Localization of Multiple Sources with a Raw Data Fusion

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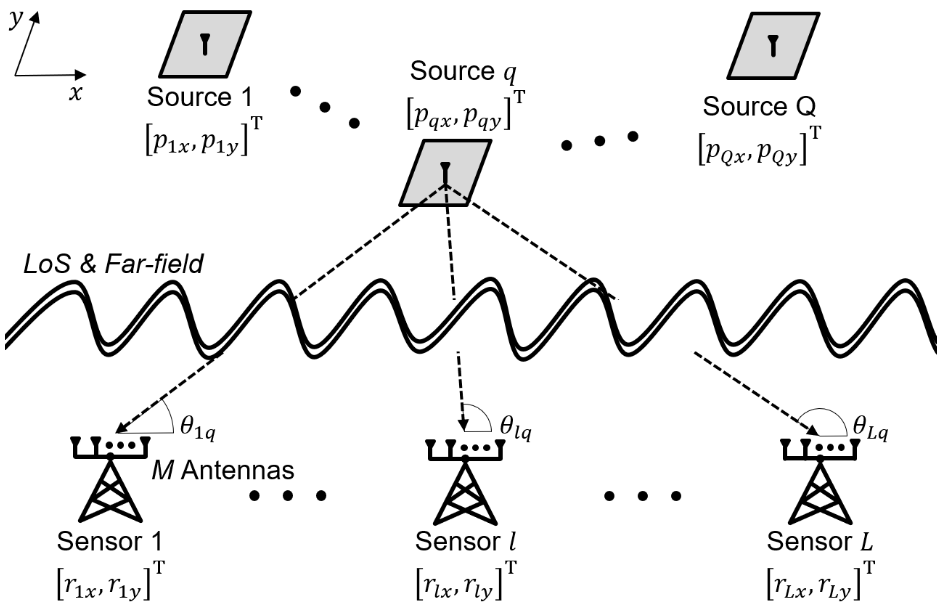
CONTENTS

Introduction

- **Backgrounds:**
  - Position localization is a fundamental signal processing task.
  - Developing a localization algorithm is crucial, especially for distinguishing highly co-located signal sources.
- **Contribution:**
  - Presentation of three distinct source localization algorithms.
  - Introduction and exploration of the Cramér-Rao Lower Bounds (CRLB) for each position estimator.

System model & Problem formulation

❖ Geometry for a scenario of multi-signal sources and sensors

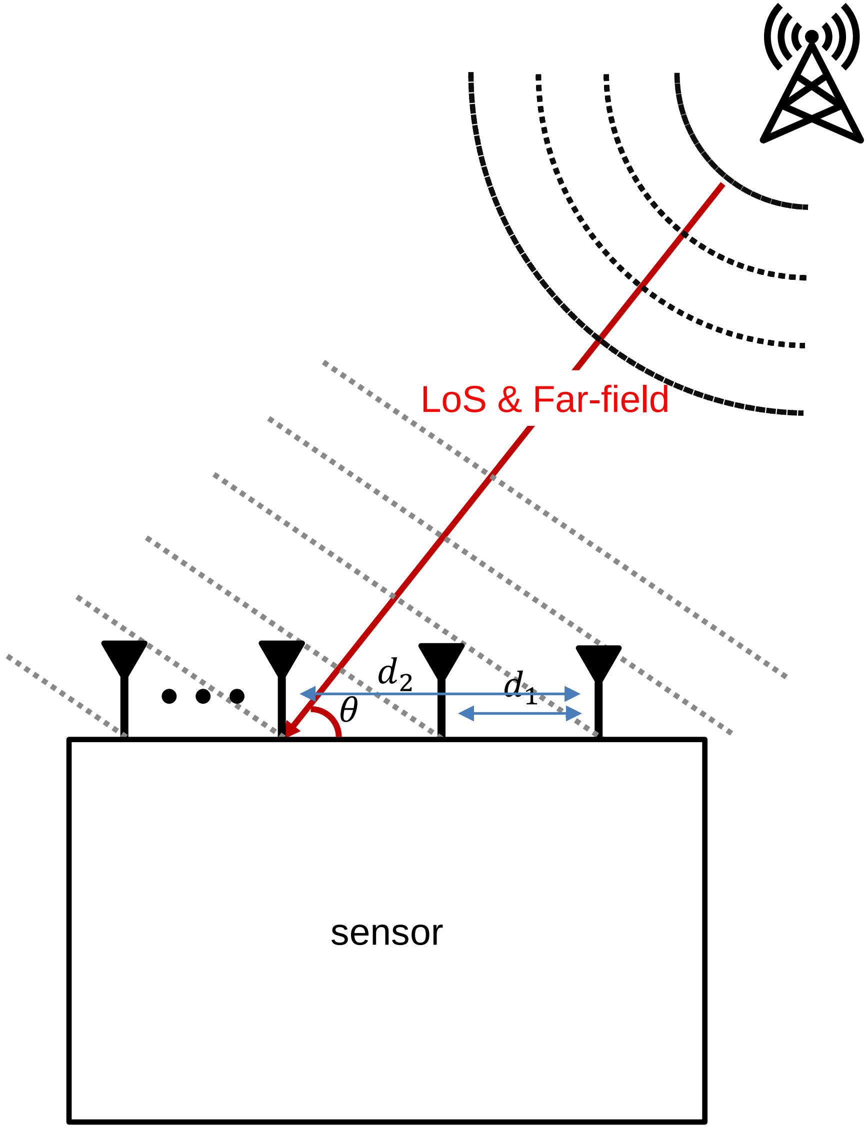


- Q number of signal sources are equipped with single antenna
- The source radiate narrowband signals
- There present LoS channel path btwn. sources and sensors
- L number of signal sources are equipped with M number of antennas

System model & Problem formulation

❖ Steering vector

$$\mathbf{a}(\theta) = \begin{bmatrix} e^{-j2\pi\left(\frac{d_0}{\lambda}\right)cos\theta} \\ e^{-j2\pi\left(\frac{d_1}{\lambda}\right)cos\theta} \\ \vdots \\ e^{-j2\pi\left(\frac{d_{M-1}}{\lambda}\right)cos\theta} \end{bmatrix}$$



❖ Received Signal at sensor

$$\mathbf{y}(k) = \mathbf{a}(\theta)s(k) + \mathbf{n}_l(k)$$

System model & Problem formulation

❖ Received signal in multi-sources scenario

$$\mathbf{y}_l(k) = \mathbf{A}_l(\boldsymbol{\theta}_l)\mathbf{s}(k) + \mathbf{n}_l(k) \in \mathbb{C}^{M\times 1}$$

❖ Steering matrix

$$\mathbf{A}_l(\boldsymbol{\theta}_l) = \begin{bmatrix} e^{-j2\pi\frac{d_0}{\lambda}\theta_{l1}} & \cdots & e^{-j2\pi\frac{d_0}{\lambda}\theta_{lQ}} \\ \vdots & \ddots & \vdots \\ e^{-j2\pi\frac{d_{M-1}}{\lambda}\theta_{l1}} & \cdots & e^{-j2\pi\frac{d_{M-1}}{\lambda}\theta_{lQ}} \end{bmatrix} \in \mathbb{C}^{M\times Q}$$

❖ Relation between DoA and position

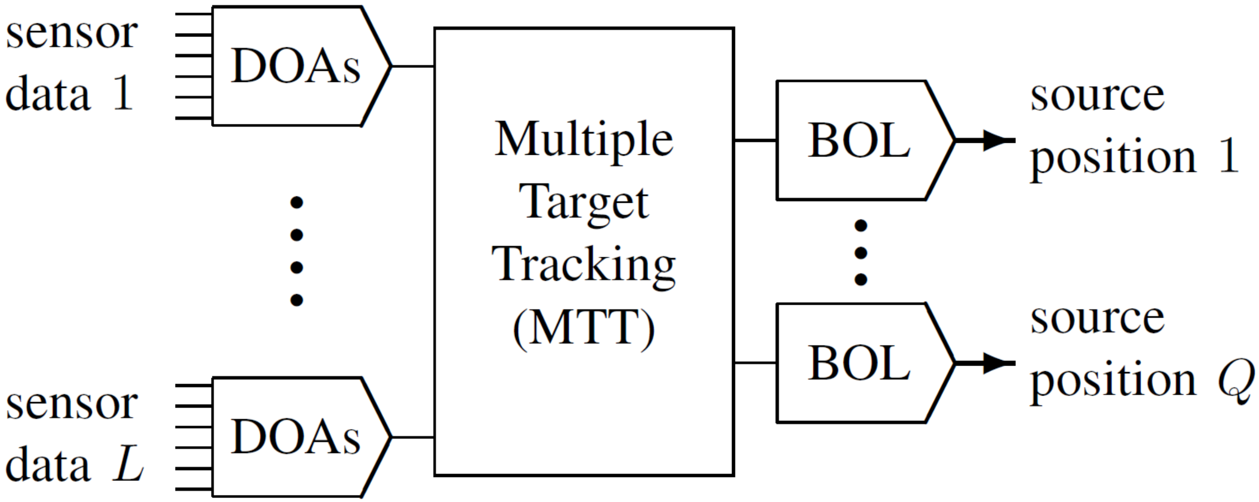
$$\theta_{lq} = \frac{p_{qx} - r_{lx}}{\left\|\mathbf{p}_q - \mathbf{r}_l\right\|_2}$$

❖ Problem formulation

- Estimate the position of the signal sources
  - It is assumed that the number of antennas M is larger than Q, i.e. $M > Q$
  - It is assumed that the number of signal sources, Q, are known to signal sensors
  - The noise is zero-mean complex Gaussian

Source localization estimator – BOL

- **Bearing only localization (BOL)**



Source localization estimator – BOL

❖ Bearing only localization (BOL)

- Step 1: Covariance matrix of Rx signals

$$\mathbf{R}_{\mathbf{y}_l} = \frac{1}{K}\mathbf{y}_l\mathbf{y}_l^H = \mathbb{E}\left[\mathbf{y}_l\mathbf{y}_l^H\right] = \mathbf{A}(\boldsymbol{\theta}_l)\mathbb{E}[\mathbf{s}\mathbf{s}^H]\mathbf{A}^H(\boldsymbol{\theta}) + \mathbb{E}[\mathbf{n}\mathbf{n}^H]$$

- Step 2: Eigenvalue decomposition (EVD)

$$EVD\left(\mathbf{R}_{y_l}\right) = \mathbf{U}_l\boldsymbol{\Lambda}_l\mathbf{U}_l^H = \mathbf{U}_l^s\boldsymbol{\Lambda}\mathbf{U}_l^{s\,H} + N_0\mathbf{U}_l^n\mathbf{U}_l^n$$

- Step 3: DoA Estimation

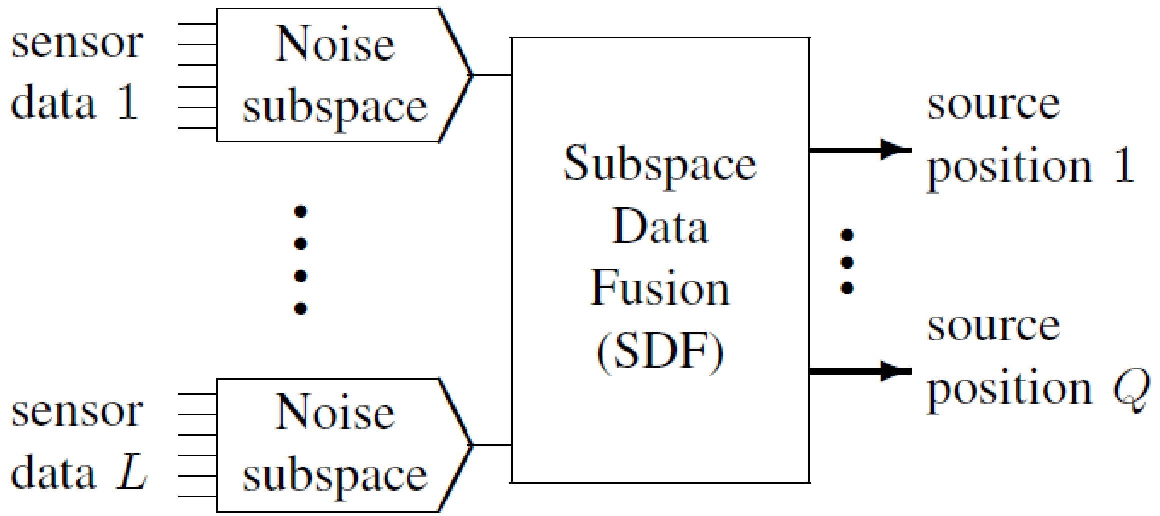
$$f_{MUSIC}^{-1}(\alpha_l) = |\mathbf{a}^H(\alpha_l)\mathbf{U}_l^n|^2$$

- Step 4: BOL

$$f_{BOL}\left(\tilde{x}_q, \tilde{y}_q\right) = \sum_{l=1}^{L}\frac{\left(\hat{\alpha}_{l,q} - \alpha_{l,q}\right)^2}{\sigma_{\alpha_{l,q}}^2}$$

Source localization estimator – SDF

- **Subspace data fusion (SDF)**



Source localization estimator – SDF

Subspace data fusion (SDF)

- Step 1: Covariance matrix of Rx signals

$$\mathbf{R}_{\mathbf{y}_l} = \frac{1}{K}\mathbf{y}_l\mathbf{y}_l^H = \mathbb{E}\left[\mathbf{y}_l\mathbf{y}_l^H\right] = \mathbf{A}(\boldsymbol{\theta}_l)\mathbb{E}[\mathbf{s}\mathbf{s}^H]\mathbf{A}^H(\boldsymbol{\theta}) + \mathbb{E}[\mathbf{n}\mathbf{n}^H]$$

- Step 2: Eigenvalue decomposition (EVD)

$$EVD\left(\mathbf{R}_{y_l}\right) = \mathbf{U}_l\boldsymbol{\Lambda}_l\mathbf{U}_l^H = \mathbf{U}_l^s\boldsymbol{\Lambda}\mathbf{U}_l^{s^H} + N_0\mathbf{U}_l^n\mathbf{U}_l^n$$

- Step 3: Source position estimation

$$f_{SDF}(x,y) = \sum_{l=1}^{L}\left|\mathbf{a}_l^H(x,y)U_l^n\right|^2$$

Source localization estimator – RDF

- **Proposed algorithm (Skip)**

Cramer Rao lower bound

CRLB

- Fundamental limit on the variance of any unbiased estimator for a parameter
- Quantifies the best achievable precision for estimating a parameter in a given statistical model
- No unbiased estimator can have a variance lower than the CRLB

CRLB for BOL and SDF

$$CRLB(\mathbf{p}) = \frac{\sigma_n^2}{2}\left(\sum_{k=1}^{K} Re\left\{\mathbf{S}^H(k)\mathbf{D}^H\mathbf{P}_{\boldsymbol{\mathcal{A}}}^{\perp}\mathbf{D}\mathbf{S}(k)\right\}\right)^{-1}$$

where

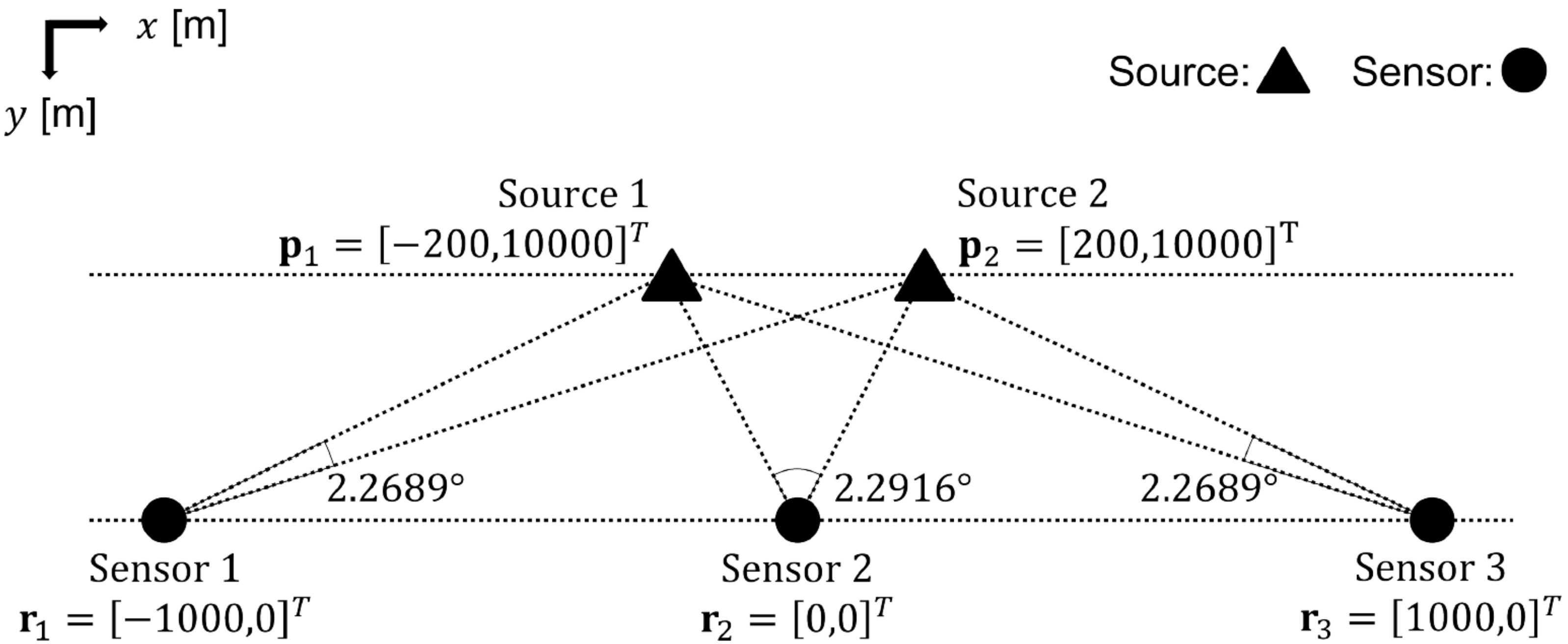
$$P_{\boldsymbol{\mathcal{A}}}^{\perp} = \mathbf{I}_{LM} - \boldsymbol{\mathcal{A}}(\boldsymbol{\mathcal{A}}^H\boldsymbol{\mathcal{A}})^{-1}\boldsymbol{\mathcal{A}}^H \in \mathbb{C}^{LM\times LM},$$

$$\mathbf{D} = \left[\frac{\partial\boldsymbol{\mathcal{A}}}{\partial x_1}\frac{\partial\boldsymbol{\mathcal{A}}}{\partial y_1}\cdots\frac{\partial\boldsymbol{\mathcal{A}}}{\partial x_Q}\frac{\partial\boldsymbol{\mathcal{A}}}{\partial y_Q}\right] \in \mathbb{C}^{LM\times 3LQ^2},$$

$$\mathbf{S}(k) = \mathbf{I}_{3Q} \otimes \mathbf{s}(k) \in \mathbb{C}^{3LQ^2\times 3Q}.$$

Simulation results

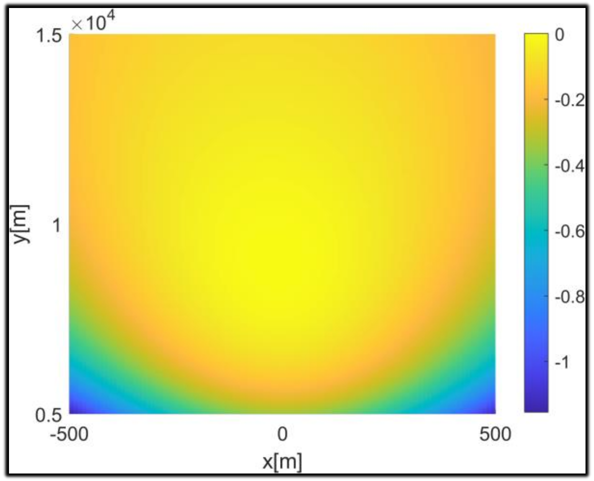
- **Geographical locations of signal sources and sensors**



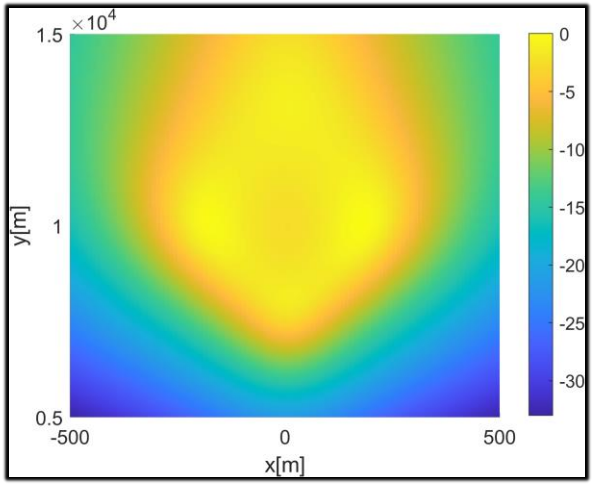
Simulation results

❖ **Inverse cost function of the position estimator**

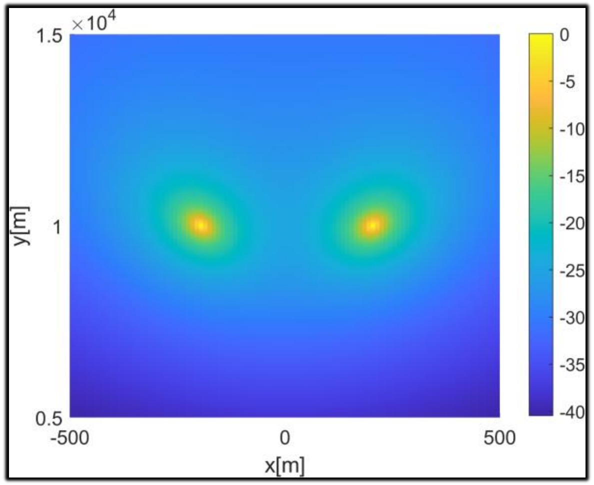
BOL



SDF

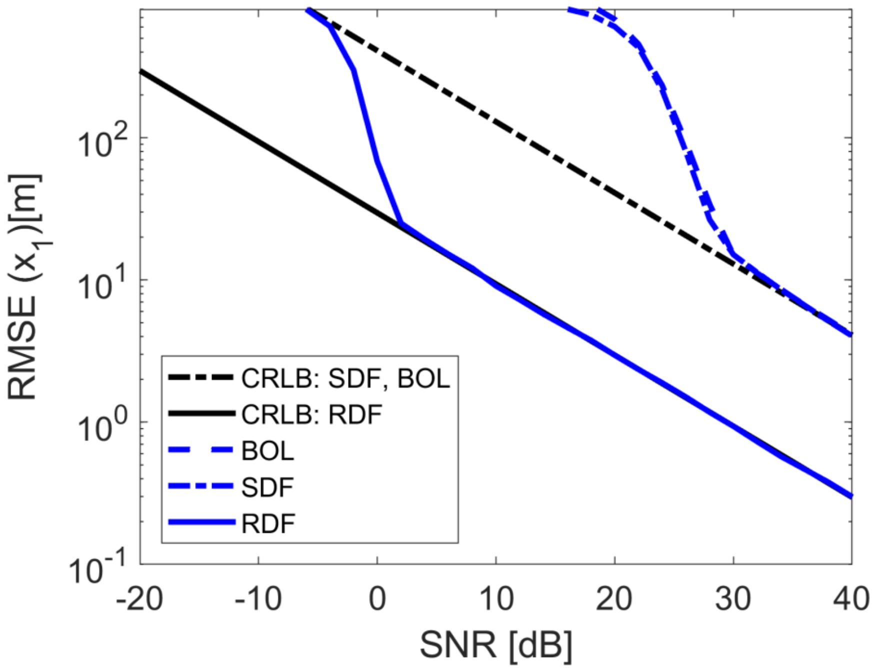


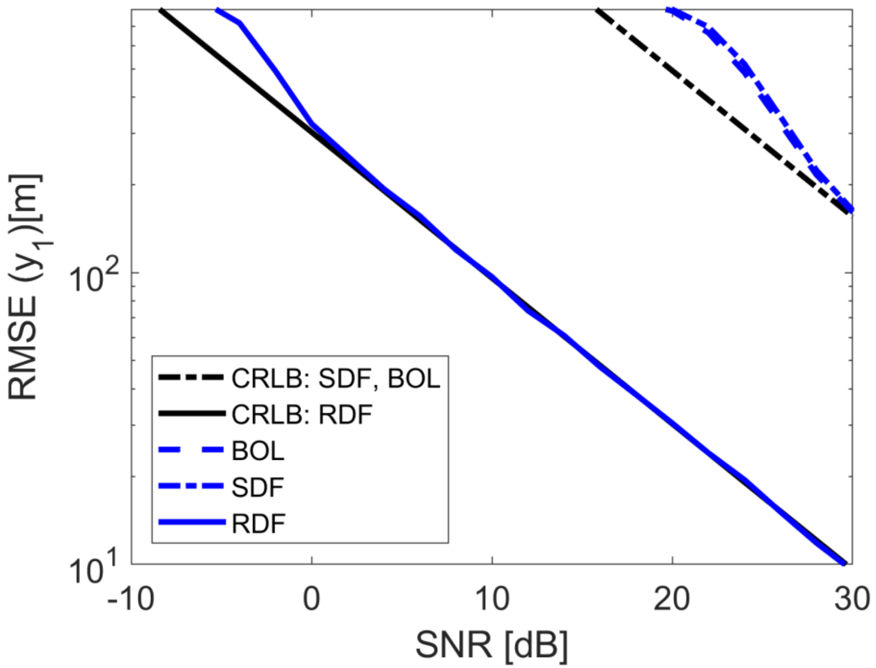
Proposed



Simulation results

❖ **Root mean squared error results**





Summary

- **Localization Estimators:**
  - Bearings only localization
  - Subspace data fusion
  - Raw data fusion
- **CRLB for BOL and SDF:**
  - CRLB to position estimators for BOL and SDF.
- **Proposed Method Performance:**
  - Exhibits more distinctive peaks in the inverse cost function graph.
- **RMSE Comparison:**
  - RDF demonstrates over 10 dB gain in Root Mean Square Error (RMSE).
- **CRLB Efficiency Confirmation:**
  - BOL, SDF, and RDF established as CRLB efficient estimators.
- **Visualization Emphasis:**
  - Consideration of graphs or visuals to illustrate distinctive peaks and RMSE comparison.
- **Key Findings:**
  - Proposed method excels in peak distinctiveness.
  - RDF stands out with a significant RMSE gain.
  - All methods prove CRLB efficiency, highlighting their accuracy.

Thank you