Identification of Heterogeneous Causal Effects

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Identifying the Causal Effects

In this section we estimate the causal effects of additional funding on students performances. Namely, our outcome variables are *certificate* (which is a dummy variable that assumes value 1 if the student gets an "A" certificate: the highest possible grade in the Flanders) and *progress school* (which is a dummy recording whether or not the student got school retention).

This analyses are performed using the probit version of the BCF-IV algorithm (Hahn et al. 2017, Starling et al., 2019).

First Outcome: Certificate

We now foucus on the effects when we sample units in a bandwidth of 0.035 (which is the optimal bandwidth for the outcome *certificate*) around the cutoff (10%).

```
students_data_randomized_03_2011 <-
  students_data_2011[which(students_data_2011$GOKpercentage >= .065
                           & students_data_2011$GOKpercentage <= .135),]
```

Moreover, from every school we sample a number students in order to increase the balance in the covariate and to guarantee an equal representation to all the schools, avoiding biases related to the over-representation of biggest schools' students. In the first case, we sample 50 students from each school.

```
schools <-
students_data_randomized_03_2011$school[which(
!duplicated(students_data_randomized_03_2011$school))]

sample_students <- as.data.frame(matrix(data = NA, nrow = 50*length(schools),
                                        ncol = ncol(students_data_randomized_03_2011)))
colnames(sample_students) <- colnames(students_data_randomized_03_2011)

for (j in (0:(length(schools)-1))){
  set.seed(j  + 123)
  sample_students[(1+(j*50)):(50+(j*50)),]  <-
  students_data_randomized_03_2011[which(
  students_data_randomized_03_2011$school %in%
  schools[j+1]),][sample(1:nrow(students_data_randomized_03_2011[which(
  students_data_randomized_03_2011$school %in% schools[j+1]),]),
  50, replace = FALSE ),]
}

sample_student <- round(sample_students[, -(1:4)], 0)
sample_students <- cbind(sample_students[,1:4], sample_student)
```

Then we run our BCF-IV algorithm on this sample of units.

```r
# Attaching the Sample and the Covariates
attach(sample_students)
x <- cbind(primary_retention , man , BULO,
           leerkracht_age , leerkracht_seniority,
           leerkracht_diploma,
           directie_age, directie_seniority
)
z <- as.matrix(eligible_dummy)
y <- as.matrix(certificate)
w <- as.matrix(GOKschool)
detach(sample_students)
```

Below we run the BCF-IV algorithm.

```r
# Running the BCF algorithm on the IV
set.seed(123)
bcf_iv(y, w, z, x, max_depth = 2, n_burn= 2000, n_sim= 2000, binary = TRUE)
```

```
## Loading required package: lattice

## Loading required package: rattle

## Rattle: A free graphical interface for data science with R.
## Version 5.2.0 Copyright (c) 2006-2018 Togaware Pty Ltd.
## Type 'rattle()' to shake, rattle, and roll your data.

## Using 9 control variables.
## Using 8 potential effect moderators.
##
## Beginning MCMC:
## iteration: 0 sigma: 1
## iteration: 100 sigma: 0.975905
## iteration: 200 sigma: 1.0126
## iteration: 300 sigma: 0.975569
## iteration: 400 sigma: 0.988494
## iteration: 500 sigma: 0.991072
## iteration: 600 sigma: 1.01119
## iteration: 700 sigma: 0.992681
## iteration: 800 sigma: 0.986909
## iteration: 900 sigma: 0.988032
## iteration: 1000 sigma: 0.984547
## iteration: 1100 sigma: 1.00403
## iteration: 1200 sigma: 0.985189
## iteration: 1300 sigma: 0.98622
## iteration: 1400 sigma: 0.995455
## iteration: 1500 sigma: 0.990805
## iteration: 1600 sigma: 0.992702
## iteration: 1700 sigma: 0.99214
## iteration: 1800 sigma: 0.993923
## iteration: 1900 sigma: 0.994163
## iteration: 2000 sigma: 0.999938
## iteration: 2100 sigma: 0.99574
## iteration: 2200 sigma: 0.978008
## iteration: 2300 sigma: 0.993844
## iteration: 2400 sigma: 0.980196
## iteration: 2500 sigma: 0.998858
```

```
## iteration: 2600 sigma: 0.988934
## iteration: 2700 sigma: 0.982662
## iteration: 2800 sigma: 1.01801
## iteration: 2900 sigma: 0.981125
## iteration: 3000 sigma: 0.999646
## iteration: 3100 sigma: 0.990349
## iteration: 3200 sigma: 0.994495
## iteration: 3300 sigma: 1.01028
## iteration: 3400 sigma: 1.01117
## iteration: 3500 sigma: 0.979865
## iteration: 3600 sigma: 1.01447
## iteration: 3700 sigma: 0.973085
## iteration: 3800 sigma: 0.988223
## iteration: 3900 sigma: 0.994196
## time for loop: 174
## The effect on the overall sample is 0.0118
## P-value 0.758342498199347
## P-value Weak-Instrument Test 3.42189860171129e-134
## Proportion of observations in the node:  1.00
##
##  node number: 2
##    root
##    xleerkracht_seniority< 3.5
## The conditional effect on the subpopulation is -0.0895
## P-value 0.168084849190633
## P-value Weak-Instrument Test 2.16988054262431e-54
## Proportion of observations in the node:  0.13953488372093
##
##  node number: 4
##    root
##    xleerkracht_seniority< 3.5
##    xleerkracht_age>=3.5
## The conditional effect on the subpopulation is -0.1815
## P-value 0.151200919936055
## P-value Weak-Instrument Test 8.00155380736828e-21
## Proportion of observations in the node:  0.104651162790698
##
##  node number: 5
##    root
##    xleerkracht_seniority< 3.5
##    xleerkracht_age< 3.5
## The conditional effect on the subpopulation is 0.0456
## P-value 0.106345602660564
## P-value Weak-Instrument Test NaN
## Proportion of observations in the node:  0.0348837209302326
##
##  node number: 3
##    root
##    xleerkracht_seniority>=3.5
## The conditional effect on the subpopulation is 0.0294
## P-value 0.542024676172768
## P-value Weak-Instrument Test 9.01273924348802e-91
## Proportion of observations in the node:  0.86046511627907
##
```

```
##  node number: 6
##    root
##    xleerkracht_seniority>=3.5
##    xdirectie_age>=7.5
## The conditional effect on the subpopulation is -0.1963
## P-value 0.175974072848397
## P-value Weak-Instrument Test 2.30698101798262e-11
## Proportion of observations in the node:  0.0813953488372093
##
##  node number: 7
##    root
##    xleerkracht_seniority>=3.5
##    xdirectie_age< 7.5
## The conditional effect on the subpopulation is 0.048
## P-value 0.376999682467482
## P-value Weak-Instrument Test 2.30358514883622e-73
## Proportion of observations in the node:  0.779069767441861
```

```
# Running the BCF algorithm on the IV
set.seed(123)
mm_bcf_iv(y, w, z, x, max_depth = 2, n_burn= 2000, n_sim= 2000, binary = TRUE)
```

```
## Using 9 control variables.
## Using 8 potential effect moderators.
##
## Beginning MCMC:
## iteration: 0 sigma: 1
## iteration: 100 sigma: 0.975905
## iteration: 200 sigma: 1.0126
## iteration: 300 sigma: 0.975569
## iteration: 400 sigma: 0.988494
## iteration: 500 sigma: 0.991072
## iteration: 600 sigma: 1.01119
## iteration: 700 sigma: 0.992681
## iteration: 800 sigma: 0.986909
## iteration: 900 sigma: 0.988032
## iteration: 1000 sigma: 0.984547
## iteration: 1100 sigma: 1.00403
## iteration: 1200 sigma: 0.985189
## iteration: 1300 sigma: 0.98622
## iteration: 1400 sigma: 0.995455
## iteration: 1500 sigma: 0.990805
## iteration: 1600 sigma: 0.992702
## iteration: 1700 sigma: 0.99214
## iteration: 1800 sigma: 0.993923
## iteration: 1900 sigma: 0.994163
## iteration: 2000 sigma: 0.999938
## iteration: 2100 sigma: 0.99574
## iteration: 2200 sigma: 0.978008
## iteration: 2300 sigma: 0.993844
## iteration: 2400 sigma: 0.980196
## iteration: 2500 sigma: 0.998858
## iteration: 2600 sigma: 0.988934
## iteration: 2700 sigma: 0.982662
## iteration: 2800 sigma: 1.01801
```

```
## iteration: 2900 sigma: 0.981125
## iteration: 3000 sigma: 0.999646
## iteration: 3100 sigma: 0.990349
## iteration: 3200 sigma: 0.994495
## iteration: 3300 sigma: 1.01028
## iteration: 3400 sigma: 1.01117
## iteration: 3500 sigma: 0.979865
## iteration: 3600 sigma: 1.01447
## iteration: 3700 sigma: 0.973085
## iteration: 3800 sigma: 0.988223
## iteration: 3900 sigma: 0.994196
## time for loop: 222
## The effect on the overall sample is 0.0071
## P-value 0.853238419607103
## P-value Weak-Instrument Test 2.47688826600989e-130
## Proportion of observations in the node:  1.00
## Intention-to-treat effect:  0.00446511627906376
## Proportion of compliers in the node:  0.627906976744186
##
##  node number: 2
##    root
##    xleerkracht_seniority< 3.5
## The conditional effect on the subpopulation is -0.025
## P-value 0.599811848685678
## P-value Weak-Instrument Test 2.39068699284151e-68
## Proportion of observations in the node:  0.13953488372093
## Intention-to-treat effect:  -0.0208333333333319
## Proportion of compliers in the node:  0.833333333333333
##
##  node number: 4
##    root
##    xleerkracht_seniority< 3.5
##    xleerkracht_age>=3.5
## The conditional effect on the subpopulation is -0.09
## P-value 0.321065853123692
## P-value Weak-Instrument Test 7.72614592604999e-30
## Proportion of observations in the node:  0.104651162790698
## Intention-to-treat effect:  -0.0699999999999986
## Proportion of compliers in the node:  0.777777777777778
##
##  node number: 5
##    root
##    xleerkracht_seniority< 3.5
##    xleerkracht_age< 3.5
## The conditional effect on the subpopulation is 0.04
## P-value 0.153761239501752
## P-value Weak-Instrument Test 0
## Proportion of observations in the node:  0.0348837209302326
## Intention-to-treat effect:  0.0400000000000003
## Proportion of compliers in the node:  1
##
##  node number: 3
##    root
##    xleerkracht_seniority>=3.5
```

```
## The conditional effect on the subpopulation is 0.02
## P-value 0.677713217337347
## P-value Weak-Instrument Test 9.92583530452573e-91
## Proportion of observations in the node:  0.86046511627907
## Intention-to-treat effect:  0.011891891891882
## Proportion of compliers in the node:  0.594594594594595
##
##  node number: 6
##    root
##    xleerkracht_seniority>=3.5
##    xdirectie_age>=7.5
## The conditional effect on the subpopulation is -0.065
## P-value 0.492346927546972
## P-value Weak-Instrument Test 9.16361844772418e-21
## Proportion of observations in the node:  0.0813953488372093
## Intention-to-treat effect:  -0.0464285714285729
## Proportion of compliers in the node:  0.714285714285714
##
##  node number: 7
##    root
##    xleerkracht_seniority>=3.5
##    xdirectie_age< 7.5
## The conditional effect on the subpopulation is 0.0342
## P-value 0.525313401993449
## P-value Weak-Instrument Test 5.03825555299004e-75
## Proportion of observations in the node:  0.779069767441861
## Intention-to-treat effect:  0.019932157394836
## Proportion of compliers in the node:  0.582089552238806
```

In the second case we sample 62 students from every school (where 62 units is the size of the smallest school).

```
# Attaching the Sample and the Covariates
attach(sample_students)
x <- cbind(primary_retention , man , BULO,
           leerkracht_age , leerkracht_seniority,
           directie_age, directie_seniority
)
z <- as.matrix(eligible_dummy)
w <- as.matrix(GOKschool)
y <- as.matrix(certificate)
detach(sample_students)
```

As we can see from the plot the most important variables detected are the same: *teacher age* and *primary retention*.

```
# Running the BCF algorithm on the IV
set.seed(123)
bcf_iv(y, w, z, x, max_depth = 2, n_burn= 2000, n_sim= 2000, binary = TRUE)
```

```
## Using 8 control variables.
## Using 7 potential effect moderators.
##
## Beginning MCMC:
## iteration: 0 sigma: 1
## iteration: 100 sigma: 0.997596
## iteration: 200 sigma: 0.988988
```

```
## iteration: 300 sigma: 0.990572
## iteration: 400 sigma: 0.996178
## iteration: 500 sigma: 0.990307
## iteration: 600 sigma: 0.984164
## iteration: 700 sigma: 0.995489
## iteration: 800 sigma: 0.996191
## iteration: 900 sigma: 0.995101
## iteration: 1000 sigma: 0.985562
## iteration: 1100 sigma: 0.988982
## iteration: 1200 sigma: 0.986299
## iteration: 1300 sigma: 0.989968
## iteration: 1400 sigma: 0.993088
## iteration: 1500 sigma: 1.00508
## iteration: 1600 sigma: 0.988674
## iteration: 1700 sigma: 0.982088
## iteration: 1800 sigma: 0.978139
## iteration: 1900 sigma: 0.980985
## iteration: 2000 sigma: 1.00269
## iteration: 2100 sigma: 0.978859
## iteration: 2200 sigma: 0.968879
## iteration: 2300 sigma: 0.989366
## iteration: 2400 sigma: 0.983936
## iteration: 2500 sigma: 0.98983
## iteration: 2600 sigma: 0.983245
## iteration: 2700 sigma: 0.984931
## iteration: 2800 sigma: 1.00104
## iteration: 2900 sigma: 0.994779
## iteration: 3000 sigma: 1.00813
## iteration: 3100 sigma: 0.993134
## iteration: 3200 sigma: 0.996132
## iteration: 3300 sigma: 0.993709
## iteration: 3400 sigma: 0.984137
## iteration: 3500 sigma: 0.960686
## iteration: 3600 sigma: 0.988459
## iteration: 3700 sigma: 0.984613
## iteration: 3800 sigma: 0.987298
## iteration: 3900 sigma: 0.974668
## time for loop: 229
## The effect on the overall sample is 0.0313
## P-value 0.361755368384603
## P-value Weak-Instrument Test 1.3368380135115e-165
## Proportion of observations in the node:  1.00
##
##  node number: 2
##    root
##    xprimary_retention>=0.5
## The conditional effect on the subpopulation is -0.3144
## P-value 0.135092277870465
## P-value Weak-Instrument Test 1.48984648736186e-09
## Proportion of observations in the node:  0.0393848462115529
##
##  node number: 3
##    root
##    xprimary_retention< 0.5
```

```
## The conditional effect on the subpopulation is 0.0494
## P-value 0.153136638317197
## P-value Weak-Instrument Test 2.34859290378679e-157
## Proportion of observations in the node:  0.960615153788447
##
##  node number: 6
##    root
##    xprimary_retention< 0.5
##    xdirectie_seniority< 3.5
##
##  node number: 7
##    root
##    xprimary_retention< 0.5
##    xdirectie_seniority>=3.5
## The conditional effect on the subpopulation is 0.0412
## P-value 0.243866667884515
## P-value Weak-Instrument Test 1.00344105946104e-143
## Proportion of observations in the node:  0.927231807951988
```

```
# Running the BCF algorithm on the IV
set.seed(123)
mm_bcf_iv(y, w, z, x, max_depth = 2, n_burn= 2000, n_sim= 2000, binary = TRUE)
```

```
## Using 8 control variables.
## Using 7 potential effect moderators.
##
## Beginning MCMC:
## iteration: 0 sigma: 1
## iteration: 100 sigma: 0.997596
## iteration: 200 sigma: 0.988988
## iteration: 300 sigma: 0.990572
## iteration: 400 sigma: 0.996178
## iteration: 500 sigma: 0.990307
## iteration: 600 sigma: 0.984164
## iteration: 700 sigma: 0.995489
## iteration: 800 sigma: 0.996191
## iteration: 900 sigma: 0.995101
## iteration: 1000 sigma: 0.985562
## iteration: 1100 sigma: 0.988982
## iteration: 1200 sigma: 0.986299
## iteration: 1300 sigma: 0.989968
## iteration: 1400 sigma: 0.993088
## iteration: 1500 sigma: 1.00508
## iteration: 1600 sigma: 0.988674
## iteration: 1700 sigma: 0.982088
## iteration: 1800 sigma: 0.978139
## iteration: 1900 sigma: 0.980985
## iteration: 2000 sigma: 1.00269
## iteration: 2100 sigma: 0.978859
## iteration: 2200 sigma: 0.968879
## iteration: 2300 sigma: 0.989366
## iteration: 2400 sigma: 0.983936
## iteration: 2500 sigma: 0.98983
## iteration: 2600 sigma: 0.983245
## iteration: 2700 sigma: 0.984931
```

```
## iteration: 2800 sigma: 1.00104
## iteration: 2900 sigma: 0.994779
## iteration: 3000 sigma: 1.00813
## iteration: 3100 sigma: 0.993134
## iteration: 3200 sigma: 0.996132
## iteration: 3300 sigma: 0.993709
## iteration: 3400 sigma: 0.984137
## iteration: 3500 sigma: 0.960686
## iteration: 3600 sigma: 0.988459
## iteration: 3700 sigma: 0.984613
## iteration: 3800 sigma: 0.987298
## iteration: 3900 sigma: 0.974668
## time for loop: 202
## The effect on the overall sample is 0.0281
## P-value 0.415704275145072
## P-value Weak-Instrument Test 3.8038046041627e-161
## Proportion of observations in the node:  1.00
## Intention-to-treat effect:  0.0176294073518326
## Proportion of compliers in the node:  0.627906976744186
##
##  node number: 2
##    root
##    xprimary_retention>=0.5
## The conditional effect on the subpopulation is -0.3089
## P-value 0.11909525304271
## P-value Weak-Instrument Test 6.15849835317179e-10
## Proportion of observations in the node:  0.0393848462115529
## Intention-to-treat effect:  -0.192712585034013
## Proportion of compliers in the node:  0.623809523809524
##
##  node number: 3
##    root
##    xprimary_retention< 0.5
## The conditional effect on the subpopulation is 0.0554
## P-value 0.110441407056893
## P-value Weak-Instrument Test 3.44911922655108e-152
## Proportion of observations in the node:  0.960615153788447
## Intention-to-treat effect:  0.0348040482380016
## Proportion of compliers in the node:  0.628074970714565
##
##  node number: 6
##    root
##    xprimary_retention< 0.5
##    xdirectie_seniority< 3.5
##
##  node number: 7
##    root
##    xprimary_retention< 0.5
##    xdirectie_seniority>=3.5
## The conditional effect on the subpopulation is 0.0475
## P-value 0.175046047032999
## P-value Weak-Instrument Test 1.47166018738745e-142
## Proportion of observations in the node:  0.927231807951988
## Intention-to-treat effect:  0.0292237976713322
```

```
## Proportion of compliers in the node:  0.614684466019418
```

Second Outcome: School Progress

We now foucus on the effects when we sample units in a bandwidth of 0.035 (which is the optimal bandwidth for the outcome *progress_school*) around the cutoff (10%).

```
students_data_randomized_03_2011 <-
  students_data_2011[which(students_data_2011$GOKpercentage >= .063
                           & students_data_2011$GOKpercentage <= .137),]
```

Moreover, from every school we sample a number students in order to increase the balance in the covariate and to guarantee an equal representation to all the schools, avoiding biases related to the over-representation of biggest schools' students. In the first case, we sample 50 students from each school.

```
schools <-
students_data_randomized_03_2011$school[which(
!duplicated(students_data_randomized_03_2011$school))]

sample_students <- as.data.frame(matrix(data = NA, nrow = 50*length(schools),
                                        ncol = ncol(students_data_randomized_03_2011)))
colnames(sample_students) <- colnames(students_data_randomized_03_2011)

for (j in (0:(length(schools)-1))){
  set.seed(j  + 123)
  sample_students[(1+(j*50)):(50+(j*50)),]  <-
  students_data_randomized_03_2011[which(
  students_data_randomized_03_2011$school %in%
  schools[j+1]),][sample(1:nrow(students_data_randomized_03_2011[which(
  students_data_randomized_03_2011$school %in% schools[j+1]),]),
  50, replace = FALSE ),]
}

sample_student <- round(sample_students[, -(1:4)], 0)
sample_students <- cbind(sample_students[,1:4], sample_student)
```

Then we run our BCF-IV algorithm on this sample of units.

```
# Attaching the Sample and the Covariates
attach(sample_students)
x <- cbind(primary_retention , man , BULO,
           leerkracht_age , leerkracht_seniority,
           directie_age, directie_seniority
)
z <- as.matrix(eligible_dummy)
w <- as.matrix(GOKschool)
y <- as.matrix(progress_school)
detach(sample_students)
```

```
# Running the BCF algorithm on the IV
set.seed(123)
bcf_iv(y, w, z, x, max_depth = 2, n_burn= 2000, n_sim= 2000, binary = TRUE)
```

```
## Using 8 control variables.
## Using 7 potential effect moderators.
```

```
##
## Beginning MCMC:
## iteration: 0 sigma: 1
## iteration: 100 sigma: 1.0038
## iteration: 200 sigma: 1.0168
## iteration: 300 sigma: 0.988621
## iteration: 400 sigma: 0.986895
## iteration: 500 sigma: 0.973616
## iteration: 600 sigma: 0.986905
## iteration: 700 sigma: 0.998089
## iteration: 800 sigma: 0.979706
## iteration: 900 sigma: 0.991207
## iteration: 1000 sigma: 0.978347
## iteration: 1100 sigma: 1.01241
## iteration: 1200 sigma: 0.989462
## iteration: 1300 sigma: 1.01009
## iteration: 1400 sigma: 1.00889
## iteration: 1500 sigma: 0.975059
## iteration: 1600 sigma: 0.999178
## iteration: 1700 sigma: 0.988179
## iteration: 1800 sigma: 1.0086
## iteration: 1900 sigma: 1.00278
## iteration: 2000 sigma: 1.0027
## iteration: 2100 sigma: 0.996183
## iteration: 2200 sigma: 0.988073
## iteration: 2300 sigma: 1.00069
## iteration: 2400 sigma: 1.00542
## iteration: 2500 sigma: 1.02719
## iteration: 2600 sigma: 0.993215
## iteration: 2700 sigma: 0.983998
## iteration: 2800 sigma: 1.01297
## iteration: 2900 sigma: 1.01192
## iteration: 3000 sigma: 0.999392
## iteration: 3100 sigma: 0.99839
## iteration: 3200 sigma: 0.99419
## iteration: 3300 sigma: 0.997401
## iteration: 3400 sigma: 0.976157
## iteration: 3500 sigma: 1.00556
## iteration: 3600 sigma: 1.00199
## iteration: 3700 sigma: 0.986853
## iteration: 3800 sigma: 1.00256
## iteration: 3900 sigma: 0.986092
## time for loop: 238
## The effect on the overall sample is -0.0031
## P-value 0.790412006787677
## P-value Weak-Instrument Test 4.19939980800448e-174
## Proportion of observations in the node:  1.00
##
##  node number: 2
##    root
##    xdirectie_seniority>=5.5
## The conditional effect on the subpopulation is -0.0368
## P-value 0.0120619336505979
## P-value Weak-Instrument Test 1.44907402008323e-107
```

```
## Proportion of observations in the node:  0.640449438202247
##
##  node number: 4
##    root
##    xdirectie_seniority>=5.5
##    xdirectie_age>=5.5
## The conditional effect on the subpopulation is -0.0392
## P-value 0.00996503818171283
## P-value Weak-Instrument Test 8.91695602138972e-98
## Proportion of observations in the node:  0.617977528089888
##
##  node number: 5
##    root
##    xdirectie_seniority>=5.5
##    xdirectie_age< 5.5
##
##  node number: 3
##    root
##    xdirectie_seniority< 5.5
## The conditional effect on the subpopulation is 0.0255
## P-value 0.232060232407332
## P-value Weak-Instrument Test 6.77803181903841e-58
## Proportion of observations in the node:  0.359550561797753
##
##  node number: 6
##    root
##    xdirectie_seniority< 5.5
##    xdirectie_age>=6.5
## The conditional effect on the subpopulation is 0.0317
## P-value 0.364820158129083
## P-value Weak-Instrument Test NaN
## Proportion of observations in the node:  0.0449438202247191
##
##  node number: 7
##    root
##    xdirectie_seniority< 5.5
##    xdirectie_age< 6.5
## The conditional effect on the subpopulation is 0.0178
## P-value 0.496672886179129
## P-value Weak-Instrument Test 9.04363937627722e-43
## Proportion of observations in the node:  0.314606741573034
```

```r
# Running the BCF algorithm on the IV
set.seed(123)
mm_bcf_iv(y, w, z, x, max_depth = 2, n_burn= 2000, n_sim= 2000, binary = TRUE)
```

```
## Using 8 control variables.
## Using 7 potential effect moderators.
##
## Beginning MCMC:
## iteration: 0 sigma: 1
## iteration: 100 sigma: 1.0038
## iteration: 200 sigma: 1.0168
## iteration: 300 sigma: 0.988621
## iteration: 400 sigma: 0.986895
```

```
## iteration: 500 sigma: 0.973616
## iteration: 600 sigma: 0.986905
## iteration: 700 sigma: 0.998089
## iteration: 800 sigma: 0.979706
## iteration: 900 sigma: 0.991207
## iteration: 1000 sigma: 0.978347
## iteration: 1100 sigma: 1.01241
## iteration: 1200 sigma: 0.989462
## iteration: 1300 sigma: 1.01009
## iteration: 1400 sigma: 1.00889
## iteration: 1500 sigma: 0.975059
## iteration: 1600 sigma: 0.999178
## iteration: 1700 sigma: 0.988179
## iteration: 1800 sigma: 1.0086
## iteration: 1900 sigma: 1.00278
## iteration: 2000 sigma: 1.0027
## iteration: 2100 sigma: 0.996183
## iteration: 2200 sigma: 0.988073
## iteration: 2300 sigma: 1.00069
## iteration: 2400 sigma: 1.00542
## iteration: 2500 sigma: 1.02719
## iteration: 2600 sigma: 0.993215
## iteration: 2700 sigma: 0.983998
## iteration: 2800 sigma: 1.01297
## iteration: 2900 sigma: 1.01192
## iteration: 3000 sigma: 0.999392
## iteration: 3100 sigma: 0.99839
## iteration: 3200 sigma: 0.99419
## iteration: 3300 sigma: 0.997401
## iteration: 3400 sigma: 0.976157
## iteration: 3500 sigma: 1.00556
## iteration: 3600 sigma: 1.00199
## iteration: 3700 sigma: 0.986853
## iteration: 3800 sigma: 1.00256
## iteration: 3900 sigma: 0.986092
## time for loop: 235
## The effect on the overall sample is -9e-04
## P-value 0.936329167800298
## P-value Weak-Instrument Test 6.01030003402249e-170
## Proportion of observations in the node:  1.00
## Intention-to-treat effect:  -0.000593008739075609
## Proportion of compliers in the node:  0.640449438202247
##
##  node number: 2
##    root
##    xdirectie_seniority>=5.5
## The conditional effect on the subpopulation is -0.0286
## P-value 0.0497498130559705
## P-value Weak-Instrument Test 1.03535150997586e-106
## Proportion of observations in the node:  0.640449438202247
## Intention-to-treat effect:  -0.0185463659147836
## Proportion of compliers in the node:  0.649122807017544
##
##  node number: 4
```

```
##    root
##    xdirectie_seniority>=5.5
##    xdirectie_age>=5.5
## The conditional effect on the subpopulation is -0.0295
## P-value 0.0465305261886397
## P-value Weak-Instrument Test 5.5713713846664e-100
## Proportion of observations in the node:  0.617977528089888
## Intention-to-treat effect:  -0.0187662337662303
## Proportion of compliers in the node:  0.636363636363636
##
##  node number: 5
##    root
##    xdirectie_seniority>=5.5
##    xdirectie_age< 5.5
##
##  node number: 3
##    root
##    xdirectie_seniority< 5.5
## The conditional effect on the subpopulation is 0.044
## P-value 0.0244386730854404
## P-value Weak-Instrument Test 6.08685778727359e-64
## Proportion of observations in the node:  0.359550561797753
## Intention-to-treat effect:  0.0275000000000029
## Proportion of compliers in the node:  0.625
##
##  node number: 6
##    root
##    xdirectie_seniority< 5.5
##    xdirectie_age>=6.5
## The conditional effect on the subpopulation is 0
## P-value 0.999999999999991
## P-value Weak-Instrument Test 3.58380679702108e-19
## Proportion of observations in the node:  0.0449438202247191
## Intention-to-treat effect:  -2.50233485602727e-16
## Proportion of compliers in the node:  0.75
##
##  node number: 7
##    root
##    xdirectie_seniority< 5.5
##    xdirectie_age< 6.5
## The conditional effect on the subpopulation is 0.0558
## P-value 0.0176726039269617
## P-value Weak-Instrument Test 2.44874068746436e-49
## Proportion of observations in the node:  0.314606741573034
## Intention-to-treat effect:  0.033859890109892
## Proportion of compliers in the node:  0.607142857142857
```

In the second case we sample 62 students from every school (where 62 units is the size of the smallest school).

```
# Attaching the Sample and the Covariates
attach(sample_students)
x <- cbind(primary_retention , man , BULO,
           leerkracht_age , leerkracht_seniority,
           directie_age, directie_seniority
)
```

```r
z <- as.matrix(eligible_dummy)
w <- as.matrix(GOKschool)
y <- as.matrix(progress_school)
detach(sample_students)
```

```r
# Running the BCF algorithm on the IV
set.seed(123)
bcf_iv(y, w, z, x, max_depth = 2, n_burn= 2000, n_sim= 2000, binary = TRUE)
```

```
## Using 8 control variables.
## Using 7 potential effect moderators.
##
## Beginning MCMC:
## iteration: 0 sigma: 1
## iteration: 100 sigma: 0.987155
## iteration: 200 sigma: 1.00036
## iteration: 300 sigma: 0.999281
## iteration: 400 sigma: 1.00682
## iteration: 500 sigma: 0.992197
## iteration: 600 sigma: 1.0171
## iteration: 700 sigma: 0.999866
## iteration: 800 sigma: 1.00742
## iteration: 900 sigma: 0.998096
## iteration: 1000 sigma: 0.993351
## iteration: 1100 sigma: 1.00161
## iteration: 1200 sigma: 0.999634
## iteration: 1300 sigma: 1.0064
## iteration: 1400 sigma: 0.986313
## iteration: 1500 sigma: 1.00687
## iteration: 1600 sigma: 1.00098
## iteration: 1700 sigma: 0.997865
## iteration: 1800 sigma: 1.00565
## iteration: 1900 sigma: 1.00823
## iteration: 2000 sigma: 1.00115
## iteration: 2100 sigma: 1.00626
## iteration: 2200 sigma: 0.997358
## iteration: 2300 sigma: 1.00588
## iteration: 2400 sigma: 0.989997
## iteration: 2500 sigma: 1.01742
## iteration: 2600 sigma: 1.00759
## iteration: 2700 sigma: 1.0043
## iteration: 2800 sigma: 1.00423
## iteration: 2900 sigma: 0.99804
## iteration: 3000 sigma: 0.9931
## iteration: 3100 sigma: 0.983317
## iteration: 3200 sigma: 0.988807
## iteration: 3300 sigma: 0.988749
## iteration: 3400 sigma: 1.00228
## iteration: 3500 sigma: 0.992145
## iteration: 3600 sigma: 0.990844
## iteration: 3700 sigma: 0.985872
## iteration: 3800 sigma: 1.01673
## iteration: 3900 sigma: 0.997594
## time for loop: 282
```

```
## The effect on the overall sample is -0.0012
## P-value 0.909200626576663
## P-value Weak-Instrument Test 3.03623945214761e-215
## Proportion of observations in the node:  1.00
##
##  node number: 2
##    root
##    xdirectie_seniority>=5.5
## The conditional effect on the subpopulation is -0.0396
## P-value 0.00199085806382
## P-value Weak-Instrument Test 2.6194269652092e-133
## Proportion of observations in the node:  0.640449438202247
##
##  node number: 4
##    root
##    xdirectie_seniority>=5.5
##    xleerkracht_seniority>=3.5
## The conditional effect on the subpopulation is -0.031
## P-value 0.0109743870545647
## P-value Weak-Instrument Test 1.05677449222627e-134
## Proportion of observations in the node:  0.606741573033708
##
##  node number: 5
##    root
##    xdirectie_seniority>=5.5
##    xleerkracht_seniority< 3.5
##
##  node number: 3
##    root
##    xdirectie_seniority< 5.5
## The conditional effect on the subpopulation is 0.0431
## P-value 0.0356270500139718
## P-value Weak-Instrument Test 2.82636087121711e-71
## Proportion of observations in the node:  0.359550561797753
##
##  node number: 6
##    root
##    xdirectie_seniority< 5.5
##    xdirectie_age>=6.5
## The conditional effect on the subpopulation is 0.0271
## P-value 0.34459233757944
## P-value Weak-Instrument Test NaN
## Proportion of observations in the node:  0.0449438202247191
##
##  node number: 7
##    root
##    xdirectie_seniority< 5.5
##    xdirectie_age< 6.5
## The conditional effect on the subpopulation is 0.038
## P-value 0.137629290294332
## P-value Weak-Instrument Test 1.47569109448061e-52
## Proportion of observations in the node:  0.314606741573034
```

```r
# Running the BCF algorithm on the IV
set.seed(123)
mm_bcf_iv(y, w, z, x, max_depth = 2, n_burn= 2000, n_sim= 2000, binary = TRUE)
```

```
## Using 8 control variables.
## Using 7 potential effect moderators.
##
## Beginning MCMC:
## iteration: 0 sigma: 1
## iteration: 100 sigma: 0.987155
## iteration: 200 sigma: 1.00036
## iteration: 300 sigma: 0.999281
## iteration: 400 sigma: 1.00682
## iteration: 500 sigma: 0.992197
## iteration: 600 sigma: 1.0171
## iteration: 700 sigma: 0.999866
## iteration: 800 sigma: 1.00742
## iteration: 900 sigma: 0.998096
## iteration: 1000 sigma: 0.993351
## iteration: 1100 sigma: 1.00161
## iteration: 1200 sigma: 0.999634
## iteration: 1300 sigma: 1.0064
## iteration: 1400 sigma: 0.986313
## iteration: 1500 sigma: 1.00687
## iteration: 1600 sigma: 1.00098
## iteration: 1700 sigma: 0.997865
## iteration: 1800 sigma: 1.00565
## iteration: 1900 sigma: 1.00823
## iteration: 2000 sigma: 1.00115
## iteration: 2100 sigma: 1.00626
## iteration: 2200 sigma: 0.997358
## iteration: 2300 sigma: 1.00588
## iteration: 2400 sigma: 0.989997
## iteration: 2500 sigma: 1.01742
## iteration: 2600 sigma: 1.00759
## iteration: 2700 sigma: 1.0043
## iteration: 2800 sigma: 1.00423
## iteration: 2900 sigma: 0.99804
## iteration: 3000 sigma: 0.9931
## iteration: 3100 sigma: 0.983317
## iteration: 3200 sigma: 0.988807
## iteration: 3300 sigma: 0.988749
## iteration: 3400 sigma: 1.00228
## iteration: 3500 sigma: 0.992145
## iteration: 3600 sigma: 0.990844
## iteration: 3700 sigma: 0.985872
## iteration: 3800 sigma: 1.01673
## iteration: 3900 sigma: 0.997594
## time for loop: 266
## The effect on the overall sample is 0.0017
## P-value 0.872085618331881
## P-value Weak-Instrument Test 2.92985224549173e-210
## Proportion of observations in the node:  1.00
## Intention-to-treat effect:  0.00109037090733918
```

```
## Proportion of compliers in the node:  0.640449438202247
##
##  node number: 2
##    root
##    xdirectie_seniority>=5.5
## The conditional effect on the subpopulation is -0.0302
## P-value 0.0175071993645751
## P-value Weak-Instrument Test 7.1750334639301e-132
## Proportion of observations in the node:  0.640449438202247
## Intention-to-treat effect:  -0.0195933381841691
## Proportion of compliers in the node:  0.649122807017544
##
##  node number: 4
##    root
##    xdirectie_seniority>=5.5
##    xleerkracht_seniority>=3.5
## The conditional effect on the subpopulation is -0.0225
## P-value 0.0669149835784016
## P-value Weak-Instrument Test 3.27931328326368e-129
## Proportion of observations in the node:  0.606741573033708
## Intention-to-treat effect:  -0.0146142686465243
## Proportion of compliers in the node:  0.648148148148148
##
##  node number: 5
##    root
##    xdirectie_seniority>=5.5
##    xleerkracht_seniority< 3.5
##
##  node number: 3
##    root
##    xdirectie_seniority< 5.5
## The conditional effect on the subpopulation is 0.0546
## P-value 0.00376106673175724
## P-value Weak-Instrument Test 7.2688803534356e-79
## Proportion of observations in the node:  0.359550561797753
## Intention-to-treat effect:  0.0341397849462326
## Proportion of compliers in the node:  0.625
##
##  node number: 6
##    root
##    xdirectie_seniority< 5.5
##    xdirectie_age>=6.5
## The conditional effect on the subpopulation is 0
## P-value 0.999999999999966
## P-value Weak-Instrument Test 1.91395572039115e-23
## Proportion of observations in the node:  0.0449438202247191
## Intention-to-treat effect:  -7.25205777466066e-16
## Proportion of compliers in the node:  0.75
##
##  node number: 7
##    root
##    xdirectie_seniority< 5.5
##    xdirectie_age< 6.5
## The conditional effect on the subpopulation is 0.0695
```

```
## P-value 0.00255442477476573
## P-value Weak-Instrument Test 9.12506907035885e-61
## Proportion of observations in the node:  0.314606741573034
## Intention-to-treat effect:  0.0421836228287855
## Proportion of compliers in the node:  0.607142857142857
```