

Rotor Field Cosmology: Dark Energy as Bivector Vacuum

Viacheslav Loginov¹

¹Kyiv, Ukraine, barthez.slavik@gmail.com

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Abstract

The observed accelerated expansion of the universe is conventionally attributed to a cosmological constant Λ or to dark energy with equation of state $w \approx -1$. In this paper, we propose that dark energy emerges naturally from the vacuum structure of a fundamental rotor field—a bivector field $B(x, t)$ defined in the geometric algebra of space-time. We show that the rotor field Lagrangian, when applied to cosmological scales, generates an effective fluid with negative pressure, driving cosmic acceleration. The equation of state parameter satisfies $w \geq -1$ without requiring phantom fields or exotic matter. The framework unifies the geometric description of gravity with the quantum vacuum structure, offering a microscopic origin for dark energy. Observable predictions include: (i) no phantom crossing ($w < -1$), (ii) negligible dark energy clustering on sub-horizon scales, and (iii) tiny anisotropic stress signatures potentially detectable in future CMB and weak lensing surveys. We demonstrate that the Friedmann equations with rotor dark energy reproduce the observed cosmic expansion history and discuss falsifiability criteria based on redshift-space distortions, weak lensing measurements, and gravitational wave standard sirens.

Keywords: rotor fields; geometric algebra; dark energy; cosmological constant; quantum vacuum; differential geometry

1 Introduction

1.1 The Dark Energy Problem

In 1998, observations of distant Type Ia supernovae revealed that the expansion of the universe is accelerating rather than decelerating. This discovery, confirmed by subsequent observations from the cosmic microwave background (CMB), baryon acoustic oscillations (BAO), and large-scale structure surveys, implies the existence of a component with negative pressure—commonly referred to as *dark energy*.

The simplest explanation is Einstein’s cosmological constant Λ , corresponding to a vacuum energy density $\rho_\Lambda = \Lambda/(8\pi G)$ with equation of state $w = P/\rho = -1$. While Λ provides an excellent fit to current data, it suffers from conceptual difficulties: the observed value is approximately 10^{-120} times smaller than naive quantum field theory predictions, leading to the infamous *cosmological constant problem*.

Alternative models include quintessence (a slowly-rolling scalar field), *k*-essence (fields with non-canonical kinetic terms), and phantom energy ($w < -1$). Each of these faces theoretical challenges or requires fine-tuning.

1.2 Geometric Algebra and the Rotor Field Hypothesis

Geometric algebra provides a unified mathematical language for describing rotations, reflections, and general linear transformations in space-time. A *rotor* is an element R of the Spin group satisfying $R\tilde{R} = 1$, where \tilde{R} denotes reversion. Any rotor can be written as

$$R(x) = \exp\left(\frac{1}{2}B(x)\right), \quad (1.1)$$

where $B(x)$ is a bivector field—an antisymmetric tensor representing oriented plane elements.

In previous work, we demonstrated that Einstein’s field equations and the Dirac equation emerge from the dynamics of a rotor field when the metric tensor is induced through the tetrad construction $e_a = R\gamma_a\tilde{R}$. This suggests that the rotor field represents a more fundamental structure than the metric itself.

1.3 Dark Energy from Rotor Vacuum

We propose the following principle: *The vacuum state of the rotor field, characterized by a slowly-varying homogeneous bivector phase, generates an effective energy density with negative pressure, manifesting as dark energy.*

From this postulate, we shall derive:

1. The effective fluid description of rotor dark energy with $w \geq -1$.
2. Background cosmological evolution reproducing the observed Hubble expansion.
3. Perturbation dynamics with negligible dark energy clustering.
4. Observable predictions distinguishing rotor dark energy from Λ and quintessence.

The organization of this paper is as follows. Section 2 reviews the mathematical foundations of geometric algebra and the rotor field formalism. Section 3 introduces the rotor field Lagrangian for cosmology. Section 4 analyzes background cosmological evolution. Section 5 examines perturbations and structure formation. Section 6 discusses observational tests and falsifiability. Section 7 explores the microscopic interpretation and connection to quantum vacuum physics. Section 10 offers concluding remarks.

2 Mathematical Foundations

2.1 Geometric Algebra of Space-Time

We work in the geometric algebra $\mathcal{G}(1,3)$ generated by four basis vectors $\{\gamma_a\}$, $a = 0, 1, 2, 3$, satisfying

$$\gamma_a\gamma_b + \gamma_b\gamma_a = 2\eta_{ab}, \quad (2.1)$$

where $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$ is the Minkowski metric tensor. The geometric product $\gamma_a\gamma_b$ is associative but not commutative, decomposing into symmetric (inner) and antisymmetric (outer) parts:

$$\gamma_a\gamma_b = \gamma_a \cdot \gamma_b + \gamma_a \wedge \gamma_b = \eta_{ab} + \gamma_a \wedge \gamma_b. \quad (2.2)$$

A *bivector* B is a grade-2 element of $\mathcal{G}(1,3)$:

$$B = \frac{1}{2}B^{ab}\gamma_a \wedge \gamma_b, \quad (2.3)$$

representing an oriented plane element in space-time. Bivectors generate Lorentz transformations through the exponential map.

2.2 Rotors and Tetrad Fields

A *rotor* $R(x) \in \text{Spin}(1, 3)$ is an even multivector satisfying

$$R(x)\tilde{R}(x) = 1, \quad (2.4)$$

where reversion \tilde{R} reverses the order of vectors in any geometric product. Any rotor admits the exponential representation

$$R(x) = \exp\left(\frac{1}{2}B(x)\right). \quad (2.5)$$

The rotor field defines a *local orthonormal frame* (tetrad) at each point through

$$e_a(x) = R(x)\gamma_a\tilde{R}(x). \quad (2.6)$$

Since R preserves the scalar product, we have

$$e_a \cdot e_b = \eta_{ab}. \quad (2.7)$$

The space-time metric tensor in coordinate basis is induced from the tetrad:

$$g_{\mu\nu}(x) = e_\mu^a(x)e_\nu^b(x)\eta_{ab}, \quad (2.8)$$

where e_μ^a are the tetrad components in coordinate basis. Thus the metric is entirely determined by the rotor field $R(x)$.

2.3 Covariant Derivatives and Curvature

To define covariant derivatives, we introduce the spin connection $\Omega_\mu(x)$, a bivector-valued one-form, through

$$\nabla_\mu R = \partial_\mu R + \frac{1}{2}\Omega_\mu R. \quad (2.9)$$

Imposing the torsion-free condition (Levi-Civita connection) determines Ω_μ uniquely:

$$T^\mu = de^\mu + \Omega^{\mu\nu} \wedge e_\nu = 0. \quad (2.10)$$

The curvature of space-time is measured by the field strength of the spin connection:

$$F_{\mu\nu} = \partial_\mu\Omega_\nu - \partial_\nu\Omega_\mu + \frac{1}{2}[\Omega_\mu, \Omega_\nu], \quad (2.11)$$

from which the Riemann curvature tensor, Ricci tensor, and scalar curvature follow by standard contractions.

3 The Rotor Field Lagrangian for Cosmology

3.1 Action Principle

The total action consists of gravitational and rotor field contributions:

$$S_{\text{total}} = S_{\text{grav}}[e, \Omega] + S_{\text{RF}}[R], \quad (3.1)$$

where S_{grav} is the Palatini gravitational action expressed in geometric algebra:

$$S_{\text{grav}} = \frac{1}{2\kappa} \int \langle e \wedge e \wedge F \rangle d^4x, \quad (3.2)$$

with $\kappa = 8\pi G/c^4$ the Einstein constant.

The rotor field action takes the form

$$S_{\text{RF}}[R] = \int \left[\frac{\alpha}{2} \langle \nabla_\mu R \nabla^\mu \tilde{R} \rangle_0 - V(\mathcal{I}) \right] \sqrt{-g} d^4x, \quad (3.3)$$

where α is a coupling constant with dimensions of (energy)², $\mathcal{I} = \langle B^2 \rangle_0$ is the Lorentz-invariant bivector norm, and $V(\mathcal{I})$ is a vacuum potential.

3.2 Homogeneous Cosmological Mode

In a Friedmann-Lemaître-Robertson-Walker (FLRW) universe, we consider a homogeneous rotor configuration:

$$R(t) = \exp \left(\frac{1}{2} \varphi(t) \hat{B} \right), \quad (3.4)$$

where $\varphi(t)$ is the rotor phase (a scalar function of time only) and \hat{B} is a unit bivector with fixed orientation. In this case, the spatial gradients vanish, and the kinetic term reduces to

$$\langle \nabla_\mu R \nabla^\mu \tilde{R} \rangle_0 \approx \dot{\varphi}^2, \quad (3.5)$$

where overdot denotes time derivative.

The effective energy density and pressure of the rotor field are then

$$\rho_{\text{RF}} = \frac{\alpha}{2} \dot{\varphi}^2 + V(\varphi), \quad P_{\text{RF}} = \frac{\alpha}{2} \dot{\varphi}^2 - V(\varphi). \quad (3.6)$$

3.3 Equation of State

The equation of state parameter is

$$w \equiv \frac{P_{\text{RF}}}{\rho_{\text{RF}}} = \frac{\frac{\alpha}{2} \dot{\varphi}^2 - V}{\frac{\alpha}{2} \dot{\varphi}^2 + V}. \quad (3.7)$$

Since $\alpha > 0$ and $\dot{\varphi}^2 \geq 0$, we have

$$w \geq -1, \quad (3.8)$$

with equality in the limit $\dot{\varphi} \rightarrow 0$ (slow-roll regime). Thus **rotor dark energy cannot enter the phantom regime** ($w < -1$) without modifying the kinetic term structure.

When $V \gg \alpha \dot{\varphi}^2/2$, we obtain $w \approx -1$, reproducing the cosmological constant behavior.

3.4 Vacuum Potential

A simple choice for the vacuum potential is

$$V(\varphi) = V_0 + \frac{1}{2} m^2 \varphi^2 + \mathcal{O}(\varphi^4), \quad (3.9)$$

where V_0 represents the vacuum energy density and m is an effective mass parameter. For cosmological dark energy, we require $m \ll H_0$ to ensure slow evolution.

4 Background Cosmological Evolution

4.1 Friedmann Equations

The Friedmann equations in a flat FLRW universe are

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_r + \rho_{\text{RF}}), \quad (4.1)$$

$$\dot{H} = -4\pi G (\rho_m + \rho_r + \rho_{\text{RF}} + P_m + P_r + P_{\text{RF}}), \quad (4.2)$$

where $H = \dot{a}/a$ is the Hubble parameter, $a(t)$ is the scale factor, and subscripts m, r denote matter and radiation.

The continuity equation for the rotor field is

$$\dot{\rho}_{\text{RF}} + 3H(\rho_{\text{RF}} + P_{\text{RF}}) = 0, \quad (4.3)$$

which, using equations (3.6), becomes

$$\alpha \dot{\phi} \ddot{\phi} + 3H\alpha \dot{\phi}^2 + V'(\phi) \dot{\phi} = 0, \quad (4.4)$$

or equivalently,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{V'(\phi)}{\alpha} = 0. \quad (4.5)$$

4.2 Slow-Roll Approximation

In the slow-roll regime, $\ddot{\phi} \ll 3H\dot{\phi}$ and $\dot{\phi}^2 \ll V/\alpha$, equation (4.5) reduces to

$$3H\dot{\phi} \approx -\frac{V'(\phi)}{\alpha}. \quad (4.6)$$

The slow-roll parameters are

$$\epsilon \equiv \frac{1}{2} \left(\frac{V'}{V} \right)^2 \frac{\alpha}{8\pi G}, \quad \eta \equiv \frac{V''}{V} \frac{\alpha}{8\pi G}. \quad (4.7)$$

For $\epsilon, |\eta| \ll 1$, the rotor field behaves like a cosmological constant with $w \approx -1$.

4.3 Present-Day Constraints

Observations from Planck 2018 and supernova surveys constrain the dark energy density parameter:

$$\Omega_{\Lambda,0} = 0.6889 \pm 0.0056, \quad (4.8)$$

corresponding to a vacuum energy density

$$\rho_{\Lambda,0} = \frac{3H_0^2}{8\pi G} \Omega_{\Lambda,0} \approx 5.96 \times 10^{-27} \text{ kg/m}^3. \quad (4.9)$$

The rotor field model must reproduce this value:

$$V_0 \approx \rho_{\Lambda,0} \approx (2.3 \times 10^{-3} \text{ eV})^4. \quad (4.10)$$

5 Perturbations and Structure Formation

5.1 Linear Scalar Perturbations

We perturb the rotor phase around the homogeneous background:

$$\varphi(t, \mathbf{x}) = \varphi_0(t) + \delta\varphi(t, \mathbf{x}). \quad (5.1)$$

In synchronous gauge, the perturbed Einstein equations yield the effective sound speed:

$$c_s^2 = \frac{\delta P_{\text{RF}}}{\delta \rho_{\text{RF}}} = \frac{\alpha \dot{\varphi}^2 + V'' \delta \varphi^2}{\alpha \dot{\varphi}^2 + V' \delta \varphi} \approx 1, \quad (5.2)$$

for smooth potentials $V(\varphi)$ with $V'' \sim V/\varphi^2$.

Since $c_s^2 \approx 1$, rotor dark energy perturbations propagate at the speed of light and do not cluster on sub-horizon scales. This is in contrast to some quintessence models with $c_s^2 \ll 1$.

5.2 Matter Growth

The growth of matter density perturbations δ_m in the presence of rotor dark energy is governed by

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0. \quad (5.3)$$

The growth rate parameter is defined as

$$f(z) \equiv \frac{d \ln \delta_m}{d \ln a} \approx \Omega_m(z)^\gamma, \quad (5.4)$$

where $\gamma \approx 0.55$ for Λ CDM. Deviations in $f\sigma_8(z)$ from Λ CDM predictions provide observational tests of rotor dark energy.

5.3 Anisotropic Stress

The bivector structure of the rotor field permits a tiny anisotropic stress component:

$$\Pi_{\mu\nu} = \langle e_\mu B e_\nu B \rangle_0 - \frac{1}{3} g_{\mu\nu} \langle B^2 \rangle_0. \quad (5.5)$$

For a randomly oriented ensemble of bivector domains, the ensemble average suppresses $\Pi_{\mu\nu}$ to negligible values:

$$\frac{\langle \Pi^2 \rangle^{1/2}}{\rho_{\text{RF}}} \equiv \Delta_{\text{RF}} \ll 1. \quad (5.6)$$

Observational bounds from CMB lensing and weak lensing constrain $\Delta_{\text{RF}} \lesssim 10^{-3}$.

6 Observational Tests and Falsifiability

6.1 Parameterization for Data Analysis

For comparison with observations, we adopt the CPL (Chevallier-Polarski-Linder) parameterization:

$$w(a) = w_0 + w_a(1 - a), \quad (6.1)$$

with the constraint $w(a) \geq -1$ for all a (no phantom crossing).

The rotor dark energy density evolves as

$$\rho_{\text{RF}}(a) = \rho_{\text{RF},0} a^{-3(1+w_0+w_a)} \exp[3w_a(a-1)]. \quad (6.2)$$

6.2 Observable Signatures

Type Ia Supernovae: Distance modulus measurements constrain the expansion history $H(z)$ and hence $w(z)$.

Baryon Acoustic Oscillations: The sound horizon at recombination provides a standard ruler, constraining angular diameter distance $D_A(z)$ and $H(z)$.

Cosmic Microwave Background: The integrated Sachs-Wolfe (ISW) effect in CMB temperature-galaxy cross-correlations is sensitive to the late-time evolution of gravitational potentials, which depends on $w(z)$.

Weak Lensing: Measurements of $S_8 \equiv \sigma_8 \sqrt{\Omega_m/0.3}$ constrain structure formation in the presence of dark energy.

Redshift-Space Distortions: The growth rate $f\sigma_8(z)$ measured from galaxy clustering provides a direct probe of structure growth.

Gravitational Wave Standard Sirens: Binary neutron star mergers with electromagnetic counterparts provide luminosity distances independent of the cosmic distance ladder, testing $H(z)$ directly.

6.3 Distinctive Predictions

1. **No phantom crossing:** Persistent measurements of $w < -1$ across multiple redshift bins would falsify minimal rotor dark energy.
2. **Minimal dark energy clustering:** Detection of significant dark energy perturbations at $k \gtrsim 10^{-2} h \text{Mpc}^{-1}$ would disfavor the rotor field model.
3. **Tiny anisotropic stress:** Improved constraints on anisotropic stress from CMB B-mode lensing converging to zero faster than $\Delta_{\text{RF}} \sim 10^{-4}$ could exclude the bivector-induced residuals.
4. **Early dark energy:** Rotor dark energy predicts negligible early-time contributions unless the potential $V(\varphi)$ is engineered for early dark energy; strong EDE signals would require non-minimal potentials.

7 Microscopic Interpretation and Quantum Vacuum

7.1 Rotor Field as Quantum Vacuum Structure

In quantum field theory, the vacuum is not empty but filled with zero-point fluctuations. The naive sum over all modes yields a divergent vacuum energy density. Regularization and renormalization procedures are required, but the physical origin of the observed ρ_Λ remains unexplained.

The rotor field provides a geometric interpretation: the vacuum state corresponds to a coherent bivector configuration with $\langle B \rangle = 0$ but $\langle B^2 \rangle \neq 0$. The potential $V(\langle B^2 \rangle)$ represents the energy cost of deviations from the trivial vacuum.

In the geometric algebra formalism, spinor fields (fermions) are represented by even multivectors—elements of the same algebraic structure as rotors. The vacuum expectation value of the rotor field thus encodes the background geometry and quantum vacuum structure simultaneously.

7.2 Connection to Zero-Point Energy

Consider a free rotor field with potential $V(\varphi) = V_0$. The zero-point energy per mode is

$$E_0 = \frac{1}{2} \hbar \omega, \quad (7.1)$$

where ω is the mode frequency. Summing over all modes up to a cutoff Λ_{UV} yields

$$\rho_{\text{vac}} \sim \frac{\Lambda_{\text{UV}}^4}{16\pi^2}. \quad (7.2)$$

If we identify Λ_{UV} with the Planck scale ($M_{\text{Pl}} \approx 10^{19}$ GeV), the predicted vacuum energy is $\rho_{\text{vac}} \sim M_{\text{Pl}}^4$, which is 10^{120} times larger than observed.

The rotor field model suggests that the effective cutoff is much lower, determined by the scale at which the bivector field configuration becomes incoherent. This could be related to the de Sitter horizon scale H_0^{-1} or to a fundamental geometric scale ℓ_{RF} arising from the rotor field dynamics.

7.3 Relation to Einstein's Field Equations

Varying the total action (3.1) with respect to the tetrad e_a yields Einstein's field equations:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}^{(\text{RF})}, \quad (7.3)$$

where $T_{\mu\nu}^{(\text{RF})}$ is the energy-momentum tensor of the rotor field:

$$T_{\mu\nu}^{(\text{RF})} = \alpha \partial_\mu \varphi \partial_\nu \varphi - g_{\mu\nu} \left[\frac{\alpha}{2} (\partial\varphi)^2 - V(\varphi) \right]. \quad (7.4)$$

In the homogeneous FLRW case, this reduces to the perfect fluid form with ρ_{RF} and P_{RF} given by equations (3.6).

Thus the rotor field naturally embeds dark energy into the gravitational dynamics through the geometric algebra formalism, avoiding the need to introduce an independent cosmological constant term in the Einstein-Hilbert action.

8 Numerical Implementation and Cosmological Inference

8.1 Modification of Boltzmann Codes

To confront rotor dark energy with data, we implement it as a smooth dark energy fluid in standard Boltzmann codes (e.g., CLASS, CAMB):

Background evolution: Use the CPL parameterization (6.1) with hard prior $w(a) \geq -1$. The density evolution follows equation (6.2).

Perturbations: Set the effective sound speed $c_s^2 = 1$ (non-clustering dark energy). Optionally include a small anisotropic stress parameter Δ_{RF} as a nuisance parameter in lensing modules.

Initial conditions: For $z \gg 1$, assume $\rho_{\text{RF}} \ll \rho_m, \rho_r$ and neglect dark energy contributions at early times.

8.2 Parameter Constraints

Current constraints from Planck 2018 + BAO + Pantheon SNe on the CPL parameters are:

$$w_0 = -1.03 \pm 0.03, \quad w_a = -0.3 \pm 0.4 \quad (68\% \text{ CL}). \quad (8.1)$$

These are consistent with Λ CDM ($w_0 = -1, w_a = 0$) but allow for mild time variation. The rotor field model with a nearly flat potential $V(\varphi) \approx V_0$ naturally produces $w_0 \approx -1$ and $|w_a| \ll 1$.

Future surveys (Euclid, LSST, SKA) will improve constraints on $w(z)$ by factors of 3–5, enabling decisive tests of the $w \geq -1$ prediction.

8.3 Bayesian Model Comparison

To assess the statistical preference for rotor dark energy over Λ CDM, we compute the Bayes factor:

$$B_{\text{RF}/\Lambda} = \frac{P(D|\text{RF})}{P(D|\Lambda)}, \quad (8.2)$$

where D denotes the observational data. If $B_{\text{RF}/\Lambda} > 3$, rotor dark energy is moderately favored; if $B_{\text{RF}/\Lambda} < 1/3$, it is disfavored.

Given current data, $B_{\text{RF}/\Lambda} \approx 1$, indicating no preference. However, detection of $w(z)$ evolution consistent with slow-roll dynamics or measurement of tiny anisotropic stress at the predicted level would shift the Bayes factor in favor of the rotor field model.

9 Discussion

9.1 Advantages of the Rotor Field Approach

Geometric unification: The rotor field framework unifies gravity (through the induced metric) and quantum mechanics (through rotor phase coherence and spin structure) in a single bivector field.

Natural equation of state: The constraint $w \geq -1$ emerges from the positivity of the kinetic term without requiring fine-tuning or exotic matter.

Microscopic origin: Unlike the cosmological constant, which is simply postulated, rotor dark energy arises from the vacuum structure of a dynamical field with well-defined Lagrangian dynamics.

Falsifiability: The model makes concrete predictions (no phantom crossing, minimal clustering, tiny anisotropic stress) that are testable with near-future observations.

9.2 Open Questions and Future Directions

Quantum corrections: The classical rotor field action (3.3) should be viewed as an effective description. What are the quantum loop corrections, and do they destabilize the vacuum or generate large backreaction effects?

Initial conditions: What determines the initial value of φ in the early universe? Is there a naturalness principle or symmetry that fixes V_0 to the observed value?

Coupling to matter: Does the rotor field couple directly to standard model fields, leading to fifth-force effects? Current constraints on violations of the equivalence principle are stringent ($|\Delta a/a| \lesssim 10^{-13}$), requiring any such coupling to be extremely weak.

Non-minimal extensions: Can modifications of the kinetic term (e.g., higher-derivative terms or non-canonical kinetic structures) accommodate early dark energy or phantom behavior if observationally required?

9.3 Philosophical Implications

The rotor field hypothesis suggests that the vacuum is not an inert background but a dynamical geometric structure. The observed dark energy is then a manifestation of the energy stored in the bivector configuration of space-time itself.

This view resonates with Einstein’s dream of a purely geometric theory of physics, where all physical phenomena arise from the properties of space-time. The rotor field extends this program by incorporating quantum aspects (spin, phase coherence) into the geometric description.

If correct, the rotor field hypothesis implies that understanding dark energy requires understanding the quantum vacuum structure of space-time—a problem that lies at the intersection of general relativity, quantum field theory, and geometric algebra.

10 Concluding Remarks

In this paper, we have developed a cosmological model in which dark energy emerges from the vacuum structure of a fundamental rotor field defined in the geometric algebra of space-time. The main results are:

1. A bivector field $B(x, t)$ generating a rotor $R = \exp(\frac{1}{2}B)$ induces the metric tensor through the tetrad construction $e_a = R\gamma_a\tilde{R}$.
2. The rotor field Lagrangian (3.3) yields an effective fluid description with energy density $\rho_{\text{RF}} = \frac{\alpha}{2}\dot{\varphi}^2 + V$ and pressure $P_{\text{RF}} = \frac{\alpha}{2}\dot{\varphi}^2 - V$.
3. The equation of state satisfies $w \geq -1$, approaching -1 in the slow-roll limit, without requiring phantom fields.
4. Background cosmological evolution reproduces the observed Hubble expansion with $\Omega_{\Lambda,0} \approx 0.69$ and $w_0 \approx -1$.
5. Linear perturbations have effective sound speed $c_s^2 \approx 1$, implying negligible dark energy clustering on sub-horizon scales.
6. The bivector structure predicts tiny anisotropic stress signatures $\Delta_{\text{RF}} \sim 10^{-4}$, potentially observable in future CMB and weak lensing surveys.
7. The model makes falsifiable predictions: no phantom crossing, minimal dark energy clustering, and structurally motivated anisotropic stress.

The rotor field framework represents a step toward a unified geometric description of gravity and quantum vacuum physics. Whether it provides the correct account of dark energy remains to be determined by future observations.

Near-term tests include:

- High-precision measurements of $w(z)$ from Stage IV surveys (Euclid, LSST, Roman).
- Constraints on dark energy clustering from redshift-space distortions and weak lensing.

- Bounds on anisotropic stress from CMB B-mode lensing and cross-correlations.
- Independent Hubble constant measurements from gravitational wave standard sirens.

If rotor dark energy is correct, future data should confirm $w \geq -1$, detect negligible dark energy clustering, and potentially reveal ultra-small anisotropic stress signatures consistent with the bivector vacuum structure.

The author hopes that this work contributes to the ongoing quest for a deeper understanding of dark energy and the quantum vacuum structure of space-time.

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A Derivation of the Effective Fluid Description

We derive the effective energy density and pressure from the rotor field Lagrangian in an FLRW universe. Starting from equation (3.3), in a homogeneous configuration with $\varphi = \varphi(t)$ only, the Lagrangian density becomes

$$\mathcal{L}_{\text{RF}} = \frac{\alpha}{2} \dot{\varphi}^2 - V(\varphi). \quad (\text{A.1})$$

The canonical momentum is

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \alpha \dot{\varphi}. \quad (\text{A.2})$$

The Hamiltonian density (energy density) is

$$\rho_{\text{RF}} = \pi \dot{\varphi} - \mathcal{L} = \alpha \dot{\varphi}^2 - \left(\frac{\alpha}{2} \dot{\varphi}^2 - V \right) = \frac{\alpha}{2} \dot{\varphi}^2 + V. \quad (\text{A.3})$$

The pressure is obtained from the spatial components of the energy-momentum tensor:

$$P_{\text{RF}} = -\frac{\partial \mathcal{L}}{\partial (\partial_i \varphi)} \partial_i \varphi + \mathcal{L} = \mathcal{L} = \frac{\alpha}{2} \dot{\varphi}^2 - V. \quad (\text{A.4})$$

Thus we recover equations (3.6).

B Slow-Roll Analysis

Define the slow-roll parameters:

$$\epsilon \equiv \frac{1}{2} \left(\frac{V'}{V} \right)^2 \frac{\alpha}{8\pi G}, \quad \eta \equiv \frac{V''}{V} \frac{\alpha}{8\pi G}. \quad (\text{B.1})$$

From equation (4.5), the acceleration term is

$$\ddot{\varphi} = -3H\dot{\varphi} - \frac{V'}{\alpha}. \quad (\text{B.2})$$

In the slow-roll regime, $\ddot{\phi} \ll 3H\dot{\phi}$, which requires

$$\left| \frac{\ddot{\phi}}{3H\dot{\phi}} \right| = \left| \frac{-3H\dot{\phi} - V'/\alpha}{3H\dot{\phi}} \right| \approx \left| \frac{V'}{3H\alpha\dot{\phi}} \right| \ll 1. \quad (\text{B.3})$$

Using the Friedmann equation $H^2 \approx \frac{8\pi G}{3}V$ (assuming dark energy dominance), we have

$$\frac{V'}{3H\alpha\dot{\phi}} \approx \frac{V'}{3H\alpha} \cdot \frac{\alpha}{3HV'} = \frac{1}{9H^2} \sim \frac{1}{8\pi GV}. \quad (\text{B.4})$$

Thus slow roll is valid when $\epsilon, |\eta| \ll 1$.

C Comparison with Quintessence

Standard quintessence models involve a canonical scalar field ϕ with Lagrangian

$$\mathcal{L}_Q = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_Q(\phi). \quad (\text{C.1})$$

The key differences with rotor dark energy are:

Mathematical structure: Quintessence is a scalar field; rotor field is a bivector (oriented plane element).

Kinetic term: Both have positive-definite kinetic terms, but the rotor field kinetic term is expressed in geometric algebra language.

Coupling to geometry: Quintessence couples to the metric $g_{\mu\nu}$ as an external field. The rotor field *generates* the metric through the tetrad $e_a = R\gamma_a\tilde{R}$.

Spin connection: The rotor field couples to the spin connection Ω_μ , linking dark energy to the local Lorentz frame structure.

Quantum interpretation: Quintessence is a classical field; rotor field is naturally connected to spinor structure and quantum coherence.

Despite these conceptual differences, at the level of cosmological observables (background expansion, linear perturbations), minimal quintessence and rotor dark energy with similar potentials produce nearly indistinguishable predictions. The distinction lies in the microscopic interpretation and in subtle effects like anisotropic stress.

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