Dark Matter as Phase-Dephased Rotor Vacuum: A Geometric Algebra Formulation and Testable Predictions

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Version 1.0 | October 15, 2025

Abstract

Observational evidence for dark matter spans rotation curves, gravitational lensing, structure formation, and the cosmic microwave background. Particle searches have yielded null results; modified gravity struggles with consistency across scales. We propose a geometric alternative: dark matter as the *phase-dephased component* of a fundamental rotor field defined in geometric algebra. Spacetime carries a bivector field $B(x)$ whose exponential $R(x) = \exp(\frac{1}{2}B)$ induces the metric and orthonormal frames. Ordinary matter corresponds to rotor orientations aligned with the electromagnetic observation plane; dark matter arises from orthogonal (dephased) components that suppress electromagnetic coupling while retaining gravitational effects through angular-gradient stresses. We derive an effective stress-energy for the dephased sector, present falsifiable predictions for rotation curves, weak lensing anisotropies, structure growth, and CMB imprints, and propose a minimal three-parameter phenomenology (ξ_0, σ_B, c_R^2) testable with current observations.

Keywords: dark matter, rotor fields, geometric algebra, weak lensing, structure formation, rotation curves

1 Introduction

1.1 The Dark Matter Problem

Galaxy rotation curves remain flat far beyond the luminous extent of stellar disks. Gravitational lensing by galaxy clusters reveals mass distributions inconsistent with visible matter. Large-scale structure formation requires non-baryonic cold dark matter seeding gravitational collapse. The cosmic microwave background acoustic peaks demand $\Omega_{\mathrm{DM}}h^2 \approx 0.12$. These observations establish that approximately 85% of the matter content of the universe is non-luminous and collisionless.

Particle physics offers candidates—weakly interacting massive particles (WIMPs), axions, sterile neutrinos—but direct detection experiments have found no signal despite decades of searches. Modified Newtonian dynamics (MOND) explains rotation curves but fails for cluster dynamics, lensing, and cosmology. The Bullet Cluster observation, where lensing mass peaks are spatially offset from baryonic gas peaks, provides strong evidence for collisionless dark matter distinct from baryons.

1.2 Geometric Algebra and the Rotor Field Framework

Clifford's geometric algebra unifies vectors, bivectors, and higher-grade multivectors within a single algebraic structure. The geometric product encodes both inner and outer products. Rotations are represented by *rotors*—exponentials of bivectors. Hestenes reformulated the Dirac equation in geometric algebra, revealing the spinor as a geometric rotor rather than an abstract Hilbert-space entity. Lasenby, Doran, and Gull showed that general relativity can be formulated as a gauge theory of the Lorentz rotor group, with the metric emerging from tetrad fields.

This suggests that spacetime geometry itself may be rotor-induced. If the fundamental field is a bivector $B(x)$ in the Clifford algebra $\mathcal{G}(1,3)$, the associated rotor $R = \exp(\frac{1}{2}B)$ defines local orthonormal frames and thus the metric. Different bivector *orientations* in $\mathcal{B}(1,3)$ may encode distinct interaction channels.

1.3 The Phase-Dephasing Hypothesis

We propose that dark matter is not a new particle species but a *geometric phase* of the rotor field:

Spacetime admits a fundamental bivector field $B(x)$,
and matter is characterized by rotor orientation relative to observational planes.
Luminous matter: $R_\parallel \sim \exp(\frac{1}{2}B_\parallel)$ aligned with EM coupling.
Dark matter: $R_\perp \sim \exp(\frac{1}{2}B_\perp)$ orthogonal (dephased) orientations.

Electromagnetic interactions couple to bivectors aligned with photon polarization planes. Matter whose rotor is predominantly $R_\parallel$ scatters and emits light. Matter whose rotor is predominantly $R_\perp$ (orthogonal orientation) remains electromagnetically silent but gravitates through its gradient energy $\alpha \left\langle (\nabla B_\perp)^2 \right\rangle$.

From this hypothesis, we shall demonstrate:

1. **Effective stress-energy:** The dephased sector behaves as a fluid with small sound speed $c_R^2 \propto \sigma_B^2$, where σ_B is the bivector orientation dispersion.
2. **Rotation curves:** Vortex-like rotor textures around disk galaxies source flat asymptotic tails $v_c(r) \to \text{const}$ when $\xi\alpha \propto r^{-2}$.
3. **Lensing quadrupoles:** Anisotropic stress produces $\epsilon_2 \propto \sigma_B^2$ quadrupolar modulation aligned with disk orientation axes.
4. **Structure suppression:** Rotor sound speed c_R^2 suppresses small-scale power at $k \gtrsim H_0/c_R$ without WDM freestreaming artifacts.
5. **CMB anisotropy:** Rotor stress modifies ISW effect, producing cross-correlation with galaxy spin-axis fields.

The remainder of this paper is organized as follows. Section 2 reviews geometric algebra preliminaries. Section 3 presents the rotor dark matter hypothesis with kinematic and dynamical postulates. Section 4 derives the effective action and stress-energy. Section 5 develops phenomenological predictions. Section 6 proposes a data pipeline for parameter inference. Section 7 discusses consistency checks and degeneracies. Section 8 addresses open questions. Section 9 offers concluding remarks.

2 Geometric Algebra Preliminaries

2.1 Clifford Algebra and the Geometric Product

Let $\{\gamma_a\}$, $a = 0, 1, 2, 3$, be an orthonormal basis of the spacetime Clifford algebra $\mathcal{G}(1, 3)$ satisfying

$$\gamma_a\gamma_b + \gamma_b\gamma_a = 2\eta_{ab}, \qquad \eta = \mathrm{diag}(+1, -1, -1, -1). \tag{2.1}$$

The geometric product decomposes as

$$\gamma_a\gamma_b = \gamma_a \cdot \gamma_b + \gamma_a \wedge \gamma_b = \eta_{ab} + B_{ab}, \tag{2.2}$$

where $B_{ab} = \frac{1}{2}(\gamma_a\gamma_b - \gamma_b\gamma_a)$ is a bivector. The space $\mathcal{B}(1, 3)$ is six-dimensional, spanned by $\{\gamma_a \wedge \gamma_b\}_{a<b}$.

A general bivector $B = \frac{1}{2}B^{ab}\gamma_a \wedge \gamma_b$ generates rotations in the planes it spans. The exponential map yields rotors.

2.2 Rotors and Frame Fields

Definition 2.1 (Rotor)**.** A *rotor* $R \in \mathrm{Spin}(1, 3)$ satisfies $R\widetilde{R} = 1$ (where $\widetilde{R}$ denotes reversion) and admits

$$R = \exp\left(\tfrac{1}{2}B\right), \qquad B \in \mathcal{B}(1, 3). \tag{2.3}$$

The rotor field $R(x)$ defines a position-dependent orthonormal tetrad:

$$e_a(x) \equiv R(x)\,\gamma_a\,\widetilde{R}(x), \qquad e_a \cdot e_b = \eta_{ab}, \tag{2.4}$$

with components $e_a = e_a{}^{\mu}\partial_\mu$. The induced spacetime metric is

$$g_{\mu\nu}(x) = e_\mu{}^a e_\nu{}^b \eta_{ab}. \tag{2.5}$$

Thus the metric emerges from the rotor field. A spin connection Ω_μ (bivector-valued one-form) defines the gauge-covariant derivative $\nabla_\mu R = \partial_\mu R + \frac{1}{2}\Omega_\mu R$. The curvature bivector is

$$F_{\mu\nu} = \partial_\mu\Omega_\nu - \partial_\nu\Omega_\mu + \tfrac{1}{2}[\Omega_\mu, \Omega_\nu]. \tag{2.6}$$

Remark 2.2. General relativity is reformulated as a gauge theory of $\mathrm{Spin}(1, 3)$, analogous to Yang–Mills gauge theories. The metric (2.5) is not fundamental but derived from rotor dynamics.

3 The Rotor Dark Matter Hypothesis

3.1 Kinematic Postulates and the Electromagnetic Decomposition

3.1.1 Physical Origin of Bivector Splitting

Let spacetime admit a rotor field $R(x)$ with bivector generator $B(x)$. The key insight is that *electromagnetic interactions induce a natural decomposition* of the bivector space.

The electromagnetic field tensor $F_{\mu\nu}$ is itself a bivector in geometric algebra: $F = \frac{1}{2}F^{\mu\nu}\gamma_\mu \wedge \gamma_\nu$. When matter couples to the electromagnetic field, the coupling Lagrangian takes the form:

$$\mathcal{L}_{\mathrm{int}} = -g_{\mathrm{EM}}\,\langle F \cdot B\rangle_0 = -g_{\mathrm{EM}}\,\eta_{\mu\nu\rho\sigma}\,F^{\mu\nu}B^{\rho\sigma}, \tag{3.1}$$

where $g_{\rm EM}$ is the electromagnetic coupling constant and $\langle \cdot \rangle_0$ extracts the scalar part.

This coupling is *maximal* when B is aligned with F (i.e., when the rotor bivector orientation matches the electromagnetic bivector plane) and *vanishes* when B is orthogonal to F. Since the six-dimensional bivector space $\mathcal{B}(1,3)$ decomposes into three two-dimensional eigenspaces under the action of F, we may write:

$$B = B_{\parallel} + B_{\perp}, \tag{3.2}$$

where:

- $B_{\parallel}$ lies in the eigenspace with *nonzero* eigenvalue under F-action, coupling electromagnetically;
- $B_{\perp}$ lies in the orthogonal complement, decoupled from F.

Physical interpretation: Electromagnetic photons propagate in a definite polarization plane (bivector orientation). Matter whose rotor generator B aligns with this plane couples strongly to light—this is *luminous matter*. Matter whose rotor generator is orthogonal remains electromagnetically silent—this is *dark matter*.

The decomposition is not postulated but *induced by the presence of electromagnetic fields* in the universe. In the absence of F, the distinction between $B_{\parallel}$ and $B_{\perp}$ would be gauge-dependent. The observed electromagnetic background (CMB photons, starlight, etc.) defines a preferred orientation in bivector space, breaking the symmetry.

3.1.2 Coarse-Grained Decomposition

In practice, electromagnetic fields vary on scales much smaller than galactic or cosmological scales. We coarse-grain over a cell of size L (e.g., $L \sim 1\,\mathrm{kpc}$ for galaxies) and define an *effective EM observation plane* by averaging the local electromagnetic bivector:

$$\hat{F}_{\rm obs} \equiv \frac{\langle F \rangle}{\|\langle F \rangle\|}. \tag{3.3}$$

The rotor field then decomposes:

$$R(x) = \exp\!\left(\tfrac{1}{2} B_{\parallel}(x)\right) \exp\!\left(\tfrac{1}{2} B_{\perp}(x)\right), \tag{3.4}$$

where $B_{\parallel}$ projects onto the plane spanned by $\hat{F}_{\rm obs}$ and $B_{\perp}$ onto its orthogonal complement in $\mathcal{B}(1,3)$.

Definition 3.1 (Dephasing fraction)**.** The *dephasing fraction* is

$$\xi(x) \equiv \frac{\left\langle \|B_{\perp}\|^2 \right\rangle}{\left\langle \|B_{\parallel}\|^2 \right\rangle + \left\langle \|B_{\perp}\|^2 \right\rangle}, \qquad 0 \le \xi \le 1, \tag{3.5}$$

where $\langle \cdot \rangle$ denotes coarse-graining over a cell of size $L \gg \ell_{\rm Planck}$.

Definition 3.2 (Bivector orientation dispersion)**.** The *orientation dispersion* is

$$\sigma_B^2 \equiv \mathrm{Var}(\hat{B}) = \left\langle \hat{B}^2 \right\rangle - \left\langle \hat{B} \right\rangle^2, \tag{3.6}$$

where $\hat{B} = B / \|B\|$ is the unit bivector orientation.

3.2 Dynamical Postulates

We postulate a rotor action with gravitational and kinetic terms:

$$S = \frac{1}{2\kappa} \int \langle e \wedge e \wedge F \rangle_0 \, \mathrm{d}^4x + \int \mathcal{L}_R \sqrt{-g} \, \mathrm{d}^4x, \tag{3.7}$$

$$\mathcal{L}_R = \frac{\alpha}{2} \left\langle \nabla_\mu R \, \widetilde{\nabla^\mu R} \right\rangle_0 - V(R; \Phi), \tag{3.8}$$

where $\kappa = 8\pi G/c^4$, $\alpha > 0$ is the rotor coupling constant with dimensions (energy)^2 or equivalently (mass)^2 in natural units, and V is a potential coupling to other fields Φ.

Variation yields Einstein's equations $G_{\mu\nu} = \kappa T_{\mu\nu}$ with stress-energy

$$T^{(R)}_{\mu\nu} = \alpha \left\langle \nabla_\mu R \, \widetilde{\nabla_\nu R} \right\rangle_0 - g_{\mu\nu} \mathcal{L}_R. \tag{3.9}$$

Remark 3.3. The stress-energy (3.9) is *anisotropic* when B has preferred orientations. This anisotropy sources gravitational effects distinct from isotropic dark matter fluids.

4 Effective Action and Phase Decomposition

4.1 Decomposition into Coherent and Dephased Sectors

Using the decomposition (3.4), the Lagrangian (3.8) expands to quadratic order in gradients:

$$\mathcal{L}_R \approx \frac{\alpha}{8} \left[\mathrm{Tr}\left((\nabla B_\parallel)^2 \right) + \mathrm{Tr}\left((\nabla B_\perp)^2 \right) \right] - V_\parallel - V_\perp. \tag{4.1}$$

Coarse-graining over cells of size L yields effective densities:

$$\rho_{\mathrm{lum}} \simeq \frac{\alpha}{8L^2} \left\langle \mathrm{Tr}((\nabla B_\parallel)^2) \right\rangle + V_\parallel, \tag{4.2}$$

$$\rho_{\mathrm{DM}} \simeq \frac{\alpha}{8L^2} \left\langle \mathrm{Tr}((\nabla B_\perp)^2) \right\rangle + V_\perp. \tag{4.3}$$

The dephasing fraction (3.5) controls the ratio when the gradient energy dominates:

$$\frac{\rho_{\mathrm{DM}}}{\rho_{\mathrm{tot}}} \approx \frac{\left\langle \mathrm{Tr}((\nabla B_\perp)^2) \right\rangle}{\left\langle \mathrm{Tr}((\nabla B_\parallel)^2) \right\rangle + \left\langle \mathrm{Tr}((\nabla B_\perp)^2) \right\rangle} \equiv \xi', \tag{4.4}$$

where $\xi' = \xi$ only if the bivector orientations are statistically independent with equal variance: $\left\langle |\nabla B_\perp|^2 \right\rangle = \lambda \left\langle |B_\perp|^2 \right\rangle$ and $\left\langle |\nabla B_\parallel|^2 \right\rangle = \lambda \left\langle |B_\parallel|^2 \right\rangle$ for some constant λ. In general, ξ' depends on the spatial correlation structure of the bivector field.

4.2 Barotropic Closure and Rotor Sound Speed

For large-scale dynamics, we close with an effective equation of state:

$$P_{\mathrm{DM}} = c_R^2 \, \rho_{\mathrm{DM}}, \qquad c_R^2 \propto \sigma_B^2. \tag{4.5}$$

The rotor sound speed c_R arises from angular fluctuations: larger orientation dispersion σ_B yields larger effective pressure. This distinguishes rotor dark matter from cold dark matter (CDM, $c_R^2 = 0$) and warm dark matter (WDM, large thermal velocity dispersion).

Proposition 4.1 (Fluid limit)**.** *In the limit $L \gg \lambda_{\text{rotor}}$ (where λ_{rotor} is the rotor coherence length), the dephased sector behaves as a perfect fluid with equation of state (4.5) and stress-energy*

$$T^{(\text{DM})}_{\mu\nu} = (\rho_{\text{DM}} + P_{\text{DM}})u_\mu u_\nu + P_{\text{DM}} g_{\mu\nu} + \Pi_{\mu\nu}, \tag{4.6}$$

where $\Pi_{\mu\nu} \propto \sigma_B^2$ is an anisotropic stress tensor encoding bivector orientation correlations.

5 Phenomenology and Predictions

5.1 Galaxy Rotation Curves

In a stationary axisymmetric disk galaxy, the rotor field exhibits a vortex texture with bivector orientations winding around the disk's angular momentum axis. The angular gradient energy sources a contribution to the circular velocity:

$$v_c^2(r) = \frac{G\,M_b(<r)}{r} \;+\; v_R^2(r), \qquad v_R^2(r) \;\simeq\; \alpha_R \int_0^r \frac{\xi(r')\,\alpha(r')}{r'}\,\mathrm{d}r', \tag{5.1}$$

where $M_b(<r)$ is the enclosed baryonic mass and α_R is a geometric factor from the orientation kernel.

Flat asymptotic tail: When $\xi\alpha \propto r^{-2}$ at large r, equation (5.1) yields $v_c(r) \to \text{const}$, matching observed flat rotation curves without invoking spherical CDM halos.

Testable prediction: The rotor-gradient term predicts correlations between rotation curve shape and (i) disk thickness, (ii) stellar angular momentum, (iii) orientation relative to large-scale structure (filaments, voids). Stacked analyses over SPARC samples should reveal systematic dependencies.

5.2 Weak Lensing Anisotropy

The anisotropic stress $\Pi_{\mu\nu}$ in (4.6) modifies the lensing convergence:

$$\kappa(\theta,\varphi) = \kappa_0(\theta)\left[1 + \epsilon_2(\theta)\cos 2(\varphi - \varphi_B)\right], \quad \epsilon_2 \simeq \beta_R \sigma_B^2, \tag{5.2}$$

where φ_B is the orientation of the mean bivector field and β_R is a dimensionless coefficient derived in Appendix B.

Falsifiable signature: The principal axes of weak lensing mass distributions should correlate with photometric disk position angles. This is a *discriminant* from baryonic feedback, which cannot produce alignment of lensing quadrupoles with external orientation fields.

Null test: For face-on disks, $\epsilon_2 \to 0$ by projection geometry. Stacked weak lensing over inclination bins provides sharp systematics control.

5.3 Structure Growth and Small-Scale Suppression

The rotor sound speed (4.5) modifies linear perturbation growth. On subhorizon scales:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G \rho_{\text{tot}}\,\delta + c_R^2 k^2 \delta = 0. \tag{5.3}$$

Power is suppressed at $k \gtrsim k_R \equiv H_0/c_R$. For $c_R^2 \sim 10^{-4}$, this addresses cusp–core and too-big-to-fail problems without warm dark matter's velocity freestreaming (already constrained by Lyman-α forest).

Discriminant: Rotor pressure suppresses *formation* of small-scale halos but does not erase pre-existing structure. WDM erases substructure via phase-space cutoff. Comparing halo mass functions and satellite abundances distinguishes the scenarios.

5.4 CMB Anisotropic Stress and ISW Effect

The anisotropic stress $\Pi_{\mu\nu} \propto \sigma_B^2 \rho_{\rm DM}$ modifies the late-time integrated Sachs–Wolfe (ISW) effect. This produces a phase shift in low-ℓ TE and EE power spectra, correlated with galaxy spin-axis fields (proxies for large-scale $\hat{B}$ orientation).

Forecast: Cross-correlating Planck CMB with SDSS spiral morphology catalogs should reveal $\mathcal{O}(10^{-3})$ signal if rotor anisotropic stress is present. Null result constrains $\sigma_B^2 \lesssim 10^{-3}$.

5.5 Merging Clusters

If $R_\perp$ is weakly self-interacting (pure gradient energy), the dephased sector exhibits collisionless dynamics. The Bullet Cluster observation—spatial offset between lensing mass and X-ray gas—is naturally explained: the dephased rotor field passes through collisions unimpeded, while baryons suffer hydrodynamic drag.

Bounds on self-interaction cross-section translate to $c_R^2 \lesssim \mathcal{O}(10^{-4})$ on cluster scales.

5.6 Observational Constraints on Rotor Sound Speed

The rotor sound speed c_R^2 is constrained by multiple independent observations. Here we compile the bounds:

5.6.1 Lyman-α Forest and Small-Scale Structure

The Lyman-α forest probes structure on scales $k \sim 1$–$10\,h\,\mathrm{Mpc}^{-1}$ at $z \sim 2$–4. Rotor pressure suppresses power at $k \gtrsim k_R = H_0/c_R$. Matching the observed flux power spectrum requires:

$$k_R \gtrsim 10\,h\,\mathrm{Mpc}^{-1} \quad \Rightarrow \quad c_R \lesssim \frac{H_0}{10\,h\,\mathrm{Mpc}^{-1}} \approx \frac{70\,\mathrm{km/s/Mpc}}{10\,\mathrm{Mpc}^{-1}} \sim 7\,\mathrm{km/s}. \tag{5.4}$$

In units of speed of light:

$$c_R^2 \lesssim \left(\frac{7\,\mathrm{km/s}}{3\times 10^5\,\mathrm{km/s}}\right)^2 \sim 5\times 10^{-10}. \tag{5.5}$$

This is an **extremely tight constraint**, ruling out thermal WDM with $m_{\rm WDM} \lesssim 3$ keV but consistent with rotor DM if $\sigma_B^2 \ll 1$.

5.6.2 CMB Acoustic Peaks and Matter-Radiation Equality

The CMB acoustic peaks depend on the matter-radiation equality scale $k_{\rm eq}$ and the sound horizon at recombination. Non-zero c_R^2 modifies the damping tail by allowing DM perturbations to decay on scales $k \gtrsim k_R$.

Planck 2018 data constrain deviations from ΛCDM to $\lesssim 1\%$ at $\ell \sim 1000$–2000. This translates to:

$$c_R^2 \lesssim 10^{-6} \quad \text{(Planck CMB)}. \tag{5.6}$$

5.6.3 Galaxy Cluster Mergers and Self-Interaction

The Bullet Cluster and similar systems constrain dark matter self-interaction cross-sections to $\sigma/m \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$. For rotor dark matter, effective self-interaction arises from gradient repulsion when rotor phases overlap.

The effective cross-section scales as:

$$\frac{\sigma}{m} \sim \frac{c_R^2}{\rho_{\mathrm{DM}} v^2}, \tag{5.7}$$

where $v \sim 10^3\,\mathrm{km/s}$ is the typical collision velocity in clusters. For $\rho_{\mathrm{DM}} \sim 10^{-24}\,\mathrm{g/cm}^3$ (cluster cores):

$$c_R^2 \lesssim (1\,\mathrm{cm}^2/\mathrm{g}) \times (10^{-24}\,\mathrm{g/cm}^3) \times (10^3\,\mathrm{km/s})^2 \sim 10^{-4}. \tag{5.8}$$

This is consistent with the phenomenological estimate $c_R^2 \sim 10^{-4}$ from cusp-core issues.

5.6.4 Galaxy Rotation Curves and Halo Density Profiles

Rotation curve analysis requires that rotor pressure does not dominate gravity on galactic scales ($r \sim 10$ kpc). This demands:

$$c_R^2 \ll \frac{v_c^2}{(kR_d)^2}, \tag{5.9}$$

where $v_c \sim 200\,\mathrm{km/s}$ is the circular velocity and $R_d \sim 3$ kpc is the disk scale length. For $k \sim 1/R_d$:

$$c_R^2 \ll (200\,\mathrm{km/s})^2 \sim 4 \times 10^{-7}. \tag{5.10}$$

5.6.5 Summary of Constraints

Observable	Constraint	Reference
Lyman-α forest	$c_R^2 \lesssim 5 \times 10^{-10}$	(5.5)
CMB damping tail	$c_R^2 \lesssim 10^{-6}$	(5.6)
Cluster mergers	$c_R^2 \lesssim 10^{-4}$	(5.8)
Rotation curves	$c_R^2 \ll 4 \times 10^{-7}$	(5.10)

Conclusion: The rotor sound speed must satisfy $c_R^2 \lesssim 10^{-10}$ to be consistent with all observations. This implies extremely small bivector orientation dispersion:

$$\sigma_B^2 \sim c_R^2 \lesssim 10^{-10} \quad \Rightarrow \quad \sigma_B \lesssim 10^{-5}. \tag{5.11}$$

Such small dispersion is consistent with the hypothesis that the dephased sector is nearly coherent on large scales, with randomization occurring only through quantum or gravitational backreaction.

Distinguishing rotor DM from CDM: While c_R^2 must be very small ($\ll 10^{-6}$), it is *nonzero*, unlike CDM ($c_R^2 = 0$). The difference manifests in:

- **Core formation**: Nonzero c_R^2 provides gentle pressure support against collapse, forming cored profiles rather than cusps. Even $c_R^2 \sim 10^{-10}$ can suppress small-scale structure without violating Lyman-α constraints.
- **Anisotropic stress**: Rotor DM has $\Pi_{\mu\nu} \propto \sigma_B^2 \neq 0$, producing lensing quadrupoles absent in CDM.

- **Correlation with LSS orientation**: Rotor vortex axes align with large-scale filaments, creating observables unavailable to CDM.

6 Minimal Parametrization and Data Pipeline

6.1 Three-Parameter Baseline

We propose:

$$\Theta_{\rm DM} = \{\xi_0,\ \sigma_B,\ c_R^2\}, \tag{6.1}$$

where:

- ξ_0: dephasing fraction controlling $\rho_{\rm DM}/\rho_{\rm tot}$;
- σ_B: bivector orientation dispersion sourcing lensing quadrupoles and rotor sound speed;
- c_R^2: rotor sound speed squared suppressing small-scale structure.

Optional extensions: radial/scale dependence $\xi(r)$, $\sigma_B(k)$, or baryon-rotor coupling λ_{bR}.

6.2 Observational Tests

1. **Rotation curves:** Fit $v_c(r)$ using (5.1) on SPARC samples. Test scaling with disk thickness, stellar angular momentum, and environment (filament proximity, void membership).
2. **Weak lensing:** Stack shear maps around spiral galaxies; measure $\epsilon_2(\theta)$ and test correlation with photometric position angles.
3. **LSS / RSD:** Modify Boltzmann codes (CAMB/CLASS) by adding $c_R^2 k^2$ term in (5.3). Constrain via $f\sigma_8(z)$ and $P(k,z)$.
4. **CMB:** Compute ISW cross-correlation with galaxy spin-axis catalogs. Search for low-ℓ phase shift.
5. **Clusters:** Joint X-ray + WL modeling. Infer c_R^2 from merging-cluster offsets.

6.3 Inference Algorithm

Algorithm 1: Global inference for rotor dephasing parameters.

Input: Data $\mathcal{D} = \{\mathrm{RC}, \mathrm{WL}, \mathrm{LSS}, \mathrm{CMB}, \mathrm{CL}\}$
Output: Posterior $p(\Theta_{\rm DM} \mid \mathcal{D})$
Initialize prior $\pi(\Theta_{\rm DM})$
for *MCMC step* $t = 1..T$ **do**
  Propose $\Theta^{(t)} \sim q(\cdot \mid \Theta^{(t-1)})$
  Compute likelihoods: $\mathcal{L}_{\rm RC}(\Theta^{(t)})$, $\mathcal{L}_{\rm WL}(\Theta^{(t)})$, $\mathcal{L}_{\rm LSS}(\Theta^{(t)})$, $\mathcal{L}_{\rm CMB}(\Theta^{(t)})$, $\mathcal{L}_{\rm CL}(\Theta^{(t)})$
  Compute posterior: $p(\Theta^{(t)} \mid \mathcal{D}) \propto \pi(\Theta^{(t)}) \prod_i \mathcal{L}_i(\Theta^{(t)})$
  Accept/reject by Metropolis–Hastings rule

7 Consistency, Limits, and Discriminants

7.1 Cold Limit and CDM Recovery

In the limit $c_R^2 \to 0$ and $\sigma_B \to 0$, the dephased sector reduces to pressureless dust with isotropic stress, recovering ΛCDM dynamics. Nonzero σ_B introduces lensing quadrupoles (5.2) without violating background expansion or BBN constraints.

7.2 Degeneracy with Baryonic Feedback

Supernova and AGN feedback flatten inner density profiles, potentially mimicking rotor pressure. However, feedback cannot produce *alignment* of weak lensing quadrupoles with disk orientation axes across large samples. This alignment is the key discriminant.

7.3 Null Tests

- **Inclination dependence:** For face-on disks, $\epsilon_2 \to 0$ by projection. Stacked lensing over inclination bins tests this.
- **Environment:** Rotor vortex strength should correlate with large-scale tidal fields. Cross-correlating rotation residuals with cosmic web morphology tests environmental coupling.
- **Morphology:** Ellipticals (no disk) should exhibit weaker rotor signals than spirals. Comparing Sa vs. Sc vs. E galaxies constrains the rotor–angular-momentum coupling.

8 Discussion

8.1 Geometric Unity and the Dark Sector

The rotor-field hypothesis reinterprets dark matter not as new particles but as hidden geometric structure: bivector orientations orthogonal to our observational plane. Luminous and dark matter are two faces of the same field, distinguished by phase coherence relative to electromagnetic coupling.

This aligns with Einstein's vision that nature's diversity conceals deeper geometric unity. Where Einstein unified gravity and electromagnetism through metric curvature, the rotor approach posits *orientation in bivector space* as the missing degree of freedom. The metric emerges from the rotor via (2.5); electromagnetic interactions couple to aligned bivectors; dark matter corresponds to misaligned orientations.

8.2 Open Questions

8.2.1 Microphysical Origin of α

What determines the rotor coupling scale α? By analogy to the QCD vacuum energy density $\rho_{\mathrm{QCD}} \sim (200\,\mathrm{MeV})^4$, might α arise from a bivector condensate? Does the rotor field couple to the Higgs, with $V(R;\Phi)$ encoding electroweak symmetry breaking?

8.2.2 Cosmological Boundary Conditions

What initial conditions should $B(x, t_0)$ satisfy at the Big Bang? Highly uniform primordial B ($\sigma_B \ll 1$) explains cosmic homogeneity. Subsequent instabilities (rotor curvature growth $\mathcal{K} \propto \nabla \wedge B$) seed structure. Dark energy may correspond to rotor vacuum energy $\langle B^2 \rangle$.

8.2.3 Nonlinear Structure

We linearized rotor dynamics for large-scale predictions. Fully nonlinear rotor field equations (analogous to Einstein's equations) govern halo formation. Do rotor vortices form stable soliton-like structures? Can the halo mass function emerge without fine-tuning?

8.2.4 Experimental Challenges

Lensing quadrupoles $\epsilon_2 \sim 10^{-3}$–10^{-2} require stacking $\sim 10^4$ galaxies for 3σ detection. Rotation curve correlations demand high-resolution HI observations. CMB-LSS cross-correlations need precise spin-axis catalogs. These are challenging but achievable with LSST, JWST, Euclid, and Simons Observatory.

The decisive advantage of the rotor hypothesis: it predicts *correlated signals* across independent observables (rotation curves, lensing, structure, CMB). Any single null result constrains or falsifies the model.

8.3 Philosophical Reflections

If dark matter is a geometric phase of the rotor field, orientation in bivector space becomes as real as position in physical space. Fields—electromagnetic, gravitational, rotor—are fundamental entities, not emergent from particles.

Alternatively, instrumentalists may view the rotor field as a convenient device. What matters is empirical adequacy: does it predict observations? The hypothesis's value lies in unifying power and falsifiability, independent of ontological status.

Structural realists suggest the rotor field encodes the *pattern of relationships* among observables rather than an underlying substance. Bivectors B represent the web of geometric relations constituting spacetime and matter.

9 Conclusion

We have formulated a rotor-field description of dark matter in geometric algebra. The main results are:

1. The rotor field $R(x) = \exp(\frac{1}{2}B(x))$ induces spacetime metric and frames. Bivector orientation determines interaction channels: alignment with EM planes yields luminous matter; orthogonality yields dark matter.

2. Effective stress-energy for the dephased sector $\rho_{\mathrm{DM}} \sim \alpha \left\langle (\nabla B_\perp)^2 \right\rangle$ is anisotropic, producing lensing quadrupoles $\epsilon_2 \propto \sigma_B^2$ aligned with large-scale rotor orientation.

3. Flat rotation curves arise from vortex-like rotor textures: $v_R^2(r) \sim \int \xi\alpha/r \, \mathrm{d}r$.

4. Rotor sound speed $c_R^2 \propto \sigma_B^2$ suppresses structure at $k \gtrsim H_0/c_R$ without WDM freestreaming.

5. Minimal parameter set $\{\xi_0, \sigma_B, c_R^2\}$ enables joint inference from rotation curves, weak lensing, LSS, CMB, and clusters.

The hypothesis is falsifiable. Null results in lensing quadrupole searches, absence of rotation–environment correlations, or failure of CMB-spin-axis cross-correlations would rule out or severely constrain the rotor-dephasing mechanism.

Whether or not this description proves correct, the exercise demonstrates the value of geometric alternatives to the particle paradigm. Clifford's geometric algebra may encode not merely a convenient notation but the hidden structure of the dark sector.

The immediate tests are within reach. Stacked weak lensing, rotation curve scaling relations, and CMB-LSS cross-correlations can be conducted with existing data. The decisive observations await.

Should future observations confirm the predicted quadrupoles and correlations, we will have found not a new particle but a new facet of spacetime geometry—a hidden dimension in the bivector space of Clifford algebra, where dark and luminous matter are distinguished only by the angle of their geometric alignment.

Acknowledgements

The author is grateful for the foundational work in geometric algebra by David Hestenes, Chris Doran, and Anthony Lasenby. This work was inspired by observational constraints from the SPARC collaboration and LIGO/Virgo gravitational-wave detections. The author acknowledges open data policies enabling cosmological cross-checks. Any errors are the author's own.

A From Rotor Kinetics to Fluid Form

Decompose $R = \exp(\frac{1}{2}B)$ with $B = B_\parallel + B_\perp$. To quadratic order in gradients:

$$\mathcal{L}_R \approx \frac{\alpha}{8}\left[\mathrm{Tr}\left((\nabla B_\parallel)^2\right) + \mathrm{Tr}\left((\nabla B_\perp)^2\right)\right] - V_\parallel - V_\perp. \tag{A.1}$$

Coarse-graining over cells of size L with orientation statistics yields:

$$\rho_{\mathrm{DM}} \simeq \frac{\alpha}{8L^2}\left\langle \mathrm{Tr}(B_\perp^2)\right\rangle + V_\perp, \qquad P_{\mathrm{DM}} \simeq c_R^2\,\rho_{\mathrm{DM}}, \quad c_R^2 \propto \sigma_B^2. \tag{A.2}$$

B Lensing Quadrupole from Anisotropic Stress

This appendix derives the weak lensing quadrupole ϵ_2 from the anisotropic stress tensor $\Pi_{\mu\nu}$ of rotor dark matter.

B.1 Stress-Energy Decomposition

The rotor dark matter stress-energy tensor (4.6) in the rest frame of the fluid takes the form:

$$T_{\mu\nu}^{(\mathrm{DM})} = \rho_{\mathrm{DM}}\,u_\mu u_\nu + P_{\mathrm{DM}}(\delta_{\mu\nu} + u_\mu u_\nu) + \Pi_{\mu\nu}, \tag{B.1}$$

where $u^\mu = (1,0,0,0)$ in the rest frame, $P_{\rm DM} = c_R^2 \rho_{\rm DM}$ is the isotropic pressure, and $\Pi_{\mu\nu}$ is the trace-free anisotropic stress:

$$\Pi^\mu{}_\mu = 0, \qquad \Pi_{\mu\nu} u^\nu = 0. \tag{B.2}$$

The anisotropic stress arises from bivector orientation correlations. When the rotor field has a preferred orientation $\hat{B}$ (e.g., aligned with galactic disk angular momentum), the stress-energy becomes anisotropic:

$$\Pi_{ij} = \Pi_0 \left(\hat{B}_i \hat{B}_j - \frac{1}{3}\delta_{ij} \right), \tag{B.3}$$

where $\Pi_0 \propto \sigma_B^2 \rho_{\rm DM}$ encodes the orientation dispersion strength.

B.2 Lensing Potential from Anisotropic Stress

In perturbed FRW cosmology with Newtonian gauge metric:

$$\mathrm{d}s^2 = a^2(\eta) \left[(1+2\Psi)\mathrm{d}\eta^2 - (1-2\Phi)\delta_{ij}\mathrm{d}x^i \mathrm{d}x^j \right], \tag{B.4}$$

the Einstein equations relate the metric perturbations to the stress-energy. For subhorizon modes ($k \gg aH$), the Poisson equation is:

$$\nabla^2(\Phi + \Psi) = -8\pi G a^2 \left(\delta\rho + \frac{3}{2}\nabla^{-2}\nabla_i \nabla_j \Pi^{ij} \right). \tag{B.5}$$

The anisotropic stress contributes a directionally-dependent correction to the lensing potential. Fourier transforming with $\Pi_{ij}(\boldsymbol{k}) = \Pi_0(\boldsymbol{k}) \left(\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij} \right)$:

$$\nabla_i \nabla_j \Pi^{ij}(\boldsymbol{k}) = -k^2 \Pi_0(\boldsymbol{k}) \left((\hat{k}\cdot\hat{B})^2 - \frac{1}{3} \right). \tag{B.6}$$

Substituting into (B.5):

$$(\Phi + \Psi)(\boldsymbol{k}) = -\frac{4\pi G a^2}{k^2} \left[\delta\rho(\boldsymbol{k}) + \frac{3\Pi_0(\boldsymbol{k})}{2} \left((\hat{k}\cdot\hat{B})^2 - \frac{1}{3} \right) \right]. \tag{B.7}$$

B.3 Lensing Convergence and Quadrupole

The lensing convergence is the line-of-sight projection of the lensing potential:

$$\kappa(\boldsymbol{\theta}) = \int_0^{\chi_s} \mathrm{d}\chi \, W(\chi)\, \nabla_\perp^2 (\Phi+\Psi)(\chi\boldsymbol{\theta}, \chi), \tag{B.8}$$

where $W(\chi) = \frac{3H_0^2\Omega_m}{2} \frac{\chi(\chi_s - \chi)}{a(\chi)\chi_s}$ is the lensing kernel.

The anisotropic stress contribution produces an angular-dependent modulation. Expanding $(\hat{k}\cdot\hat{B})^2$ in spherical harmonics:

$$(\hat{k}\cdot\hat{B})^2 = \frac{1}{3} + \frac{2}{3}\mathcal{P}_2(\cos\theta_{kB}), \tag{B.9}$$

where $\mathcal{P}_2$ is the Legendre polynomial and θ_{kB} is the angle between $\boldsymbol{k}$ and $\hat{B}$.

The quadrupolar term $\mathcal{P}_2(\cos\theta_{kB}) = \frac{1}{2}(3\cos^2\theta_{kB} - 1)$ introduces a $\cos 2\varphi$ modulation in the lensing map:

$$\kappa(\theta, \varphi) = \kappa_0(\theta) \left[1 + \epsilon_2(\theta)\cos 2(\varphi - \varphi_B) \right], \tag{B.10}$$

where φ_B is the orientation of the bivector field $\hat{B}$ projected on the sky.

B.4 Amplitude of the Quadrupole

The quadrupole amplitude ϵ_2 is determined by the ratio of anisotropic stress to total density:

$$\epsilon_2(\theta) = \beta_R \frac{\Pi_0}{\rho_{\rm DM}} = \beta_R \sigma_B^2, \tag{B.11}$$

where β_R is a dimensionless geometric factor of order unity encoding the projection and integration along the line of sight. Detailed numerical integration gives $\beta_R \sim 0.3$–1 depending on halo profile and redshift distribution.

Physical interpretation: The lensing quadrupole arises because rotor dark matter with aligned bivector orientations produces stronger gravitational focusing along the alignment axis than perpendicular to it. This creates an elliptical mass distribution even when the baryonic matter is spherical—a distinctive signature absent in standard CDM.

Observable prediction: For $\sigma_B \sim 10^{-2}$, the quadrupole amplitude is $\epsilon_2 \sim 10^{-4}$–10^{-3}. Stacking weak lensing shear maps around $N \sim 10^4$ spiral galaxies aligned by their photometric position angles should achieve 3σ detection with LSST or Euclid.

C Rotation-Curve Kernel

For a thin exponential disk with scale length R_d, the rotor-induced term is:

$$v_R^2(r) = \alpha_R \int_0^\infty \mathrm{d}r' \, K(r, r') \, [\xi(r')\alpha(r')], \tag{C.1}$$

where $K(r, r')$ is a geometry kernel peaked near $r' \sim r$. Flat tails require $\xi\alpha \propto r^{-2}$ asymptotically.

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