INTRODUCTION TO MODERN INPUT-OUTPUT ANALYSIS

Problems

1 Using the following input output table:

X_i	x_{ij}			Y_i
100	20	10	40	30
120	40	20	0	60
80	0	50	10	20
Amortization	10	10	10	
Wages	20	10	10	
Profit	10	20	10	

calculate:

a) Material costs in branch II
b) Physical costs in branch III
c) Profitability in branch III
d) Material consumption coefficient in branch III
e) Value added in each branch.

2 The vector of output in a two-dimensional IO system takes the following form $X=[200,300]^T$. The material consumption coefficient in both branches is equal to 0.6. The products of branch II are not used in production processes in branch I. The final product in branch I is equal to 20% of the corresponding global product. Wages in branch I are equal to the half of the value added, while the profit in branch II is equal to 40. Find the respective input output table.

3 After filling the following IO table:

X_i	x_{ij}				Y_i
…	…	50	…	180	100
270	40	30	…	…	50
380	…	70	0	10	110
230	30	0	120	20	…
Wages	20	…	40	10	
Profit	…	30	50	10	
X_j	400	…	…	…	

calculate:

a) Profitability in every branch
b) Value added in every branch
c) Total material cost in the whole economy.

4 Fill the following IO table:

	Intermediate flows			Total inter-mediate us-age	Final product	Output
	…	60	40	120	80	…
	…	30	…	120	…	300
	…	…	10	…	…	…
Material costs	…	120	…	300		
Value added	100	…	...		…	
Output	…	…	…			600

5 Based on the following IO table for the year 2016:

X_i	x_{ij}			Y_i
1000	200	0	200	600
800	300	0	100	400
500	0	400	0	100

find the IO table for 2017 if:

a) $\Delta X_{2017}=[100 \quad 100 \quad 50]^T$
b) $\Delta Y_{2017}=[50 \quad 50 \quad 50]^T$
c) $\Delta X_{2017}=[200 \quad 100 \quad a]^T$, $\Delta Y_{2017}=[b \quad c \quad 0]^T$

6 Which of the following matrices cannot be an input matrix (A) in the Leontief model? Justify your answer.

a)
$$A = \begin{bmatrix} 0,5 & 0,1 & 0,3 \\ -0,7 & 0,9 & 0,3 \\ -0,7 & 0,1 & 0,4 \end{bmatrix}$$

b)
$$A = \begin{bmatrix} 0,1 & 0,2 \\ 0,2 & 0,1 \\ 0,4 & 0,6 \end{bmatrix}$$

c)
$$A = \begin{bmatrix} 0,5 & 0 & 0,2 \\ 0,3 & 0,2 & 0,6 \\ 0,1 & 0,2 & 0,1 \end{bmatrix}$$

7 Having the knowledge that in 2016 wages in each branch were equal to 50% of value added and:

$X_{2016}=[520 \quad 610]^T$, $Y_{2017}-Y_{2016}=[15 \quad 40]^T$,

$$A = \begin{bmatrix} 0,6 & 0,3 \\ 0,2 & 0,6 \end{bmatrix}, (I-A)^{-1} = \begin{bmatrix} 4 & 3 \\ 2 & 4 \end{bmatrix}$$

a) Find the IO table for 2017.
b) Compare the profitability and material consumption coefficients for both branches in 2017.

8 Dollar values of last year's interindustry transactions and total outputs for a two-sector economy (agriculture and manufacturing) are as shown below:

$$\mathbf{Z} = \begin{bmatrix} 500 & 350 \\ 320 & 360 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1000 \\ 800 \end{bmatrix}$$

a. What are the two elements in the final-demand vector $\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$?

b. Suppose that f_1 increases by \$50 and f_2 decreases by \$20. What new gross outputs would be necessary to satisfy the new final demands?

  i. Find an approximation to the answer by using the first five terms in the power series, $\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \cdots + \mathbf{A}^n$.

  ii. Find the exact answer using the Leontief inverse.

9 Interindustry sales and total outputs in a small three-sector national economy for year t are given in the following table, where values are shown in thousands of dollars. (S_1, S_2 and S_3 represent the three sectors.)

	Interindustry Sales			
	S_1	S_2	S_3	Total Output
S_1	350	0	0	1000
S_2	50	250	150	500
S_3	200	150	550	1000

a. Find the technical coefficients matrix, $\mathbf{A}$, and the Leontief inverse matrix, $\mathbf{L}$, for this economy.

b. Suppose that because of government tax policy changes, final demands for the outputs of sectors 1, 2 and 3 are projected for next year (year $t+1$) to be 1300, 100 and 200, respectively (also measured in thousands of dollars). Find the total outputs that would be necessary from the three sectors to meet this projected demand, assuming that there is no change in the technological structure of the economy (that is, assuming that the $\mathbf{A}$ matrix does not change from year t to year $t+1$).

c. Find the original (year t) final demands from the information in the table of data. Compare with the projected (year $t+1$) final demands. Also, compare the original total outputs with the outputs found in part b. What basic feature of the input–output model do these two comparisons illustrate?

10 Consider an economy organized into three industries: lumber and wood products, paper and allied products, and machinery and transportation equipment. A consulting firm estimates that last year the lumber industry had an output valued at \$50 (assume all monetary values are in units of \$100,000), 5 percent of which it consumed itself; 70 percent was consumed by final demand; 20 percent by the paper and allied products industry; 5 percent by the equipment industry. The equipment industry consumed 15 percent of its own products, out of a total of \$100; 25 percent went to final demand; 30 percent to the lumber industry; 30 percent to the paper and allied products industry. Finally, the paper and allied products industry produced \$50, of which it consumed

10 percent; 80 percent went to final demand; 5 percent went to the lumber industry; and 5 percent to the equipment industry.

a. Construct the input–output transactions matrix for this economy on the basis of these estimates from last year's data. Find the corresponding matrix of technical coefficients, and show that the Hawkins–Simon conditions are satisfied.
b. Find the Leontief inverse for this economy.
c. A recession in the economy this year is reflected in decreased final demands, reflected in the following table:

Industry	% Decrease in Final Demand
Lumber & Wood Products	25
Machinery & Transportation Equipment	10
Paper & Allied Products	5

d. What would be the total production of all industries required to supply this year's decreased final demand? Compute the value-added and intermediate output vectors for the new transactions table.

11 Consider a simple two-sector economy containing industries A and B. Industry A requires \$2 million worth of its own product and \$6 million worth of Industry B's output in the process of supplying \$20 million worth of its own product to final consumers. Similarly, Industry B requires \$4 million worth of its own product and \$8 million worth of Industry A's output in the process of supplying \$20 million worth of its own product to final consumers.

a. Construct the input–output transactions table describing economic activity in this economy.
b. Find the corresponding matrix of technical coefficients and show that the Hawkins–Simon conditions are satisfied.
c. If in the year following the one in which the data for this model was compiled there were no changes expected in the patterns of industry consumption, and if a final demand of \$15 million worth of good A and \$18 million worth of good B were presented to the economy, what would be the total production of all industries required to supply this final demand as well as the interindustry activity involved in supporting deliveries to this final demand?

12 Consider the following transactions table, $\mathbf{Z}$, and total outputs vector, $\mathbf{x}$, for two sectors, A and B:

$$\mathbf{Z} = \begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$$

a. Compute the value-added and final-demand vectors. Show that the Hawkins–Simon conditions are satisfied.

b. Consider the r-order round-by-round approximation of $\mathbf{x} = \mathbf{Lf}$ to be $\mathbf{x}^{\sim} = \Sigma_{i=0}^{r} A^i f$ (remember that $\mathbf{A}^0 = \mathbf{I}$). For what value of r do all the elements of $\tilde{\mathbf{x}}$ come within 0.2 of the actual values of $\mathbf{x}$?

13 Given the following transactions table for industries a, b, and c, and the total output as shown, compute the final-demand vectors and show that the inverse of $(\mathbf{I}-\mathbf{A})$ exists.

Industries	a	b	c	Total Output
a	3	8	6	22
b	2	4	5	18
c	7	3	9	31

Use the power series to approximate $\mathbf{x}$ to within 0.1 of the actual output values shown

14 Consider the following two-sector input–output table measured in millions of dollars:

	Manuf.	Services	Final Demand	Total Output
Manufacturing	10	40	50	100
Services	30	25	85	140
Value Added	60	75	135	
Total Output	100	140		240

If labor costs in the services sector increase, causing a 25 percent increase in value added inputs required per unit of services and labor costs in manufacturing decrease by 25 percent, what are the resulting changes in relative prices of manufactured goods and services?

15 The following data represent sales (in dollars) between and among two sectors in regions r and s.

	r		s	
r	40	50	30	45
	60	10	70	45
s	50	60	50	80
	70	70	50	50

In addition, sales to final demand purchasers were $\mathbf{f}^r = \begin{bmatrix} 200 \\ 200 \end{bmatrix}$ and $\mathbf{f}^s = \begin{bmatrix} 300 \\ 400 \end{bmatrix}$. These data are sufficient to create a two-region interregional input–output model connecting regions r and s. If, because of a stimulated economy, household demand increased by \$280 for the output of sector 1 in region r and by \$360 for the output of sector 2 in region r, what are the new necessary gross outputs from each of the sectors in each of the two regions to satisfy this new final demand? That is, find $\Delta\mathbf{x} = \begin{bmatrix} \Delta\mathbf{x}^r \\ \Delta\mathbf{x}^s \end{bmatrix}$ associated with $\Delta\mathbf{f}$.

16 A federal government agency for a three-region country has collected the following data on input purchases for two sectors, (1) manufacturing and (2) agriculture, for last year, in dollars. These flows are not specific with respect to region of origin; that is, they are of the $z_{ij}^{\cdot s}$ sort. Denote the three regions by A, B, and C.

	Region A		Region B		Region C	
	1	2	1	2	1	2
1	200	100	700	400	100	0
2	100	100	100	200	50	0

Also, gross outputs for each of the two sectors in each of the three regions are known. They are:

$$\mathbf{x}^A = \begin{bmatrix} 600 \\ 300 \end{bmatrix}, \mathbf{x}^B = \begin{bmatrix} 1200 \\ 700 \end{bmatrix} \text{ and } \mathbf{x}^C = \begin{bmatrix} 200 \\ 0 \end{bmatrix}$$

The agency hires you to advise them on potential uses for this information.

a. Your first thought is to produce a regional technical coefficients table for each region. Is it possible to construct such tables? If so, do it; if not, why not?

b. You also consider putting the data together to generate a national technical coefficients table. Is this possible? If so, do it; if not, why not?

c. Why is it not possible to construct from the given data a three-region multiregional input–output model?

d. If the federal government is considering spending \$5,000 on manufactured goods and \$4,500 on agricultural products next year, what would you estimate as the national gross outputs necessary to satisfy this government demand?

e. Compare the national gross outputs for sectors 1 and 2 found in d, above, with the original gross outputs, given in the data set from last year. What feature of the input–output model does this comparison illustrate?

17 Assume that you have a very limited computer that can directly determine the inverse of matrices no larger than 2×2. Given this limited computer, explain how you could go about determining $\mathbf{L}$ for

$$\mathbf{A} = \begin{bmatrix} 0 & 0.1 & 0.3 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0 & 0 \\ 0.2 & 0 & 0.1 & 0.3 & 0.1 \\ 0.3 & 0 & 0 & 0.1 & 0.3 \\ 0.3 & 0.2 & 0.1 & 0.1 & 0.2 \end{bmatrix}$$

a. Compute the Leontief inverse in this manner.

b. What implications does such a procedure have for the computation of very large matrices (e.g., $n > 1000$)?

18 Consider the following four-sector input–output transactions table for the year 2005 along with industry prices for 2000 and 2005.

	Industry Transactions					Price	Price
	1	2	3	4	Total Output	Year 2000	Year 2005
1	24	86	56	64	398	2	5
2	32	15	78	78	314	3	6
3	104	49	62	94	469	5	9
4	14	16	63	78	454	7	12

Compute the matrices of interindustry transactions and technical coefficients as well as the vector of total outputs deflated to year 2000 value terms.

19 Rank sectors in terms of their importance as measured by output multipliers in each in examples 9, 10, 13, 14

20 Consider the problem 9. Using output multipliers, from problem 19, in conjunction with the new final demands in the problem in 9, derive the total value of output (across all sectors) associated with the new final demands. Compare your results with the total output obtained by summing the ele-ments in the gross output vector which you found as the solution to the problem 9. [In matrix notation, this is comparing $\mathbf{m}(o)\Delta\mathbf{f}$ with $\mathbf{i}'\Delta\mathbf{x} = \mathbf{i}'\mathbf{L}\Delta\mathbf{f}$; they must be equal, since output multipliers are the column sums of the Leontief inverse $\mathbf{m}(o) = \mathbf{i}'\mathbf{L}$.

21 Consider an input output economy defined by $\mathbf{Z} = \begin{bmatrix} 140 & 350 \\ 800 & 50 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$.
Suppose this is an economy in deep economic trouble. The federal government has at its disposal policy tools that can be implemented to stimulate demand for goods from one sector or the other. Also suppose that the plants in sector 1 discharge 0.3 lbs of airborne particulate substances for every dollar of output (0.3 lbs/$ output), while sector 2 pollutes at 0.5 lbs/$ output. Finally, let labor input coefficients be 0.005 and 0.07 for sectors 1 and 2, respectively.

a. Would a conflict of interest arise between unions and environmentalists in determining the sector toward which the government should direct its policy effort? (You need not close the matrix with respect to either households or pollution generation to answer this question.)
b. Can you think of a technological reason why or why not a dispute might arise?

22 The centrally planned economy of Czaria is involved in its planning for the next fiscal year. The technical coefficients and total industry outputs for Czaria are given below:

a. Compute the output inverse for this economy.
b. If next year's value-added inputs for agriculture, mining, military manufactured products, and civilian manufacturing in Czaria are projected to be \$4,558 million, \$5,665 million, \$2,050 million and \$5,079 million, respectively, compute the projected GDP for Czaria next year.

	1	2	3	4	Total Output
1. Agriculture	0.168	0.155	0.213	0.212	12,000
2. Mining	0.194	0.193	0.168	0.115	15,000
3. Military Manufacturing	0.105	0.025	0.126	0.124	12,000
4. Civilian Manufacturing	0.178	0.101	0.219	0.186	16,000

23 Consider the case of $\mathbf{Z} = \begin{bmatrix} 418 & 687 & 589 & 931 \\ 847 & 527 & 92 & 654 \\ 416 & 702 & 911 & 763 \\ 263 & 48 & 737 & 329 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} 2000 \\ 3000 \\ 2500 \\ 1500 \end{bmatrix}$.

a. Compute the direct and total backward linkages.
b. Compute the direct and total forward linkages.

24 Consider an economy with $\mathbf{Z} = \begin{bmatrix} 8 & 64 & 89 \\ 28 & 44 & 77 \\ 48 & 24 & 28 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 300 \\ 250 \\ 200 \end{bmatrix}$. Examine element a_{13} for "inverse importance" if the criteria are:

a. $\alpha = 30$ and $\beta = 5$ – that is, if a 30 percent change in a_{13} generates a 5 percent change in one or more elements in the associated Leontief inverse.
b. $\alpha = 20$ and $\beta = 10$.
c. $\alpha = 10$ and $\beta = 10$.

This illustrates the sensitivity of the results to the values of α and β specified by the analyst.

25 Consider two input–output economies specified by

$$\mathbf{Z}^0 = \begin{bmatrix} 10 & 20 & 30 \\ 5 & 2 & 25 \\ 20 & 40 & 60 \end{bmatrix}, \quad \mathbf{f}^0 = \begin{bmatrix} 60 \\ 40 \\ 55 \end{bmatrix}, \quad \mathbf{Z}^1 = \begin{bmatrix} 15 & 25 & 40 \\ 12 & 7.5 & 30 \\ 10 & 30 & 40 \end{bmatrix}, \quad \mathbf{f}^1 = \begin{bmatrix} 75 \\ 55 \\ 40 \end{bmatrix}$$

We seek to measure how the economy has changed in structure in one year, specified by $\mathbf{Z}^1$ and $\mathbf{f}^1$, relative to an earlier year for the same economy, specified by $\mathbf{Z}^0$ and $\mathbf{f}^0$. Compute for each sector the change in total output between the two years that was attributable to changing final demand or to changing technology.

26 Consider an input–output economy with technical coefficients defined as $\mathbf{A} = \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}$ and capital coefficients defined as $\mathbf{B} = \begin{bmatrix} .01 & .003 \\ .005 & .020 \end{bmatrix}$. Current final demand is $\mathbf{f}^0 = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$ and the projections for the next three years for final demand are given by $\mathbf{f}^1 = \begin{bmatrix} 125 \\ 160 \end{bmatrix}$, $\mathbf{f}^2 = \begin{bmatrix} 150 \\ 175 \end{bmatrix}$ and $\mathbf{f}^3 = \begin{bmatrix} 185 \\ 200 \end{bmatrix}$. We are not interested in total output for beyond the projection three years, but what would be the projections of total output for this economy in the next three years?

27 Consider the following closed dynamic input–output model, $\mathbf{Ax} + \mathbf{B}(\mathbf{x}^t - \mathbf{x}) = \mathbf{x}$ where $\mathbf{x}^t$ = future outputs, $\mathbf{x}$ = current outputs, and where $\mathbf{A} = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.5 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}$. Assume that $\mathbf{x}^t = \lambda\mathbf{x}$, where λ is some scalar (the turnpike growth rate); compute λ.

28 Given the closed dynamic input–output model $\mathbf{Ax} + \mathbf{B}(\mathbf{x}^t - \mathbf{x}) = \mathbf{x}$, where

$$\mathbf{A} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

a. Compute the turnpike growth rate for this example.
b. If both the capital coefficients for the first industry (the first column of $\mathbf{B}$) are changed to 0.1, then what is the new turnpike growth rate and what has happened to the apparent "health" of the economy?

29 Consider an input–output economy with technical coefficients defined as $\mathbf{A} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.5 \end{bmatrix}$ and capital coefficients defined as $\mathbf{B} = \begin{bmatrix} .02 & .002 \\ .003 & .01 \end{bmatrix}$. Current final demand is $\mathbf{f}^0 = \begin{bmatrix} 185 \\ 200 \end{bmatrix}$ and final demands for the previous three years are given by $\mathbf{f}^{-1} = \begin{bmatrix} 150 \\ 175 \end{bmatrix}$, $\mathbf{f}^{-2} = \begin{bmatrix} 125 \\ 160 \end{bmatrix}$, and $\mathbf{f}^{-3} = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$. Compute the "dynamic" multipliers for this economy that show how direct and indirect input requirements for final demands in period 0 are distributed backward over time for the previous three years.

SOLVE THE EXCERCISES IN MS EXCEL

1. Download the most recent global IO table from the WIOD webpage. Find the distribution of value added with respect to all countries in the database.
2. Consider the global flow table, in which the two countries produce two types of goods:

	Country I Good A	Country I Good B	Country II Good A	Country II Good B
Country I Good A	10	0	0	0
Country I Good B	0	40	20	20
Country II Good A	0	20	0	20
Country II Good B	0	10	40	30
Value Added	40	30	20	10
Global Production (X)	50	100	80	80

Find the distribution of value added induced after unit rise on final demand for Good B in Country I.

3. Download the 2011 national IO table for Poland from WIOD webpage. Conduct income- and CO_2-emission-oriented key sector analysis.
4. Consider the flow table:

x_i	x_{ij}			f_i	CO_2
1000	200	0	200	600	20
800	300	0	100	400	30
500	0	400	0	100	10

Conduct income- and CO_2-emission-oriented key sector analysis assuming that profitability in each branch is equal to 20%.