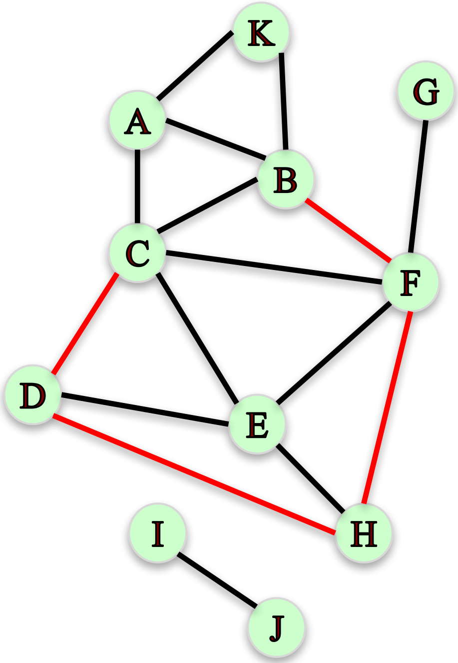
Triadic Closure

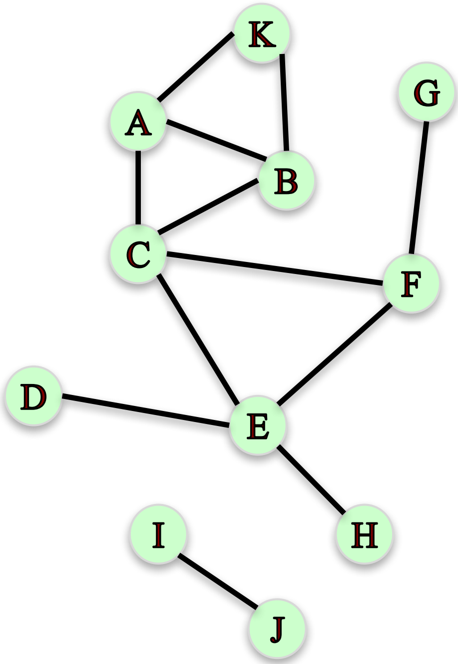
Triadic closure: The tendency for people who share connections in a social network to become connected.

How can we measure the prevalence of triadic closure in a network?



Local Clustering Coefficient

Local clustering coefficient of a node:
Fraction of pairs of the node's friends that are friends with each other.



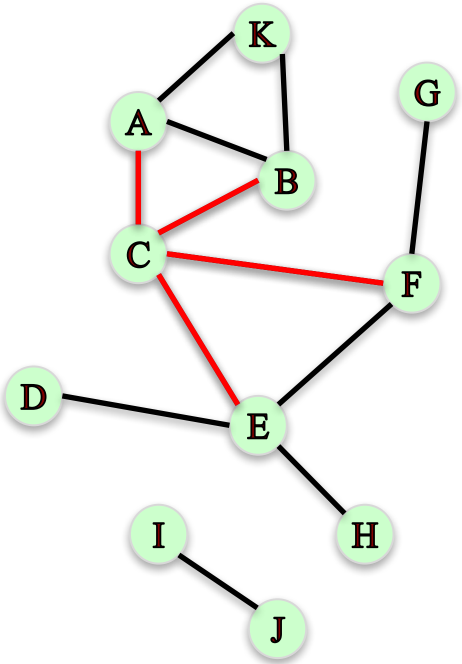
Local Clustering Coefficient

Compute the local clustering coefficient of node C:

$$\frac{\text{\# of pairs of C's friends who are friends}}{\text{\# of pairs of C's friends}}$$

# of C's friends = $d_c = 4$ (the "degree" of C)

# of pairs of C's friends =



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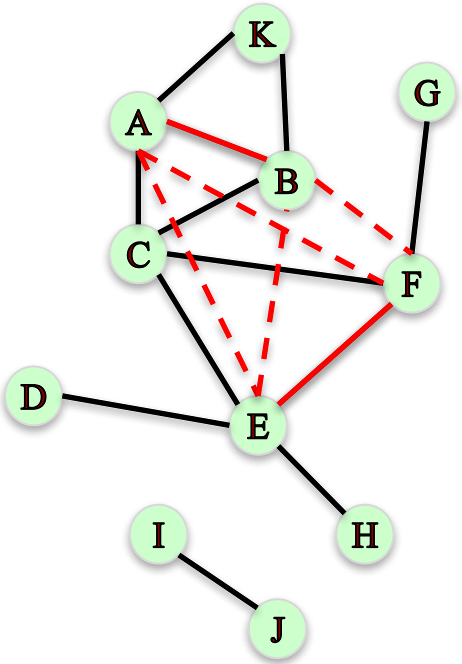
# of C's friends = $d_c = 4$ (the "degree" of C)

# of pairs of C's friends = $\dfrac{d_c(d_c-1)}{2}$



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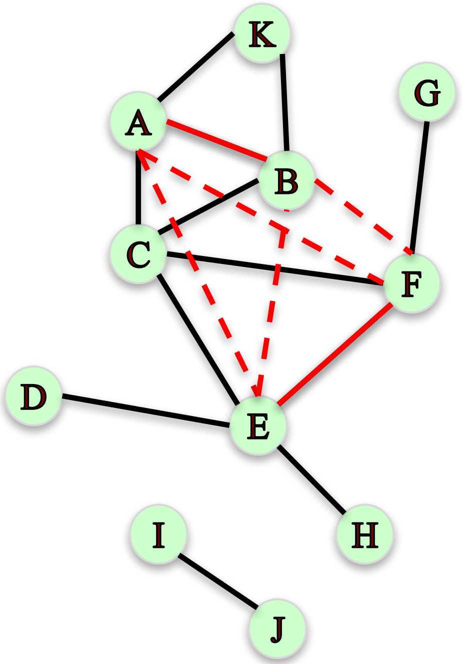
# of C's friends = $d_c = 4$ (the "degree" of C)

# of pairs of C's friends = $\dfrac{d_c(d_c-1)}{2} = \dfrac{12}{2} = 6$



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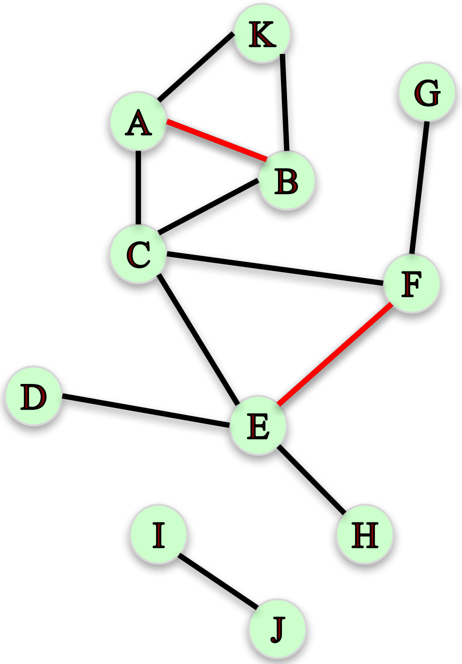

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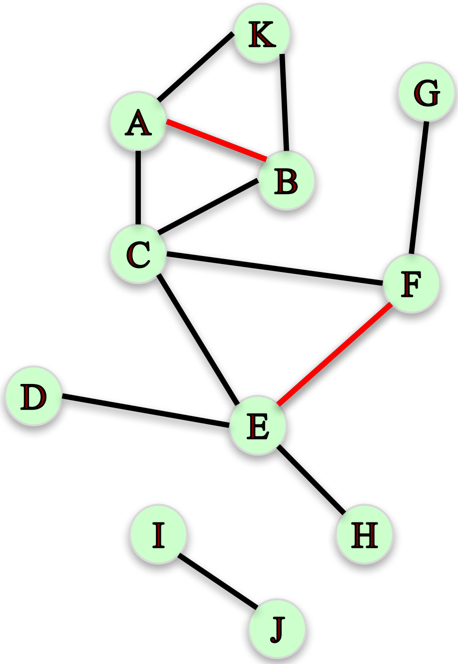
# of pairs of C's friends who are friends = 2



Local Clustering Coefficient

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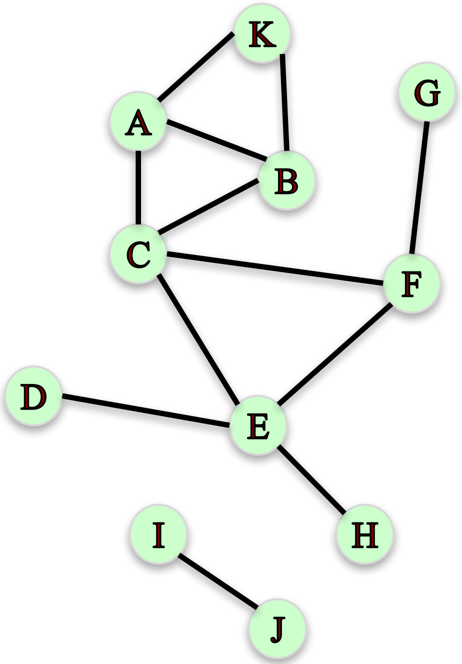
\# of pairs of C's friends = $\dfrac{d_c(d_c-1)}{2} = \dfrac{12}{2} = 6$

\# of pairs of C's friends who are friends = 2

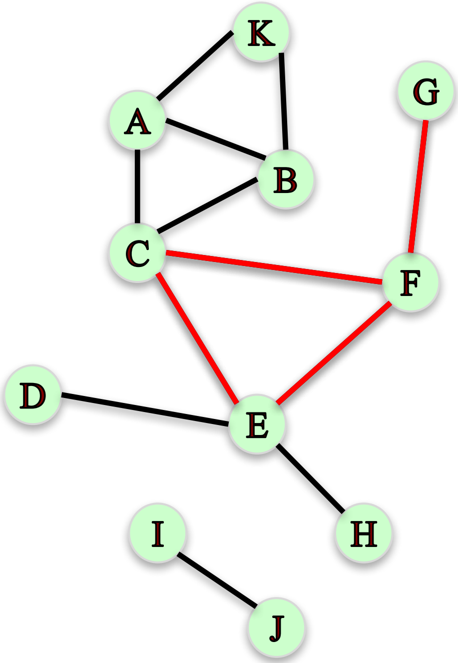
Local clustering coefficent of C = $\dfrac{2}{6} = \dfrac{1}{3}$



Local Clustering Coefficient

Compute the local clustering coefficient of node F:

$$\frac{\text{\# of pairs of F's friends who are friends}}{\text{\# of pairs of F's friends}}$$



$d_F = 3$

$\text{\# of pairs of F's friends} = \frac{d_F(d_F - 1)}{2} = \frac{6}{2} = 3$

$\text{\# of pairs of F's friends who are friends} = 1$

$\text{Local clustering coefficent of F} = \frac{1}{3}$

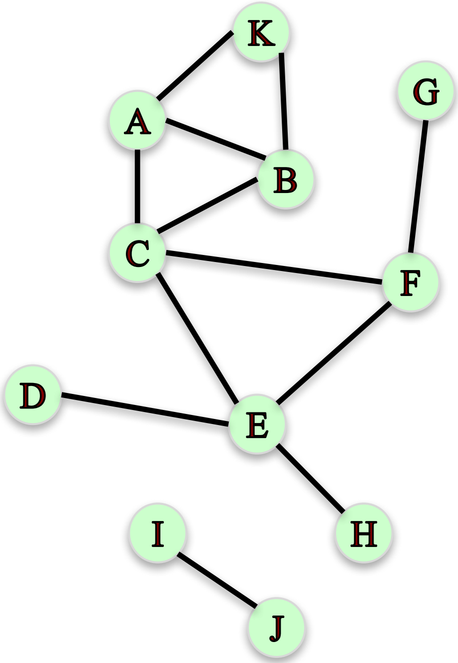
Local Clustering Coefficient

Compute the local clustering coefficient of node J:

$$\frac{\text{\# of pairs of J's friends who are friends}}{\text{\# of pairs of J's friends}}$$

# of pairs of J's friends = 0 (Can not divide by 0)

We will assume that the local clustering coefficient of a node of degree less than 2 to be 0.



Local Clustering Coefficient

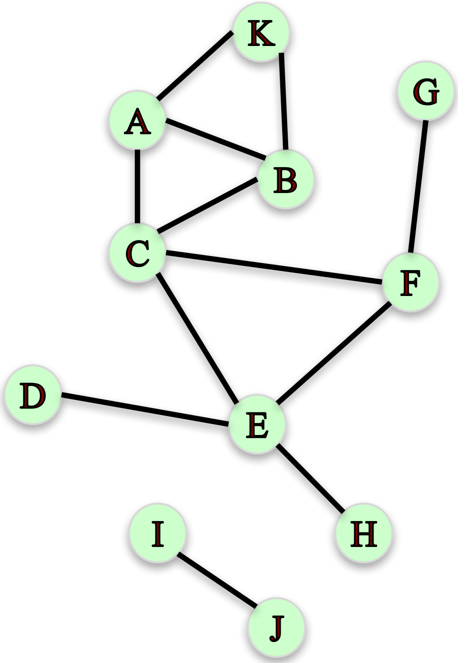
Local clustering coefficient in NetworkX:

```
G = nx.Graph()
G.add_edges_from([('A', 'K'), ('A', 'B'), ('A', 'C'), ('B', 'C'), ('B', 'K'),
('C', 'E'), ('C', 'F'), ('D', 'E'), ('E', 'F'), ('E', 'H'), ('F', 'G'), ('I', 'J')])

In: nx.clustering(G, 'F')
Out: 0.3333333333333333

In: nx.clustering(G, 'A')
Out: 0.6666666666666666

In: nx.clustering(G, ’J')
Out: 0.0
```

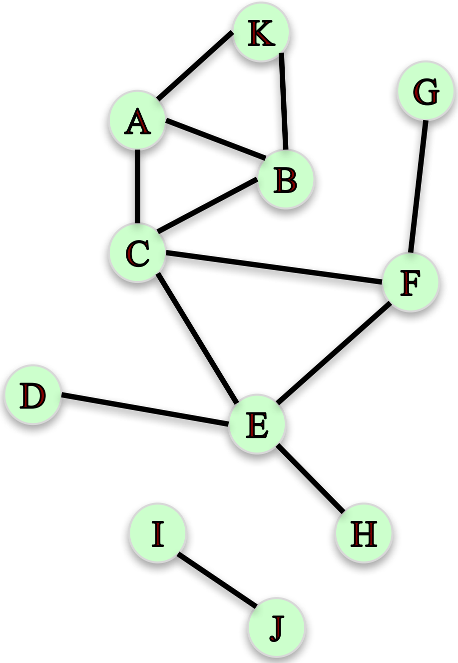


Global Clustering Coefficient

Measuring clustering on the whole network:

Approach 1: Average local clustering coefficient over all nodes in the graph.

```
In: nx.average_clustering(G)
Out: 0.28787878787878785
```

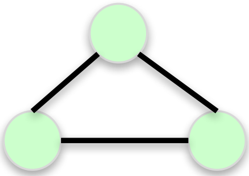


Global Clustering Coefficient

Measuring clustering on the whole network (Approach 2):

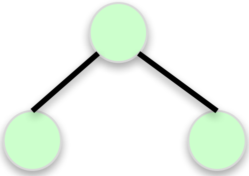
Percentage of "open triads" that are triangles in a network.

Triangles:



$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Open triads:

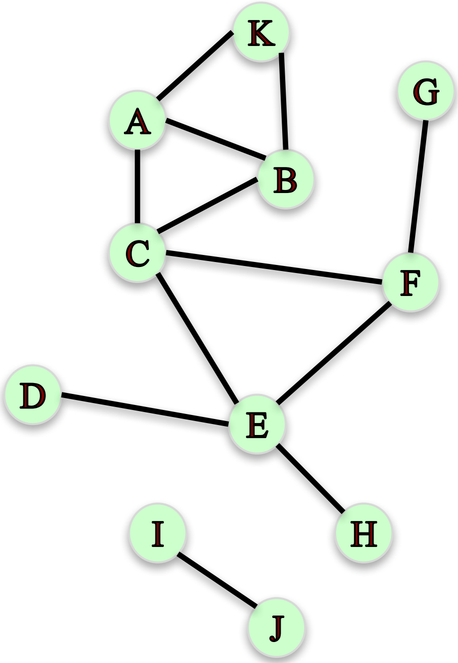


Global Clustering Coefficient

Measuring clustering on the whole network:

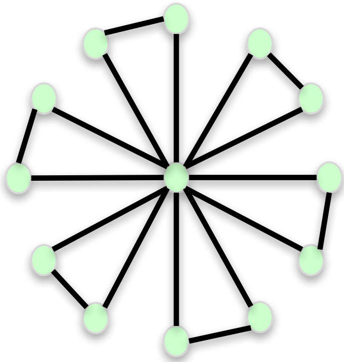
Transitivity: Ratio of number of triangles and number of "open triads" in a network.

In: nx.transitivity(G)
Out: 0.409090909091



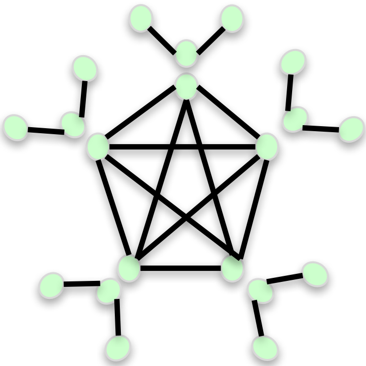
Transitivity vs. Average Clustering Coefficient

Both measure the tendency for edges to form triangles.
Transitivity weights nodes with large degree higher.



- Most nodes have high LCC
- The high degree node has low LCC

Ave. clustering coeff. = 0.93
Transitivity = 0.23



- Most nodes have low LCC
- High degree node have high LCC

Ave. clustering coeff. = 0.25
Transitivity = 0.86

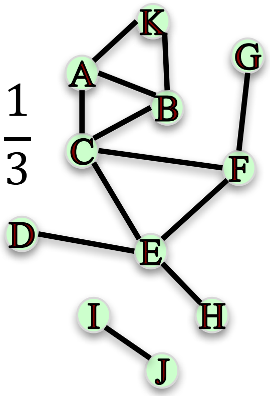
Summary

Clustering coefficient measures the degree to which nodes in a network tend to "cluster" or form triangles.

Local Clustering Coefficient

Fraction of pairs of the node's friends that are friends with each other.

LCC of C = $\frac{2}{6} = \frac{1}{3}$



Global Clustering Coefficient

Average Local Clustering Coefficient

nx.average_clustering(G)

Transitivity

Ratio of number of triangles and number of "open triads".

Puts larger weight on high degree nodes.

nx.transitivity(G)