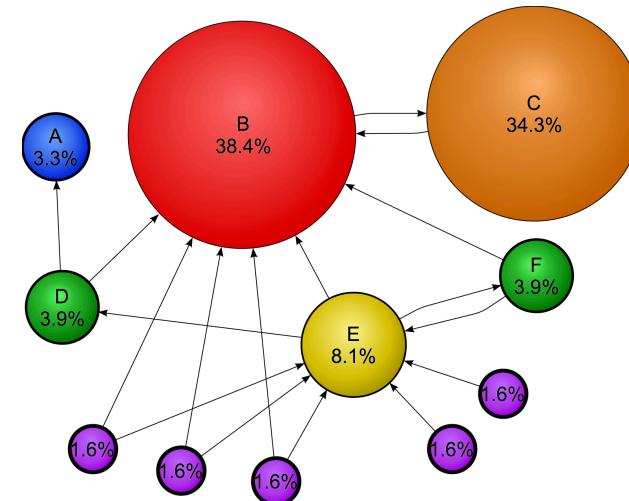


# PageRank

Developed by Google founders to measure the importance of webpages from the hyperlink network structure.

PageRank assigns a score of importance to each node. Important nodes are those with many in-links from important pages.

PageRank can be used for any type of network, but it is mainly useful for directed networks.



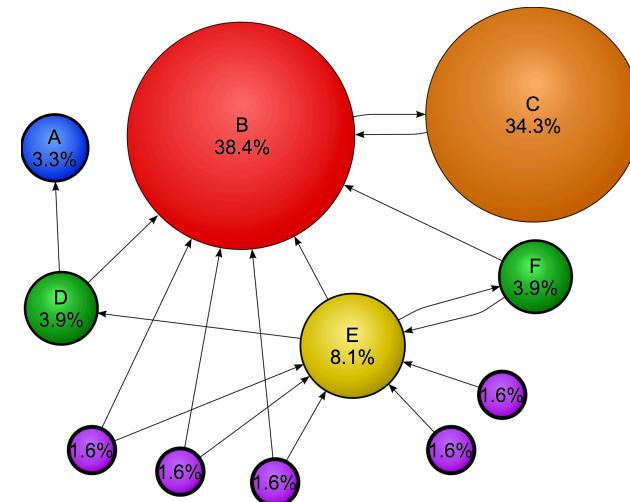
A node's PageRank depends on the PageRank of other nodes (Circular definition?).

# PageRank

$n$  = number of nodes in the network

$k$  = number of steps

1. Assign all nodes a PageRank of  $1/n$
2. Perform the *Basic PageRank Update Rule*  $k$  times.



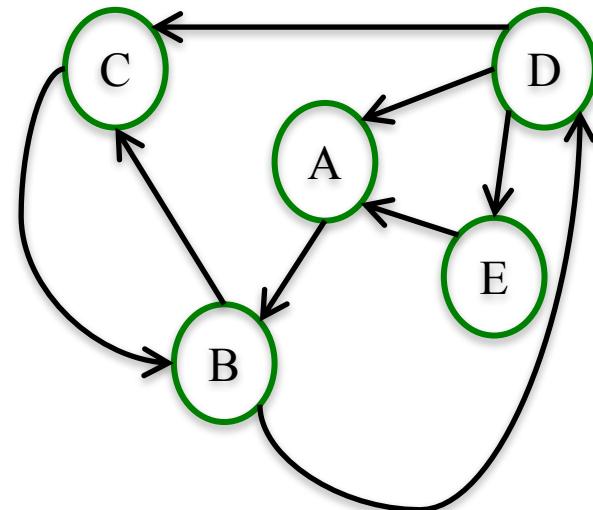
**Basic PageRank Update Rule:** Each node gives an equal share of its current PageRank to all the nodes it links to.

The new PageRank of each node is the sum of all the PageRank it received from other nodes.

# PageRank

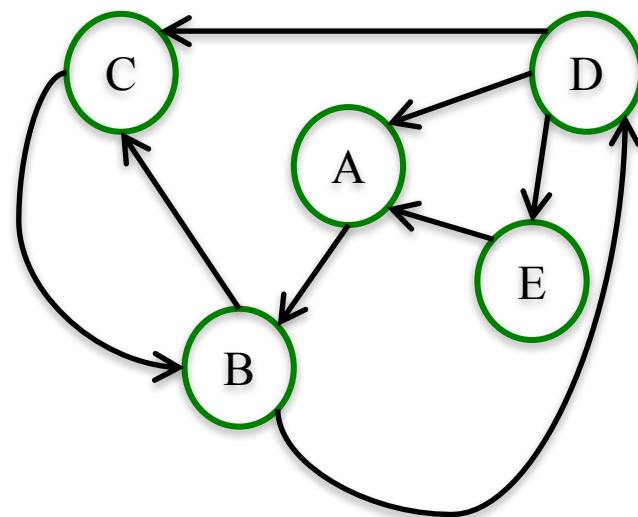
Who should be the most “important” node in this network?

Calculate the PageRank of each node after 2 steps of the procedure ( $k = 2$ ).



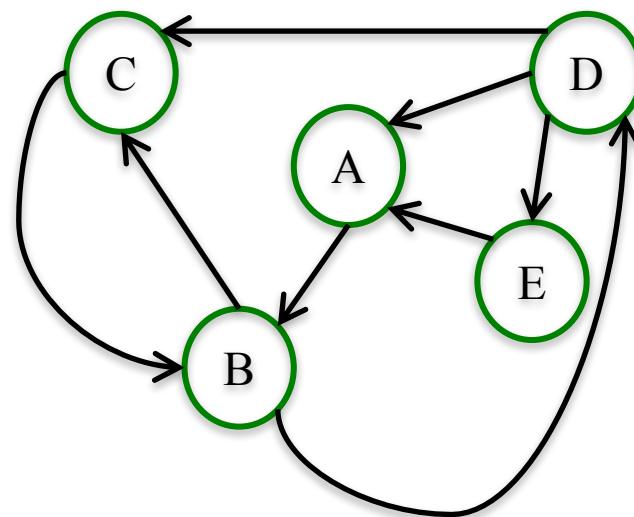
# PageRank

Page Rank					
	A	B	C	D	E
	1/5	1/5	1/5	1/5	1/5



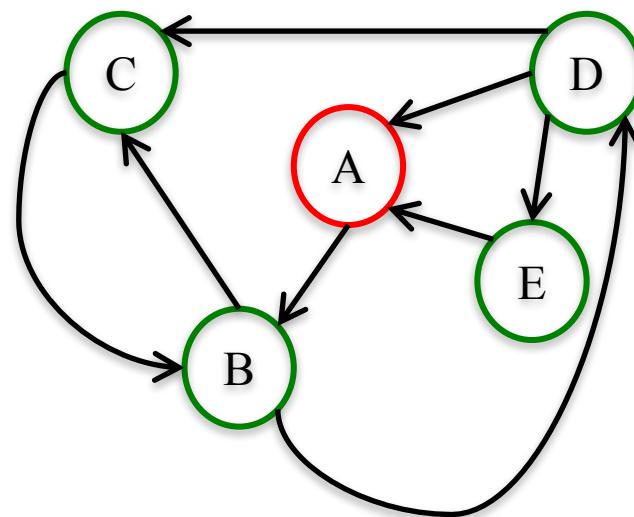
# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New					



# PageRank – Step 1

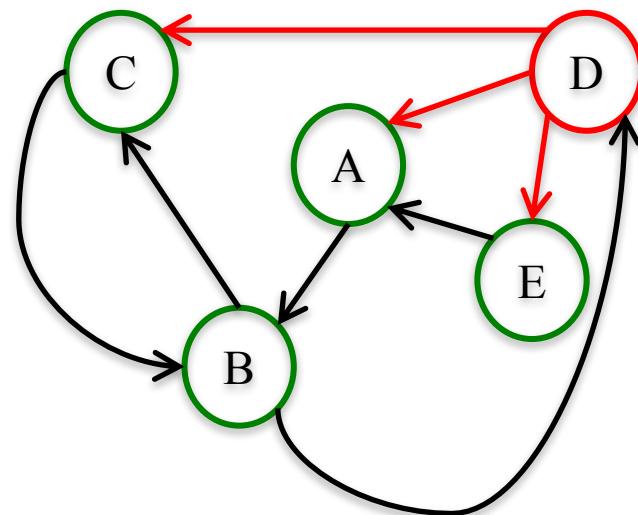
Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New					



# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New					

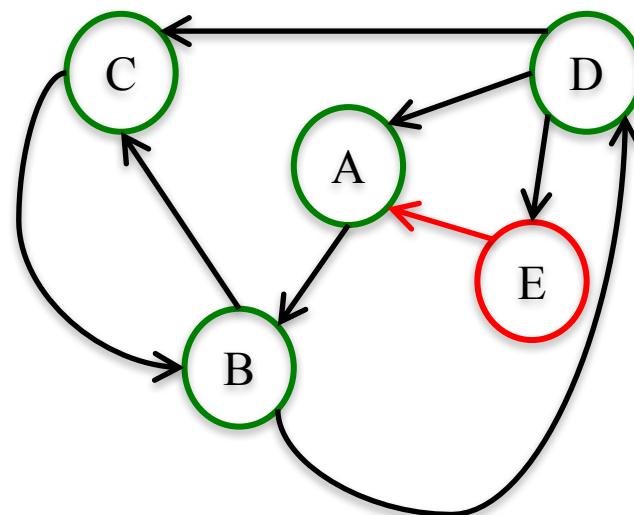
$$A: (1/3) * (1/5)$$



# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New					

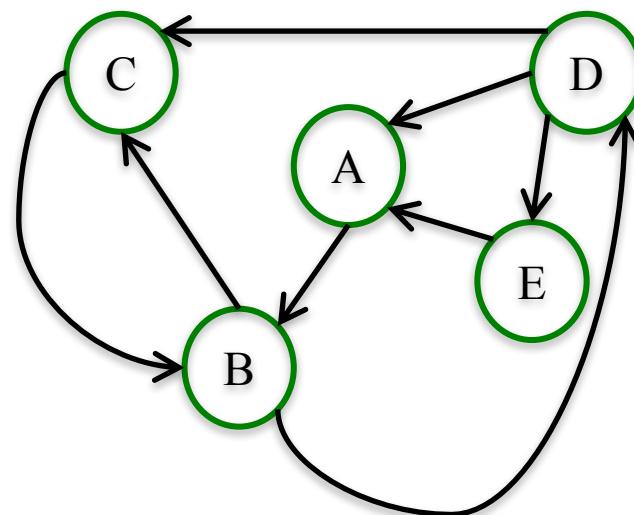
$$A: (1/3)*(1/5) + 1/5$$



# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New					

$$A: (1/3)*(1/5) + 1/5 = 4/15$$

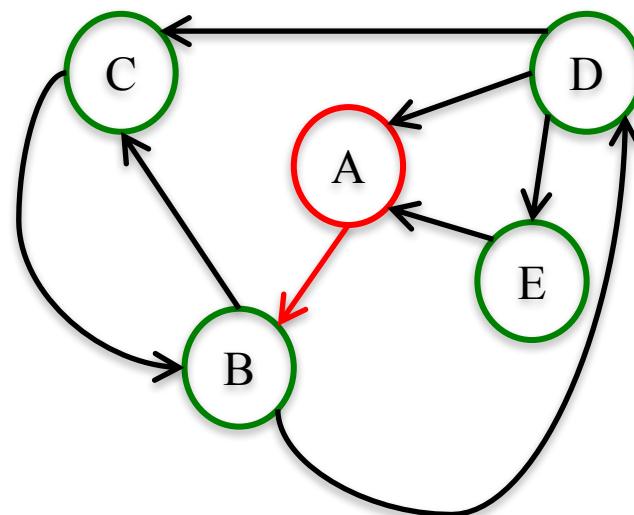


# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New	4/15				

$$A: (1/3)*(1/5) + 1/5 = 4/15$$

$$B: 1/5$$

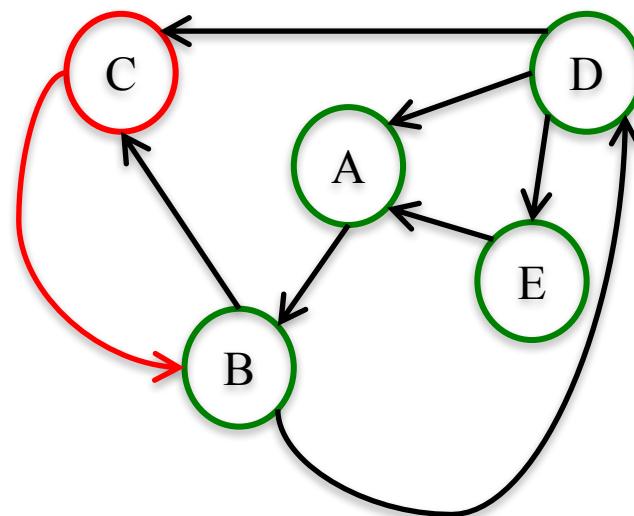


# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New	4/15				

$$A: (1/3)*(1/5) + 1/5 = 4/15$$

$$B: 1/5 + 1/5$$

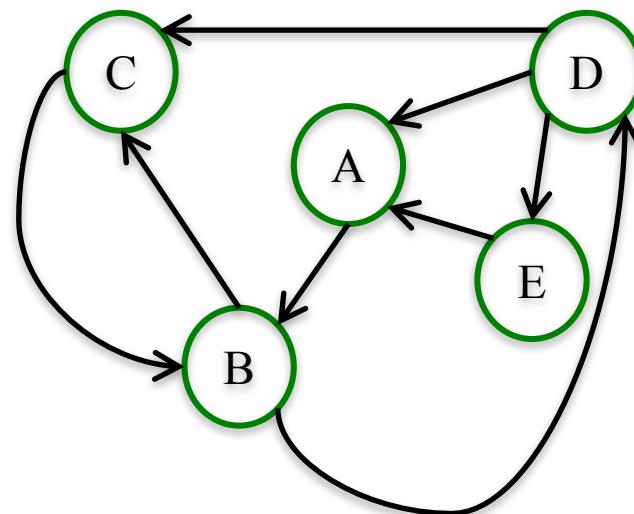


# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New	4/15	2/5			

$$A: (1/3)*(1/5) + 1/5 = 4/15$$

$$B: 1/5 + 1/5 = 2/5$$



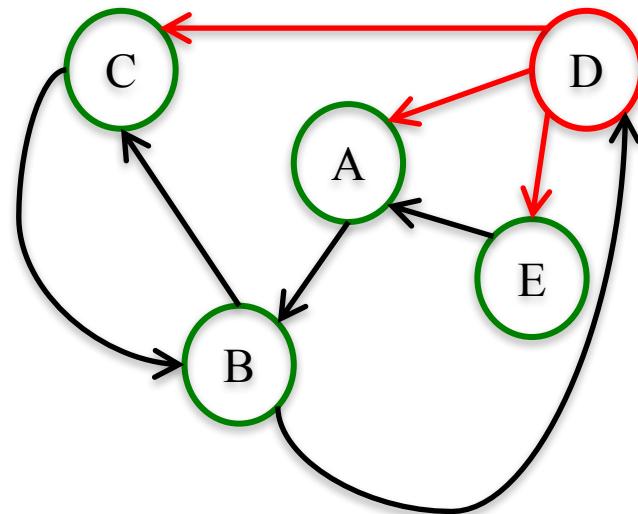
# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New	4/15	2/5			

$$A: (1/3)*(1/5) + 1/5 = 4/15$$

$$B: 1/5 + 1/5 = 2/5$$

$$C: (1/3)*(1/5)$$



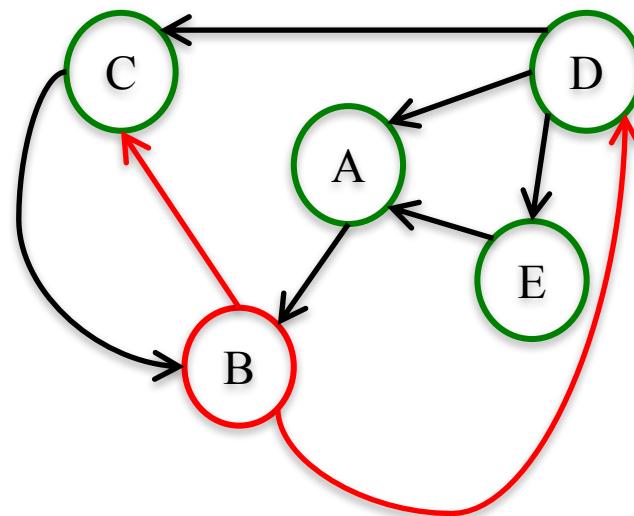
# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New	4/15	2/5			

$$A: (1/3)*(1/5) + 1/5 = 4/15$$

$$B: 1/5 + 1/5 = 2/5$$

$$C: (1/3)*(1/5) + (1/2)*(1/5)$$



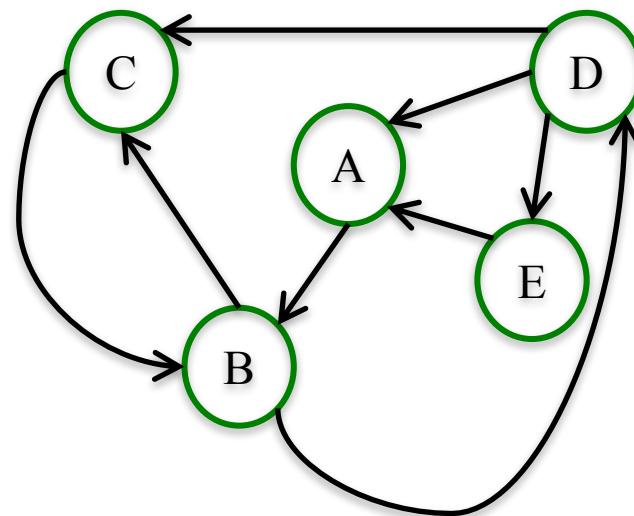
# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New	4/15	2/5	1/6		

$$A: (1/3)*(1/5) + 1/5 = 4/15$$

$$B: 1/5 + 1/5 = 2/5$$

$$C: (1/3)*(1/5) + (1/2)*(1/5) = 5/30 = 1/6$$



# PageRank – Step 1

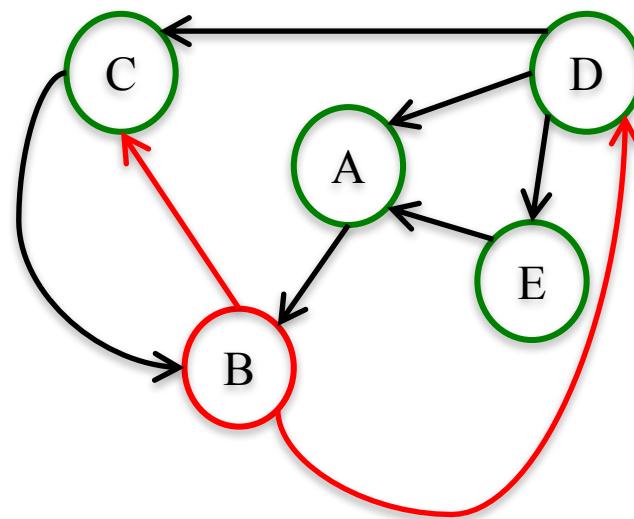
Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New	4/15	2/5	1/6		

$$A: (1/3)*(1/5) + 1/5 = 4/15$$

$$B: 1/5 + 1/5 = 2/5$$

$$C: (1/3)*(1/5) + (1/2)*(1/5) = 5/30 = 1/6$$

$$D: (1/2)*(1/5)$$



# PageRank – Step 1

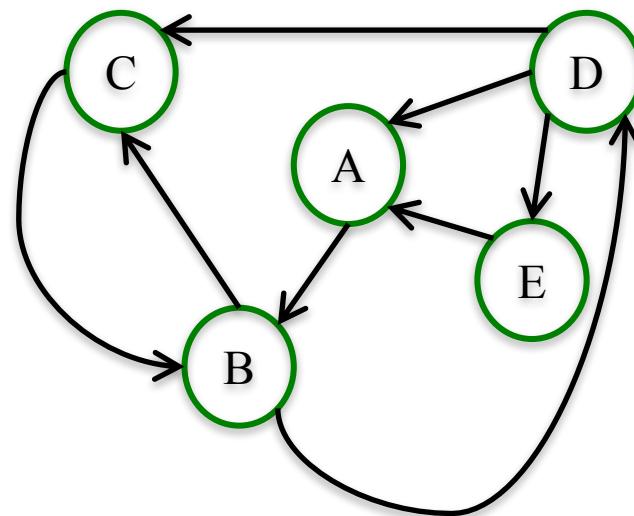
Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New	4/15	2/5	1/6	1/10	

$$A: (1/3)*(1/5) + 1/5 = 4/15$$

$$B: 1/5 + 1/5 = 2/5$$

$$C: (1/3)*(1/5) + (1/2)*(1/5) = 5/30 = 1/6$$

$$D: (1/2)*(1/5) = 1/10$$



# PageRank – Step 1

Page Rank ( $k = 1$ )					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New	4/15	2/5	1/6	1/10	

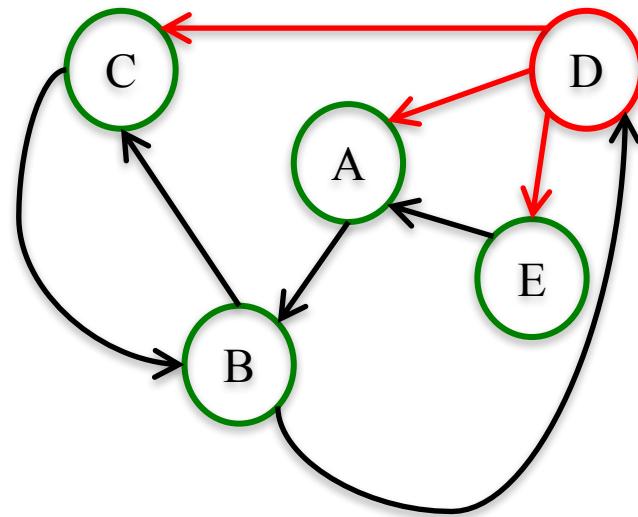
$$A: (1/3)*(1/5) + 1/5 = 4/15$$

$$B: 1/5 + 1/5 = 2/5$$

$$C: (1/3)*(1/5) + (1/2)*(1/5) = 5/30 = 1/6$$

$$D: (1/2)*(1/5) = 1/10$$

$$E: (1/3)*(1/5)$$



# PageRank – Step 1

Page Rank (k = 1)					
	A	B	C	D	E
Old	1/5	1/5	1/5	1/5	1/5
New	4/15	2/5	1/6	1/10	1/15

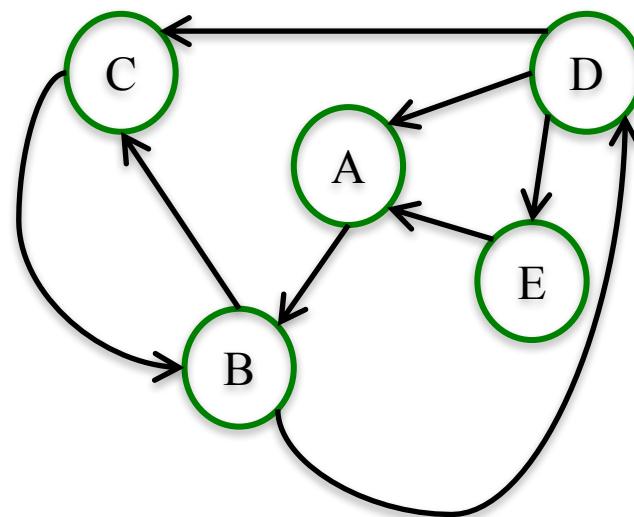
$$A: (1/3)*(1/5) + 1/5 = 4/15$$

$$B: 1/5 + 1/5 = 2/5$$

$$C: (1/3)*(1/5) + (1/2)*(1/5) = 5/30 = 1/6$$

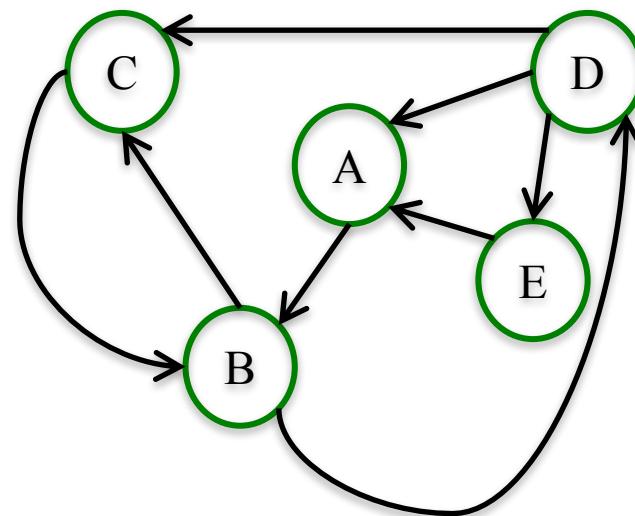
$$D: (1/2)*(1/5) = 1/10$$

$$E: (1/3)*(1/5) = 1/15$$



# PageRank – Step 2

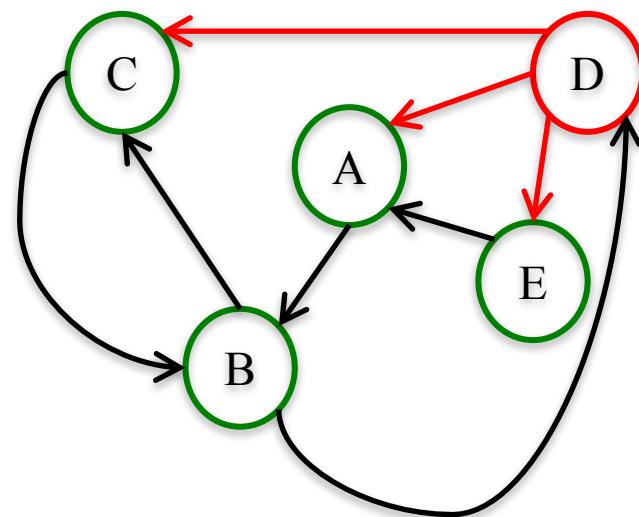
Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New					



# PageRank – Step 2

Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New					

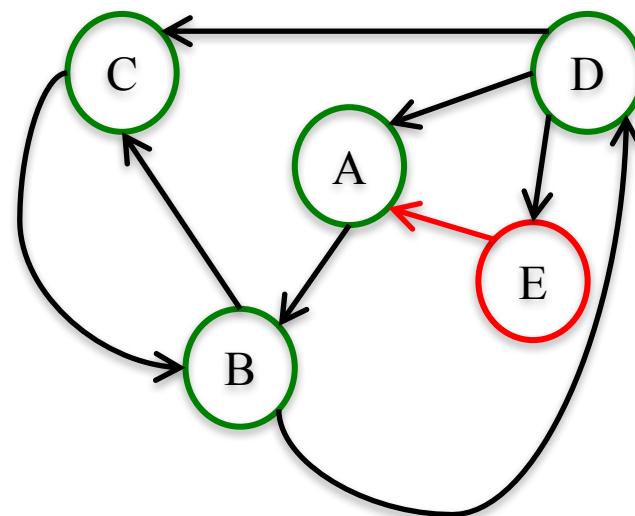
$$A: (1/3) * (1/10)$$



# PageRank – Step 2

Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New					

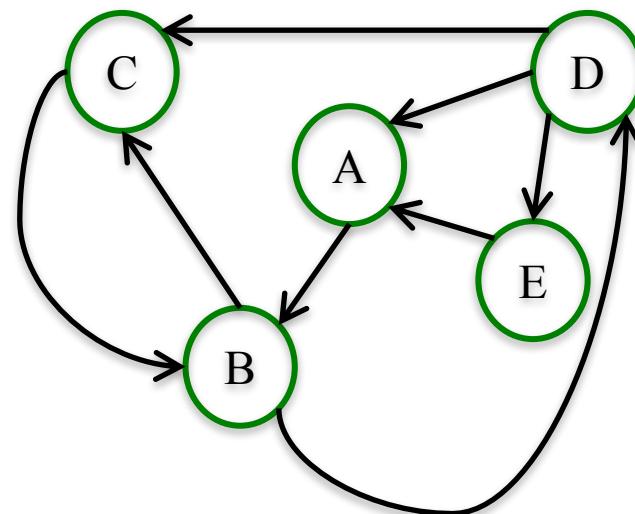
$$A: (1/3)*(1/10) + 1/15$$



# PageRank – Step 2

Page Rank (k = 2)					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10				

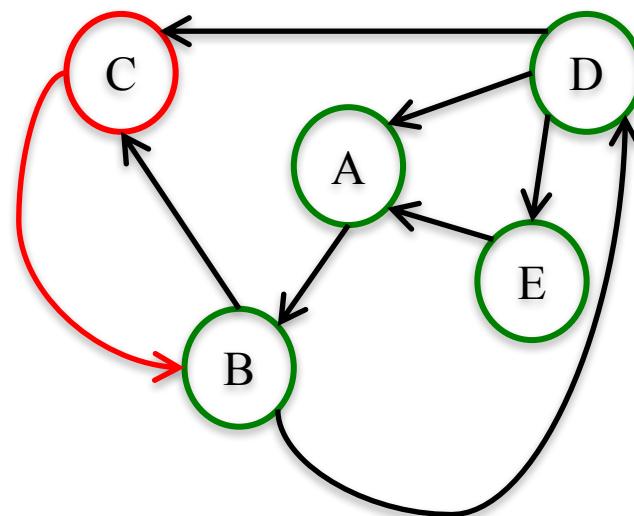
$$A: (1/3)*(1/10) + 1/15 = 1/10$$



# PageRank – Step 2

Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10				

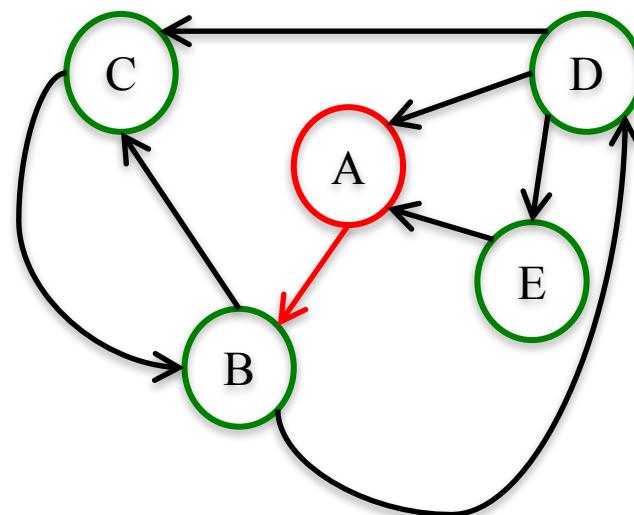
$$\begin{aligned} A: & (1/3)*(1/10) + 1/15 = 1/10 \\ B: & 1/6 \end{aligned}$$



# PageRank – Step 2

Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10				

$$\begin{aligned}A: & (1/3)*(1/10) + 1/15 = 1/10 \\B: & 1/6 + 4/15\end{aligned}$$

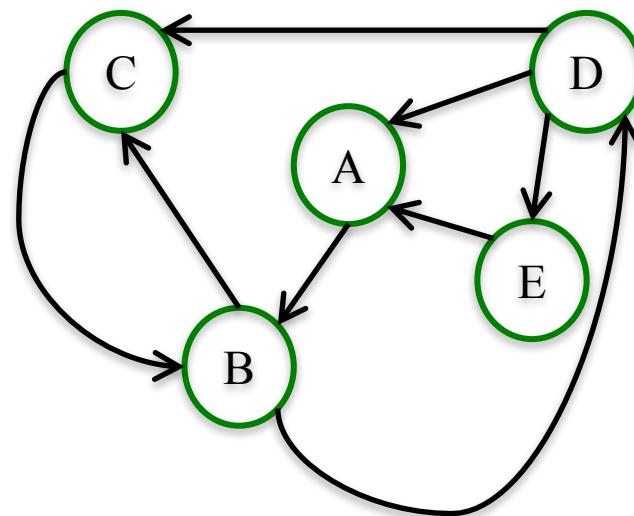


# PageRank – Step 2

Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10	13/30			

$$A: (1/3)*(1/10) + 1/15 = 1/10$$

$$B: 1/6 + 4/15 = 13/30$$



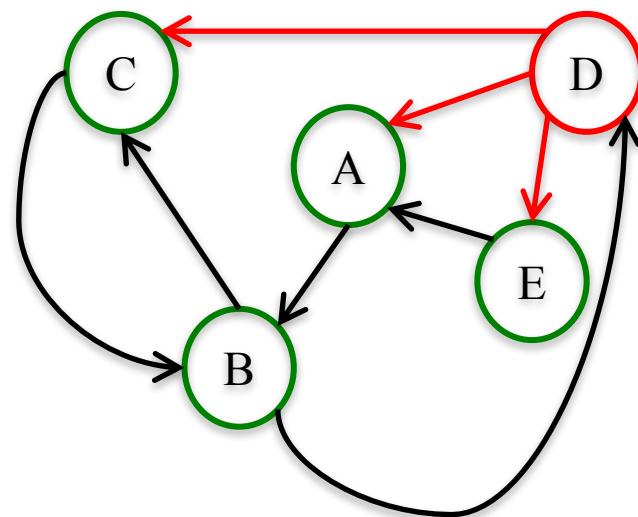
# PageRank – Step 2

Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10	13/30			

$$A: (1/3)*(1/10) + 1/15 = 1/10$$

$$B: 1/6 + 4/15 = 13/30$$

$$C: (1/3)*(1/10)$$



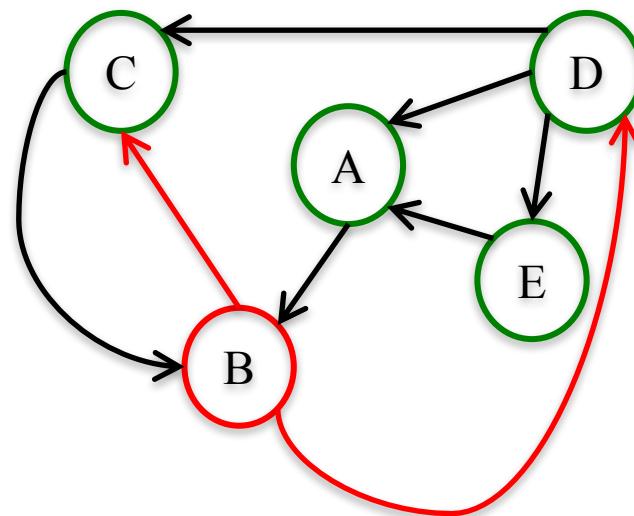
# PageRank – Step 2

Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10	13/30			

$$A: (1/3)*(1/10) + 1/15 = 1/10$$

$$B: 1/6 + 4/15 = 13/30$$

$$C: (1/3)*(1/10) + (1/2)*(2/5)$$



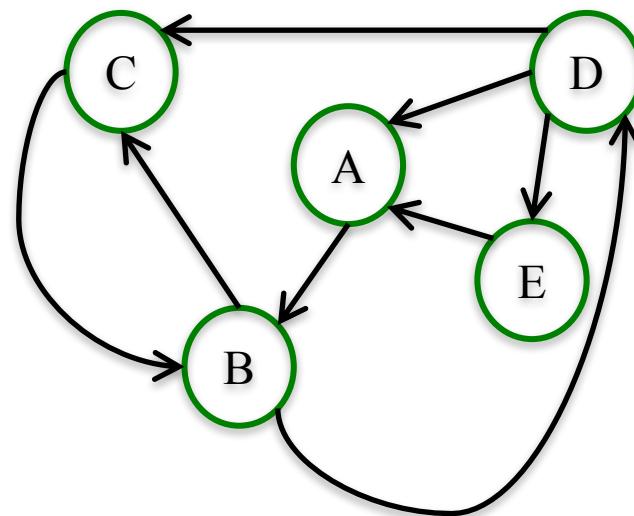
# PageRank – Step 2

Page Rank (k = 2)					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10	13/30	7/30		

$$A: (1/3)*(1/10) + 1/15 = 1/10$$

$$B: 1/6 + 4/15 = 13/30$$

$$C: (1/3)*(1/10) + (1/2)*(2/5) = 7/30$$



# PageRank – Step 2

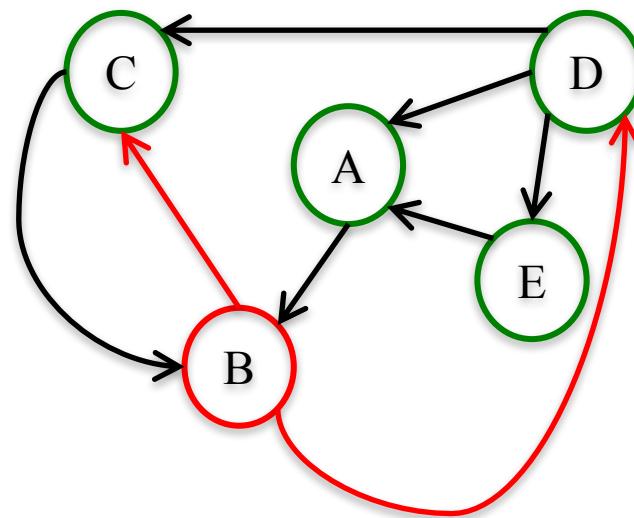
Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10	13/30	7/30		

$$A: (1/3)*(1/10) + 1/15 = 1/10$$

$$B: 1/6 + 4/15 = 13/30$$

$$C: (1/3)*(1/10) + (1/2)*(2/5) = 7/30$$

$$D: (1/2)*(2/5)$$



# PageRank – Step 2

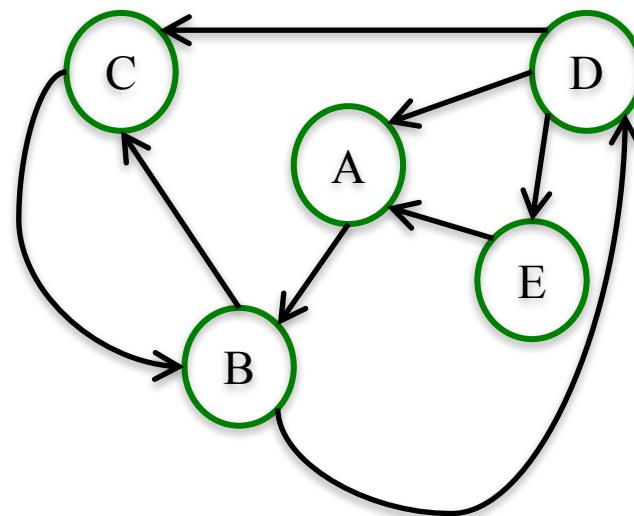
Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10	13/30	7/30	2/10	

$$A: (1/3)*(1/10) + 1/15 = 1/10$$

$$B: 1/6 + 4/15 = 13/30$$

$$C: (1/3)*(1/10) + (1/2)*(2/5) = 7/30$$

$$D: (1/2)*(2/5) = 2/10$$



# PageRank – Step 2

Page Rank ( $k = 2$ )					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10	13/30	7/30	2/10	

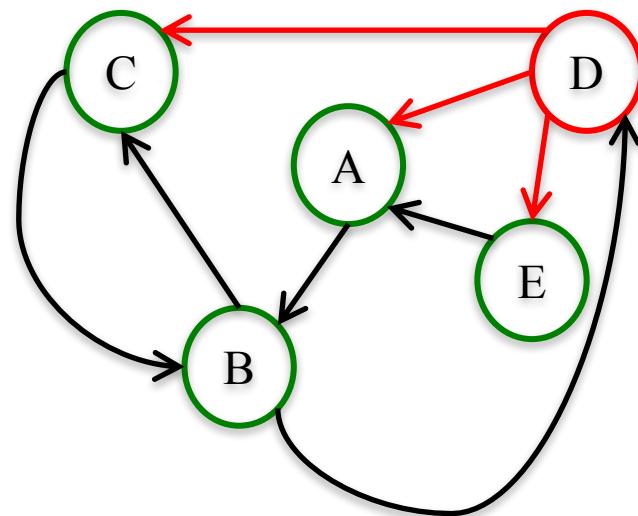
$$A: (1/3)*(1/10) + 1/15 = 1/10$$

$$B: 1/6 + 4/15 = 13/30$$

$$C: (1/3)*(1/10) + (1/2)*(2/5) = 7/30$$

$$D: (1/2)*(2/5) = 2/10$$

$$E: (1/3)*(1/10)$$



# PageRank – Step 2

Page Rank (k = 2)					
	A	B	C	D	E
Old	4/15	2/5	1/6	1/10	1/15
New	1/10	13/30	7/30	2/10	1/30

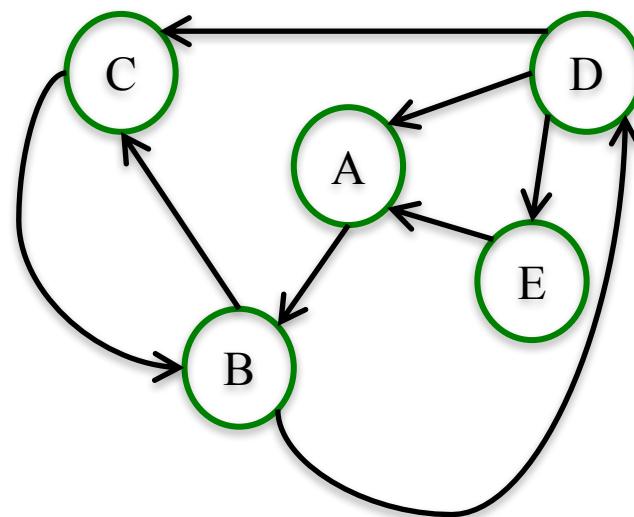
$$A: (1/3)*(1/10) + 1/15 = 1/10$$

$$B: 1/6 + 4/15 = 13/30$$

$$C: (1/3)*(1/10) + (1/2)*(2/5) = 7/30$$

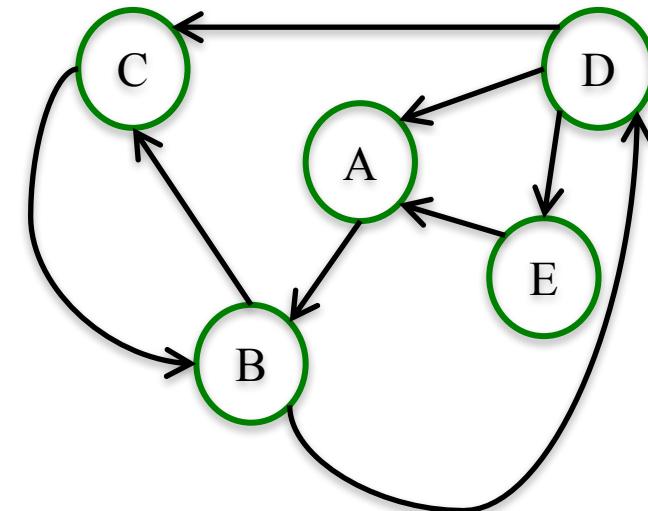
$$D: (1/2)*(2/5) = 2/10$$

$$E: (1/3)*(1/10) = 1/30$$



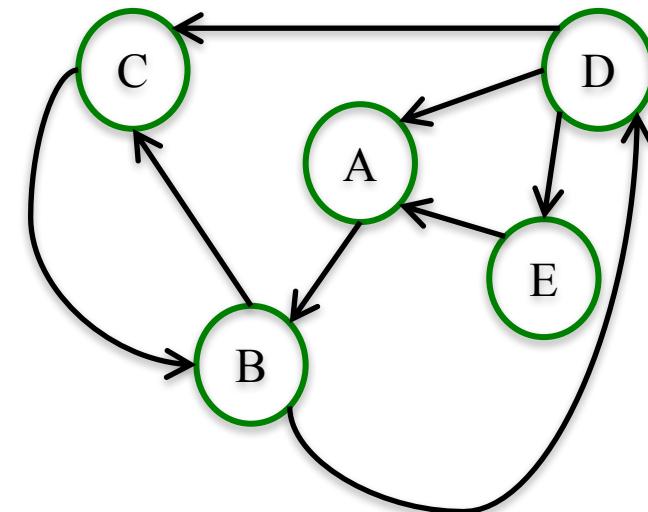
# PageRank

	Page Rank				
	A	B	C	D	E
k=2	1/10	13/30	7/30	2/10	1/30
k=2	.1	.43	.23	.20	.03



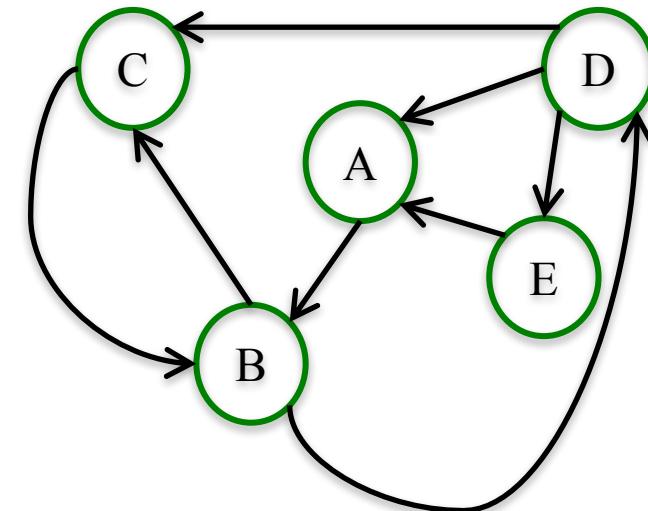
# PageRank

	Page Rank				
	A	B	C	D	E
k=2	1/10	13/30	7/30	2/10	1/30
k=2	.1	.43	.23	.20	.03
k=3	.1	.33	.28	.22	.06



# PageRank

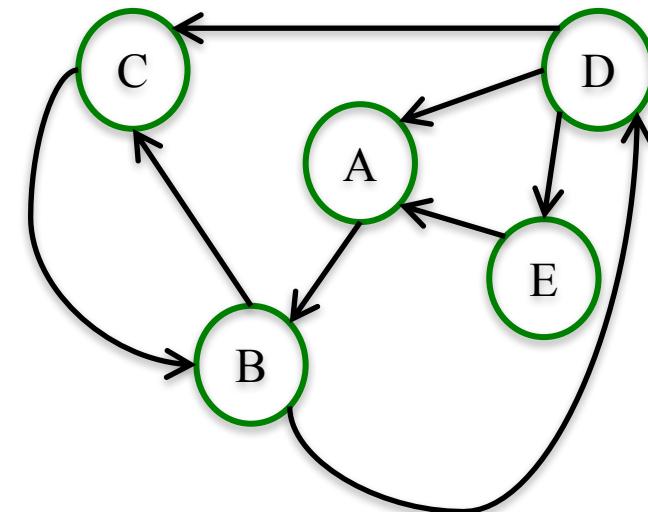
	Page Rank				
	A	B	C	D	E
k=2	1/10	13/30	7/30	2/10	1/30
k=2	.1	.43	.23	.20	.03
k=3	.1	.33	.28	.22	.06



What if continue with  $k = 4, 5, 6, \dots$ ?

# PageRank

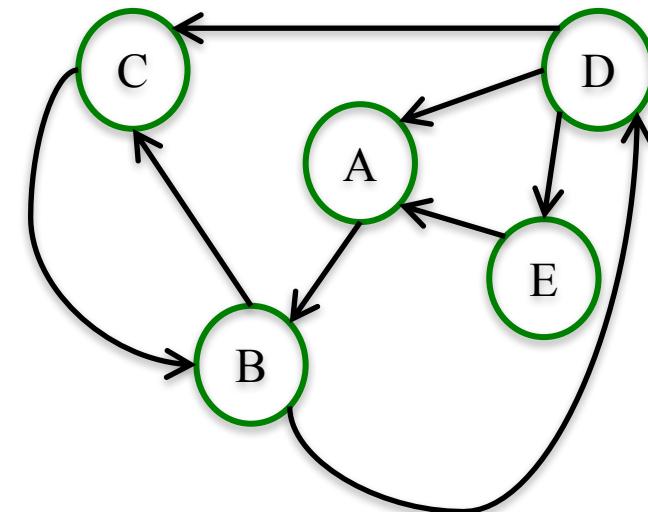
	Page Rank				
	A	B	C	D	E
k=2	1/10	13/30	7/30	2/10	1/30
k=2	.1	.43	.23	.20	.03
k=3	.1	.33	.28	.22	.06
k= $\infty$	.12	.38	.25	.19	.06



What if continue with  $k = 4, 5, 6, \dots$ ?

# PageRank

	Page Rank				
	A	B	C	D	E
k=2	1/10	13/30	7/30	2/10	1/30
k=2	.1	.43	.23	.20	.03
k=3	.1	.33	.28	.22	.06
k= $\infty$	.12	.38	.25	.19	.06



What if continue with  $k = 4, 5, 6, \dots$ ? For most networks, PageRank values converge.

# Summary

Steps of Basic PageRank:

1. All nodes start with PageRank of  $1/n$
2. Perform the *Basic PageRank Update Rule*  $k$  times:
  - **Basic PageRank Update Rule:** Each node gives an equal share of its current PageRank to all the nodes it links to.
  - The new PageRank of each node is the sum of all the PageRank it received from other nodes.

For most networks, PageRank values converge as  $k$  gets larger ( $k \rightarrow \infty$ )