Three Logistic Models for the Two-Species Interactions: Symbiosis, Predator-Prey and Competition

Prepared for :

NLDC Project

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2018B5A70672G

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ABSTRACT

If one isolated species (corporation) is supposed to evolve following the logistic mapping, then we are tempted to think that the dynamics of two species (corporations) can be expressed by a coupled system of two discrete logistic equations. As three basic relationships between two species are present in Nature, namely symbiosis, predator-prey and competition, three different models are obtained.

Each model is a cubic two-dimensional discrete logistic-type equation with its own dynamical properties: stationary regime, periodicity, quasiperiodicity and chaos. We also propose that these models could be useful for thinking in the different interactions happening in the economic world, as for instance for the competition and the collaboration between corporations. Furthermore, these models could be considered as the basic ingredients to construct more complex interactions in the ecological and economic networks.

BACKGROUND

Around 1798, Malthus claimed human population p, grows faster than food supply.

Inevitable poverty and human population will be wiped out.

$$\frac{dp}{dt} = kp \qquad \text{k-growth rate}$$

Thomas Robert Malthus

But it Didn't Happen!!!

Flaw

Population doesn't increase exponentially as there are other inhibitory forces at work.

LOGISTIC EQUATIONS

Verhulst included inhibitory term and gave famous Logistic-Equation.

$$\frac{dp}{dt} = kp - np^2$$

For until a century, this work of art was not given any importance.

Robert May(1976) Biologist said understanding the discrete logistics model should be considered as a milestone in **Non -Linear Phenomenon.**

Later, these equations became the basis of modern **Chaos Theory.**

Paradigm for **period Doubling Cascade**(Feigenbaum,1978)

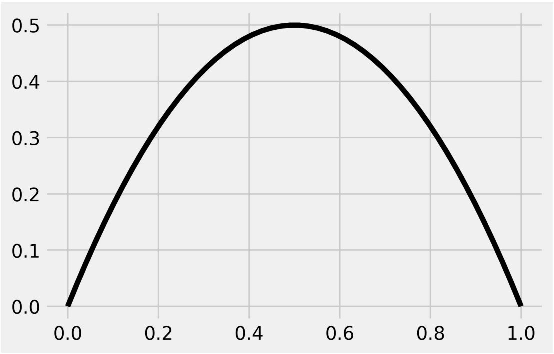
Pierre François Verhulst (1845)

DISCRETE LOGISTICS EVOLUTION EQUATIONS

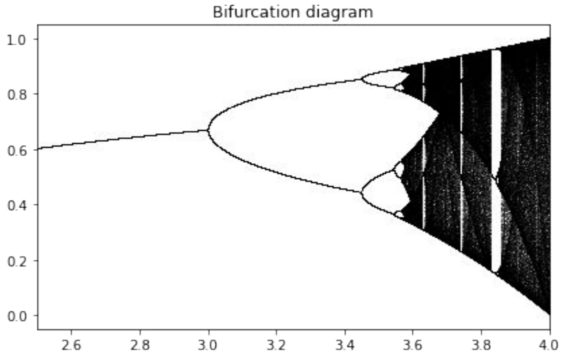
$$x_{n+1} = \mu x_n (1 - x_n)$$

x_n : population of species of **isolated species** after n generation

Modified growth rate as we change parameter can understood in biological terms.



$$\text{take } 0 < \mu < 4 \text{ to ensure } 0 < x_n < 1$$



AIM OF PROJECT

In our project, we are verifying the results for coupled logistic equations for different types of interaction of two species, **x** and **y.**

Assume each of them evolves with logistics dynamics.

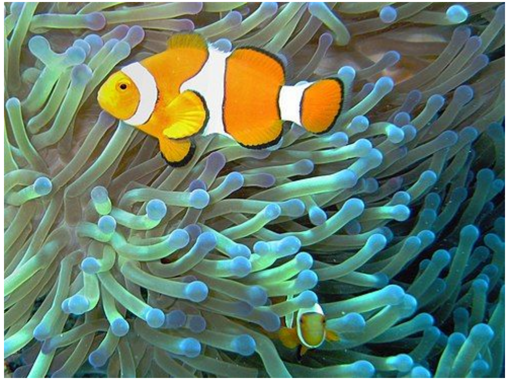
$$x_{n+1} = \mu_x(y_n)x_n(1 - x_n) \qquad \mu \in (1,4)$$
$$y_{n+1} = \mu_y(x_n)y_n(1 - y_n)$$

Here $\mu(z)$ depends on population size of others and a parameter λ.

Simplest Choice is taking parameter to increase or decrease the parameter at point where is shows activity

$$\mu_1(z) = \lambda(3z + 1) \qquad \mu_1 \text{ increasing}$$
$$\mu_2(z) = \lambda(-3z + 4) \qquad \mu_2 \text{ decreasing}$$

TYPES OF INTERACTIONS



Symbiosis

$$\mu_x = \mu_y = \mu_1$$

Predator-Prey

$$\mu_x = \mu_1 \text{ and } \mu_y = \mu_2$$

Competition

$$\mu_x = \mu_y = \mu_2$$

Symbiosis Model

SYMBIOSIS

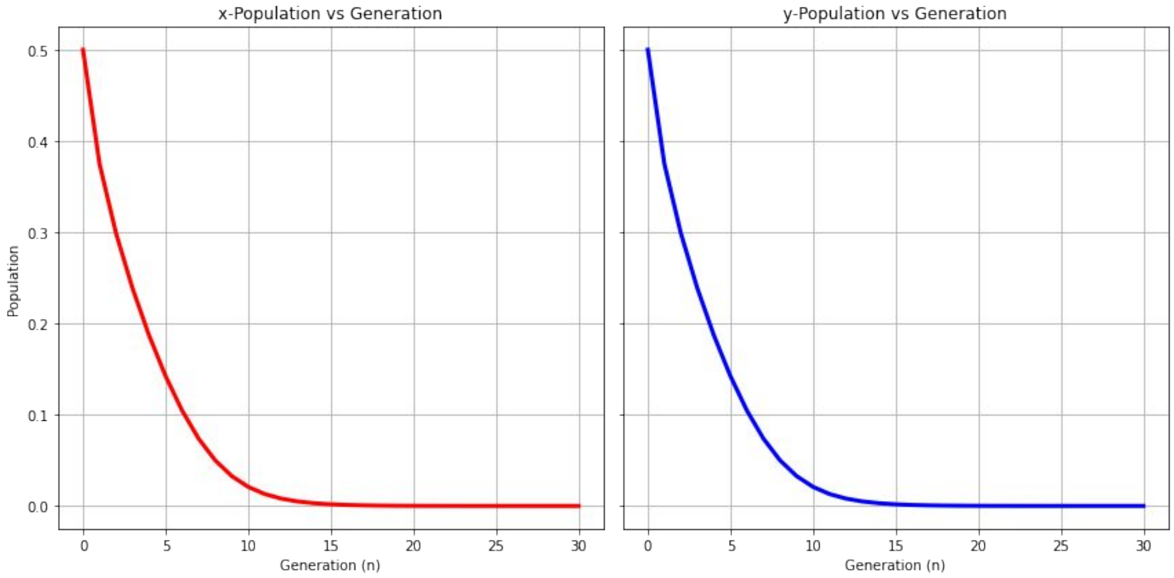
When two species (x_n, y_n) interact symbiotically the equations takes the form

$$x_{n+1} = \lambda(3y_n + 1)x_n(1 - x_n)$$
$$y_{n+1} = \lambda(3x_n + 1)y_n(1 - y_n)$$

$\lambda -\ the\, mutual\, benefit$

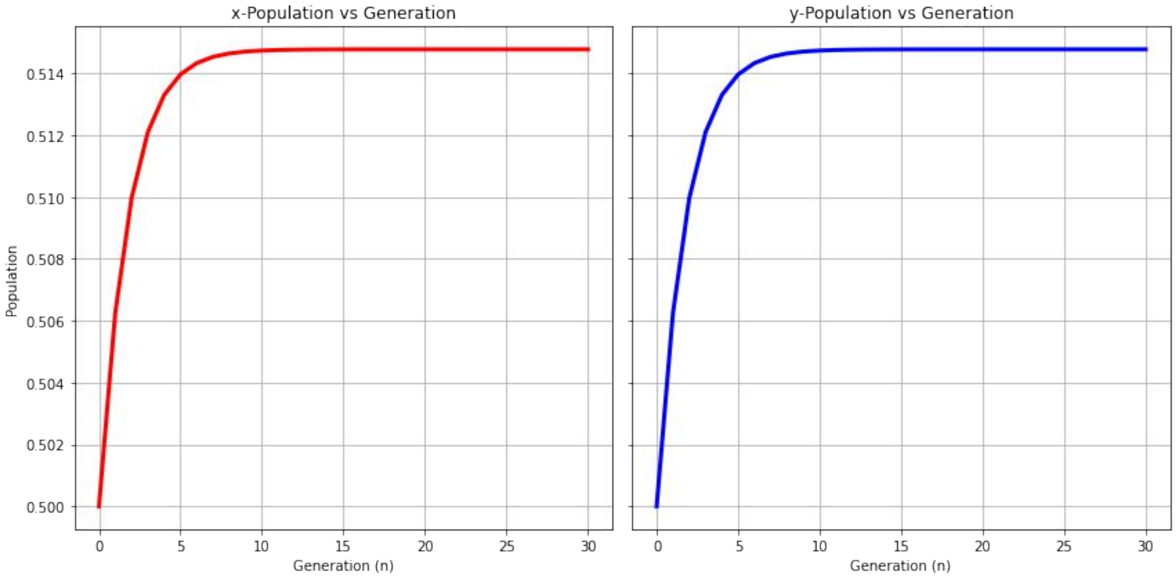
(i) $0 < \lambda < 0.75$

The mutual benefit is not big enough to allow a stable coexistence of both species and they will disappear eventually.



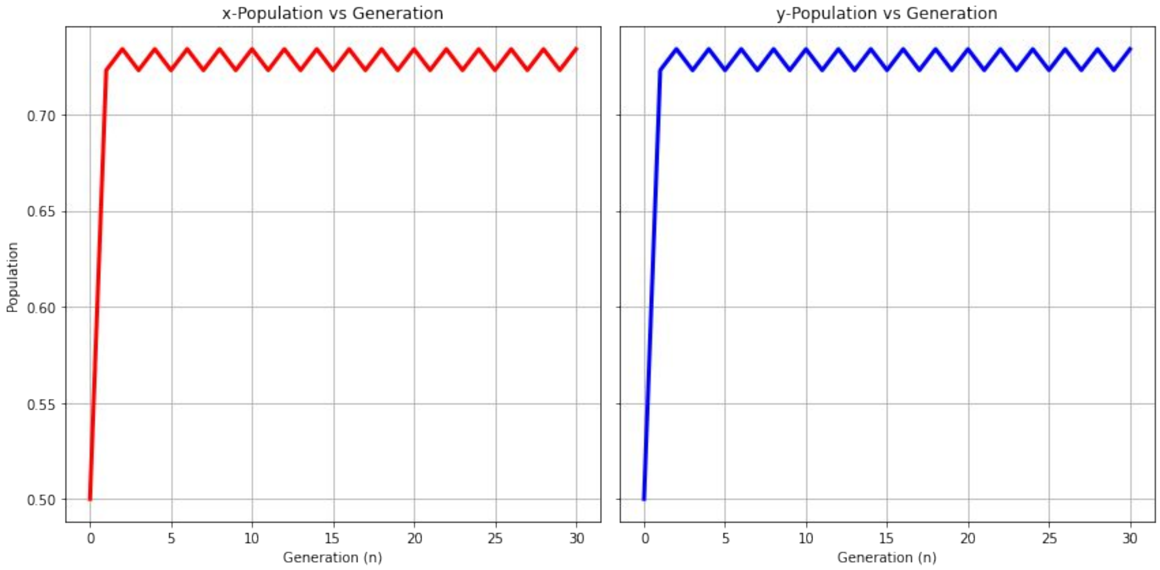
(ii) 0.75 < λ < 1.156

Both populations are synchronized to a stable non-vanishing fixed quantity.



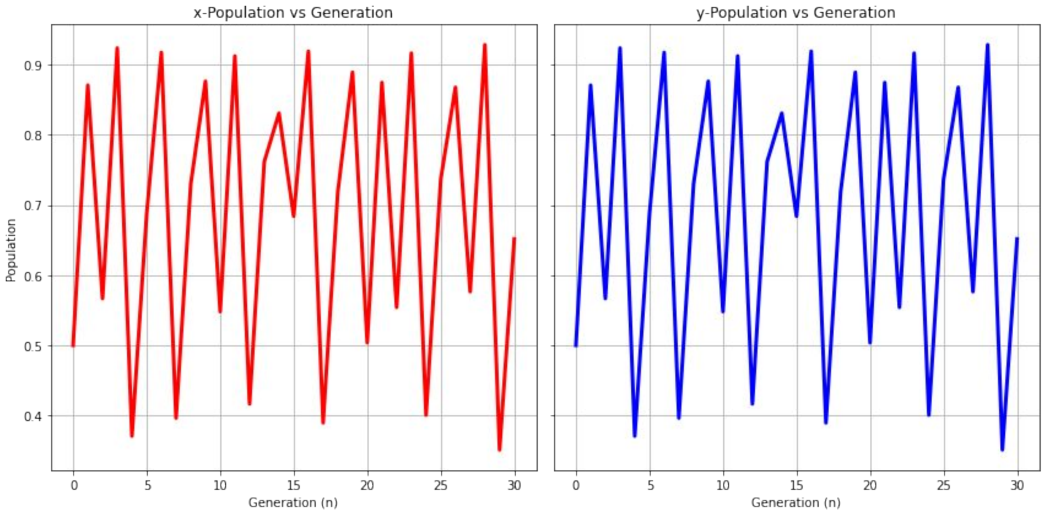
(iii) $1.156 < \lambda < 1.158$

Each one of the species oscillates between two fixed values . This is a stable 2-period orbit.



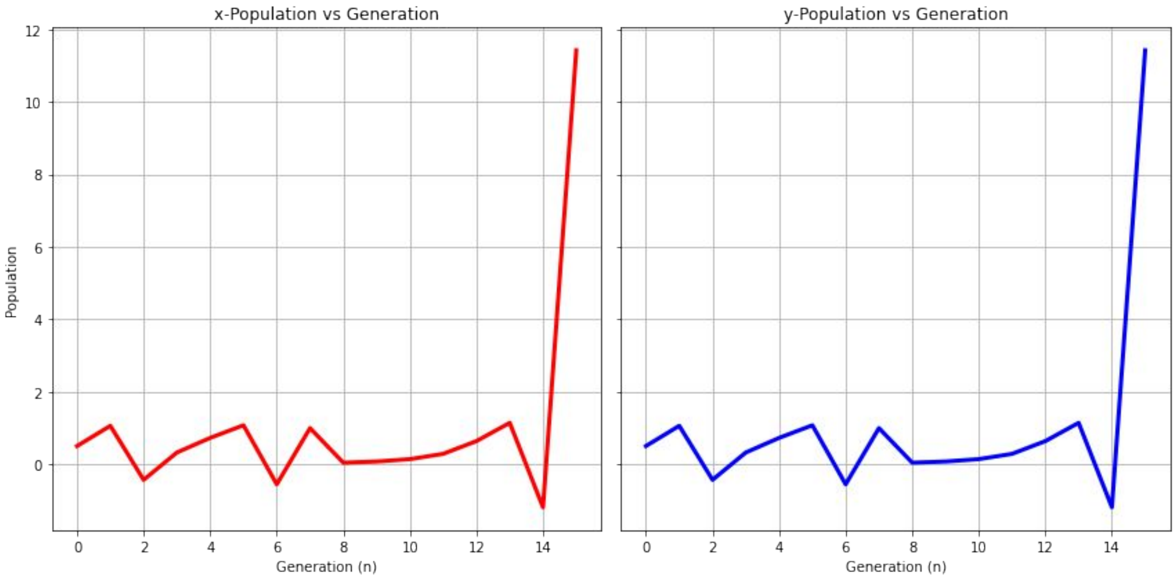
(iv) 1.158 < λ < 1.69

Both populations oscillate among infinitely many different states.



(v) $\lambda > 1.69$

The system ***evolves toward infinity***. This is interpreted as some kind of catastrophe provoking the extinction of species



Predator-Prey Model

PREDATOR-PREY

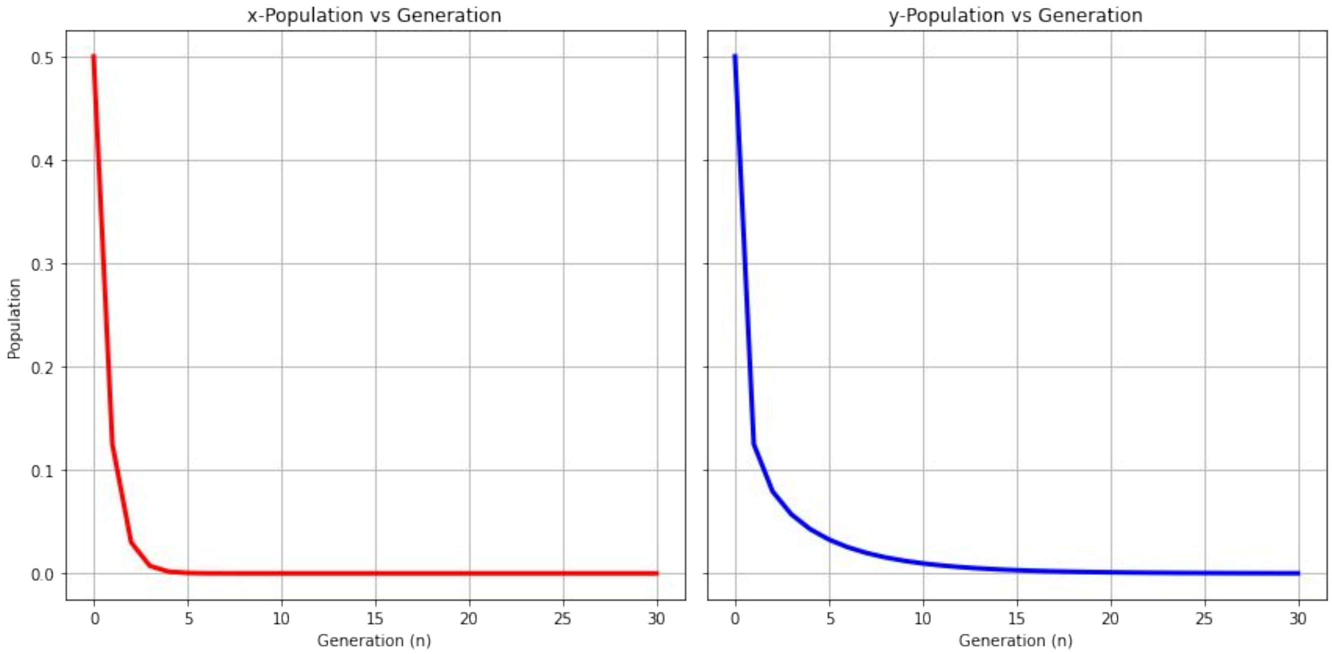
Both species are governed by an ***attack-defence*** interaction which takes place between them.

Suppose $\mathbf{x_n}$ **is predator population** and $\mathbf{y_n}$ **is prey population**.

$$x_{n+1} = \lambda(3y_n + 1)x_n(1 - x_n)$$
$$y_{n+1} = \lambda(-3x_n + 4)y_n(1 - y_n)$$

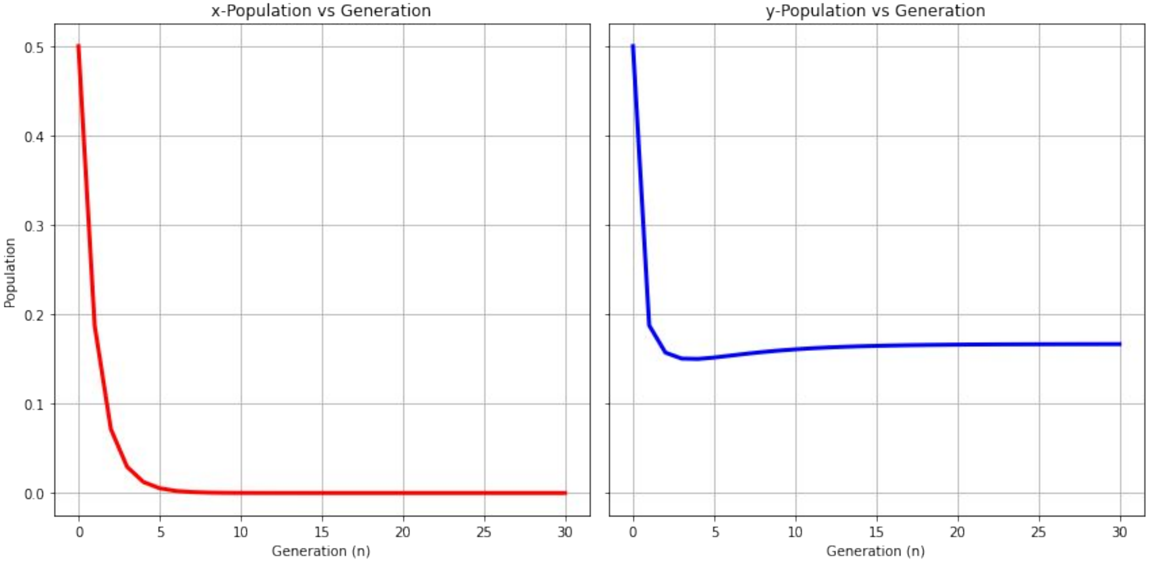
(i) $0 < \lambda < 0.25$

The reproductive force of the preys (y) is smaller than the combination of its natural death rate and the effect of predator(x) attacks. Hence preys(y) can not survive and the predators(x) become also extinct.



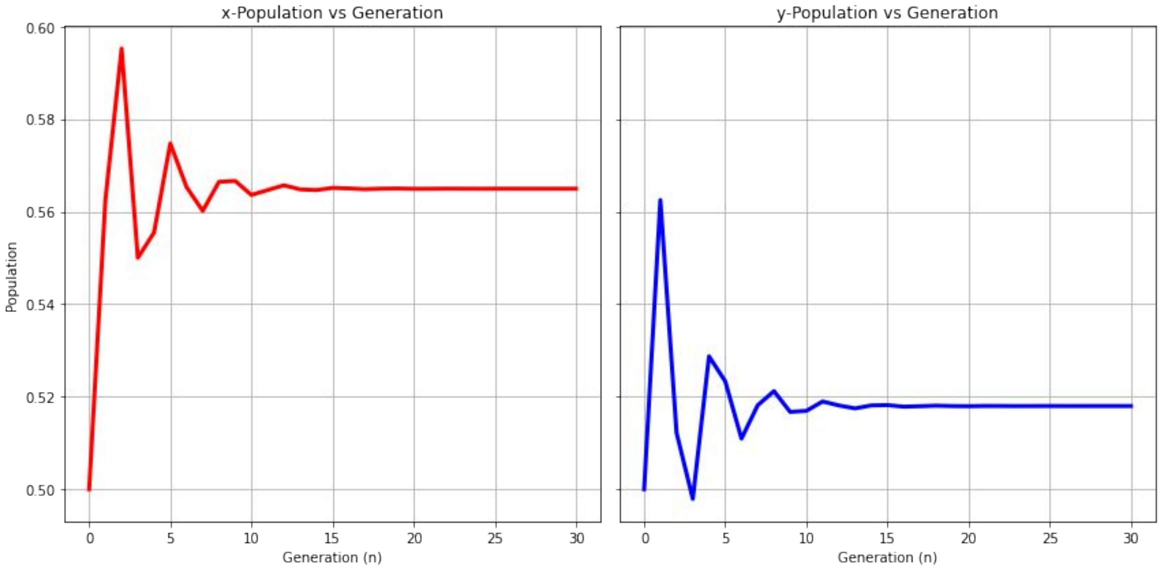
(ii) 0.25 < λ < 0.4375

Prey(y) population can survive in a small quantity but predators(x) do not have enough food to be self-sustained and become extinct.



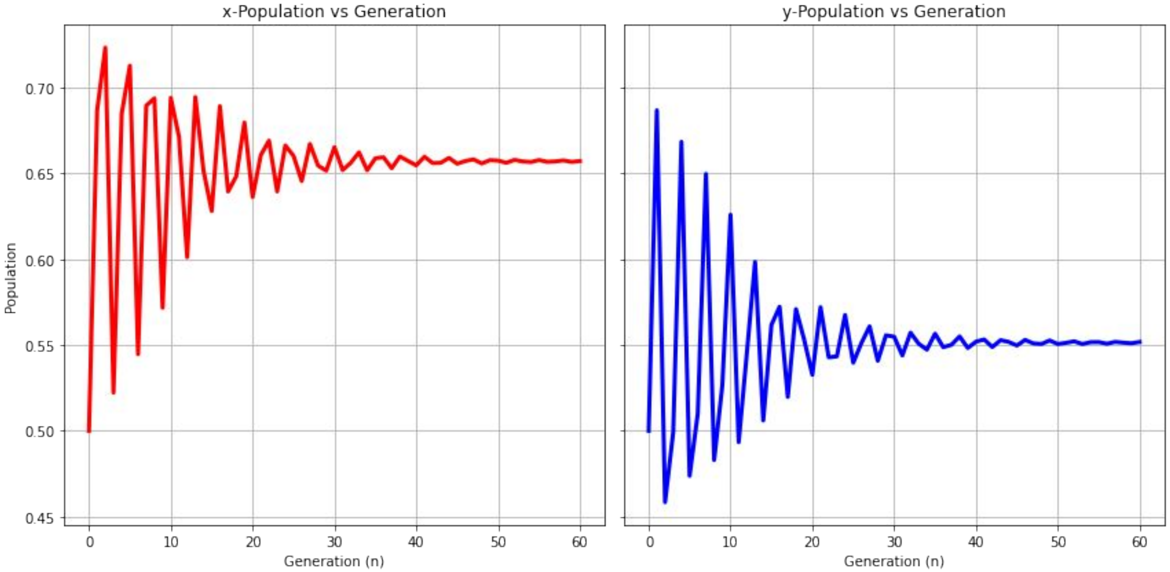
(iii) 0.4375 < λ < 1.0851

The system settles down to an equilibrium. The increasing of λ allows the coexistence of bigger populations.



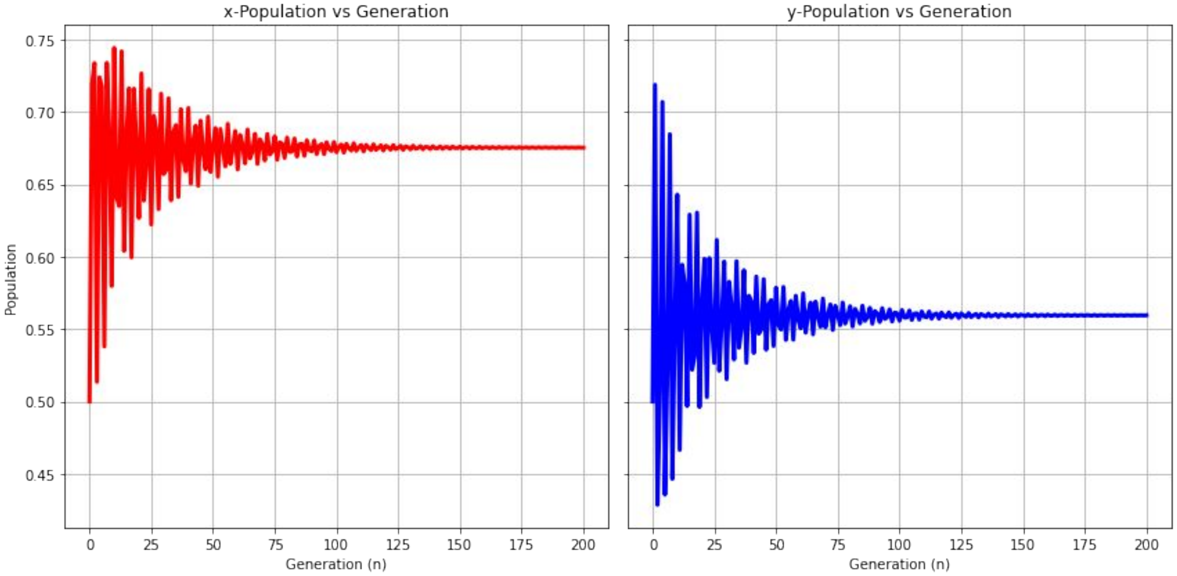
(v) 1.0851 < λ < 1.0991

The system oscillates with irregular oscillations and then achieves a stable equilibrium populations.



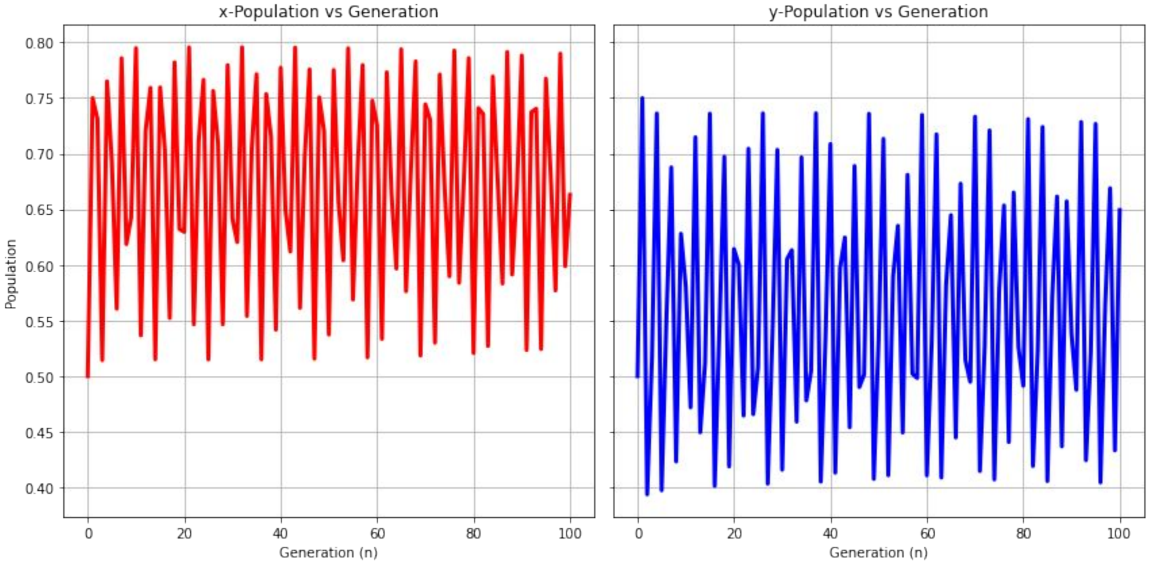
(vi) $1.0991 < \lambda < 1.1758$

Populations reach to an equilibrium, after a chaotic transient.



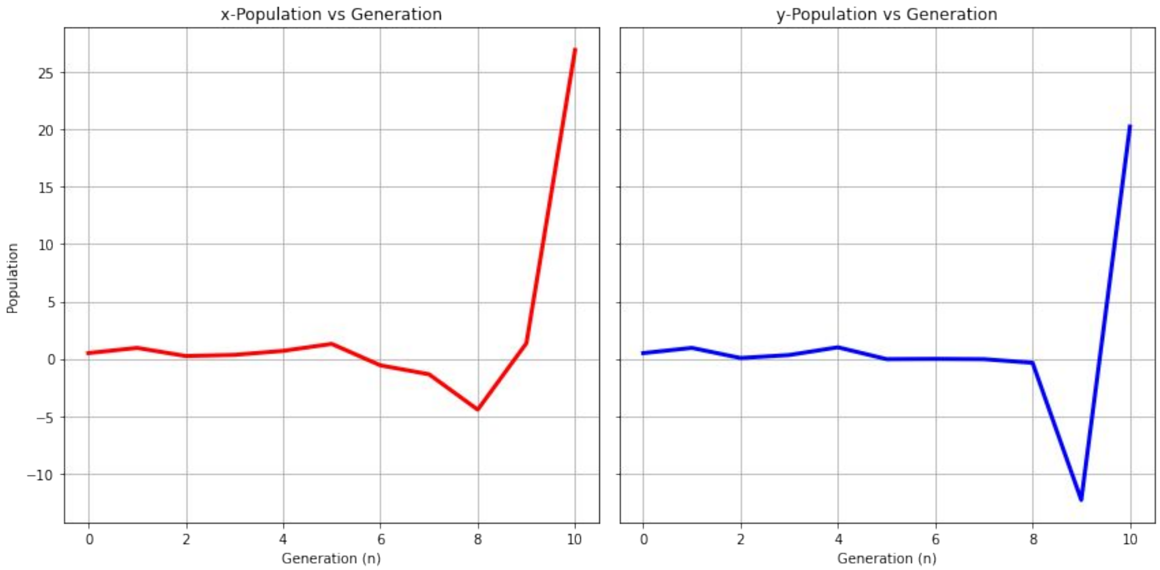
(vii) $1.1758 < \lambda < 1.211$

The populations oscillate among a continuum of possible states located on the invariant curve.



(viii) $\lambda > 1.211$

System evolve towards infinity. This can be interpreted as some kind of catastrophe provoking the extinction of the species.



Competition Model

COMPETITION

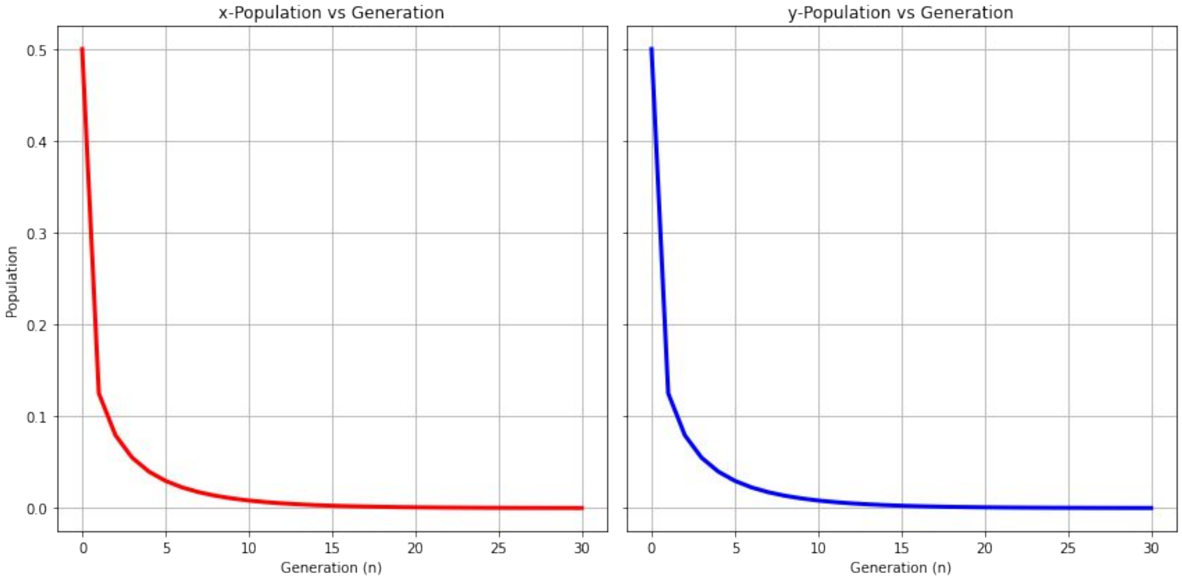
Suppose now two species (x_n, y_n) evolving under a competitive interaction (Darwin, 1859).

$$x_{n+1} = \lambda(-3y_n + 4)x_n(1 - x_n)$$
$$y_{n+1} = \lambda(-3x_n + 4)y_n(1 - y_n)$$

$$\lambda - mutual\ competitive\ interaction$$

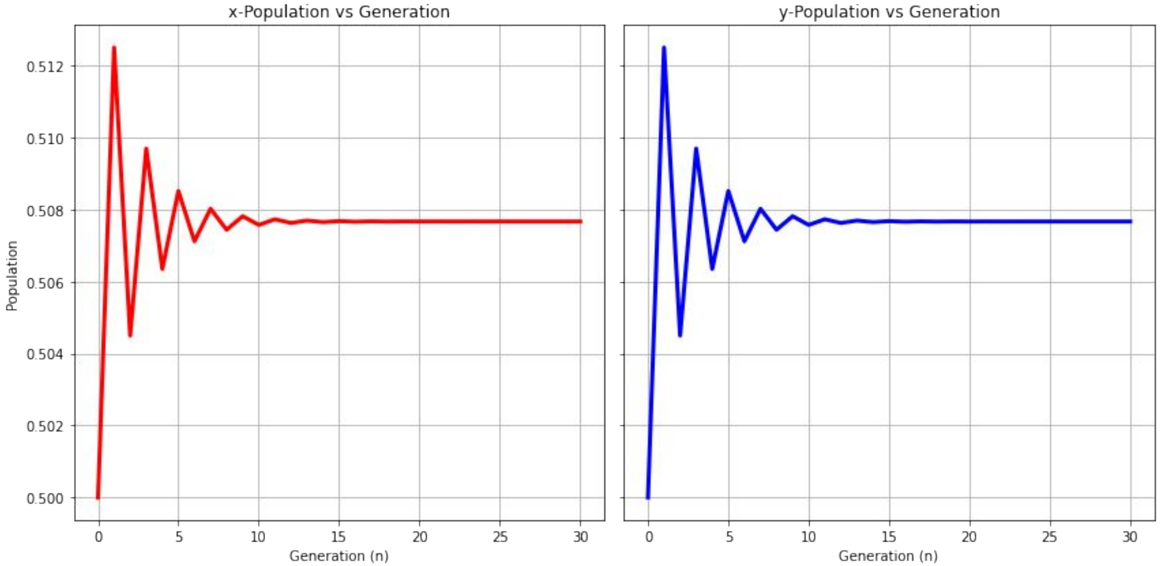
(i) 0 < λ < 0.25

Competition is an important force for survival in a pure competitive interaction and its absence or its weakness can cause extinction of both the competing species .



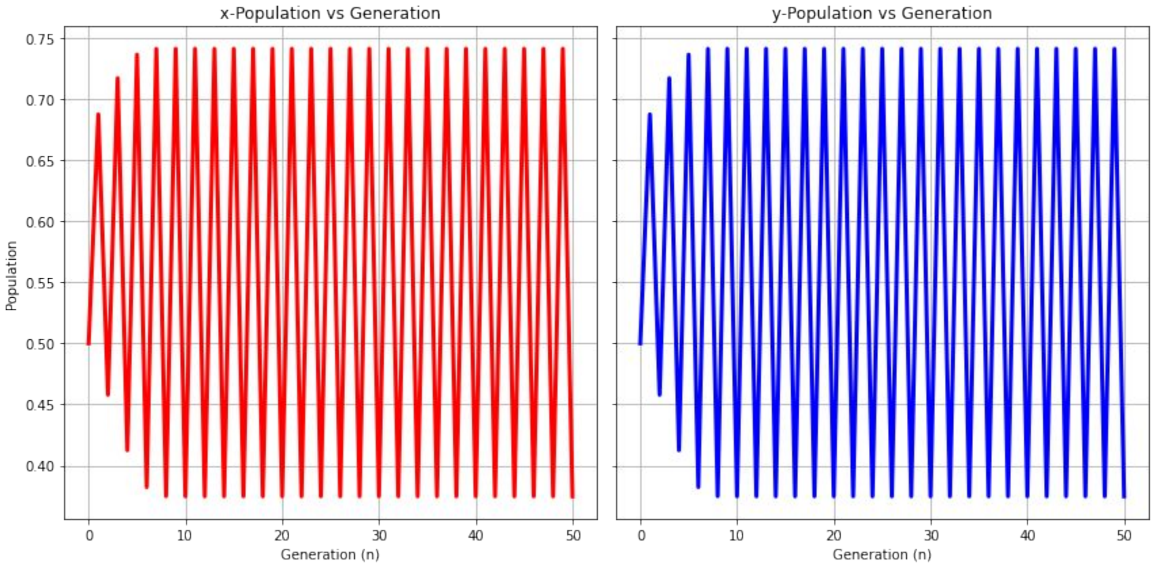
(ii) $0.25 < \lambda < 0.9811$

Both populations can survive and the system settles down in a stationary symmetrical equilibrium .



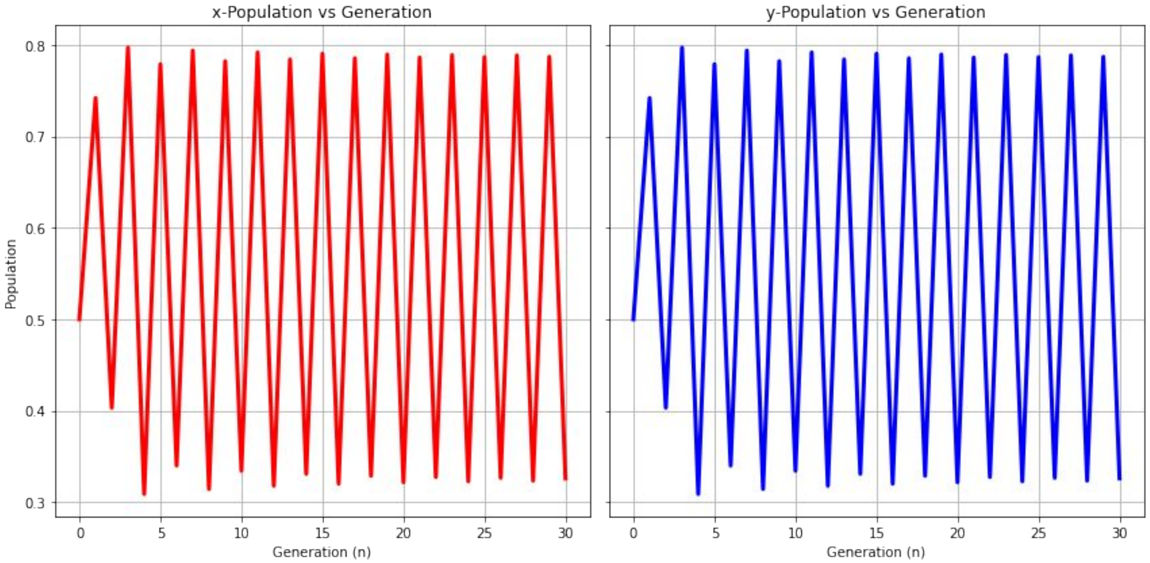
(iii) 0.9811 < λ < 1.1743

The populations oscillate synchronously between the two values of the orbit.



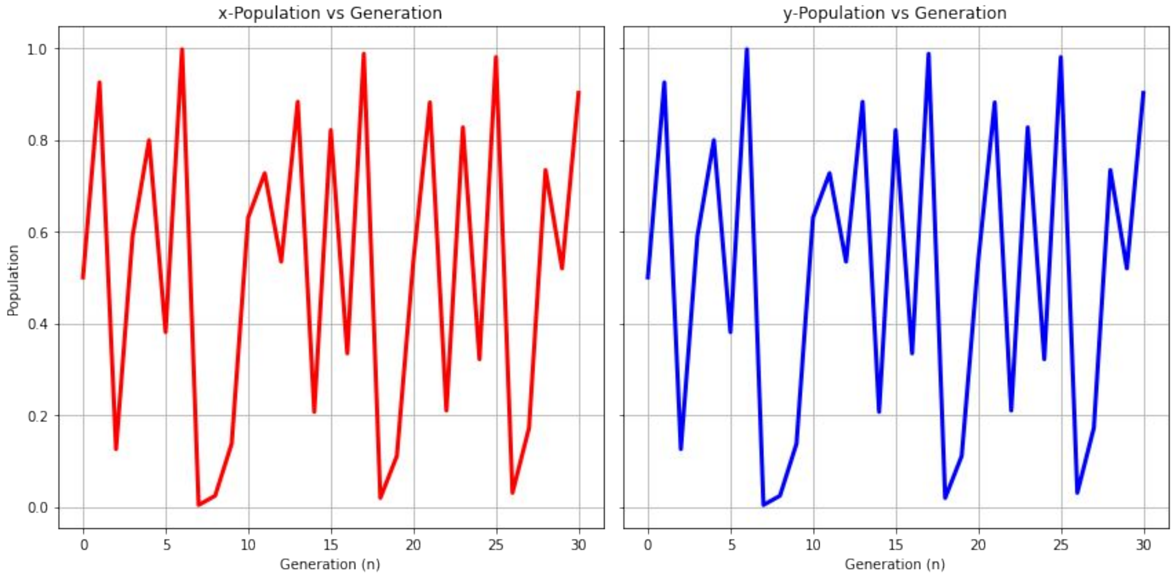
(iv) $1.1743 < \lambda < 1.1875$

The two cycle period becomes unstable.



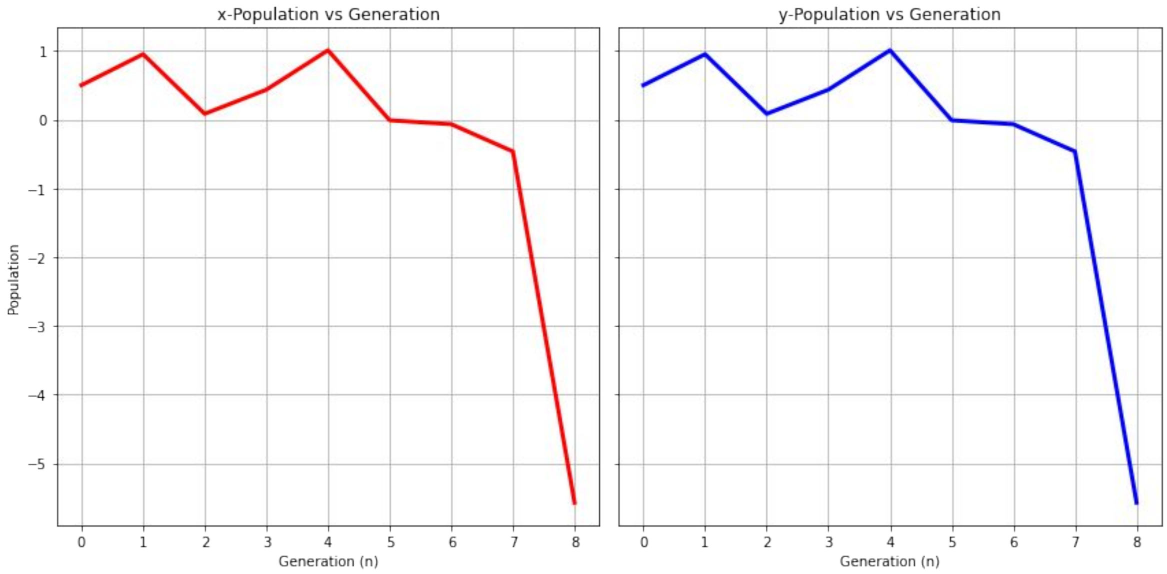
(v) $1.1875 < \lambda < 1.52$

The dynamics becomes complex.



(vi) λ > 1.52

The system evolve towards infinity. This can be interpreted as some kind of catastrophe provoking the extinction of species.



CONCLUSION

Three main types of interaction between two species, namely the symbiosis, predator-prey situation, competition, have been proposed as basic bricks for understanding how the populations evolve in our ecosystem.

Thus, we have interpreted three cubic two-dimensional coupled logistic equations as three discrete models to explain either the evolution of two symbiotically interacting species, either a predator-prey system or two species under competitive interaction. Extinction, stable coexistence, periodic or chaotic oscillations are found in the dynamics of the three systems.

FUTURE TRENDS

1. **Development of more Complex Models**

Following the path of technology and using some of these new tools, different models with a large number of interacting elements will appear and will be one of the future trends in the field of complex systems.

2. **Better Understanding of Ecological Systems**

It is for sure that these models can contribute and help to understand and to increase our insight about the evolution of ecological systems.

APPENDIX-1

Python Code : Simulation of Coupled Logistic Equation for different Ecological Interaction

```
import numpy as np
import matplotlib.pyplot as plt

## defining logistic function
## r = mu = rate
## x(n+1) = μ*x(n)*(1 − x(n))

def logistic(r, x):
    return r * x * (1 - x)

## l = lambda

def mu1(l,y):
    return l*(3*y+1)

def mu2(l,y):
    return l*(-3*y+4)

## SYMBIOSIS ##

# x0=initial pop. species 1
x0=0.5

# y0=initial pop. species 2
y0=0.5

# l=mutual benefit=lambda
l=1.393

# generations
n=30

# function for plotting population vs generation

x=np.zeros(n+1)
y=np.zeros(n+1)

def pop_gen_plot(x0, y0, l, n):

    x[0]=x0
    y[0]=y0

    for i in range(n):
        temp=x[i]
        x[i+1]=logistic(mu1(l,y[i]),x[i])
        y[i+1]=logistic(mu1(l,temp),y[i])

## Function calling

pop_gen_plot(x0, y0, l, n)

fig, (ax1,ax2)=plt.subplots(1, 2, figsize=(12,6), sharey=True)

ax1.plot(x, 'k', lw=3, color='red')
ax1.set_title("x-Population vs Generation")
ax1.set_xlabel('Generation (n)')
ax1.set_ylabel('Population')
ax1.grid()


ax2.plot(y, 'k', lw=3, color='blue')
ax2.set_title("y-Population vs Generation")
ax2.set_xlabel('Generation (n)')
ax2.grid()


plt.tight_layout()
```

```
## PREDATOR-PREY ##

# x=Predator
# y=Prey

# x0=initial pop. species 1
x0=0.5

# y0=initial pop. species 2
y0=0.5

# l=mutual benefit=lambda
l=1.099

# generations
n=60

# function for plotting population vs generation

x=np.zeros(n+1)
y=np.zeros(n+1)

def pop_gen_plot(x0, y0, l, n):

    x[0]=x0
    y[0]=y0

    for i in range(n):
        temp=x[i]
        x[i+1]=logistic(mu1(l,y[i]),x[i])
        y[i+1]=logistic(mu2(l,temp),y[i])
```

```
## Function calling

pop_gen_plot(x0, y0, l, n)

fig, (ax1,ax2)=plt.subplots(1, 2, figsize=(12,6), sharey=True)

ax1.plot(x, 'k', lw=3, color='red')
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ax2.plot(y, 'k', lw=3, color='blue')
ax2.set_title("y-Population vs Generation")
ax2.set_xlabel('Generation (n)')
ax2.grid()


plt.tight_layout()
```

```
## PREDATOR-PREY ##

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```

```
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ax1.set_title("x-Population vs Generation")
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ax1.set_ylabel('Population')
ax1.grid()

ax2.plot(y, 'k', lw=3, color='blue')
ax2.set_title("y-Population vs Generation")
ax2.set_xlabel('Generation (n)')
ax2.grid()


plt.tight_layout()
```

```
## COMPETITION ##

# x0=initial pop. species 1
x0=0.5

# y0=initial pop. species 2
y0=0.5

# l=mutual benefit=lambda
l=1.48

# generations
n=30

# function for plotting population vs generation

x=np.zeros(n+1)
y=np.zeros(n+1)

def pop_gen_plot(x0, y0, l, n):

    x[0]=x0
    y[0]=y0

    for i in range(n):
        temp=x[i]
        x[i+1]=logistic(mu2(l,y[i]),x[i])
        y[i+1]=logistic(mu2(l,temp),y[i])
```

```
## Function calling

pop_gen_plot(x0, y0, l, n)

fig, (ax1,ax2)=plt.subplots(1, 2, figsize=(12,6), sharey=True)

ax1.plot(x, 'k', lw=3, color='red')
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ax2.plot(y, 'k', lw=3, color='blue')
ax2.set_title("y-Population vs Generation")
ax2.set_xlabel('Generation (n)')
ax2.grid()


plt.tight_layout()
```