Quartic dispersion

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Consider the semi-classical energy $E(p_x, p_y)$. If we enforce inversion symmetry, then the most general E we can write up to an additive constant is

$$E(\mathbf{p}) = \sum_{k \text{ even}} E_k(\mathbf{p}),$$

where E_k is a homogeneous polynomial over $\mathbb{R}$ of degree k. Define

$$\varepsilon_k(p_x) = E_k(p_x, 1),$$

then

$$E_k(p_x, p_y) = p_y^k \varepsilon_k(p_x/p_y).$$

ε_k is a polynomial of degree k in one variable over $\mathbb{R}$,and every polynomial in one variable over $\mathbb{R}$ is reducible to a product of polynomials of degree at most 2. [1] (This corresponds to the fact that we can decompose a polynomial into products of factors for each of its roots, but a real polynomial might have paired complex roots.) Thus there are M possible forms of ε_k,

$$\varepsilon_k^M(p) = \prod_{i=1}^{M} \left(A_2^{M,i}x^2 + A_1^{M,i}x + A_0^{M,i}\right) \prod_{j=1}^{k-2M} \left(B_1^{M,j}x + B_0^{M,j}\right)$$

with $0 \leq M \leq \lceil k/2 \rceil$. For $k = 4$, there are 3 possibilities for ε_4: 2 quadratic factors, 1 quadratic factor and 2 linear factors, or 4 linear factors.

References

[1] https://en.wikipedia.org/wiki/Irreducible_polynomial

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