CSIT 5th Semester
Simulation and Modeling

Unit-1: Introduction to simulation

Syllabus:

1. System concept, Boundary, environment
2. Continuous and Discrete System, Real time simulation
3. Types of simulation model(Static Physical, Dynamic Physical, Static Mathematical)
4. Principles used in modeling, Distributed lag model
5. Phases and steps in simulation study
6. Advantages and Disadvantages of Simulation
7. Areas of Application.

System and System Environment:

System:

A *system* is a group of objects that are joined together in some regular interaction or interdependence toward the accomplishment of some purpose. Or in another words a system is a group of entities that coexist and interact, usually towards the accomplishment of some goal.
An example is a production system which is manufacturing automobiles. The machines, component parts, and workers operate jointly along an assembly line to produce the high-quality vehicles.
Systems are embedded within a larger environment, called the system environment. There is typically an exchange of information and resources between the system and its environment.
A system is usually considered as a set of inter–related factors, which are described as entities, activities and have properties or attributes. Processes that cause system changes are called activities. The state of a system is a description of all entities, attributes and the activities at any time.

Hence System is ..

- An assembly of objects joined in some regular interaction or interdependence.
- It exists and operates in time and space.
- It is bounded inside system boundary

Figure below illustrates the fact that a system is embedded within an environment and that typically a system requires inputs and produces output using internal components.

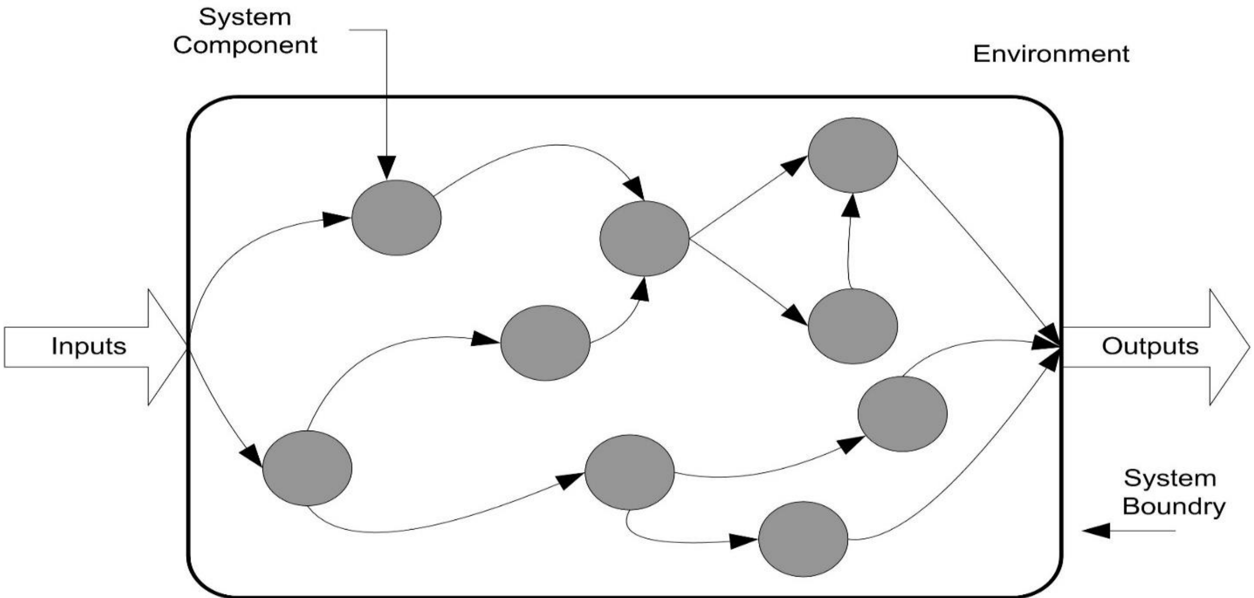


Figure : Conceptualization of a system

Examples of System:

Example 1: A factory system shown as follows:

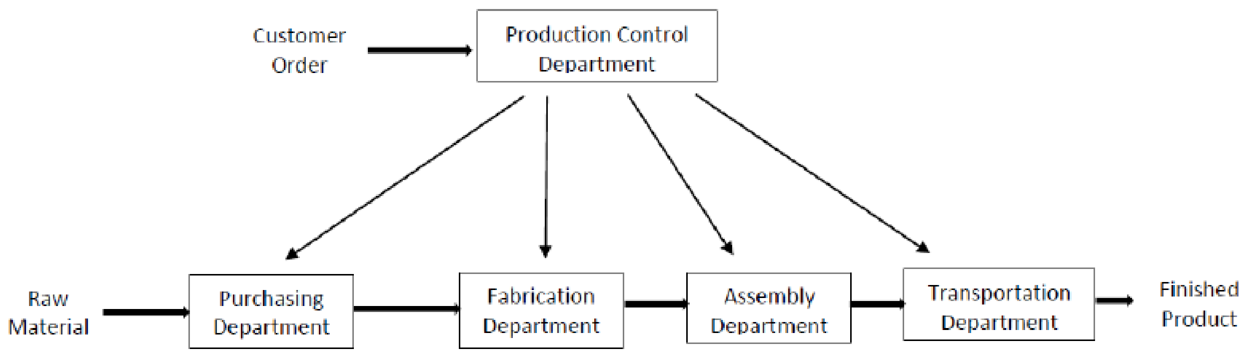


Fig: Factory System

Consider a factory system that takes orders from customers and produces a finished product. In the above system, Production Control Department takes orders direct from customers and Purchasing Department to purchase raw material to carry out order. The Fabrication Department fabricates (*Process*) the raw material and the fabricated raw materials are assembled by Assembly Department and are sent to Shipping Department for the dispatch of finished product.

System Environment:

The external components which interact with the system and produce necessary changes are said to be system environment. Or in another words the components outside the boundary are system environment which can affect the system. In modeling a system, it is necessary to decide on the *boundary* between the system and its environment. This decision may depend on the purpose of the study.

Example 1: When studying cache memory using simulation, one has to decide, where is the boundary of the system? It can be simply the CPU and cache, or it can include main memory, disk, O.S., compiler, or even user programs. The components outside the boundary are system environment which can affect the system.

Example 2: In a factory system, the factors controlling arrival of orders may be considered to be outside the factory but yet a part of the system environment. When, we consider the demand and supply of goods, there is certainly a relationship between the factory output and arrival of orders. This relationship is considered as an activity of the system.

Components of System:

Entity: An *entity* is an object of interest in the system. E.g. customers in a bank.
Attribute: The property or the characteristics of an entity is called attribute.
Activity: The time-bound process, task, or operation that occurs within a system and causes a change in the system's state over a specified period. E.g. deposit money into the checking account at a specified date and time.
Event: The instantaneous occurrence that may change the state of a system is called event. There are 2 types of event:

a) ***Endogenous Event:*** The event that occurs within the system and change the state of the system is called endogenous event.
b) ***Exogenous Event:*** The event that occurs outside the system but affects the state of the system is called exogenous event.

Example:

System	Entities	Attributes	Activities
Traffic System	Car, Bike, Pedestrian, Road, Traffic Lights, Traffic Police	Car Speed, Bike Model, Bike Number, Road Turnings, Light Colors, Traffic police name	Driving, Riding, Lightning, Turning, Walking, Traffic Managing
Bank System	Customer, Tailors	Name, balance, Duty time	Depositing, Withdrawing, Account opening
Canteen System	Manager, Cook, Server, Customer	Manager_name, Customer_order, Menu	Ordering, Cooking, Serving, Paying

In the Bank example, an ***endogenous event*** is the beginning of service of the customer since that is within the system being simulated. An ***exogenous event*** is the arrival of a customer for service since that occurrence is outside of the simulation.

State Variables:

State variables are the set of variables used to describe the current state or condition of a system at any given time. These variables capture the essential characteristics or properties of the system that can change over time.

In another words, it is a collection of variables necessary to describe the system at any time, relative to the objectives of the study.

Features of State Variables

- Descriptive: They describe the system's state at a particular point in time.
- Dynamic: They may change as the system evolves due to activities or events.
- Unique Representation: Together, the state variables provide a complete and unique representation of the system's state.

In the study of a bank, possible state variables are:

- The number of busy tellers,
- The number of customers waiting in line or being served
- The arrival time of the next customer

So the system state variables are the collection of all information needed to define what is happening within the system to a sufficient level (i.e., to attain the desired output) at a given point in time.

Table: Example of systems and components

System	Entities	Attributes	Activities	Events	State Variables
Banking	Customers	Checking account balance	Making deposits	Arrival; departure	Number of busy tellers; number of customers waiting
Rapid rail	Riders	Origination; destination	Traveling	Arrival at station; arrival at destination	Number of riders waiting at each station; number of riders in transit
Production	Machines	Speed; capacity; breakdown rate	Welding; stamping	Breakdown	Status of machines (busy, idle, or down)
Communications	Messages	Length; destination	Transmitting	Arrival	Number waiting to be transmitted
Inventory	Warehouse	Capacity	Withdrawing	Demand	Levels of inventory; backlogged demands

Open System/Close System:

Open System:

A system with exogenous activities is considered as open system.
Example: Bank System

Close System:

A system with strict endogenous activities is called a closed system.
Example: Water in a thermos.

Endogenous System:

The term endogenous is used to describe activities and events occurring within a system.
Example: Drawing cash in a bank.

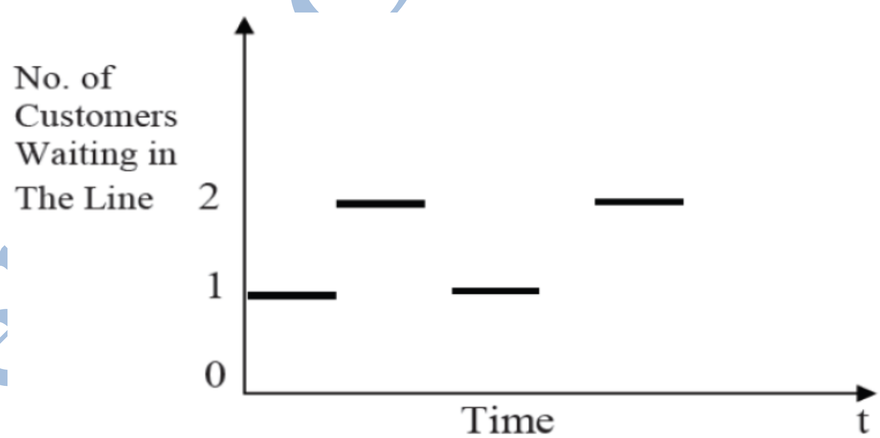
Exogenous System:

The term exogenous is used to describe activities and events in the environment that affect the system. Example: Arrival of customers.

Discrete and Continuous System:

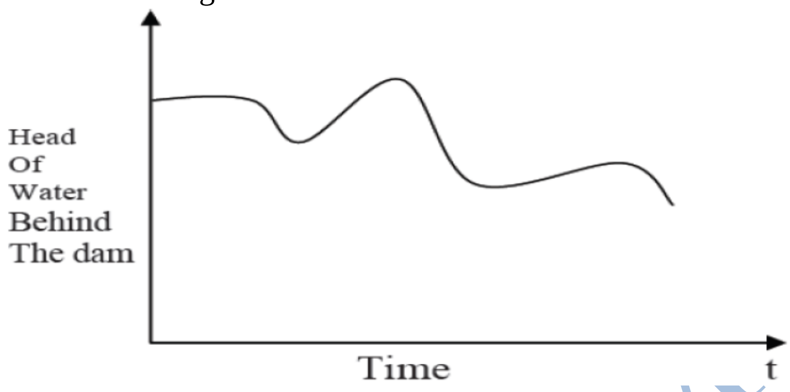
Discrete System:

Discrete system is one in which the state variables changes only at a discrete set of time. Systems in which the changes are predominantly (mostly) discontinuous are called discrete systems.
For example: banking system in which no of customers (state variable) changes only when a customer arrives or service provided to customer i.e. customer depart form system.
The figure below show how no of customer changes only at discrete points in time.



Continuous System:

Continuous system is one in which the state variables change continuously over time.
For example, during winter seasons, level of water decreases gradually and during rainy season level of water increase gradually. The change in water level is continuous.
The figure below shows the change of water level over time.



System Simulation:

Simulation

Simulation is the imitation (*copying, artificial*) of the operation of a real-world process or system over time. Simulation involves the generation of an artificial history of the system, and the observation of that artificial history to draw inferences concerning the operating characteristics of the real system.

Simulation is ***used before an existing system*** is altered or a ***new system built***, to reduce the chances of **failure** to meet specifications, to eliminate unforeseen bottlenecks, to prevent under or over-utilization of resources, and to optimize system performance.

Simulation is the numerical technique for conducting experiments on digital computer, which involves logical and mathematical relationships that interact to describe the behavior and the structure of a complex real world system over extended period of time.

The process of designing a model of a real system, implementing the model as a computer program, and conducting experiments with the model for the purpose of understanding the behavior of the system, or evaluating strategies for the operation of the system.

The behavior of a system as it evolves over time is studied by developing a simulation model. This model usually takes the form of a set of assumptions concerning the operation of the system.

These assumptions are expressed in:

- Mathematical
- Logical
- Symbolic

Real Time Simulation:

Real time simulation refers to a computer model of a physical system that can execute at the same rate as actual "wall clock" time. In other words, the computer model runs at the same rate as the actual physical system.

For example, if a tank takes 10 minutes to fill in the real-world, the simulation would take 10 minutes as well.

Real-time simulation occurs commonly in computer gaming, statistical power grid protection tests, aircraft design and simulation, motor drive controller design methods and space robot integration are a few examples of real-time simulator technology applications.

Real time simulation is an approach where an actual device (hardware or software) can be used rather than constructing a model. With this techniques actual devices which are part of a system are used in conjunction with either a digital or hybrid computer, providing a simulation of the parts of the system that do not exist or that cannot continently used in an experiment. Real time simulation will often involve interaction with a human being, there by avoiding the need to design and validate a model of human behavior.

Real time simulation requires computers that receive signals and respond to it which are sent from physical devices and transfer output signals at specific points in time.

For Example: Devices for training pilots by giving them the impression they are at the controls of an aircraft in such an environment where gravity is 1/6th part of earth and trained to perform different activities

Hybrid Simulation:

Hybrid Simulation combines multiple simulation techniques or models to achieve a more comprehensive or realistic representation of a system. It integrates elements of **discrete-event simulation (DES)**, **continuous simulation**, and sometimes **agent-based simulation (ABS)** to analyze complex systems that involve both discrete and continuous processes.

For most studies, the system under study is clearly either of continuous or discrete nature and it is the determining factor in deciding whether to use an analog or digital computer for system simulation. If the system being simulated is an interconnection of continuous and discrete subsystem, then such system simulation is known as hybrid simulation. Such hybrid system can be digital computer being linked together. Hybrid simulation required high speed converters to transform signals from analog to digital from and vice –versa.

What is system modeling?

System model:

A model is defined as a representation of a system for the purpose of studying that system. In practice, what is meant by "the system" depends on the objectives of a particular study. It is necessary to consider only those aspects of the system that affect the problem under investigation. These aspects are represented in a model, and by definition it is a simplification of the system. The aspect of system that affect the problem under investigation, are represented in a model of the system. Therefore model is the simplification of the real system.

Or in another words, we define a model as a body of information about a system gathered for the purpose of studying the system. In case of physical model, the information is embodied in the properties of the model, in contrast to the symbolic representation in a mathematical model. Model is also a simplification of the system, because it is not necessary to consider all the details of a system in most system studies.

Hence, System modeling in simulation and modeling refers to the process of creating a simplified representation of a real-world system. This representation typically involves mathematical equations, algorithms, diagrams, or other forms of abstraction that capture the behavior and interactions of the components within the system. The purpose of system modeling is to gain insights into how the system functions, predict its behavior under different conditions, and facilitate decision-making.

There is no unique model of a system. Different models of the same system will be produced by different system analysts who are interested in different aspect of system.
Models can take various forms, including mathematical equations, graphical representations, simulation software, and computational algorithms. The choice of representation depends on the nature of the system being modeled and the objectives of the modeling effort.

The task of deriving a model of a system may be divided broadly into two subtasks:

- Establishing model parameter
- Supplying data

Establishing model structure determines system boundary and identifies the entities, attributes, activities and events of a system.
Supplying data provides value contained an attribute and define relationships involved in the activities.

Types of Model:

- Static Model
- Dynamic Model
- Analytical Model
- Numerical Model
- Deterministic Model
- Stochastic Model
- Discrete Model
- Continuous Model

Static Model:

Static models can only show the values that the system attributes value does not change over time. In another words a static simulation model , sometimes called Monte Carlo simulation, represents a system at a particular point in a time.

An example could be a snapshot of a city's population distribution at a particular moment, without considering population changes over time.

Dynamic Model:

Dynamic models follow the changes over time that result from system activities. The mechanical and electrical systems are the example of dynamic system. Generally, dynamic models involve the computation of variable value over time and hence they are represented by differential equations.

A dynamic model represents how a system changes over time. An example could be a model of population growth that considers birth rates, death rates, and migration patterns over multiple time periods.

Analytical Models:

In mathematical model, we can differentiate the model on the basis of solution technique used to solve the model. Analytical technique means using deductive reasoning of mathematical theory to solve a model. Such models are known as analytical model.

An analytical model uses explicit mathematical equations to describe the behavior of a system. An example could be Newton's laws of motion, which describe the motion of objects based on forces acting upon them.

Numerical models:

Numerical models involve applying computational (*theoretical*) process to solve equations. Or in another words; A numerical model approximates the behavior of a system using computational techniques.

For example: we may solve differential equation numerically when the specific limit of variable is given. The analytical methods to produce solution may take situation numerical methods are preferred.

Another example could be a weather forecasting model that uses numerical methods to solve the equations governing atmospheric dynamics.

Deterministic Model:

It contains no random variables. They have a known set of inputs which will result in a unique set of outputs. A deterministic model assumes that the system's behavior is entirely predictable based on its initial conditions and parameters.

An example could be a model of planetary motion using Newton's laws, where the future positions of planets can be precisely calculated. Similarly another example could be, Arrival of patients to the Dentist at the scheduled appointment time.

Stochastic Model:

It has one or more random variable as inputs. Random inputs leads to random outputs.
A stochastic model incorporates randomness or uncertainty into the system's behavior. An example could be a financial market model that includes random fluctuations in stock prices to capture market volatility. Similarly another example, Simulation of a bank involves random inter-arrival and service times.

Each of these models are used to categorize the major there models

- Physical Model
- Mathematical Model
- Computer Model

Physical Model:

A physical model is a larger or smaller version of an object such as the enlargement of an atom or scale-down version of the solar system. These models are based on some analogy between mechanical and electrical system.

- The system attributes are represented by physical measures such as voltage.
- The system activities are represented by physical laws.

Physical models are of two types:

- **Static Physical Model**
- **Dynamic Physical Model**

Static physical model:

It represents a system at a particular point of time. In this model, the measurements are taken to represent attributes of the system being study under one set of equilibrium (balance) condition. In this case, the measurements do not translate directly into system attribute values. Well known laws of similitude are used to convert measurement on the scale model to the values that will occur in the real system.

For example, an architect before constructing a building makes a scaled down model of the building, which reflects all it rooms, outer design and other important features. This is an example of static physical model.

Similarly for conducting trials in water, we make small water tanks, which are replica of sea, and fire small scaled down shells in them. This tank can be treated as a static physical model of ocean.

The flow of heat and the distribution of electric charge through space can be related by common equations.

Dynamic physical models:

Dynamic physical models are ones which change with time or which are functions of time. This model is rely upon an analogy between the system being studied and some other system of different nature, the analogy usually depending upon an underlying similarities in the forces governing the behavior of the systems.

In wind tunnel, small aircraft models (static models) are kept and air is blown over them with different velocities and pressure profiles are measured with the help of transducers embedded in the model. Here wind velocity changes with time and is an example of dynamic physical model. A model of a hanging wheel of vehicle is another case of dynamic physical model .

To illustrate this type of physical model, consider the two systems shown in following figures.

Example-1:- Wheel suspension system (Mechanical System).

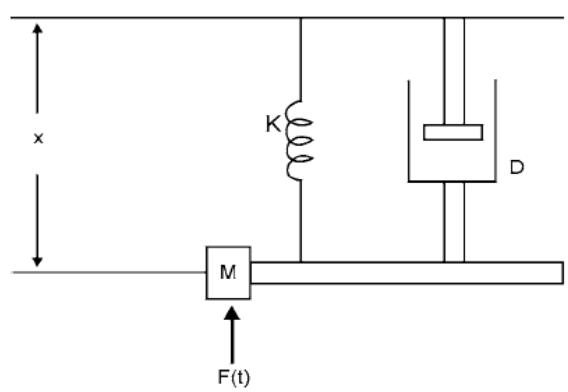


Figure (a): Mechanical System

Let us take an example of hanging wheel of a stationary truck and analyze its motion under various forces. Fig (a) above shows the wheel suspension system which is Mechanical System. It has a wheel of mass *M*, suspended in vertical direction, a force *F(t)*, which varies with time, is acting on it. Mass is connected with a spring of stiffness *K*, and a piston with damping factor *D*. When force *F* (t), is applied, mass *M* oscillates (*swing*) under the action of these three forces. This model can be used to study the oscillations in a motor wheel.

The motion of the system is described by the following differential equation:

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

Where,

M = Mass
D = Damping factor of the shock absorber
K = Stiffness of string
X = Distance moved

This is a discrete physical static model. Discrete in a sense, that one can give discrete values F and observe the oscillations of wheel with some measuring equipment. When force is applied on it, which is a function of time, this discrete physical static model becomes dynamic model. Parameters K and D can also be adjusted in order to get controlled oscillations of the wheel. This type of system is called spring-mass system. Load on the beams of a building can be studied by the combination of spring mass system.

Example-2: Electrical System

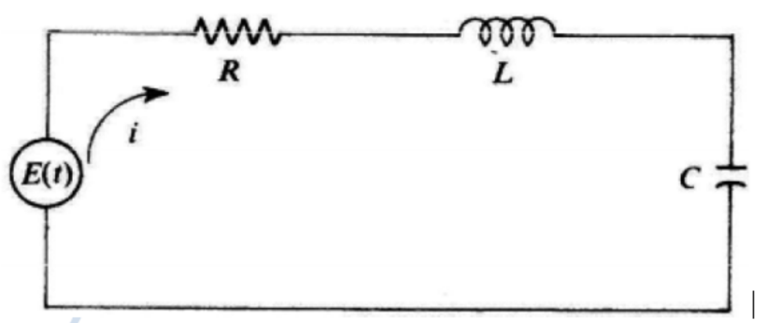


Fig (b): Electrical System

Fig (b) represents an electrical circuit with an inductance L, a resistance R, and a capacitance C, connected in series with a voltage source that varies in time according to the function E(t). If q is the charge on the capacitance, it can be shown that the behavior of the circuit is governed by the following differential equation:

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = \frac{E(t)}{C}$$

Where,
q=charge
R=resistance
C= capacitance

Inspection of these two equations shows that they have exactly same form and the following equivalences occur between the quantities in the two systems:

Displacement x	Charge q
Velocity $\dot{x}$	Current $I(=\dot{q})$
Force F	Voltage E
Mass M	Inductance L
Damping factor D	Resistance R
Spring stiffness K	1/Capacitance $1/C$

The mechanical system and the electrical system are analogs of each other, and the performance of either can be studied with the other. In practice, it is simpler to modify the electrical system than to change the mechanical system, so it is more likely that the electrical system will have been built to study the mechanical system.

Mathematical Model:

A mathematical model uses symbolic notation and mathematical equation to represent system. The system attributes are represented by variables and the activities are represented by mathematical function.

Most of the systems in general can be transformed into mathematical equations. These equations are called the mathematical model of that system.

Example: Example of a Mathematical Model

The equation of a line: $f(x)=mx+c$
Where,
m= Slope of the line.
c= Intercept on the y-axis.

This model describes the relationship between x (independent variable) and f(x) (dependent variable).

Mathematical model further divide in to two types:

- Static Mathematical Model
- Dynamic Mathematical Model

Static Mathematical Model:

If mathematical model does not involve time i.e., system does not change with time, it is called a static mathematical model of the system. A static model gives relationships between the system attributes when the system is in equilibrium.

Example: Static Market Model

Generally there should be a balance between the supply and demand of any product in the market. Supply increases if the price is higher. This is because shopkeeper gets more commission on that product and tries to push the product to the customers even if quality is not excellent. Customer generally feels that more cost means better quality. But on the other hand demand decreases with the increase of price. Aim is to find the optimum price with which demand can match the supply.

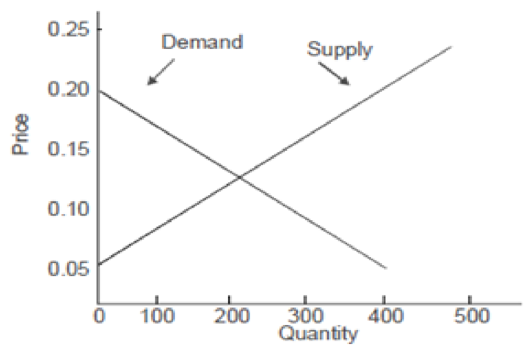


Figure a: Market Model

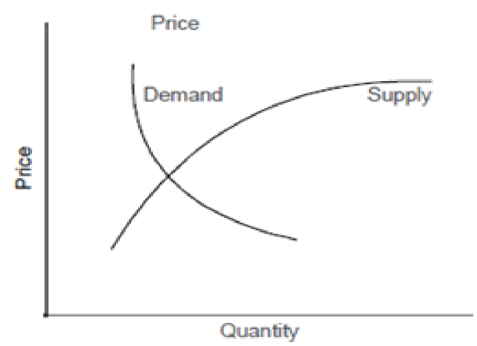


Figure (b): Non-linear market model

Let us model this situation mathematically. In this scenario, we are modeling the balance between **supply (S)** and **demand (D)** in a market to determine the **optimum price (P)** where supply equals demand.

If we denote price by *P*, supply by *S* and demand by *D*, and assuming the price equation to be linear we have

Demand Function:

$D=a-bP$

Where, 'a' and 'b'are coefficients representing parameters of the demand functions.

- a is the intercept of the demand curve. It represents the quantity demanded when the price (P) is zero. It indicates the maximum quantity consumers are willing to buy at a price of zero.

- b is the slope of the demand curve. It represents how much the quantity demanded changes for a unit change in price (P). A higher absolute value of b indicates a steeper demand curve, meaning quantity demanded changes more significantly with price changes.

Supply Function:

S=c+dP

Where, ‘c’ and ‘d’are coefficients representing parameters of the supply functions.

- c is the intercept of the supply curve. It represents the quantity supplied when the price (P) is zero. It indicates the minimum quantity producers are willing to supply at a price of zero.
- d is the slope of the supply curve. It represents how much the quantity supplied changes for a unit change in price (P). A higher absolute value of d indicates a steeper supply curve, meaning quantity supplied changes more significantly with price changes.

Equilibrium Condition:
At equilibrium, supply equals demand:

S=D

Substituting the equations for D and S:

a−bP=c+dP

The coefficients a,b,c and d are determined based on factors such as consumer behavior, producer behavior, market conditions, production costs, preferences, and external influences. They essentially quantify the relationships between price and quantity demanded (in the case of demand) and price and quantity supplied (in the case of supply). These parameters are computed based on previous market data.

Solving for Price P

a−bP=c+dP

Rearranging terms:

$$a - c = bP + dP$$

$$a - c = P(b + d)$$

$$P = \frac{a - c}{b + d}$$

Example-1:
a = 500,
b = 2000,
c = –50 and
d =1500.

Value of *c* is taken negative, since supply cannot be possible if price of the item is zero. In this case no doubt equilibrium market price will be…

$$P = \frac{a-c}{b+d} = \frac{550}{3500} = 0.1571$$
$$S = 186$$

Example-2:

Suppose:
a=100: Maximum demand when P=0.
b=2: Demand decreases by 2 units for every unit increase in price.
c=20: Minimum supply when P=0.
d=3: Supply increases by 3 units for every unit increase in price.

Step 1: Calculate Equilibrium Price

$$P = \frac{a-c}{b+d} = \frac{100-20}{2+3} = \frac{80}{5} = 16$$

Step 2: Calculate Equilibrium Quantity

Using the demand equation:
D=a−bP
=100−2(16)
=100−32
=68

Or the supply equation:
S=c+dP
=20+3(16)
=20+48
=68

At equilibrium:
P=16,
Q=68

This model provides the optimum price (P) and quantity (Q) where supply matches demand, ensuring a balanced market.

Nonlinear Supply and Demand Functions

In real-world markets, the relationships between supply, demand, and price are often nonlinear due to complex factors like diminishing returns, consumer behavior, or production constraints.

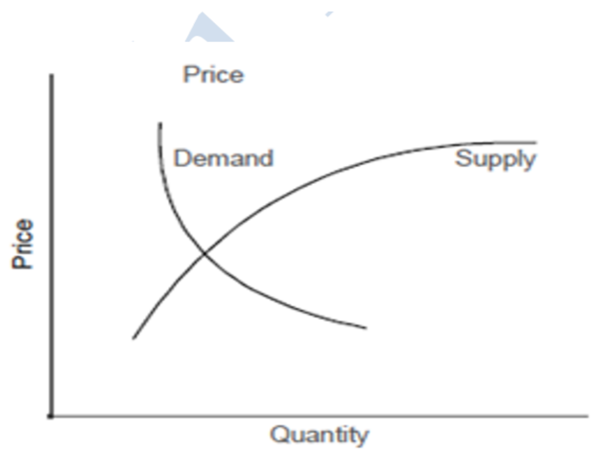


Figure (b): Non-linear market model

Nonlinear Demand Function

Instead of a linear demand function (D=a−bP), consider a **quadratic demand function**:

D=a−bP^2

$$D = a - bP^2$$

- a: Maximum demand at P=0.
- b: Nonlinear sensitivity of demand to price.
- As P increases, the rate at which demand decreases becomes steeper.

Nonlinear Supply Function

Similarly, instead of a linear supply function (S=c+dP), consider an **exponential supply function**:

$S=c\cdot e^{dP}$

$$S = c \cdot e^{dP}$$

- c: Minimum supply at P=00.
- d: Growth rate of supply with price.
- Supply increases exponentially with price.

Equilibrium Condition

At equilibrium, S=D

$$a - bP^2 = c \cdot e^{dP}$$

This equation is **nonlinear** and typically requires numerical methods or approximation techniques to solve for P. Once P is found, substitute it into either S or D to find the equilibrium quantity Q.

Example

Suppose:

- a=100: Maximum demand.
- b=0.5: Nonlinear sensitivity of demand to price.
- c=20: Minimum supply.
- d=0.1: Exponential growth rate of supply.

At equilibrium:

$$100 - 0.5P^2 = 20 \cdot e^{0.1P}$$

$$f(P) = 100 - 0.5P^2 - 20 \cdot e^{0.1P} = 0$$

Dynamic Mathematical Model:

If mathematical model of a system are derived as a function of time, such model is dynamic mathematical model, such as model of Wheel Suspension System. The derivation may be made with an **analytical solution or with a numerical computation**, depending upon the complexity of the model.

The equation that was derived to describe the behavior of a car wheel is an example of a dynamic mathematical model, in this case an equation that can be solved analytically

$$M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx = KF(t)$$

It is customary to write the equation in the form

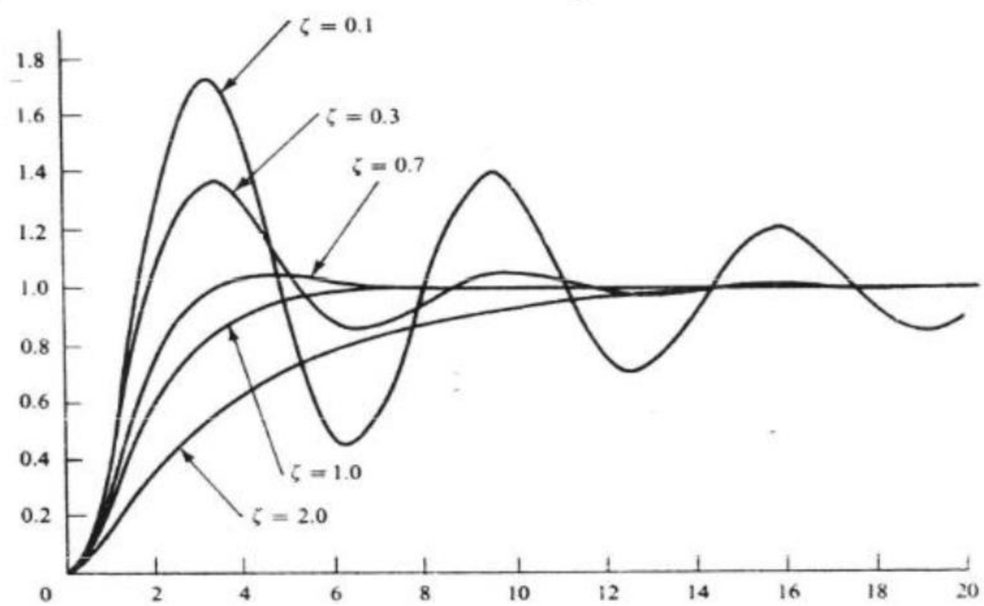
$$\frac{d^2x}{dt^2} + 2\zeta\omega\frac{dx}{dt} + \omega^2x = \omega^2F(t)$$

where $2\zeta\omega = D/M$ and $\omega^2 = K/M$.

- ζ(zeta)
- Ω(omega)

Expressed in this form, solution can be given in terms of the variable wt. figure bellow shows how x varies in response to a steady force applied at time t =0, as would occur, for instance, if a load were suddenly placed on the automobile.

Solutions are shown for several values of ζ, and it can be seen that when ζ is less than 1, the motion is oscillatory.



"The factor ζ(zeta) is called the damping ratio and, when the motion is oscillatory, the frequency of oscillation is determined from the formula.

$$\omega = 2\pi f$$

Where f is the number of cycles per second."

Suppose a case is selected is representing a satisfactory frequency and damping. The relationship given above between ζ(zeta), ω(omega), M, k and D show how to select the spring and shock absorber to get that type of motion. For example, the condition for the motion to occur without oscillation requires that $\zeta \geq 1$. It can be deduced from the definition of ζ that the condition requires that D^2 ≥4MK.

Distributed lag model:

Models that have the properties of changing only at fixed intervals of time, and of basing current values of the variables on other current values and values that occurred in previous intervals, are called distributed lag models. In another words it is a form of econometric model that incorporates the effects of past values of an independent variable on the current value of a dependent variable.

These are a type of dynamic models, because time factor is involved in them. They are extensively used in econometric studies where the uniform steps correspond to a time interval, such as a month or a year, over which some economic data are collected. As a rule, these models consist of linear, algebraic equations. They represent a continuous system, but the one in which data is only available at fixed points in time.

The features of distributed lag model:

- **Time Factor:**
  Distributed lag models are dynamic models that explicitly incorporate the time dimension. The influence of an independent variable is distributed across several time periods.
- **Fixed Intervals:**
  Changes in variables are observed at fixed intervals, such as monthly or yearly, corresponding to how the data is collected.
- **Dependence on Past Values:**
  Current values of the dependent variable are influenced not only by current values of independent variables but also by their values in previous time periods.
- **Linear Structure:**
  These models typically consist of linear algebraic equations, simplifying the relationships between variables.
- **Data Characteristics**
  Although they represent continuous systems, data for these models is available only at discrete time points.

As an example, consider the following simple dynamic mathematical model of the national economy.
Let,
C be consumption,
I be investment,
T be taxes,
G be government expenditure and
Y be national income.

Then

$$\left.\begin{aligned} C &= 20 + 0.7(Y - T) \\ I &= 2 + 0.1Y \\ T &= 0 + 0.2Y \\ Y &= C + I + G \end{aligned}\right\} \quad \text{..........(1)}$$

All quantities are expressed in billions of rupees.

This is a static model, but it can be made dynamic by picking a fixed time interval, say one year and expressed the current value of the variable in terms of values of the previous year. Any variable that appears in the form of its current and one or more previous year's values is called logged variables. Value of the previous year is denoted by the suffix with -1.

The static model can be made dynamic by logging all the variables, as follows

$$\left.\begin{aligned} C &= 20 + 0.7(Y_{-1} - T_{-1}) \\ I &= 2 + 0.1Y_{-1} \\ T &= 0.2Y_{-1} \\ Y &= C_{-1} + I_{-1} + G_{-1} \end{aligned}\right\} \quad \text{...(2)}$$

In these equations if values for the previous year (with –1 subscript) is known, then values for the current event can be computed. Taking these values as the input, values for the next year can also be computed. In equation (2) we have four equations in five unknown variables.
It is however not necessary to lag all the variable like it is done in equation (2). Only one of the variables can be lagged and others can be expressed in terms of this variable.
We solve equation for *Y* in equation (1) as:

$$Y = 20 + 0.7(Y - 0.2Y) + I + G$$
$$= 20 + 0.56Y + I + G$$

or
$$Y = 45.45 + 2.27(I + G)$$

Thus we have,

$$\left.\begin{aligned} I &= 2.0 + 0.1Y_{-1} \\ Y &= 45.45 + 2.27(I + G) \\ T &= 0.2Y \\ C &= 20 + 0.7(Y - T) \end{aligned}\right\} \quad \text{...........(3)}$$

In equations (3) only lagged parameter is *Y*. Assuming that government expenditure is known for the current year, we first compute *I*. Knowing *I* and *G*, *Y* and *T* for the current year is known, and thus *C* is computed from the last equation. In this problem, lagged model is quite simple and can be computed with hand calculator. But national economic models are generally not that simple and require long computations with number of parameters.

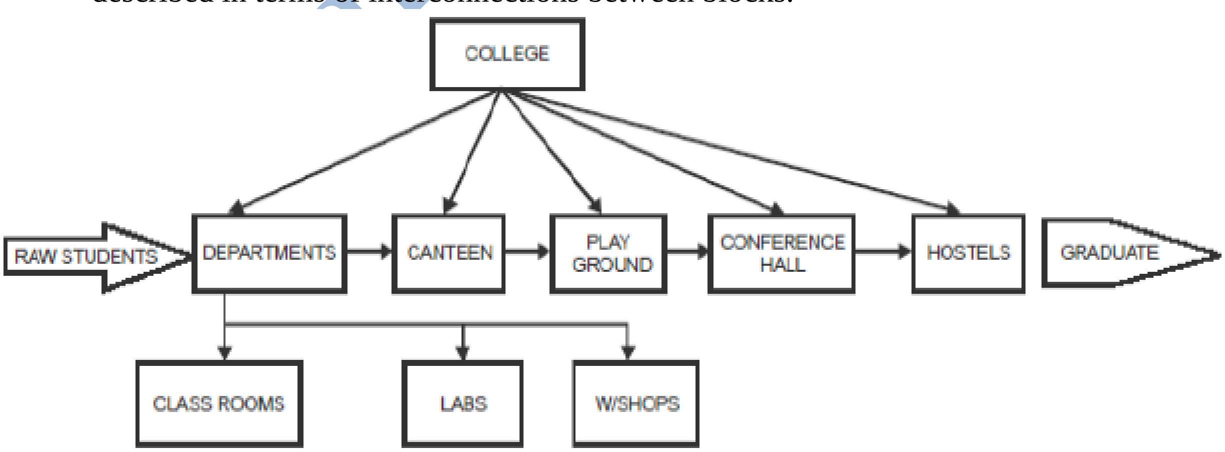
Principles used in Modeling:

Following are the principle guidelines used in modeling

1. **Block Building:**

   The description of system should be organized as a sequence of blocks. It simplifies the interaction between block within system. Then it will be easy to describe the whole system in terms of interaction between the block and can be represented graphically as simple block diagram.

   Each block represents a part of the system and the system as a whole can be described in terms of interconnections between blocks.



2. **Relevance:**
   The model should only include relevant information. For example, if the factory system study aims to compare the different operating rules efficiency, it is not relevant to consider the mining of employee as an activity. Irrelevant information should not include despite of being no harm because it increases the complexity of model and takes more time and effort to solve model.
3. **Accuracy:**
   The gathered information should be accurate as well. For example in aircraft system the accuracy as movement of the aircraft depends upon the representations of airframe such as a rigid body.
4. **Aggregation:**
   It should be considered that to which numbers of individual entities can be grouped into a block. For example in factory system, different department are grouped together handled by production manager.

Steps on simulation Study:

Step 1: Problem formulation
Every study begins with a statement of the problem, provided by policy makers. Analyst ensures it is clearly understood.

Step 2: Setting of objectives and overall project plan
The objectives indicate the questions to be answered by simulation. At this point a determination should be made concerning whether simulation has the appropriate methodology. Assuming it is appropriate, the overall project plan should include

- A statement of the alternative systems
- A method for evaluating the effectiveness of these alternatives
- Plans for the study in terms of the number of people involved
- Cost of the study
- The number of days required to accomplish each phase of the work with the anticipated results.

Step 3: Model conceptualization
The construction of a model of a system is probably as much art as science. The art of modeling is enhanced by ability:

- To abstract the essential features of a problem.
- To select and modify basic assumptions that characterizes the system.
- To enrich and elaborate the model until a useful approximation results.

Thus, it is best to start with a simple model and build toward greater complexity. Model conceptualization enhances the quality of the resulting model and increases the confidence of the model user in the application of model.

Step 4: Data collection
There is a constant interplay between the construction of model and the collection of needed input data. It is done in the early stages. Objective kinds of data are collected.

Step 5: Model translation
Real-world systems result in models that require a great deal of information storage and computation. It can be programmed by using simulation languages or special purpose simulation software. Simulation languages are powerful and flexible. Simulation software models development time can be reduced.

Step 6: Verified
It provide to the computer program and checking the performance. If the input parameters and logical structure are correctly represented, verification is completed.

Step 7: Validated
It is the determination that a model is an accurate representation of the real system. It is achieved through calibration of the model. The calibration of model is an iterative process of comparing the model to actual system behavior and the discrepancies between the two.

Step 8: Experimental Design
The alternatives that are to be simulated must be determined. Which alternatives to simulate may be a function of runs? For each system design, decisions need to be made concerning

- Length of the initialization period
- Length of simulation runs
- Number of replication to be made of each run

Step 9: Production runs and analysis
They are used to estimate measures of performance for the system designs that are being simulated.

Step 10: More runs
Based on the analysis of runs that have been completed, the analyst determines if additional runs are needed and what design those additional experiments should follow.

Step 11: Documentation and reporting
Two types of documentation:

- Program documentation
- Process documentation

Program documentation:
Can be used again by the same or different analysts to understand how the program operates. Further modification will be easier. Model users can change the input parameters for better performance.

Process documentation:
It gives the history of a simulation project. The result of all analysis should be reported clearly and concisely in a final report. This enables to review the final formulation and alternatives, results of the experiments and the recommended solution to the problem. The final report provides a vehicle of certification.

Step 12: Implementation
Success depends on the previous steps. If the model user has been thoroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced. The simulation model building can be broken into 4 phases.

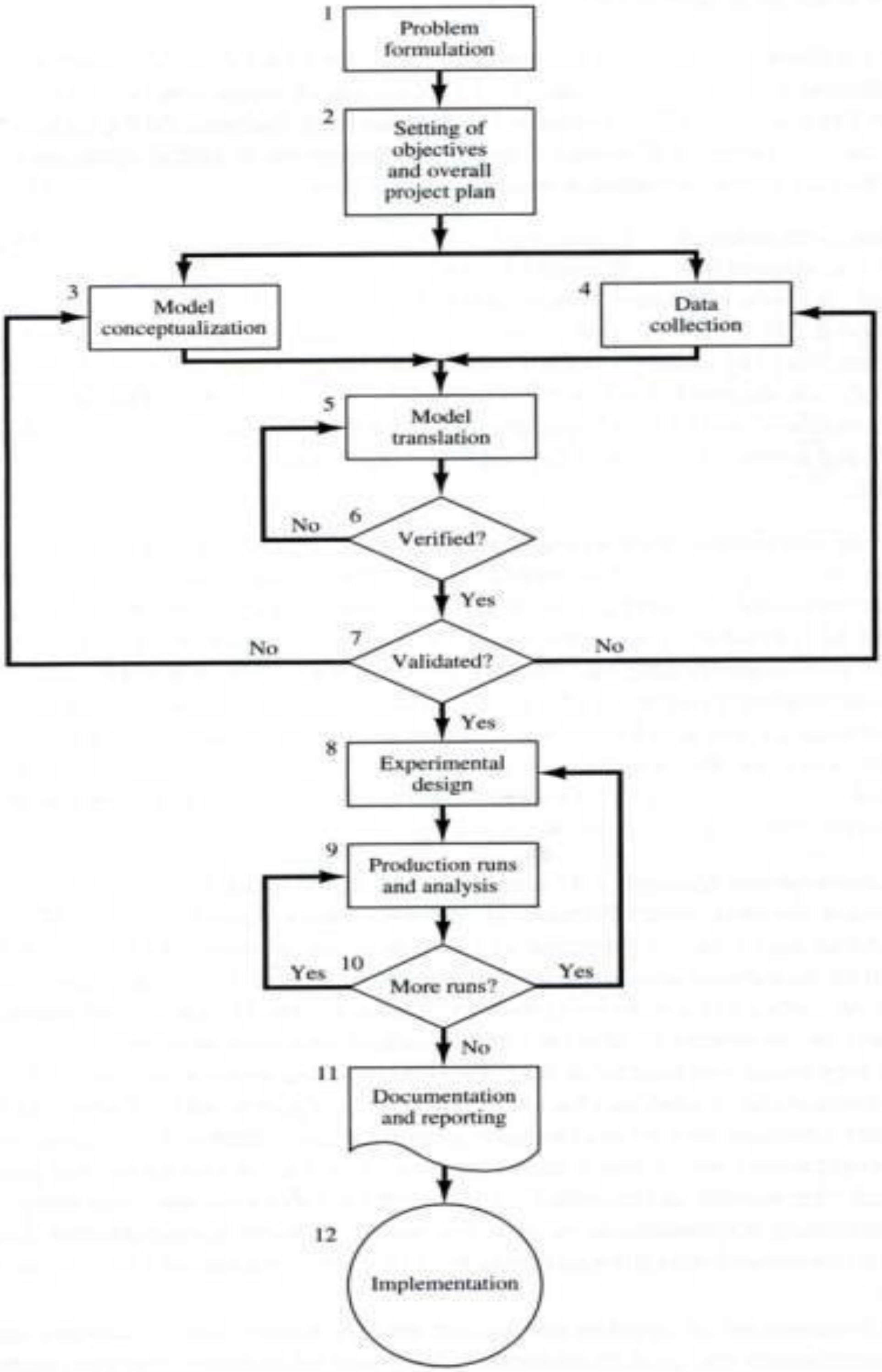


Figure 1.3. Steps in a simulation study.

Phase of Simulation Study:

I Phase:

- Consists of steps 1 and 2
- It is period of discovery/orientation
- The analyst may have to restart the process if it is not fine-tuned
- Recalibrations and clarifications may occur in this phase or another phase.

II Phase:

- Consists of steps 3,4,5,6 and 7
- A continuing interplay is required among the steps
- Exclusion of model user results in implications during implementation

III Phase:

- Consists of steps 8,9 and 10
- Conceives a thorough plan for experimenting
- Discrete-event stochastic is a statistical experiment
- The output variables are estimates that contain random error and therefore proper statistical analysis is required.

IV Phase:

- Consists of steps 11 and 12
- Successful implementation depends on the involvement of user and every steps successful completion.

Advantages of simulation:

1. Simulation can also be used to study systems in the design stage.
2. Simulation models are run rather than solver.
3. New policies, operating procedures, decision rules, information flow, etc can be explored without disrupting the ongoing operations of the real system.
4. New hardware designs, physical layouts, transportation systems can be tested without committing resources for their acquisition.
5. Hypotheses about how or why certain phenomena occur can be tested for feasibility.
6. Time can be compressed or expanded allowing for a speedup or slowdown of the phenomena under investigation.
7. Insight can be obtained about the interaction of variables.
8. Insight can be obtained about the importance of variables to the performance of the system.
9. Bottleneck analysis can be performed indication where work-in process, information materials and so on are being excessively delayed.
10. A simulation study can help in understanding how the system operates rather than how individuals think the system operates.
11. “what-if” questions can be answered. So it is useful in the design of new systems.

Disadvantage of simulation:

1. Model building requires special training.
2. Simulation results may be difficult to interpret.

3. Simulation modeling and analysis can be time consuming and expensive.
4. Simulation is used in some cases when an analytical solution is possible or even preferable.

Limitations of Simulation:

1. Simulation does not produce optimum results. When the model deals with uncertainties, the results of simulation are only reliable approximations subject to statistical errors.
2. Quantification of the variables is another difficulty. In a number of situations it is not possible to quantify all the variables that affect the behavior of the system.
3. In very large and complex problems, the large number of variables and the inter-relationships between them make the problem very unwieldy and hard to program. The number of variables may be too large and may exceed the capacity of the available computer.
4. Simulation is, by no means a cheap method of analysis. In a number of situations, simulation is comparatively costlier and time consuming.

When Simulation is appropriate:

- Simulation can be used as a pedagogical (academic) device to reinforce (highlight) analytic solution methodologies.
- Simulation can be used to experiment with new designs or policies prior implementation, so as to prepare for what may happen.
- Simulation can be used to verify analytic solution.
- By simulating different capabilities for a machine, requirements can be determined.
- Simulation models designed for training, allow learning without the cost and disruption of on-the-job learning.
- Animation shows a system in simulated operation so that the plan can be visualized.
- The modern system (factory, water fabrication plant, service organization etc.) is so complex that the interactions can be treated only through simulation.

When Simulation is not appropriate:

- Simulation should not be used when the problem can be solved using common sense.
- Simulation should not be used if the problem can be solved analytically (logically).
- Simulation should not be used if it is easier to perform direct experiments.
- Do not to use simulation, if the costs exceed the savings
- Simulation should not be performed if the resources or time are not available. if a decision in needed is two weeks and a simulation will take a month, the simulation study is not advised
- If managers have unreasonable expectations say, too much too soon or the power of simulation is overestimated, simulation may not be appropriate
- If system behavior is too complex or can't be defined, simulation is not appropriate.

CSIT 5th Semester
Simulation and Modeling

Applications of Simulation:

Manufacturing Applications

1. Analysis of electronics assembly operations
2. Design and evaluation of a selective assembly station for high precision scroll compressor shells.
3. Determining optimal lot size for a semiconductor backend factory.
4. Optimization of cycle time and utilization in semiconductor test manufacturing.
5. Analysis of storage and retrieval strategies in a warehouse.
6. Investigation of dynamics in a service oriented supply chain.
7. Model for an Army chemical munitions disposal facility.

Military Applications

1. Modeling leadership effects and recruit type in an Army recruiting station.
2. Design and test of an intelligent controller for autonomous underwater vehicles.
3. Modeling military requirements for non-war fighting operations.
4. Battle field simulation for army training

Construction Engineering

1. Construction of a dam embankment
2. Trenchless renewal of underground urban infrastructures
3. Activity scheduling in a dynamic, multi project setting
4. Investigation of the structural steel erection process
5. Special-purpose template for utility tunnel construction

Logistics, Transportation, and Distribution Applications

1. Evaluating the potential benefits of a traffic light planning algorithm
2. Evaluating strategies to improve railroad performance
3. Parametric modeling in rail-capacity planning
4. Analysis of passenger flows in an airport terminal
5. Proactive flight-schedule evaluation
6. Logistics issues in autonomous food production systems for extended- duration space exploration
7. Design of a toll plaza
8. Choosing between rental-car locations
9. Quick-response replenishment (refill)

Business Process Simulation

- Impact of connection bank redesign on airport gate assignment
- Product development program planning
- Reconciliation of business and systems modeling
- Personnel forecasting and strategic workforce planning

Use of Differential and Partial Differential equations in modeling:

Continuous Model:

- When continuous system is modeled mathematically, the variables of model representing the attribute of system are controlled by continuous functions.
- The distributed lag model is an example of a continuous model. Since in continuous system, the relationship between variables describe the rate at which the value of variable change, these system consist of differential equations.
- Continuous system simulation uses the notation of differential equation to represent the change on the basic parameter of the system with respect to time.
- Hence the Mathematical model for continuous system simulation is usually represented by differential and partial differential equations.

Differential equation models are used in many fields of applied physical science to describe the dynamic aspects of systems. The typical dynamic variable is time, and if it is the only dynamic variable, the analysis will be based on an ordinary differential equation (ODE) model. When, in addition to time, geometrical considerations are also important, partial differential equation (PDE) models are used.

Dependent and Independent Variables:

A dependent variable is a variable whose value depends upon independent variables.
The dependent variable is what is being measured in an experiment or evaluated in a mathematical equation. The dependent variable is sometimes called “the outcome variable”. In a simple mathematical equation, for example:

a = b/c

Here, the dependent variable, **a,** is determined by the values of independent variables **b** and **c**.

Differential Equation:

A differential equation is a mathematical equation that relates some function with its derivatives. The equation that consists of the higher order derivatives of the dependent variable is known as differential equations.

The differential equation is said to be linear if any of the dependent variables and its derivatives have power of one and are multiplied by the constant.
An example of a linear differential equation with constant coefficients to describe the wheel suspension system of an automobile can be given as

$$M x'' + D x' + K x = KF(t)$$

Where,

- M, D and K are constants;
- F(t) is the input to the system depending upon the independent variable t;
- x’’ and x’ are second and first order derivatives of dependent variable x.

The differential equation is said to be non-linear if the dependent variable or any of its derivatives are raised to a power (power is greater than one) or are combined in other way like multiplication.

A linear equation is always a polynomial of degree 1
For example: x+2y+3=0.
A Non-linear equation is always a polynomial of degree greater than 1
For example: $x^2+3x+2=0$.

Partial Differential Equation (PDE):

The differential equation is said to be partial if more than one independent variable occur in a differential equation. It involves the derivative of the same dependent variable with respect to each of the independent variable.

Partial differential equations (PDEs) are mathematical equations used to model a wide range of phenomena in engineering. PDEs describe the relationship between a function and its partial derivatives, and they are used to model many physical phenomena, including fluid (liquid) dynamics, heat transfer, and structural mechanics.

For Example 1: Equation of flow of heat in three dimensional body. It consists of four independent variables (Three dimensions and time) and one dependent variable (temperature).

$$\frac{\partial u}{\partial t} - \alpha\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 0$$

Here, the dependent variable is $\boldsymbol{u}$ and independent variable is $\boldsymbol{x, y, z}$ *and* $\boldsymbol{t}$.

Example 2: Wheel Suspension system

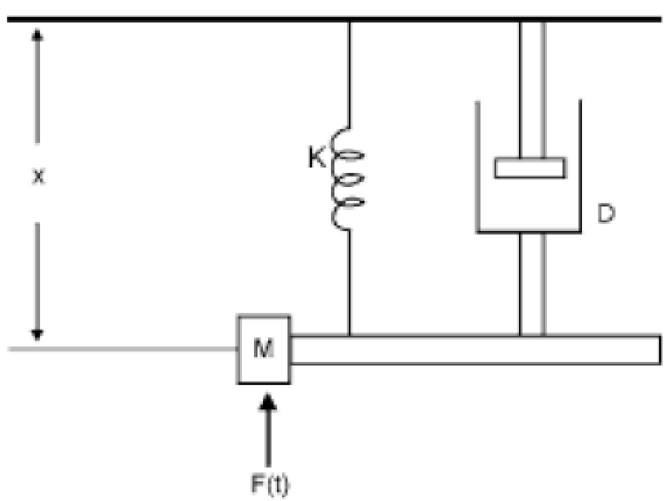


Fig. (a) Suspended weight attached with spring and piston of Mechanical System

Fig (a) represents a mass that is subjected to an applied force $F(t)$ varying with time, a spring whose force is proportional to its extension or contraction, and shock absorber (Damper) that exerts a damping force proportional to the velocity of the mass. It can be shown that the motion of the system is described by the differential equation

$$M\ddot{x} + D\dot{x} + kx = kF(t) \qquad \text{....................} \ (i)$$

Where, x is distance moved

M is the mass,

K is the stiffness of spring,

D is the damping factor of Damper

Differential equation, both linear and non-linear occurs repeatedly in scientific and engineering studies. It shows the rate of change of physical and chemical process in term of mathematical equations. Differential coefficient can also represent growth rate.

Let us illustrate how differential equation can represent engineering problems. We will show how the equation describing the automobile wheel suspension system is derived from the mechanical principle.

- x' represent velocity
- x'' represent acceleration (*the rate of change of velocity per unit of time*)

In absence of other process, Mx''= (t). However the shock absorber exerts a resisting force that depend on the velocity and increases as velocity rises, it is represented by Dx'. Similarly the spring exert a resistance force which depends upon the extent to which it has been compressed, and represented by Kx. Since both of these forces opposes the motion they are subtracted from the applied force to give the equation of motion.
Mx'' = (t) – Dx'– Kx
It is a linear differential equation with constant coefficient representing the equation for suspension of automobile wheel as mentioned before. It can be also solved analytically.

Necessity of differential equations:

1. Most physical and chemical process occurring in the nature involves rate of change, which requires differential equations to provide mathematical model.
2. It can be used to understand general effects of growth trends as differential equations can represent a growth rate.

Here are some of the applications of PDEs in engineering:

- **Fluid dynamics:** PDEs are used to describe the flow of fluids in engineering, including water, air, and gases. The Navier-Stokes equations, for example, are a set of PDEs that describe the motion of fluids.
- **Heat transfer:** PDEs are used to describe the transfer of heat in engineering. The heat equation, for example, is a PDE that describes the flow of heat in a solid object.
- **Electromagnetics:** PDEs are used to describe the behavior of electromagnetic fields in engineering. The Maxwell's equations, for example, are a set of PDEs that describe the behavior of electromagnetic fields.
- **Image processing:** PDEs are used in image processing to smooth out noisy images or to extract features from images. The Perona-Malik equation, for example, is a PDE used for image de-noising.
- **Optimization:** PDEs are used in optimization problems to find the optimal solution for a given system. The Pontryagin maximum principle, for example, is a PDE-based approach to optimize the control of dynamic systems.

Overall, PDEs play a significant role in modeling, simulation, and optimization of complex systems in engineering. The ability to describe complex physical phenomena using PDEs has significantly impacted the development of modern engineering systems, making them more efficient, reliable, and cost-effective.

End of Unit-1