Introduction

Method
- TEBD^2
- DMRG^2

Results
- Bosons $\nu = 1/2$
- Fermions $\nu = 1/3$
- Scaling

Conclusion

Characterizing fractional quantum Hall states using isometric tensor networks

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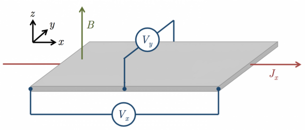
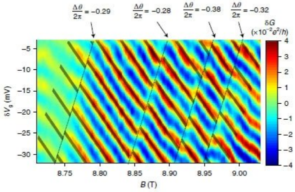
SPS Annual Meeting, Basel

September 7, 2023

Overview of the FQHE

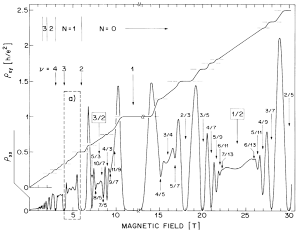


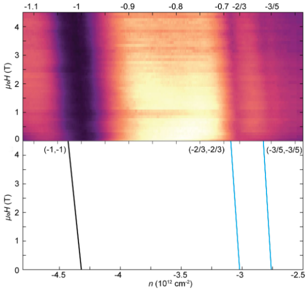
Hall bar
adapted from R. Roy website (2023)



anyon statistics

resistivity plateaux
Willett *et al.*, PRL **59**, 1776 (1987)





FCI Landau fan

→ quantum regime $k_B T \ll \hbar\omega$

→ pure sample $V_d \ll V_C \ll \hbar\omega$

→ partially-filled energy bands

→ fractional excitations

→ lattice generalizations

- anyons observed!

– Nakamura *et al.*,

Nat. Phys. **16**, 931-936 (2020)

– Bartolomei *et al.*,

Science **368**, 6487, 173-177 (2020)

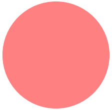
- zero-field FCIs observed?!

– Cai *et al.*,

arXiv:2304.08470 (2023)

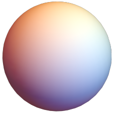
Progress in FQH simulation: a journey through geometries

disk



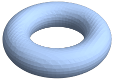
Laughlin, PRL **50**, 1395 (1983)
MacDonald, PRB **30**, 3550(R) (1984)

sphere



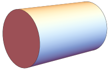
Haldane, PRL **51**, 605 (1983)
He, Simon, Halperin, PRB **50**, 1823 (1994)

torus



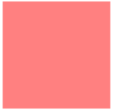
Möller and Cooper, PRL **103**, 105303 (2009)
Neupert, Santos, Chamon, Mudry, PRL **106**, 236804 (2011)

cylinder



Zaletel, Mong, Pollmann, PRL **110**, 236801 (2013)
Grushin, Motruk, Zaletel, Pollmann, PRB **91**, 035136 (2015)

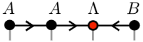
rectangle



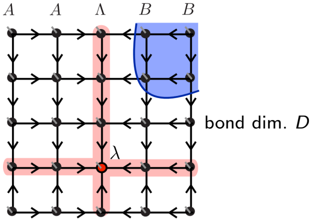
Gerster, Rizzi, Silvi *et al.*, PRB **96**, 195123 (2017)
This talk...

Isometric tensor networks (isoTNS)

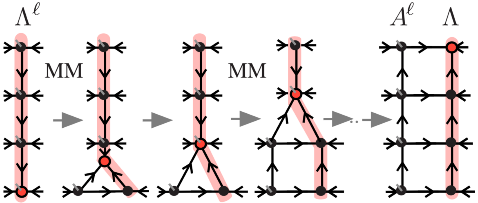
- **Subset of PEPS** representing states with exponential correlations (and more...)
- Gauge freedom in 1D tensor networks fixed via **canonical form**



- isoTNS allows us to generalize the canonical form to 2D: orthogonality center $\rightarrow$ **orthogonality cross**
- Expectation values on the orthogonality cross **reduce to 1D** MPS overlaps
- Orthogonality cross can be shifted using the **Moses move** algorithm



isoPEPS



Moses move algorithm

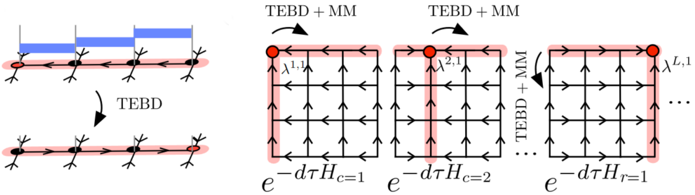
Zaletel and Pollmann, PRL **124**, 037201 (2020)

2D time-evolving block decimation (TEBD^2)

Unitary (real or imaginary) time evolution $|\Psi(\tau)\rangle = e^{-H\tau}|\Psi(0)\rangle$

1. Split the Hamiltonian into columns and rows $H = \sum_c H_c + \sum_r H_r$
2. Trotterize the columns and rows separately $e^{-H\tau}|0\rangle \approx \prod_c e^{-H_c d\tau} \prod_r e^{-H_r d\tau}|0\rangle$
3. Sweep TEBD through the columns, then sweep through the rows
4. Orthogonality cross follows sweeps to ensure SVD is globally optimal

Note: controlled errors from both Moses move and TEBD



isoTNS TEBD^2 $\sim O(D^7)$ vs. full PEPS $\sim O(D^{10})$ [assuming $D_b \sim D^2$]

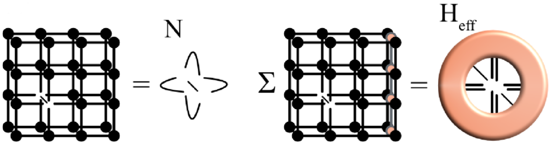
Zaletel and Pollmann, PRL **124**, 037201 (2020)

2D density matrix renormalization group (DMRG^2)

Variational ground-state search minimizing $E[\Psi] = \langle\Psi|H|\Psi\rangle \,/\, \langle\Psi|\Psi\rangle$

1. Obtain the effective Hamiltonian MPO for each column $H_{\text{eff}} = H_{\text{eff,L}} + H_{\text{eff,R}}$
2. Store intermediate effective Hamiltonians in a cache $\{\{H_{\text{eff,L,1}}, H_{\text{eff,R,1}}\}, \ldots\}$
3. Sweep (single-site) DMRG through the columns, and then back again
4. Orthogonality cross follows sweeps to ensure SVD is globally optimal

Note: controlled errors from both Moses move and DMRG



isoPEPS contraction at λ effective Hamiltonian

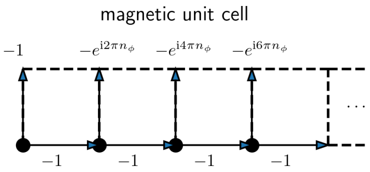
$$\text{isoTNS DMRG}^2 \sim O(D^{10}) \text{ vs. full PEPS} \sim O(D^{12}) \quad [\text{assuming } D_b \sim D^2]$$

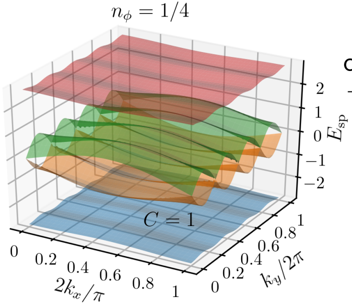
Lin, Zaletel, Pollmann, PRB **106**, 245102 (2022)

Hofstadter model

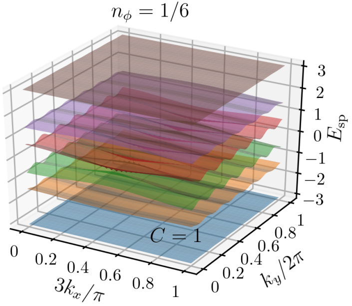
$$H = -\sum_{\langle ij\rangle} e^{\mathrm{i}\theta_{ij}} c_i^\dagger c_j - \mu \sum_i \rho_i + \sum_{ij} V_{ij}\rho_i\rho_j$$

- Peierls phase $\theta_{ij} \sim \int_i^j \mathbf{A}\cdot \mathrm{d}\mathbf{l}$
- Landau gauge $\mathbf{A} = Bx\hat{\mathbf{e}}_y$
- $V_{ij} = \delta_{ij}(\delta_{\langle ij\rangle})$ for bosons(fermions)





continuum limit $\longrightarrow$

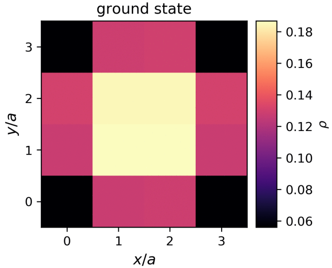


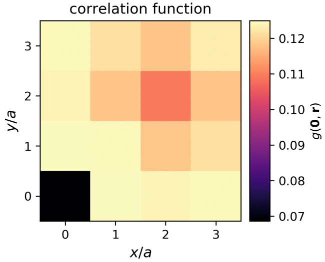
Δ/W	$\sigma_{\mathcal{B}}/\langle\mathcal{B}\rangle$	$\langle\mathcal{T}\rangle$
7.14	0.434	0.0831

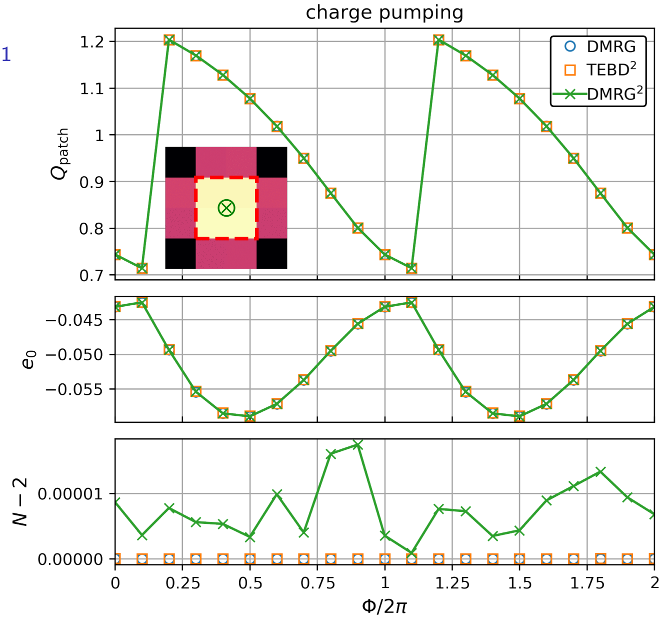
Δ/W	$\sigma_{\mathcal{B}}/\langle\mathcal{B}\rangle$	$\langle\mathcal{T}\rangle$
77.3	0.0774	0.0172

Bosonic $\nu = 1/2$ state

$n_\phi = 1/4,\ L_x \times L_y = 4 \times 4,\ \mu = -2.11$

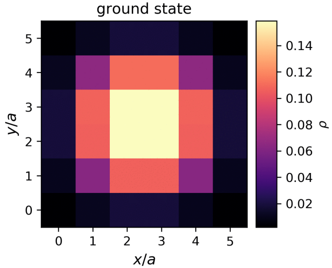


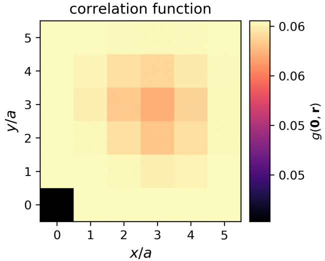


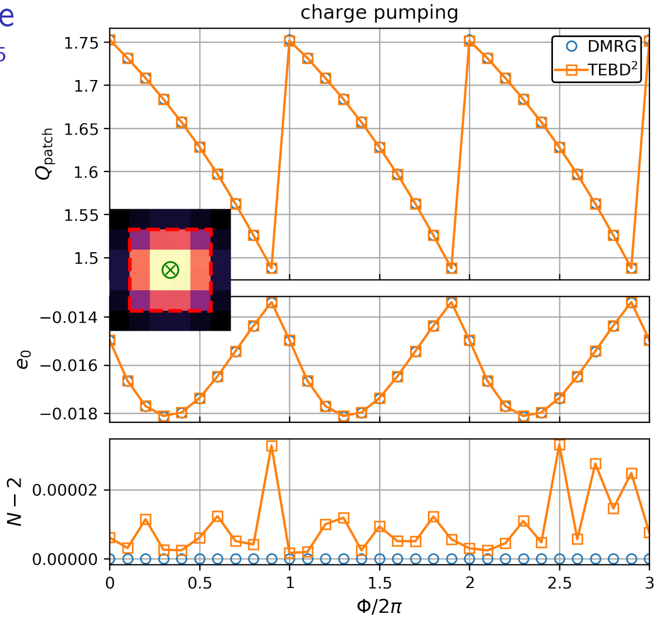


Fermionic $\nu = 1/3$ state

$n_\phi = 1/6$, $L_x \times L_y = 6 \times 6$, $\mu = -2.75$

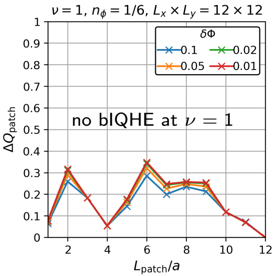




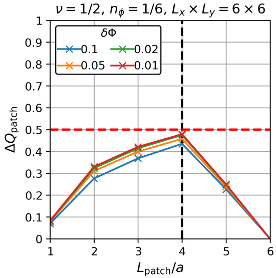


Patch-size scaling

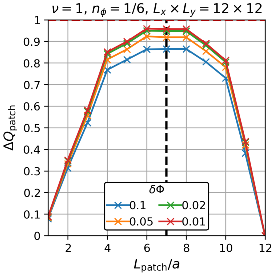
bosons



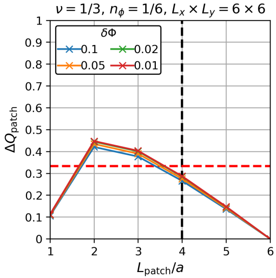
fractionalization $\longrightarrow$



fermions



fractionalization $\longrightarrow$



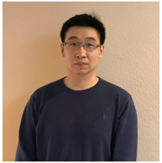
Conclusion

- isoTNS is a useful restriction of the PEPS ansatz that can represent an interesting subset of strongly-correlated systems – including FQH states (in practice)!
- isoTNS allows for natural generalizations of the TEBD and DMRG algorithms to 2D, which are more scalable than snake MPS variants

Future work:

- large-scale simulations, entanglement, infinite TEBD^2/DMRG^2, real materials, . . .

Special thanks to my collaborators:



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Funded by the SNSF Postdoc Mobility Fellowship 203168.

FNSNF Swiss National Science Foundation

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