EM Response for Quartic Hamiltonian (etc.)—Direct Numerics

In this document, we calculate expectation values directly rather than using the nested sums as before, which should be faster and more accurate.

1 Current per Orbital

We recall that the $2n$ Hamiltonian (in David's notation) is

$$\hat{H}_{2n} = \frac{1}{2n}\left[\hat{k}_x^{2n} + \hat{k}_y^{2n}\right],$$

where

$$\begin{aligned}\hat{k}_y &= -\sqrt{\frac{B}{2}}\left(a + a^\dagger\right) \\ \hat{k}_x &= -i\sqrt{\frac{B}{2}}\left(a - a^\dagger\right).\end{aligned}$$

To calculate the current per orbital, we require the current operator, which we write as

$$\hat{I}_y = \partial H / \partial k_y.$$

Noting that

$$\partial_{k_y} a = \partial_{k_y} a^\dagger = -\frac{1}{\sqrt{2B}}$$

we see that

$$\begin{aligned}\partial_{k_y}\hat{k}_x &= 0 \\ \partial_{k_y}\hat{k}_y &= 1\end{aligned}$$

(as expected) and so

$$\partial_{k_y}\hat{H}_{2n} = \hat{k}_y^{2n-1}.$$

The pth term in the external potential is

$$c_p(2B)^{-p/2}\left(a + a^\dagger\right)^p = c_p\frac{(-1)^p}{B^p}\hat{k}_y^p.$$

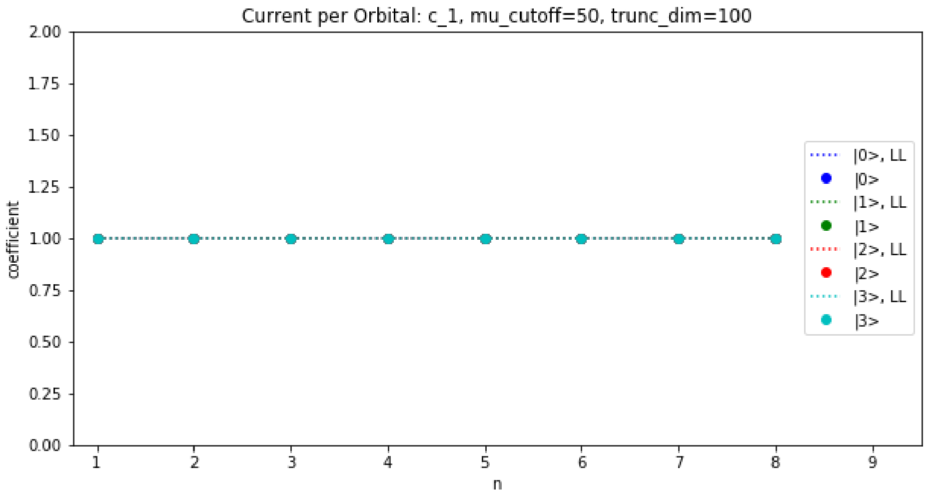
Then, the current per orbital expression becomes

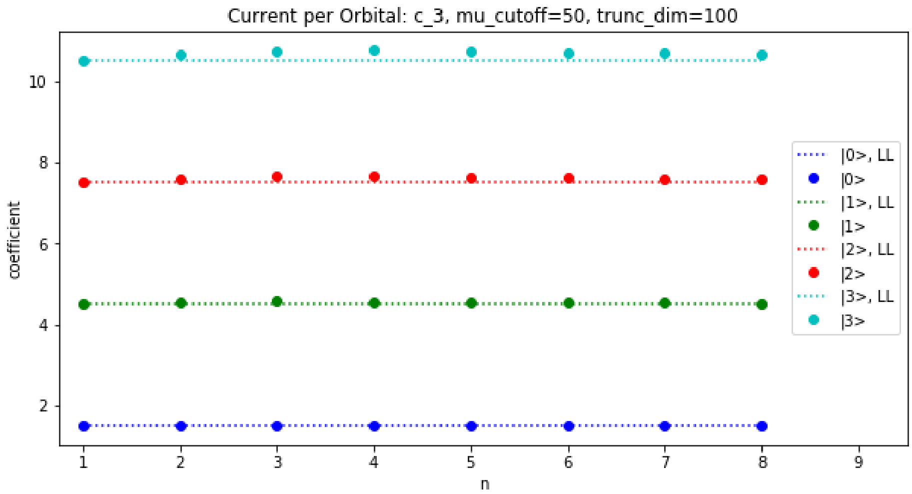
$$\begin{aligned}\left\langle \hat{I}_y \right\rangle &= \sum_{\mu\neq\lambda}\langle\lambda|\,\hat{I}_y\,|\mu\rangle\frac{\langle\mu|\,\hat{V}(x)\,|\lambda\rangle}{E_\lambda - E_\mu} + \text{H.c.} \\ &= 2\sum_{\mu\neq\lambda}\sum_p c_p\frac{(-1)^p}{B^p}\langle\lambda|\,\hat{k}_y^{2n-1}\,|\mu\rangle\frac{\langle\mu|\,\hat{k}_y^p\,|\lambda\rangle}{E_\lambda - E_\mu}.\end{aligned}$$

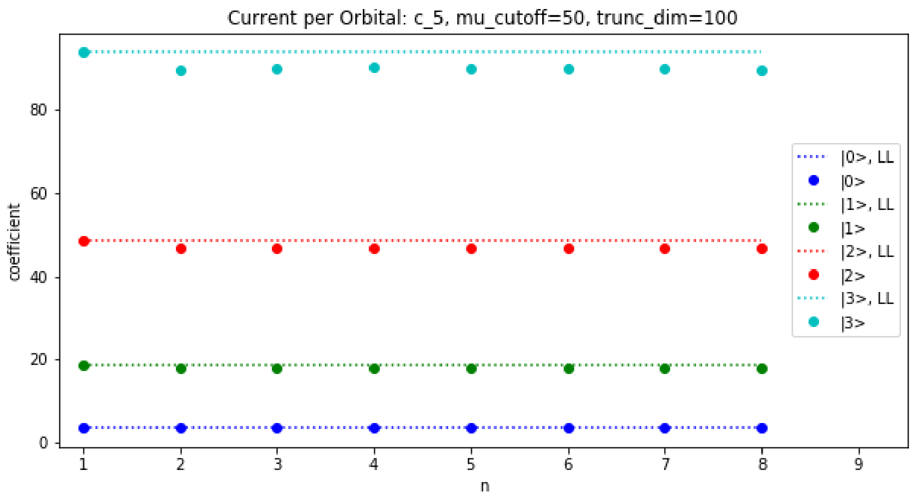
For each value of λ, p, we can calculate these matrix elements and sum over μ directly. We aim to find the numerical factor that multiplies c_p (setting $B = 1$).

1.1 Numerical Results

This is now extremely fast so we calculate up to $n = 8$. We plot the results below:







These agree with the values calculated using the previous method. The values for c_1 are equal to the LL value (topologically protected).

2 Current Density

The current density is given by the expression

$$\begin{aligned}\left\langle \hat{J}_y \right\rangle_\lambda^{2n} &= \frac{B}{2\pi}\sum_{r,s}\sum_{\mu\neq\lambda} d_{rs}\langle\lambda|\,\hat{I}_y\hat{x}^s\,|\mu\rangle\frac{\langle\mu|\,\hat{x}^r\,|\lambda\rangle}{E_\lambda-E_\mu}+\text{H.c.}\\ &= \frac{B}{2\pi}\sum_{r,p}\sum_{\mu\neq\lambda} d_{r,p-r}\langle\lambda|\,\hat{I}_y\hat{x}^{p-r}\,|\mu\rangle\frac{\langle\mu|\,\hat{x}^r\,|\lambda\rangle}{E_\lambda-E_\mu}\end{aligned}$$

where we recall that

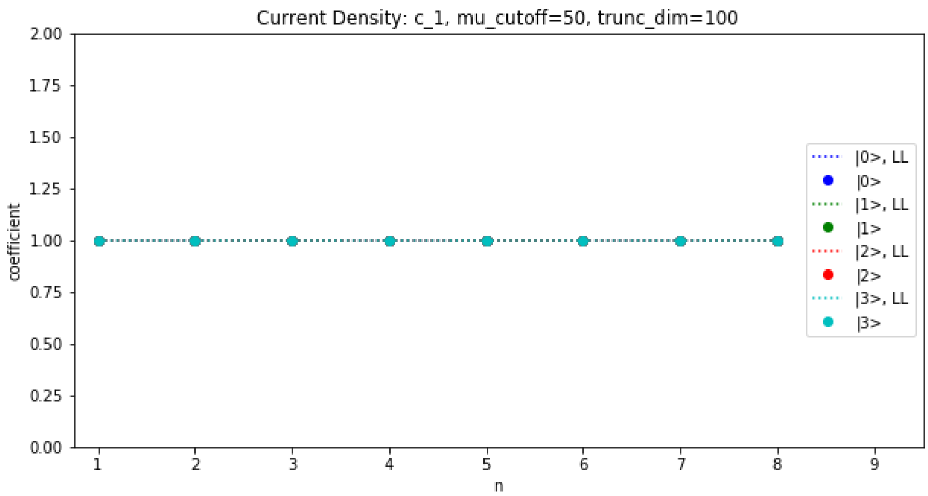
$$\begin{aligned} d_{rs} &= (-1)^s\begin{pmatrix} r+s \\ s\end{pmatrix} c_{r+s}\\ d_{r,p-r} &= (-1)^{p-r}\begin{pmatrix} p \\ p-r\end{pmatrix} c_p.\end{aligned}$$

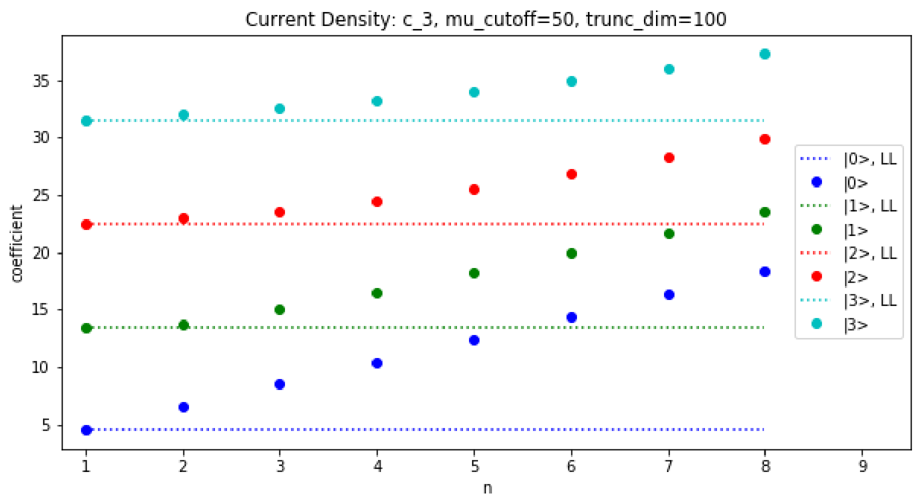
Making the same substitutions as before, this becomes

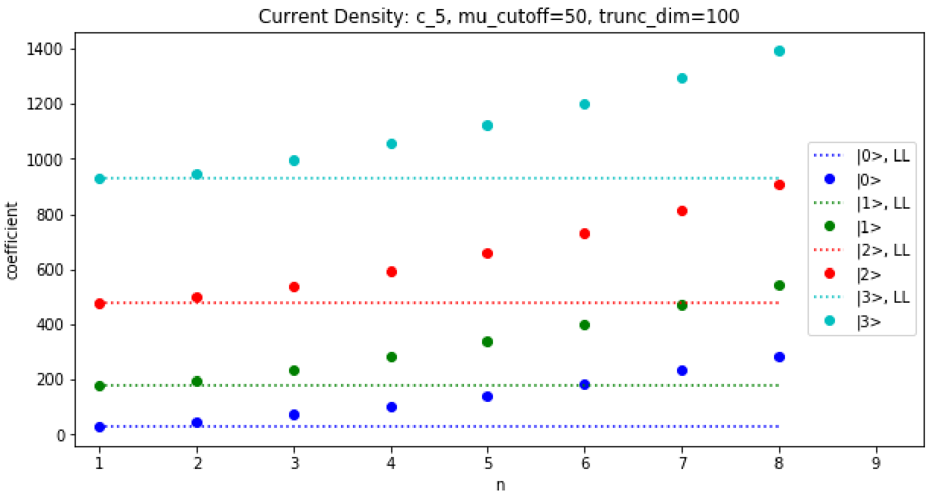
$$\left\langle \hat{J}_y \right\rangle_\lambda^{2n} = \frac{B}{2\pi}2\sum_{\mu\neq\lambda}\sum_{p,r} d_{r,p-r}\frac{(-1)^p}{B^p}\langle\lambda|\,\hat{k}_y^{2n-1+p-r}\,|\mu\rangle\frac{\langle\mu|\,\hat{k}_y^r\,|\lambda\rangle}{E_\lambda-E_\mu}.$$

We ignore the factor of $B/2\pi$ (which can be put in by hand later).

2.1 Numerical Results







Again, the coefficient of c_1 is unchanged (topologically protected), but the others all vary with n. The differences from the Landau level case are much larger this time. These values agree with those calculated using the previous method, but the calculation is much faster.