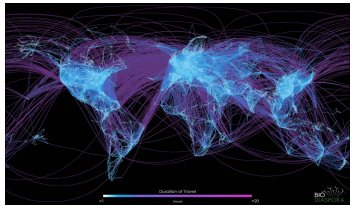
Connectivity and Robustness in Networks

Network robustness: the ability of a network to maintain its general structural properties when it faces failures or attacks.

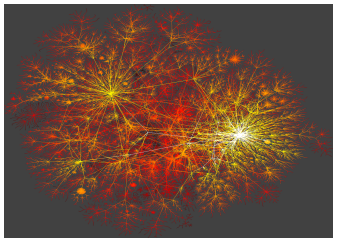
Type of attacks: removal of nodes or edges.

Structural properties: connectivity.

Examples: airport closures, internet router failures, power line failures.



Network of direct flights around the world [Bio.Diaspora]



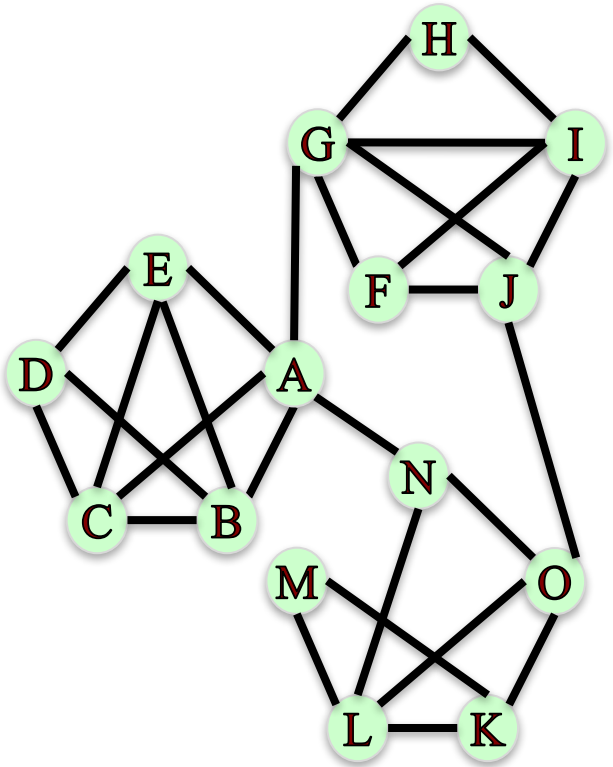
Internet Connectivity [K. C. Claffy]

Disconnecting a Graph

What is the smallest number of nodes that can be removed from this graph in order to disconnect it?

In: nx.node_connectivity(G_un)
Out: 1

Which node?
In: nx.minimum_node_cut(G_un)
Out: {'A'}

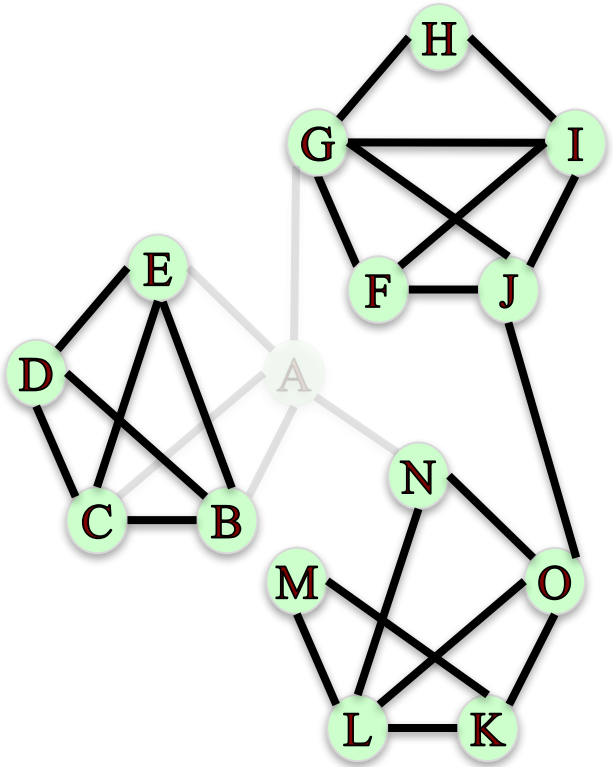


Disconnecting a Graph

What is the smallest number of nodes that can be removed from this graph in order to disconnect it?

In: nx.node_connectivity(G_un)
Out: 1

Which node?
In: nx.minimum_node_cut(G_un)
Out: {'A'}

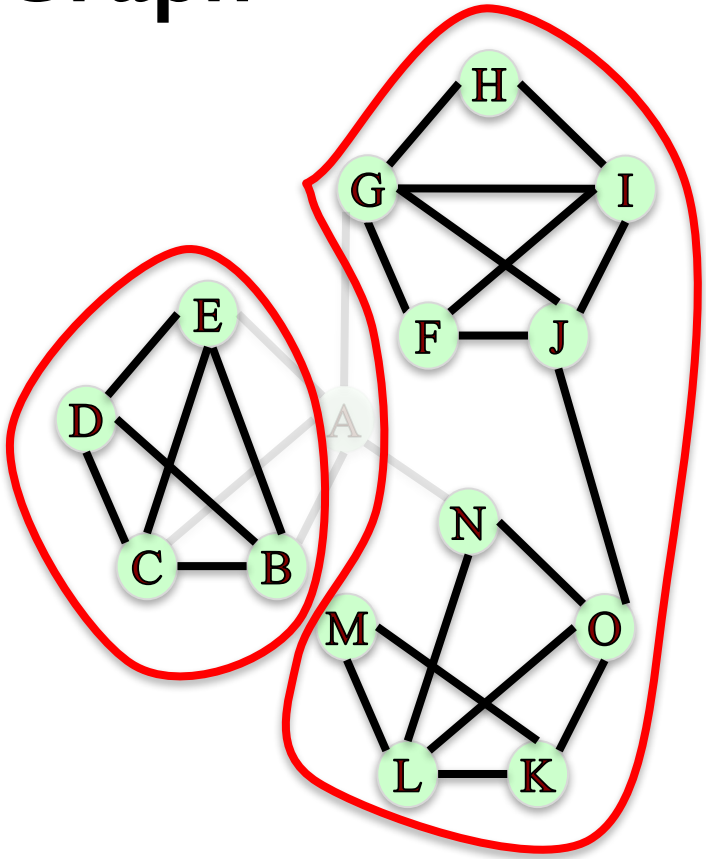


Disconnecting a Graph

What is the smallest number of **nodes** that can be removed from this graph in order to disconnect it?

In: nx.node_connectivity(G_un)
Out: 1

Which node?
In: nx.minimum_node_cut(G_un)
Out: {'A'}

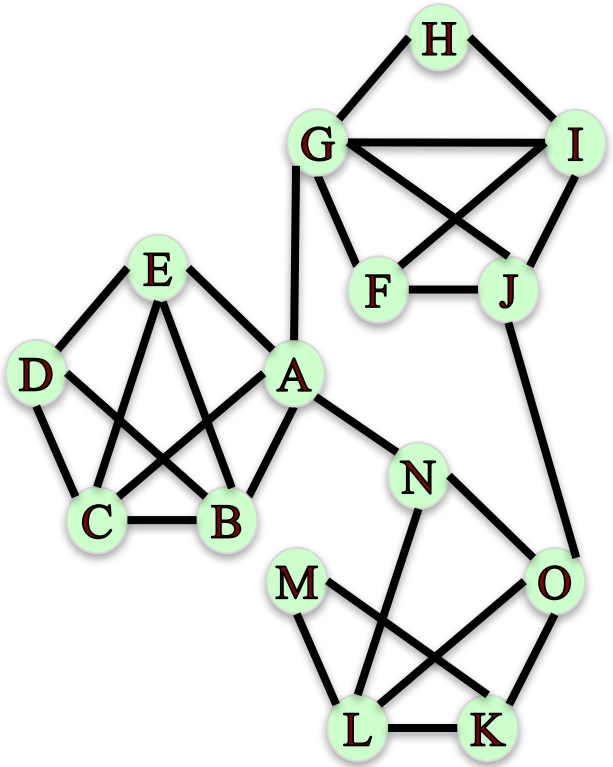


Disconnecting a Graph

What is the smallest number of **edges** that can be removed from this graph in order to disconnect it?

In: nx.edge_connectivity(G_un)
Out: 2

Which edges?
In: nx.minimum_edge_cut(G_un)
Out: {('A', 'G'), ('O', 'J')}

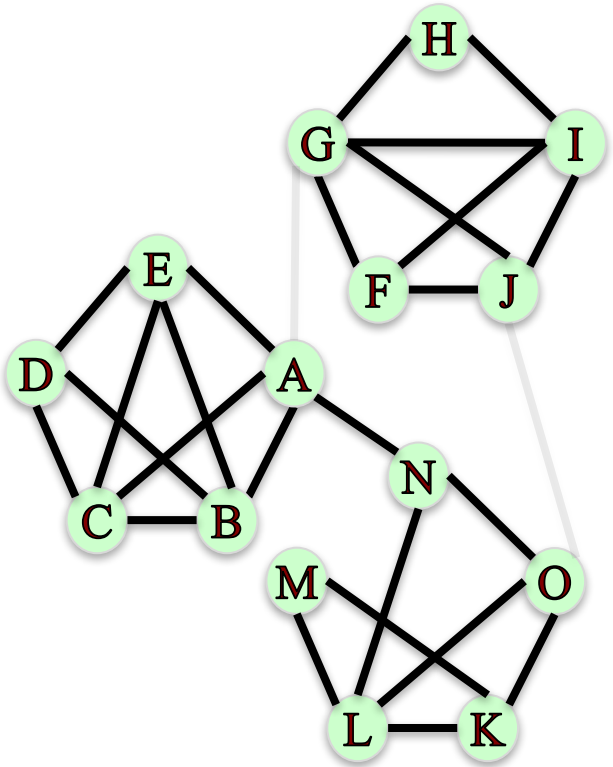


Disconnecting a Graph

What is the smallest number of **edges** that can be removed from this graph in order to disconnect it?

In: nx.edge_connectivity(G_un)
Out: 2

Which edges?
In: nx.minimum_edge_cut(G_un)
Out: {('A', 'G'), ('O', 'J')}



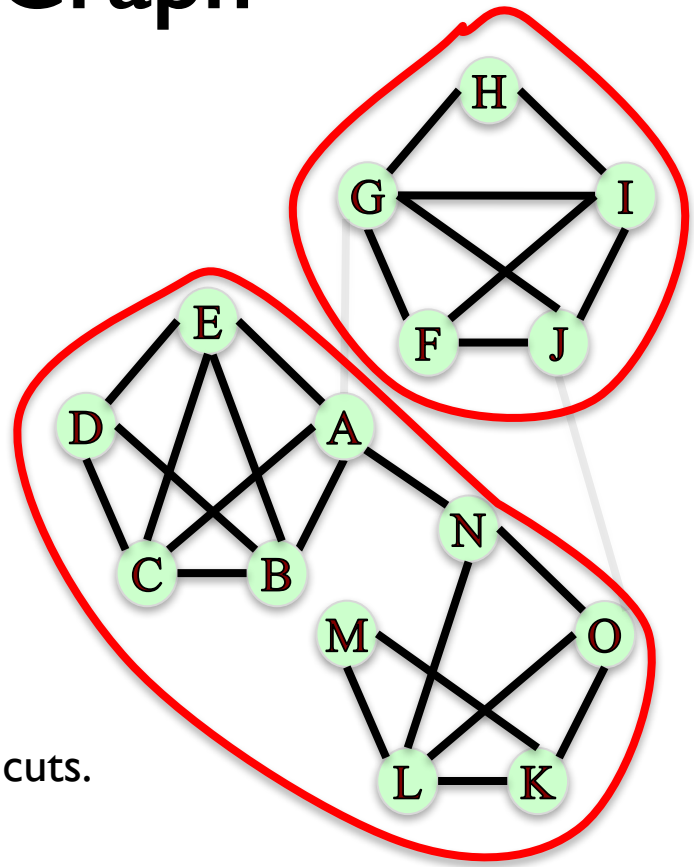
Disconnecting a Graph

What is the smallest number of **edges** that can be removed from this graph in order to disconnect it?

In: nx.edge_connectivity(G_un)
Out: 2

Which edges?
In: nx.minimum_edge_cut(G_un)
Out: {('A', 'G'), ('O', 'J')}

Robust networks have large minimum node and edge cuts.

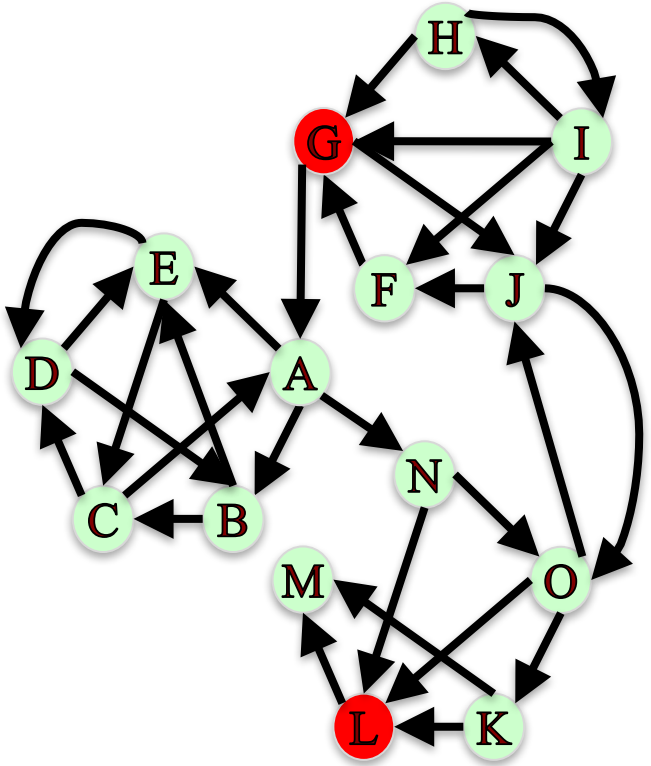


Simple Paths

Imagine node G wants to send a message to node L by passing it along to other nodes in this network.

What options does G have to deliver the message?

```
In: sorted(nx.all_simple_paths(G, 'G', 'L'))
Out: [['G', 'A', 'N', 'L'],
['G', 'A', 'N', 'O', 'K', 'L'],
['G', 'A', 'N', 'O', 'L'],
['G', 'J', 'O', 'K', 'L'],
['G', 'J', 'O', 'L']]
```

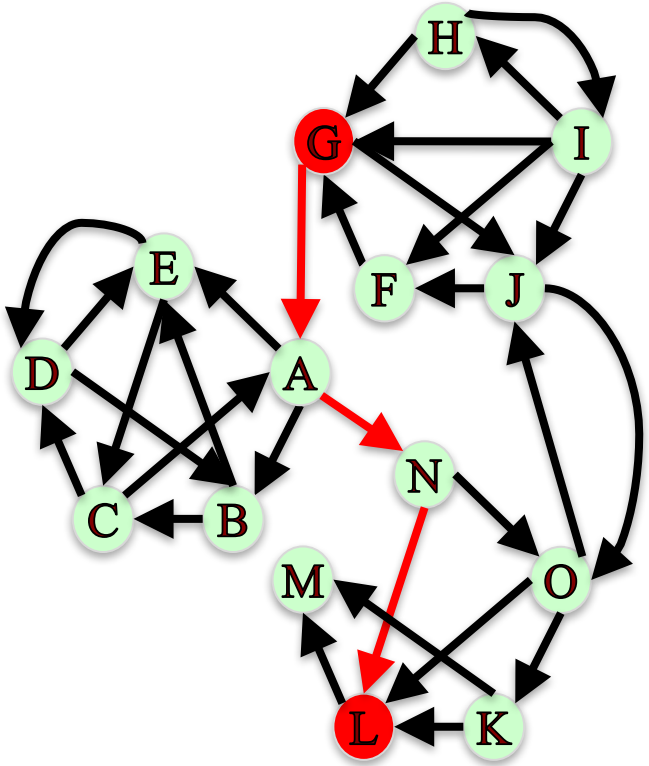


Simple Paths

Imagine node G wants to send a message to node L by passing it along to other nodes in this network.

What options does G have to deliver the message?

```
In: sorted(nx.all_simple_paths(G, 'G', 'L'))
Out: [['G', 'A', 'N', 'L'],
['G', 'A', 'N', 'O', 'K', 'L'],
['G', 'A', 'N', 'O', 'L'],
['G', 'J', 'O', 'K', 'L'],
['G', 'J', 'O', 'L']]
```

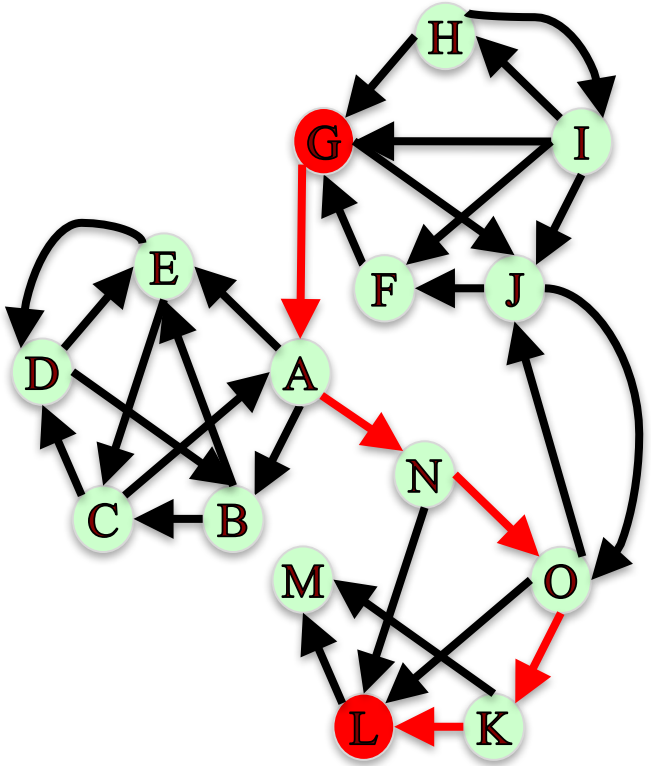


Simple Paths

Imagine node G wants to send a message to node L by passing it along to other nodes in this network.

What options does G have to deliver the message?

```
In: sorted(nx.all_simple_paths(G, 'G', 'L'))
Out: [['G', 'A', 'N', 'L'],
['G', 'A', 'N', 'O', 'K', 'L'],
['G', 'A', 'N', 'O', 'L'],
['G', 'J', 'O', 'K', 'L'],
['G', 'J', 'O', 'L']]
```

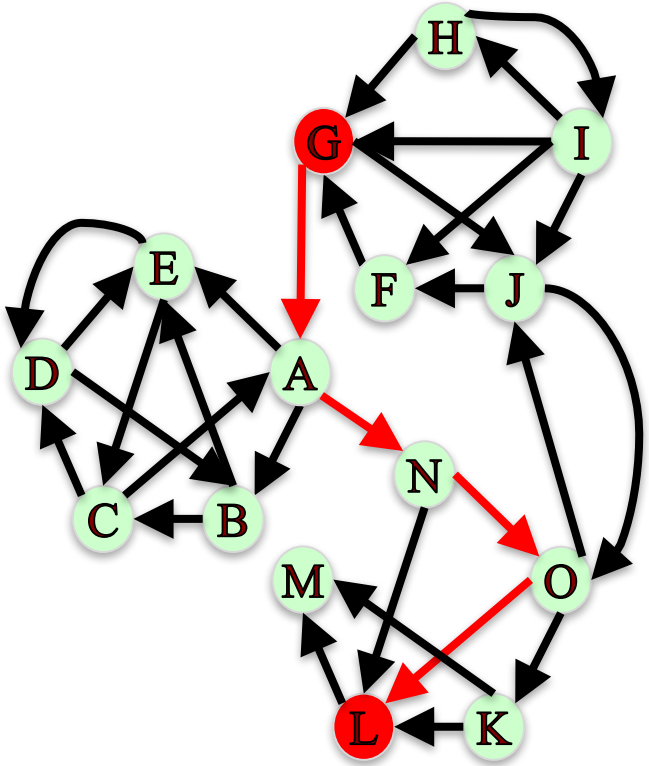


Simple Paths

Imagine node G wants to send a message to node L by passing it along to other nodes in this network.

What options does G have to deliver the message?

```
In: sorted(nx.all_simple_paths(G, 'G', 'L'))
Out: [['G', 'A', 'N', 'L'],
['G', 'A', 'N', 'O', 'K', 'L'],
['G', 'A', 'N', 'O', 'L'],
['G', 'J', 'O', 'K', 'L'],
['G', 'J', 'O', 'L']]
```

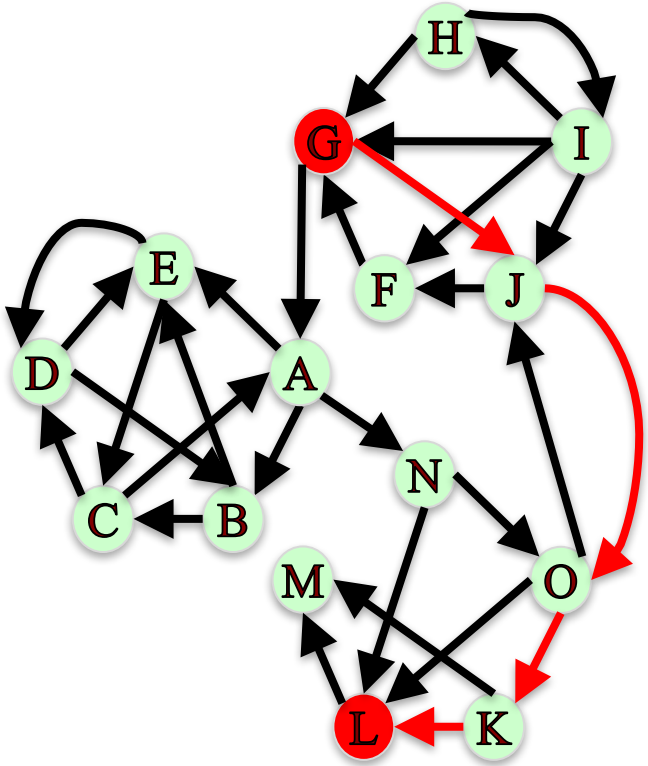


Simple Paths

Imagine node G wants to send a message to node L by passing it along to other nodes in this network.

What options does G have to deliver the message?

```
In: sorted(nx.all_simple_paths(G, 'G', 'L'))
Out: [['G', 'A', 'N', 'L'],
['G', 'A', 'N', 'O', 'K', 'L'],
['G', 'A', 'N', 'O', 'L'],
['G', 'J', 'O', 'K', 'L'],
['G', 'J', 'O', 'L']]
```

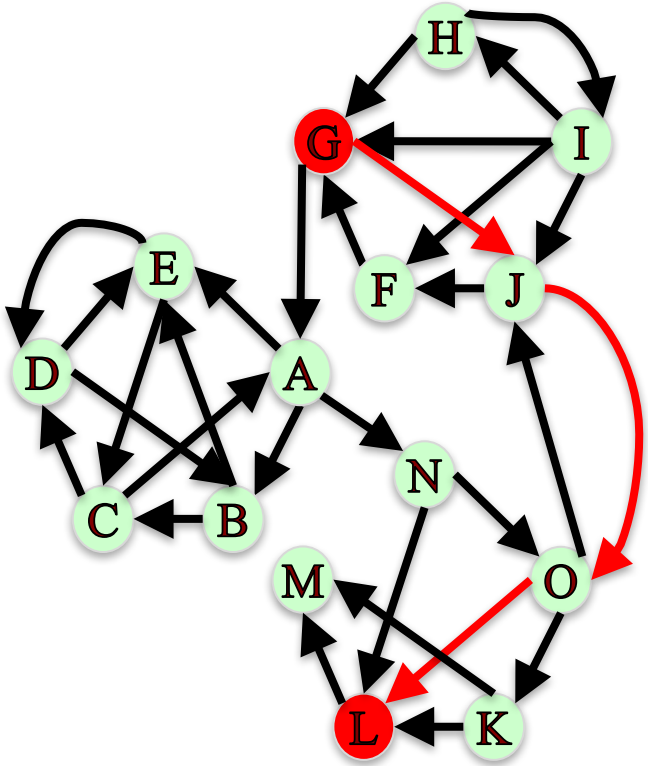


Simple Paths

Imagine node G wants to send a message to node L by passing it along to other nodes in this network.

What options does G have to deliver the message?

```
In: sorted(nx.all_simple_paths(G, 'G', 'L'))
Out: [['G', 'A', 'N', 'L'],
['G', 'A', 'N', 'O', 'K', 'L'],
['G', 'A', 'N', 'O', 'L'],
['G', 'J', 'O', 'K', 'L'],
['G', 'J', 'O', 'L']]
```



Node Connectivity

If we wanted to block the message from G to L by removing **nodes** from the network, how many nodes would we need to remove?

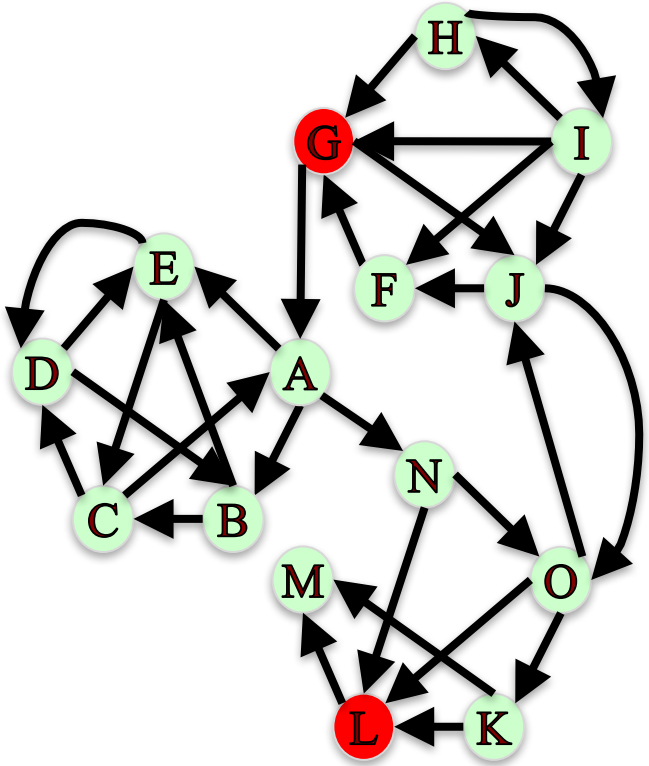
In: nx.node_connectivity(G, 'G', 'L')

Out: 2

Which nodes?

In: nx.minimum_node_cut(G, 'G', 'L')

Out: {'N', 'O'}



Node Connectivity

If we wanted to block the message from G to L by removing **nodes** from the network, how many nodes would we need to remove?

In: nx.node_connectivity(G, 'G', 'L')

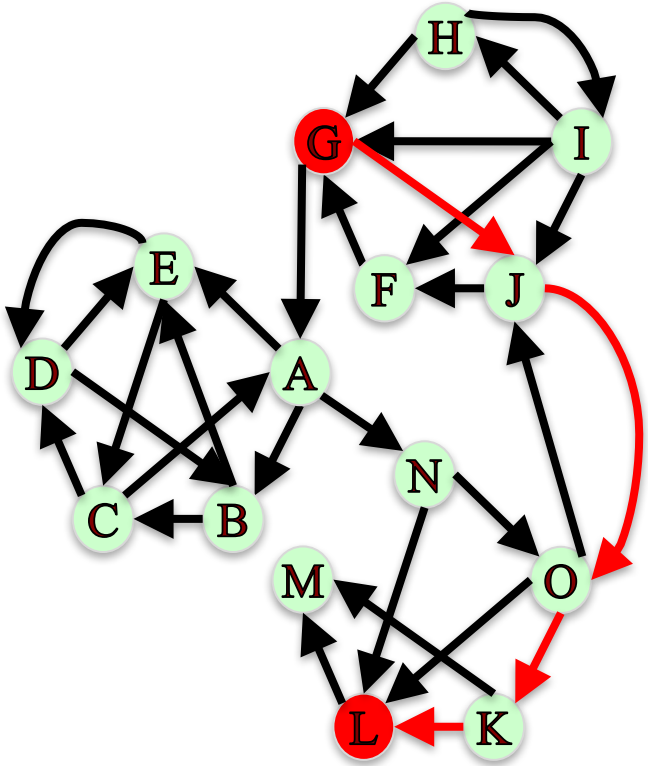
Out: 2

Which nodes?

In: nx.minimum_node_cut(G, 'G', 'L')

Out: {'N', 'O'}

If we only remove node N, message can go on path G → J → O → K → L.



Node Connectivity

If we wanted to block the message from G to L by removing **nodes** from the network, how many nodes would we need to remove?

In: nx.node_connectivity(G, 'G', 'L')

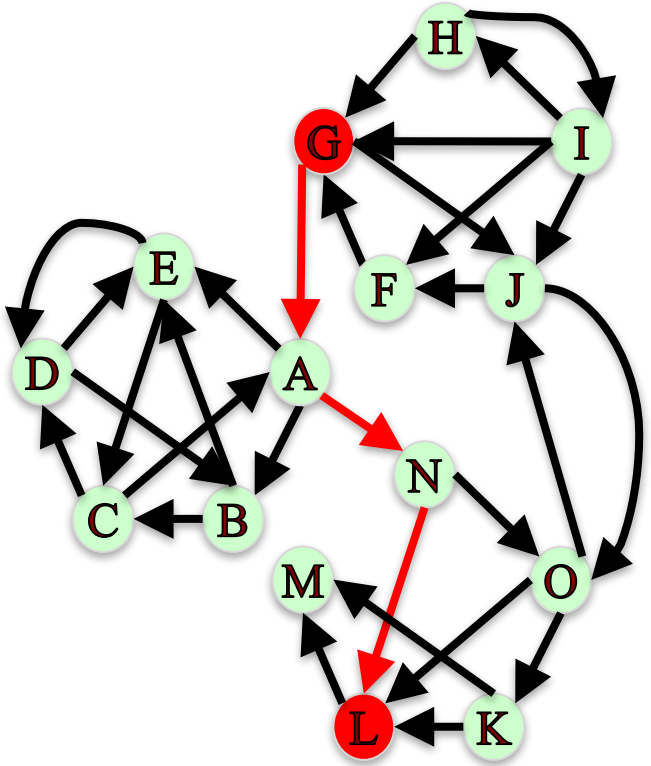
Out: 2

Which nodes?

In: nx.minimum_node_cut(G, 'G', 'L')

Out: {'N', 'O'}

If we only remove node O, message can go on path G → A → N → L.



Edge Connectivity

If we wanted to block the message from G to L by removing **edges** from the network, how many edges would we need to remove?

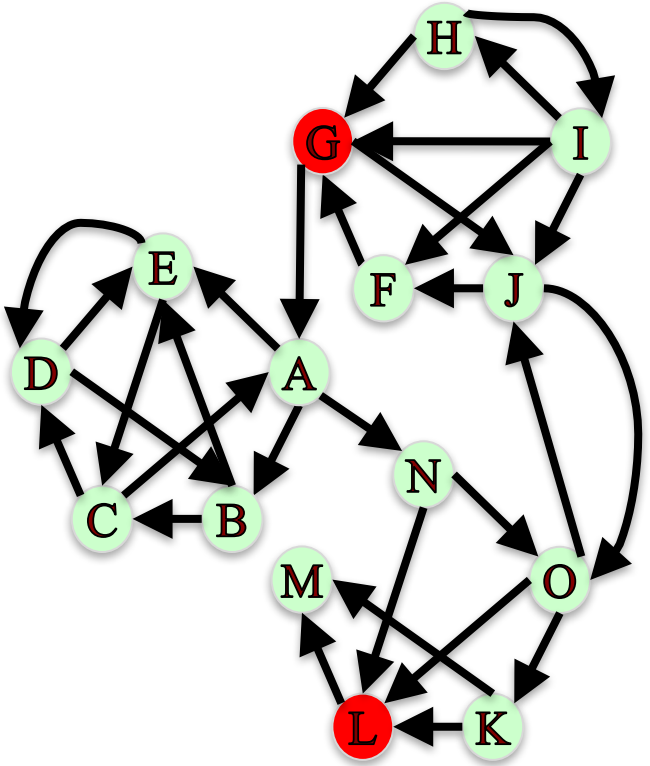
In: nx.edge_connectivity(G, 'G', 'L')

Out: 2

Which edges?

In: nx.minimum_edge_cut(G, 'G', 'L')

Out: {('A', 'N'), ('J', 'O')}



Edge Connectivity

If we wanted to block the message from G to L by removing **edges** from the network, how many edges would we need to remove?

In: nx.edge_connectivity(G, 'G', 'L')

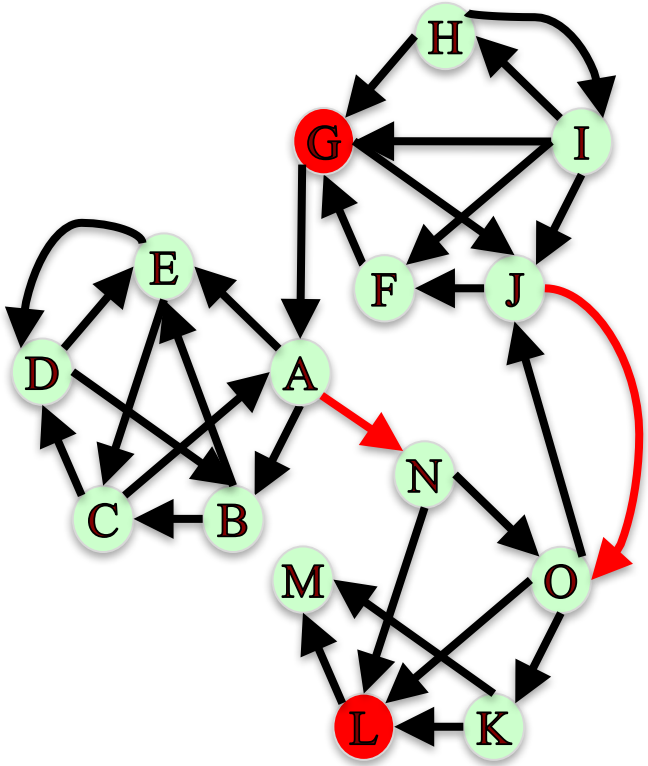
Out: 2

Which edges?

In: nx.minimum_edge_cut(G, 'G', 'L')

Out: {('A', 'N'), ('J', 'O')}

We need to remove A → N an J → O to block messages from G to L.



Summary

Node connectivity: Minimum number of *nodes* needed to disconnect a graph or pair of nodes.

nx.node_connectivity(G, 'G', 'L')

nx.minimum_node_cut(G, 'G', 'L')

Edge connectivity: Minimum number of *edges* needed to disconnect a graph or pair of nodes.

nx.edge_connectivity(G, 'G', 'L')

nx.minimum_edge_cut(G, 'G', 'L')

Graphs with large node and edge connectivity are more robust to the loss of nodes and edges.