	24.21	24.4	25.51	4.255	23.318
5	15.38	16.07	15.83	15.82	15.49	14.95	15.44	16.51	15.89	15.59	16.04	15.87	16.27	3.094	14.875
6	6.23	6.46	6.05	6.26	6.5	6.35	6.2	6.4	6.43	6.03	6.49	6.19	6.43	1.893	5.994
7	1.55	1.57	1.67	1.77	1.75	1.71	1.71	1.6	1.68	1.56	1.73	1.65	1.68	0.861	1.606
8	0.2	0.26	0.27	0.17	0.24	0.27	0.24	0.26	0.27	0.27	0.25	0.26	0.24	0.168	0.241
9	0.01	0.02	0	0.02	0.01	0.02	0.02	0.01	0.02	0	0.02	0.01	0.02	0.01	0.014
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Simulated for Spades

Frequency of Cards of a Given Rank (Columns) per Number of Cards (Rows) in a Suit (100 sets of 335 random hands)

	2	3	4	5	6	7	8	9	T	J	Q	K	A	standard_dev	mean
1	2.16	2.08	2.1	2.02	2.11	1.87	2.02	1.88	1.91	2.1	1.96	2.07	1.93	1.409	1.973
2	10.61	10.74	10.39	10.4	9.86	10.58	10.43	10.95	10.92	10.36	10.5	10.26	10.68	2.992	9.977
3	22.35	21.74	21.98	22.32	22.08	22.15	22.6	22.29	22.09	21.85	21.71	22.76	22.71	4.174	20.915

	2	3	4	5	6	7	8	9	T	J	Q	K	A	standard_dev	mean
4	24.71	24.67	25.14	24.41	24.28	24.57	25.17	24.12	25.04	25.18	24.59	24.98	24.66	4.216	23.267
5	16.15	16.01	16.57	15.96	16.33	15.98	15.87	16.33	15.49	16.06	16.02	16.21	16.72	3.22	15.209
6	6.62	6.43	6.59	6.95	6.67	6.64	6.39	6.71	6.65	6.55	6.45	6.34	6.69	1.903	6.256
7	1.49	1.57	1.51	1.62	1.46	1.65	1.55	1.53	1.6	1.53	1.71	1.57	1.72	0.812	1.523
8	0.25	0.28	0.27	0.28	0.25	0.26	0.32	0.28	0.28	0.29	0.32	0.33	0.27	0.192	0.277
9	0.02	0.03	0.01	0.02	0.01	0.03	0.02	0.02	0.03	0.03	0.01	0.02	0.02	0.014	0.02
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

SECTION I.C.ii. Observed Cumulative Statistical Frequency of Cards of a Given Rank per Number of Cards in a Suit

Observed Hearts.vex for Clubs

Frequency of Cards of a Given Rank(Columns) per Number of Cards (Rows) in a Suit (335 hearts.vex hands)

	2	3	4	5	6	7	8	9	T	J	Q	K	A	mean	standard_dev
1	0	3	1	0	3	2	1	4	1	2	0	1	3	1.615	1.325
2	16	9	10	5	10	9	13	11	6	12	15	6	6	9.846	3.532
3	14	20	20	26	20	17	21	24	14	11	21	12	20	18.462	4.557
4	39	30	26	26	24	30	19	33	28	27	22	19	29	27.077	5.53
5	20	24	27	19	26	19	23	27	31	20	28	20	26	23.846	3.976
6	1	7	4	7	6	8	4	6	5	5	8	6	5	5.538	1.898
7	2	2	1	3	3	2	3	2	4	2	3	5	3	2.692	1.032
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Observed Hearts.vex for Diamonds

Frequency of Cards of a Given Rank(Columns) per Number of Cards (Rows) in a Suit (335 hearts.vex hands)

	2	3	4	5	6	7	8	9	T	J	Q	K	A	mean	standard_dev
1	3	3	3	4	0	2	1	0	2	1	3	2	1	1.923	1.256
2	15	13	8	13	9	10	10	12	11	10	12	19	10	11.692	2.898
3	28	30	22	27	26	23	26	27	15	21	17	24	23	23.769	4.304
4	22	21	34	24	33	25	26	22	24	29	29	36	23	26.769	4.986
5	14	12	11	12	13	12	14	11	6	13	9	12	11	11.538	2.145
6	3	4	5	4	3	2	5	5	3	1	4	4	5	3.692	1.251
7	1	0	1	1	1	1	0	1	1	0	0	0	0	0.538	0.519
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Observed Hearts.vex for Hearts

Frequency of Cards of a Given Rank(Columns) per Number of Cards (Rows) in a Suit (335 hearts.vex hands)

	2	3	4	5	6	7	8	9	T	J	Q	K	A	mean	standard_dev
1	1	6	3	4	3	0	2	1	2	4	2	4	2	2.615	1.609
2	9	12	11	9	10	8	9	8	13	7	9	10	9	9.538	1.664

	2	3	4	5	6	7	8	9	T	J	Q	K	A	mean	standard_dev
3	17	18	23	19	25	21	22	20	23	23	30	23	27	22.385	3.595
4	26	18	17	14	28	38	25	18	36	28	31	26	23	25.231	7.259
5	19	18	13	11	11	18	19	13	19	20	20	15	14	16.154	3.412
6	6	7	3	6	6	5	7	7	7	4	2	6	6	5.538	1.613
7	2	1	1	1	0	0	2	2	1	1	2	1	0	1.077	0.76
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Observed Hearts.vex for Spades

Frequency of Cards of a Given Rank(Columns) per Number of Cards (Rows) in a Suit (335 hearts.vex hands)

	2	3	4	5	6	7	8	9	T	J	Q	K	A	mean	standard_dev
1	1	2	2	2	3	1	2	2	4	2	4	2	4	2.385	1.044
2	10	19	11	7	13	9	11	8	11	9	11	10	11	10.769	2.92
3	25	19	22	18	13	15	23	18	23	25	23	19	24	20.538	3.843
4	26	23	28	31	26	29	24	26	31	28	30	15	27	26.462	4.235
5	12	16	17	17	10	15	20	17	18	11	19	13	20	15.769	3.345
6	7	6	6	10	6	10	5	8	2	6	7	6	5	6.462	2.106
7	1	0	2	1	1	2	1	1	0	1	1	2	1	1.077	0.641
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

SECTION I.C.iii. Simulated Co-occurrence of Face Cards in a Single Player's 13 Card Hand

Simulated for Clubs

Counts (Columns) of Face Cards (10, J, Q, K, A)

within the Same Player's Hand per Number of Cards (Rows) in a Suit (100 sets of 335 random hands)

	0	1	2	3	4	5
1	10.34	0	0	0	0	0
2	35.77	8.46	0	0	0	0
3	47.24	26.78	3.59	0	0	0
4	31.45	31.51	9.7	0.63	0	0
5	11.65	17.98	8.58	1.23	0.02	0
6	2.07	5.6	4.66	1.11	0.03	0
7	0.32	0.96	1.13	0.53	0.03	0
8	0	0.07	0.16	0.12	0.07	0
9	0	0	0	0.03	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0

Simulated for Diamonds

Counts (Columns) of Face Cards (10, J, Q, K, A)

within the Same Player's Hand per Number of Cards (Rows) in a Suit (100 sets of 335 random hands)

	0	1	2	3	4	5
1	10.53	0	0	0	0	0
2	34.8	8.9	0	0	0	0
3	46.84	26.42	3.3	0	0	0
4	31.61	31.92	9.04	0.46	0	0
5	10.96	18.82	9.17	1.12	0.02	0
6	2.29	5.27	4.42	1	0.05	0
7	0.17	0.85	1.13	0.45	0.05	0
8	0	0.15	0.27	0.15	0.04	0
9	0	0	0	0	0.01	0
10	0	0	0	0.01	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0

Simulated for Hearts

Counts (Columns) of Face Cards (10, J, Q, K, A)

within the Same Player's Hand per Number of Cards (Rows) in a Suit (100 sets of 335 random hands)

	0	1	2	3	4	5
1	9.94	0	0	0	0	0
2	35.87	8.81	0	0	0	0
3	47.95	26.71	3.03	0	0	0
4	31.65	31.3	8.76	0.51	0	0
5	10.68	18.25	8.94	1.39	0.02	0
6	2.3	5.54	4.25	1.26	0.08	0
7	0.26	1.03	1.24	0.49	0.06	0
8	0	0.07	0.21	0.08	0.04	0
9	0	0	0.01	0.01	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0

Simulated for Spades

Counts (Columns) of Face Cards (10, J, Q, K, A)

within the Same Player's Hand per Number of Cards (Rows) in a Suit (100 sets of 335 random hands)

	0	1	2	3	4	5
1	9.97	0	0	0	0	0
2	35.36	8.68	0	0	0	0
3	46.78	27.37	3.2	0	0	0
4	31.57	31.43	9.26	0.56	0	0
5	11.26	18.28	8.96	1.4	0.04	0
6	2.38	5.9	4.63	1.04	0.09	0
7	0.18	0.91	1.21	0.5	0.1	0
8	0.01	0.09	0.18	0.14	0.04	0
9	0	0	0.01	0.02	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0

SECTION I.C.iv. Observed Co-occurrence of Face Cards in a Single Player's 13 Card Hand

Observed Hearts.vex for Clubs
 Counts (Columns) of Face Cards (10, J, Q, K, A)
 within the Same Player's Hand per Number of Cards (Rows) in a Suit (335 hearts.vex hands)

	0	1	2	3	4	5
1	7	0	0	0	0	0
2	37	4	0	0	0	0
3	40	16	2	0	0	0
4	38	27	11	0	0	0
5	15	25	16	3	0	0
6	1	6	4	1	0	0
7	0	1	1	3	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0

Observed Hearts.vex for Diamonds
 Counts (Columns) of Face Cards (10, J, Q, K, A)
 within the Same Player's Hand per Number of Cards (Rows) in a Suit (335 hearts.vex hands)

	0	1	2	3	4	5
1	9	0	0	0	0	0
2	42	10	0	0	0	0
3	48	23	2	0	0	0
4	37	29	14	1	0	0
5	8	14	5	0	0	0
6	1	2	4	0	0	0
7	1	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0

Observed Hearts.vex for Hearts
 Counts (Columns) of Face Cards (10, J, Q, K, A)
 within the Same Player's Hand per Number of Cards (Rows) in a Suit (335 hearts.vex hands)

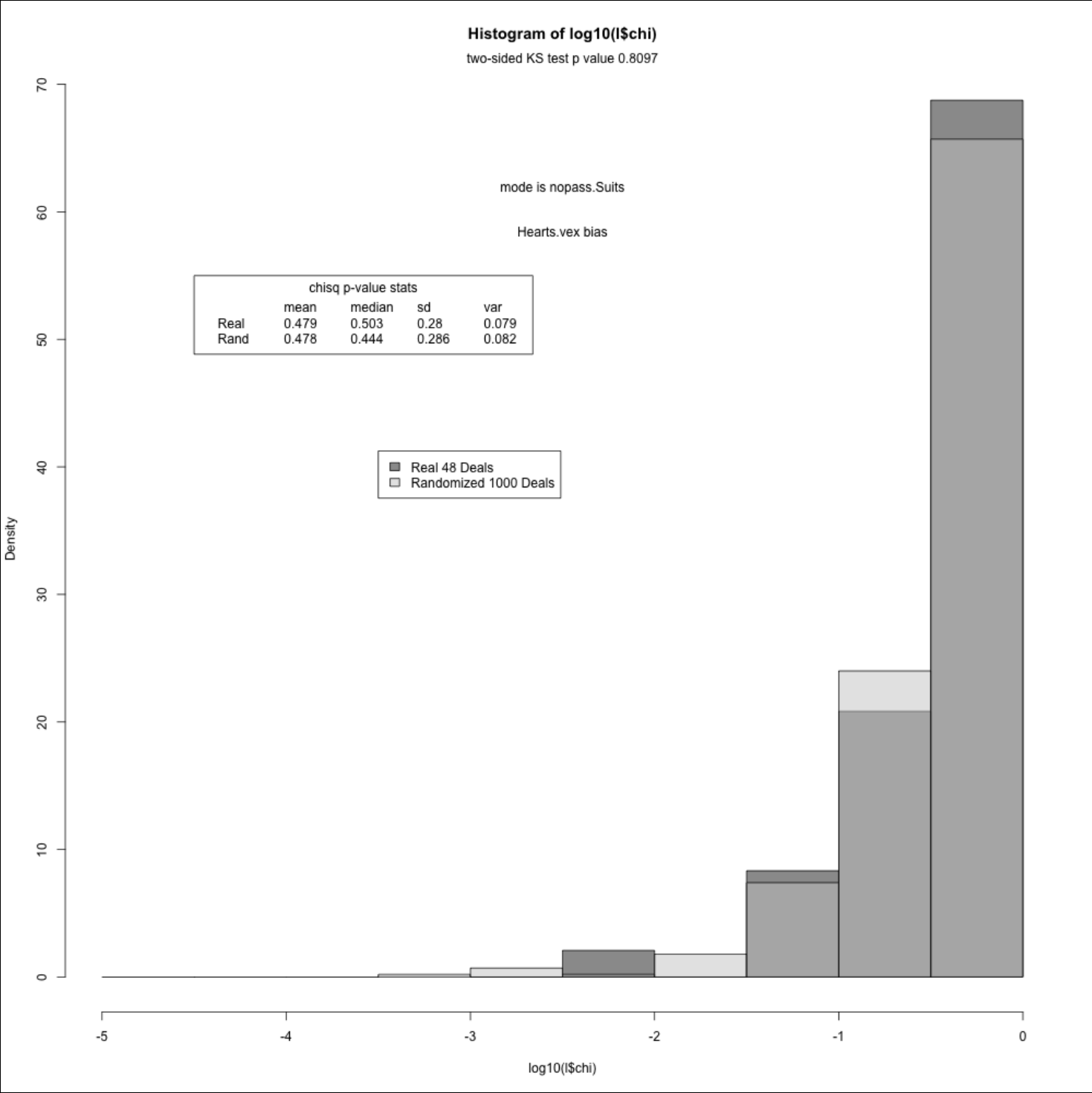
	0	1	2	3	4	5
1	14	0	0	0	0	0
2	32	8	0	0	0	0
3	52	28	6	0	0	0
4	24	42	12	0	0	0
5	10	19	8	4	0	0
6	2	7	3	0	0	0
7	0	1	1	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0

Observed Hearts.vex for Spades
 Counts (Columns) of Face Cards (10, J, Q, K, A)
 within the Same Player's Hand per Number of Cards (Rows) in a Suit (335 hearts.vex hands)

	0	1	2	3	4	5
1	16	0	0	0	0	0
2	32	10	0	0	0	0
3	33	36	3	0	0	0
4	33	31	12	0	0	0
5	11	15	8	4	0	0
6	2	6	4	0	0	0
7	0	1	1	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0

SECTION II. DISTRIBUTIONS ACROSS ALL 4 PLAYERS'S HANDS

SECTION II.A. Distribution of Suits of Cards: Observed versus Theoretical (Obtained by Simulation)



SECTION II.B. Distribution of Ranks of Cards: Observed versus Theoretical (Obtained by Simulation)

